HARMONICS AND POWER PHENOMENA

Electrical quantities such as voltages, currents, or magnetic fluxes with a waveform that repeats in time cycle after cycle are called *periodic quantities*. Usually the shortest cycle of the repetition is called a *period*. The number of periods per second is referred to as the *fundamental frequency*. The voltage produced by power plants usually changes as a sinusoidal function of time or it is very close to such a waveform. Periodic quantities that do not vary as a sinusoidal function are referred to as *nonsinusoidal* quantities.

The waveform of a periodic quantity can be specified by a uniformly spaced sequence of instantaneous values of this quantity over a single cycle. Such a sequence describes the quantity in the *time-domain*. The number of samples needed for the waveform specification depends on the variability of the quantity. This number increases with the increase of the quantity's variability.

Periodic quantities in electrical systems can be expressed as the sum of an infinite number of sinusoidal components, each having a frequency equal to an integer multiple of the fundamental frequency, called *harmonics*. Such a sum is referred to as a *Fourier series*. It is named after Jean B. Fourier, who developed the concept in 1822.

Harmonics are artificial, mathematical entities, convenient for handling periodic quantities and systems with such quantities, in particular, electrical systems with nonsinusoidal voltages and currents. The decomposition of voltages and currents into harmonics is a decomposition into components that do not exist physically, therefore, harmonics must be used very carefully. Some phenolmena − for example, current flow in linear circuits − can be studied successfully with a harmonic-by-harmonic approach, because such circuits satisfy the superposition principle. Such an approach may lead to substantial errors, however, when applied to systems that contain devices with a nonlinear voltage-current relationship. The same applies to analysis of power phenomena, since powers are products of voltages and currents. Products of the voltage and current individual harmonics may have no physical sense.

A single harmonic is specified in terms of three numbers: (1) the ratio of the frequency of the harmonic to the fundamental frequency, referred to as the *harmonic order*, (2) the root mean square (rms) value or *amplitude* of the harmonic and (3) the phase with respect to a time reference, common to all harmonics of the same quantity. The description of a quantity in terms of its harmonics (their order, rms value and phase) is re-ferred to as the description in the *frequency-domain*. A periodic current, i, that contains the fundamental harmonic, i_1, of the rms value 100 A and the seventh order harmonic, i_7, of the rms value of 15 A is shown in Fig. 1.

Harmonics in symmetrical three-phase circuits have a speci-fied *sequence*. Terminals of three-phase devices are ordered and tags−for example, R, S and T−are attributed to each of them. A three-phase quantity is of a *positive sequence*, if a particular phase of this quantity (e.g., a zero-crossing or maximum) is observed sequentially at the terminal R, next at S and after that at T. Harmonics of order *higher* by one than any multiplicity of three are of positive sequence. However, when the order of a harmonic is *lower* by one than any multiplicity of three, then a particular phase, after it is observed at terminal R, is not observed at terminal S but at terminal T. Harmonics of such orders are referred to as *negative sequence* harmonics. The same phase is observed simultaneously at terminals R, S and T, however, for harmonics of the order equal to any multiplicity of three. These are *zero sequence* harmonics. Voltage harmonics of negative sequences when applied to a three-phase winding of a motor create magnetic fields rotating in the opposite direction to that created by harmonics of the positive sequence. Harmonics of the zero sequence do not create a rotating field in such a winding at all.

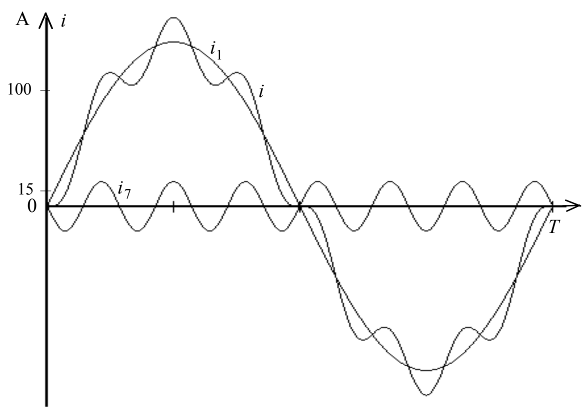

Figure 1. Plot of a periodic current, i, which contains the fundamental harmonic, i_1, of rms value of 100 A and the 7^{th}-order harmonic, i_7, of rms value of 15 A.

Harmonics may cause various harmful effects in electrical systems, both on the customers' and on the utilities' side. When customers are adversely affected by voltage harmonics, one refers it to deterioration of the *supply quality*. When utilities are affected by load-originated current harmonics, the *loading quality* is degraded by harmonics. Therefore, harmonic-related problems in electrical systems have been the subject of exten-sive studies. Several books (1-4) and several thousand articles on harmonics in power systems have been published. *IEEE Transactions on Power Delivery*, *on Industry Applications*, *on Power Electronics* and *on Instrumentation and Measurements* are the main American journals where publications on harmo-nics can be found. *Electrical Power Quality and Utilitation Journal, Archiv fur Elektrotechnik, Proceedings IEE,* and *European Transactions on Electrical Power, ETEP*, are impor-tant European sources of publications on harmonics and power phenomena. There is also a biannual International Conference on Harmonics and Power Quality in Power Systems, organized by the IEEE Power Society and the International Workshop on Power Definitions and Measurements in Nonsinusoidal Sys-tems, organized by the Italian Chapter of IEEE Instrumentation and Measurement

Society. Moreover, a lot of information can be found in IEEE Standards (5-7) and CIGRE Reports (8).

Harmonic-related issues can by subdivided into several categories. For readers interested in particular subject, a few references are provided below along with their classification.

1. Generation of harmonics by electrical and, in particular, by power electronics equipment and their propagation (6,8-12)
2. Equipment and power system modeling (13-17) to enable the determination of the level of harmonic distortion
3. Harmful effects of harmonics on the power system and customer equipment (18-23)
4. Measurement of harmonic content (24-26) and system parameters for harmonic frequencies (27)
5. Recommended limits of harmonics (6, 8)
6. Harmonics related power phenomena, power definitions and compensation, referred to as a *power theory* of systems with nonsinusoidal voltages and currents (28-51, 90-92)
7. Tariffs for electric energy (52, 53)
8. Reduction of harmonics and compensation with *reactive compensators* and in particular, *harmonic filters* (42, 52-53)
9. Reduction of harmonics and compensation with *switching compensators* (57-62, 85, 86, 94, 95)
10. Development of power electronic equipment with reduced current harmonics (63-65)

There are three major applications of the description of electrical quantities in terms of harmonics. (1) circuit analysis, (2) characterization of waveform distortion, (3) analysis of power phenomena; fundamentals of the power factor improvement and reduction of waveform distortion.

The first application is confined to linear circuits, that is, circuits that fulfill the *superposition principle*. Such a circuit at nonsinusoidal voltages and currents can be analyzed harmonic by harmonic as a circuit with sinusoidal voltages and currents. However, that is not possible in nonlinear circuits.

The second application is related to the *supply quality* in distribution systems, since the presence of the voltage harmonics means degradation of the quality of the supply. Also, generation of current harmonics by the load means degradation of the *loading quality* of customers' loads.

The third application of harmonics provides fundamentals for power theory of electrical systems with nonsinusoidal volta-ges and currents, meaning explanation of physical phenomena that accompany energy delivery. It contributes to developing definition of power quantities and description of energy flow in power terms. Power theory also provides fundamentals for methods of improving the effectiveness of energy delivery and reduction of waveform distortion. Harmonic filters and swit-ching compensators are used for that.

Distribution of windings in individual slots of the power plant generators is the primary cause of the voltage distortion in electrical power system. Therefore, generators are built to provide a voltage that is as close as possible to a sinusoidal voltage, and for synchronous generator the voltage harmonics are usually below 1% of the fundamental and are negligible. The energy to ac power systems is provided also from dc systems and from variable frequency generators, such as–for example–wind generators, through power electronics convertes. Unfortunately, such converters do not provide sinusoidal vol-tage. Voltage distortion occurs also as a result of the current distortion. It is because a distorted current causes distorted voltage drop on the power system impedances.

Current harmonics occur in electrical circuits due to three reasons: (1) nonsinusoidal supply voltage, (2) nonlinearity of electric equipment and (3) periodic time-variance of electrical parameters, usually caused by fast periodic switching.

The first reason, nonsinusoidal supply voltage, is the only cause of the current distortion in linear, time-invariant circuits. When harmonics occur in such a circuit, they can be moreover amplified by a resonance. Capacitors installed in distribution systems for improving the power factor or/and the capacitance of cable grids may resonate with the inductance of power system transformers. Even distributed capacitance and inductance of an overhead distribution line or a cable may contribute to amplification of the current and voltage harmonic when the length of such a line is comparable with the quarter-wave length of the electromagnetic wave of such a harmonic.

Nonlinearity of the voltage-current relationship of electrical devices and/or periodic switching are the main causes of current harmonics in electrical systems. Some devices such as transformers, are essentially linear devices. They generate current harmonics only due to saturation of the magnetic core, which can happen when its size is excessively reduced in order to reduce its cost. Nonlinearity is necessary, however, for the operation of some devices. Rectifiers are such devices; that use the nonlinearity of diodes for conversion of an *alternating current* (ac) into a *direct current* (dc). Periodic switching of *thyristors* makes energy flow control by *ac-dc converters* possible. Such devices cannot operate without generating current harmonics. They generate current harmonics, referred to as *characteristic harmonics,* which have an order that is specific for a particular type of equipment. Rectifiers and controlled ac-dc converters are *power electronics* devices, and development of power electronics is one of the main causes of an increase of harmonic distortion in electric distribution systems. Characteristic harmonics for three-phase rectifiers and ac-dc converters are of the order equal to a multiple of six plus and minus one: the 5th, 7th, 11th, 13th and so on.

Nonlinearity can also be not a necessary, but an intrinsic property of some devices–for example, fluorescent lamps or devices, such as arc furnaces, that use electric arcs. Generation of current harmonics cannot be avoided in such devices. Nonetheless, there are usually some possibilities for reducing current harmonics generated by nonlinear or switched devices by a proper choice of their structure. Loads that cause current distortion are generally referred to as *harmonic generating loads* (HGLs). Magnetic or electronic ballasts for fluorescent bulbs, and rectifiers in computers and video equipment, are the most common examples of low-power but numerous HGLs.

Rectifiers or ac-dc converters used for adjustable speed drive supplies are the most common industrial HGLs.

Current harmonics which occur due to HGLs propagate throughout the whole system, causing voltage distortion. Frequency properties and the system structure affect propagation of harmonics and waveform distortion. Frequency properties depend on the distribution of inductances in the system, mainly of transformers and overhead lines, and the distribution of capacitances, mainly of capacitor banks and cable grids. Due to resonances, harmonics can be attenuated or amplified. Also, *harmonic filters* installed in the system for suppressing harmonics complicate the frequency properties of the system substantially and may cause unexpected resonances and harmonic amplification.

The structure of the system, in particular the type of transformers, affects the propagation of harmonics substantially. Harmonics of the positive and negative sequences are essentially not affected by the structure of transformers, but harmonics of the zero sequence cannot go through transformers with windings that are connected in a delta configuration. In contrast, single-phase systems are coupled for harmonics of the zero sequence through the impedance of the neutral conductor.

The waveform distortion and harmonic contents caused by harmonic-generating loads can be calculated analytically only in very simple circuits. Moreover, such an analysis usually requires substantial simplifications of a circuit's properties. Therefore, computer modeling is the main tool for analysis of circuits with HGLs. Dedicated programs optimized for particular purposes or commercially available software can be used for modeling. In particular, software such as the Electromagnetic Transients Program (EMTP), HARMFLOW, or PSpice can be used for that purpose. Programs that model nonlinear devices describe the circuit in terms of nonlinear differential equations and integrate them numerically. Voltage and current waveforms are usually the outputs of such programs. They provide the waveforms in the transient state of the circuit, when they are usually nonperiodic, and in the steady state, when the transient components disappear. In the steady state waveforms are perio-dic and can be described in terms of harmonics.

A *Discrete Fourier Transform* (DFT) is the main mathematical tool for calculating the rms values and phases of the voltage and current harmonics when the values of the voltage and current at discrete instants of time, referred to as *samples*, are known. The amount of calculations needed by the DFT can be substantially reduced by using the *Fast Fourier Transform* (FFT) algorithm. The number of samples per waveform cycle for the DFT has to be more than twice as large as the order of the harmonic of the highest frequency. If this condition, known as the *Nyquist criterion*, is not fulfilled, harmonics are calculated with an error caused by the *spectrum-aliasing* phenomenon. This applies both when the samples are calculated by a circuit modeling program and when they are measured in a physical systems.

There are two approaches to measuring harmonics. Before the digital signal processing (DSP) technology was developed, *analog filters* were used for measurement of harmonic content. Such filters, tuned to frequencies of particular harmonics, were capable of measuring only their rms value. It was not possible to measure the harmonic phase; therefore, such measurements were useful only in situations where the harmonic phase was irrelevant. Digital meters of harmonics, known also as *harmonic analyzers*, are built of a *signal-conditioning* circuit, which normalizes the signal magnitude to a level that can be handled by digital devices; a *sample-and-hold* circuit, which takes analog samples of a continuous analyzed quantity; an *analog-to-digital converter*, which converts the analog samples to a digital form; a *digital data storage* device; and a *digital signal-processing unit*, which performs the FFT algorithm calculations needed for the DFT. Such meters provide both the rms value and the phase of harmonics for a single quantity or for several different quantities. Simultaneous sampling of all quantities may in fact be needed in such a case. Such a meter may be built as a separate dedicated device. A personal computer equipped with an additional board for digital data acquisition and DSP software may serve as a harmonic analyzer as well.

Harmful effects caused by harmonic distortion in customers' and power utilities' equipment are the main reason for the concern with harmonics. These effects differ substantially in their predictability. An increase in the current rms value, an increase in the loss of active power or a reduction of the mechanical torque of three-phase motors due to harmonics is easy to predict. Temperature increase and reduction of the lifetime expectation of motors and transformers due to an additional heat release are much more difficult to anticipate. The least predictable are disturbances of harmonic-sensitive devices, such as digital equipment and measuring, control and communication systems. They can be disturbed by harmonics on the supply lines (mainly on the neutral conductor, since this conductor is a collector of the zero-sequence current harmo-nics), as well as by capacitive and inductive coupling with other sources of the voltage and/or current harmonics. A current disturbing a device through capacitive coupling with a distorted voltage is proportional to the derivative of this voltage, and this derivative increases with the harmonic order. The same is true of a voltage induced in such a device by inductive coupling with a distorted current. This voltage is proportional to the derivative of the current, thus it also increases with the harmonic order. Apart from direct harmful effects, harmonics also make power factor improvement with capacitor banks less effective, since harmonic resonances and amplification of some harmonics may occur. More complex compensating devices are needed in the presence of harmonics. Consequently, the direct harmful effects of harmonics as well as the cost of various preventive methods make energy distribution and utilization in the presence of harmonics more expensive. The supply may also be less reliable because the failure rate of the distribution equipment increases with harmonic distortion.

Guidance with respect to the acceptable level of harmonics is provided by standards. IEEE Standard 519 (6) is a recommended standard for the US power system. It specifies the level of voltage harmonics for various voltage levels. This acceptable level declines as the voltage level increases. Also, it specifies the value of the current harmonics that can be injected into a system by HGLs. This level depends on the short-circuit power, meaning the system impedance at the HGL terminals. The lower the short-circuit power is (i.e., the higher the impedance), the lower the acceptable value of injected current harmonics is. These acceptable levels of harmonics are based on a consensus regarding a balance between the cost of harmful effects of harmonics and the cost of their reduction. However, this consensus applies to common situations. For specific situ-ations these recommended limits can be too liberal or too stringent.

When voltage distortion is unacceptably high, voltage harmonics can be reduced by reduction of the current harmonics injected by HGLs or by an increase in the short-circuit power at the bus where HGLs are installed–that is, by reducing the sys-tem impedance. There have been various attempts to develop power electronic equipment, mainly ac-dc converters, so that they generate as low current harmonics as possible. The injected current harmonics can also be reduced by additional equipment. *Harmonic filters* (HFs) or *switching compensators* (SCs) connected in parallel with the load, can be used for that purpose. HFs provide a low-impedance path for the dominating current harmonics generated by the load, so that they do not flow to the supply source. They consist of a few resonant LC and high-pass RLC branches connected in parallel with the load. Switching compensators, consisting of fast switches and an inductor or a capacitor for energy storage, can produce a current of the opposite sign to the load-generated current harmonics. Thus, these current harmonics concel. Such compensators are commonly known under the name of *active power filters*. However, they are neither active devices nor filters. Both HFs and SCs are also usually utilized for compensating the reactive power of the fundamental harmonic, meaning for the power factor improvement.

POWER THEORY OF SYSTEMS WITH NONSINUSOIDAL VOLTAGES AND CURRENTS

Of all harmonic related issues, the power phenomena at nonsinusoidal voltages and currents are the most controversial and confusing. Electric energy is very often conveyed at nonsinusoidal voltages and currents; consequently, a comprehension of power phenomena in such situations is both a scientific and a practical imperative. Therefore, they will be discussed in much more detail than other issues in this article. Comprehension of power phenomena may contribute to progress in methods of compensation and power factor improvement, to an improvement of tariffs for energy in the presence of harmonics, and to methods of improvement of the supply and loading qual-ity in distribution systems.

A set of power-related definitions, equations and interpretations of the power phenomena that remain valid irrespective of distortion level is referred to as a *power theory*. The reasons for the difference between the *active power* (the average value of energy delivered to the load over a period) and the *apparent power* (the product of the supply source voltage and current rms values) is a prime concern of power theory.

P.C. Stainmetz was the first to observe, in 1892 (48), that the *power factor* that is, the ratio of the active to the apparent powers, declines due to the waveform distortion caused by an electric arc, without any phase shift between the voltage and the current. This means that devices that cause waveform distortion cannot be described in terms of powers defined for systems with sinusoidal waveforms. After more than a century, the ques-tion on how the powers should be defined in the presence of waveform distortion still remains controversial.

There are two main approaches to defining powers and formulating power equations, namely, with and without the use of harmonics. The approach based on decomposition of the voltage and current into harmonics is referred to as a *frequency-domain* approach. The most disseminated frequency-domain power theory was developed by C.I. Budeanu (49) in 1927. The power definitions based on this theory are in the present *IEEE Standard Dictionary of Electrical and Electronics Terms* (5). Unfortunately, as was proven in Ref. (33) in 1987, this theory misinterprets power phenomena. The *reactive power Q* defined by Budeanu is not a measure of the apparent power increase due to energy oscillation as it is in the case of circuits with sinu-soidal waveforms. Also, the *distortion power D* defined by Budeanu is not a measure of the apparent power increase due to the waveform distortion. Attempts at formulating power theory without harmonic decomposition, (i.e., in the time-domain), were initiated by Fryze in 1931 (50). This approach requires much more simple instrumentation and provides algorithms for compensator control (30, 31, 59); however, it does not provide a physical interpretation of power phenomena.

Currently, the most advanced power theory is based on the concept of Currents' Physical Components (CPC), developed by Czarnecki (32, 34-36, 67, 69, 72). It provides physical interpretation of power phenomena in single-phase and unbalanced three-phase, three-wire systems under nonsinusoidal conditions, with linear, time-invariant loads and with harmonic generating loads (HGLs). It also provides fundamentals for reactive power compensation (32, 36) and the load balancing (42, 82-84) in such systems with reactive compensators and fundamentals for control of switching compensators (62, 84, 85, 94). The CPC- based algoritms can supersed algoritms based on the Instantane-ous Reactive Power (IRP) p-q Theory, developed in 1984 by Nabae, Akagi and Kanazawa (59). This theory misinterprets (80-81) power phenomena in systems with unbalanced loads.

FOURIER SERIES

The fundamentals of Fourier series and of the harmonic concept are presented in detail in Ref. (66). Some elements of this con-cept that are relevant to electrical circuits and the symbols used are explained below.

Electrical quantities such as voltages $u(t)$, currents $i(t)$ or fluxes $\phi(t)$, denoted generally by $x(t)$ or $y(t)$, are periodic if for any instant of time t they satisfy the relation

$$x(t) = x(t \pm nT) \tag{1}$$

where n is any integer and T, called a *period*, is a non zero real number. An example of a periodic quantity, $x(t)$, is shown in Fig. 2. Mathematically, the period T is the smallest number that satisfies Eq. (1). This condition is often neglected in electrical engineering. In particular, the period T of a power system voltage is usually considered to be the period of other periodic quantities in such a system, even if Eq. (1) is satisfied also for a shorter time. For example, the output voltage of a six pulse ac-dc converter satisfies Eq. (1) also for $T/6$, but it is usually con-sidered as a periodic quantity with the period T not $T/6$.

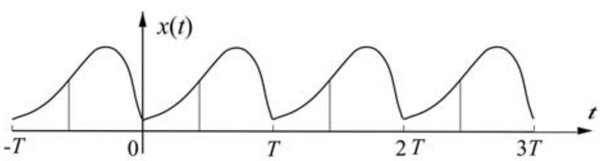

Figure 2. Periodic quantity $x(t)$ of the period T.

Periodic quantities in electrical systems are of a *finite power*, this means they are integrable with square, i.e.,

$$\frac{1}{T}\int_0^T x^2(t)\, dt \; < \; \infty \tag{2}$$

Quantities of the same period T and of a finite power form a linear space, denoted by $\boldsymbol{L}_T^2$, so that if $x(t)$ and $y(t)$ belong to this space, i.e., $x(t) \in \boldsymbol{L}_T^2$ and $y(t) \in \boldsymbol{L}_T^2$, then their linear form also belong to this space, i.e., $\alpha\, x(t) + \beta\, y(t) \in \boldsymbol{L}_T^2$, where α and β are any real numbers. The following functionals are defined in the space $\boldsymbol{L}_T^2$. The *norm*

$$||x|| \; = \; \sqrt{\frac{1}{T}\int_0^T x^2(t)\, dt} \tag{3}$$

referred to as the *root mean square* (rms) value in electrical engineering. The *distance*

$$d \; = \; ||x-y|| \; = \; \sqrt{\frac{1}{T}\int_0^T [x(t)-y(t)]^2\, dt} \tag{4}$$

and the *scalar product*

$$(x,y) \; = \; \frac{1}{T}\int_0^T x(t)\, y(t)\, dt \tag{5}$$

The rms value of the sum of two quantities $x(t)$, $y(t) \in \boldsymbol{L}_T^2$, is equal to

$$||z|| \; = \; ||x+y|| \; = \; \sqrt{||x||^2 + 2(x,y) + ||y||^2} \; = \; \sqrt{||x||^2 + ||y||^2} \tag{6}$$

only if $(x,y) = 0$

The quantities that have a zero scalar product are said to be mutually *orthogonal*. Thus, Eq. (6) holds true only for ortho-gonal quantities. In particular, quantities $x(t)$, $y(t) \in \boldsymbol{L}_T^2$ that have one of the following properties

$$x(t)\, y(t) = 0, \quad \text{for each } \; t \tag{7}$$

$$x(t) = \sqrt{2}\, X \sin(n\omega_1 t + \alpha)\,, \qquad y(t) = \sqrt{2}\, Y \cos(n\omega_1 t + \alpha) \tag{8}$$

$$x(t) = \sqrt{2} X \sin(k\omega_1 t + \alpha)\,, \qquad y(t) = \sqrt{2}\, Y \sin(n\omega_1 t + \beta)\,, \quad k \neq n \tag{9}$$

are mutually orthogonal.

If $x(t) \in \boldsymbol{L}_T^2$ then it has the Fourier series

$$f(t) \; = \; a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_1 t \; + \; \sum_{n=1}^{\infty} b_n \sin n\omega_1 t \tag{10}$$

with coefficients

$$a_0 \; = \; \frac{1}{T}\int_0^T x(t)\, dt \tag{11}$$

$$a_n \; = \; \frac{2}{T}\int_0^T x(t) \cos(n\,\omega_1 t)\, dt \tag{12}$$

$$b_n \; = \; \frac{2}{T}\int_0^T x(t) \sin(n\,\omega_1 t)\, dt \tag{13}$$

At each point t, where quantity $x(t)$ is continuous, $f(t) = x(t)$. If the quantity $x(t)$ has a discontinuity at a point $t = t_1$, then at such a point

$$f(t_1) \; = \; \frac{1}{2}[\lim_{t \to t_1^+} x(t) \; + \lim_{t \to t_1^-} x(t)] \tag{14}$$

that is, the Fourier series $f(t)$ converges to the mean value of the discontinuity of $x(t)$, or to the half of its left-side and right-side limits. Remembering this, it is a common custom to write the Fourier series neglecting the difference between $x(t)$ and $f(t)$

$$x(t) \; = \; a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_1 t \; + \; \sum_{n=1}^{\infty} b_n \sin n\omega_1 t \tag{15}$$

The term

$$x_n(t) = a_n \cos n\omega_1 t + b_n \sin n\omega_1 t = \sqrt{2} X_n \cos(n\omega_1 t - \alpha_n) \tag{16}$$

is referred to as the *harmonic of the order n* of quantity $x(t)$. The parameter X_n is equal to

$$X_n \; = \; \sqrt{\frac{a_n^2 + b_n^2}{2}}\,, \quad \text{for} \quad n \neq 0 \quad \text{and} \quad X_0 = a_0 \tag{17}$$

and denotes the rms value of the harmonic, while the parameter α_n, equal to

$$\alpha_n = \tan^{-1}(\frac{b_n}{a_n}) \tag{18}$$

is its phase. With the use of harmonics, the Fourier series of $x(t)$ can be written as

$$x(t) = \sum_{n=0}^{\infty} x_n(t) \tag{19}$$

When a quantity $x(t)$ has a limited number of harmonics of the order n from a set N, i.e.,

$$x(t) = \sum_{n \in N} x_n(t) \tag{20}$$

such a sum is called a *trigonometric polynomial* of $x(t)$. Periodic quantities in electrical engineering are usually approximated by trigonometric polynomials, and therefore, by a limited number of harmonics.

The Fourier series in the expression (15), referred to as a *classical form*, is badly suited for linear circuits analysis, since the circuit analyzed has to be described in terms of a set of differential equations and integrated numerically. The *complex form* of the Fourier series is more convenient for that purpose, namely

$$x(t) = X_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{X}_n e^{jn\omega_1 t} \tag{21}$$

with

$$\boldsymbol{X}_n = X_n e^{j\alpha_n} = \frac{a_n - jb_n}{\sqrt{2}} = \frac{\sqrt{2}}{T}\int_0^T x(t) e^{-jn\omega_1 t} dt \tag{22}$$

referred to as a *complex rms* (crms) value of the n^{th}-order harmonic. The set of the crms $\boldsymbol{X}_n$ values of all harmonics of the quantity $x(t)$ is called a *harmonic spectrum* of $x(t)$. The set of all rms X_n values is called a *harmonic rms spectrum.*

The rms value, the distance and the scalar product of periodic quantities defined by (3), (4) and (5) in the *time-domain*, can be calculated with the crms values of harmonics, that is, in the *frequency-domain*. They are equal to

$$||x|| = \sqrt{\sum_{n=0}^{\infty} X_n^2} \tag{23}$$

$$d = ||x - y|| = \sqrt{\sum_{n=0}^{\infty} |\boldsymbol{X}_n - \boldsymbol{Y}_n|^2} \tag{24}$$

$$(x, y) = \mathrm{Re}\sum_{n=0}^{\infty} \boldsymbol{X}_n \boldsymbol{Y}_n^* \tag{25}$$

The asterisk in the last formula denotes the complex conjugate number. Observe, that while these functionals were calculated, the integration in the time-domain was superseded with algebraic operations on the crms values, that means in a frequency-domain.

PROPERTIES OF HARMONICS' CRMS VAUES

The crms values $\boldsymbol{X}_n$ of harmonics have some properties that facilitate their calculation. The most useful properties are compiled below.

1. *The CRMS Values of Harmonics of a Linear Form.* If quantities $x(t)$ and $y(t) \in \boldsymbol{L}_T^2$ and their harmonics have the crms values $\boldsymbol{X}_n$ and $\boldsymbol{Y}_n$, then harmonics of their linear form equal to $z(t) = \alpha\, x(t) + \beta\, y(t)$ have the crms value

$$\boldsymbol{Z}_n = \alpha \boldsymbol{X}_n + \beta \boldsymbol{Y}_n \tag{26}$$

2. *The CRMS Values of Harmonics of a Shifted Quantity.* If harmonics of the quantity $x(t)$ have the crms values $\boldsymbol{X}_n$, then harmonics of the quantity $y(t)$ shifted as shown in Fig. 3, with respect to $x(t)$ by time τ, i.e., $y(t) = x(t - \tau)$ have the crms value

$$\boldsymbol{Y}_n = e^{-jn\omega_1\tau} \boldsymbol{X}_n \tag{27}$$

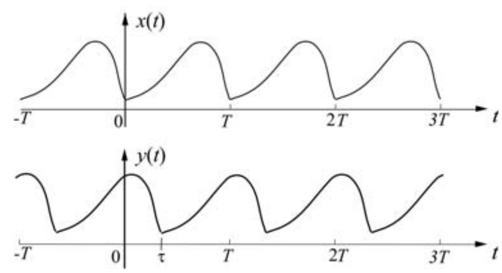

Figure 3. Quantities $x(t)$ and $y(t)$ shifted mutualy by time τ,

3. *The CRMS Values of Harmonics of a Reflected Quantity.* If harmonics of the quantity $x(t)$ have the crms values $\boldsymbol{X}_n$, then harmonics of the quantity $y(t)$ reflected with respect to $x(t)$, as shown in Fig. 4, i.e., $y(t) = x(-t)$ have the crms value

$$\boldsymbol{Y}_n = \boldsymbol{X}_n^* \tag{28}$$

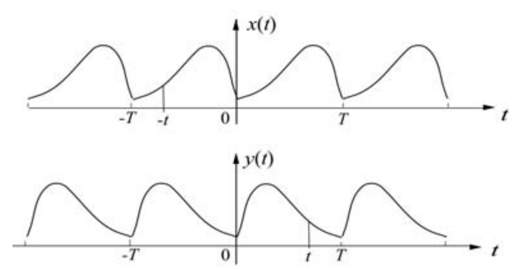

Figure 4. Quantity $x(t)$ and reflected quantity $y(t) = x(-t)$.

4. *The CRMS Values of Harmonics of an Even Quantity*. If the quantity $x(t)$ is symmetrical with respect to the time reference point, $t = 0$, meaning $x(t) = x(-t)$, then for each harmonic

$$\mathrm{Im}\{\boldsymbol{X}_n\} = 0 \tag{29}$$

5. *The CRMS Values of Harmonics of an Odd quantity*. If the quantity $x(t)$ is asymmetrical with respect to the time reference point, $t = 0$, meaning $x(t) = -x(-t)$, then for each harmonic

$$\mathrm{Re}\{\boldsymbol{X}_n\} = 0 \qquad (30)$$

6. *The CRMS Values of Harmonics of a Quantity Odd with Respect to the Values Shifted by Half the Period.* If the quantity $x(t)$ is asymmetrical with respect to the values shifted by half of the period, as shown in Fig. 5, [i.e., $x(t-T/2) = -x(t)$], then

$$\boldsymbol{X}_{2k} = 0 \qquad (31)$$

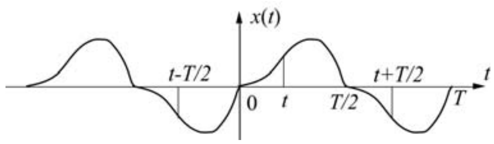

Figure 5. Quantity which is odd with respect to the values shifted by the half of the period.

which means the quantity can have harmonics only of an odd order.

7. *The CRMS Values of Harmonics of the Derivative a Quantity.* If the harmonics of the quantity $x(t)$ have the crms value $\boldsymbol{X}_n$ and the derivative $y(t) = dx(t)/dt \in \boldsymbol{L}_T^2$, then harmonics of the quantity $y(t)$ have the crms values

$$\boldsymbol{Y}_n = jn\omega_1 \boldsymbol{X}_n \qquad (32)$$

8. *The CRMS Values of Harmonics of the Integral a Quantity.* If the harmonics of the quantity $x(t)$ have the crms value $\boldsymbol{X}_n$ and the integral $y(t) = \int x(t)\,dt \in \boldsymbol{L}_T^2$, then hamonics of the quantity $y(t)$ have the crms values

$$\boldsymbol{Y}_n = \frac{1}{jn\omega_1}\boldsymbol{X}_n \qquad (33)$$

The rms value of the zero-order harmonic, X_0, can have any real value.

Application of these properties for calculating harmonics' crms values is illustrated with the following example.

Example of Application. Figure 6a shows a trapezoidal approximation of the supply current $i(t)$ of a six-pulse ac-dc converter with inductive filtering of the output current. The commutation angle of the converter $\mu = \omega_1 \tau = 10^0$. Let us find the formula for the calculation of the crms values of the current harmonics for $I = 100$ A.

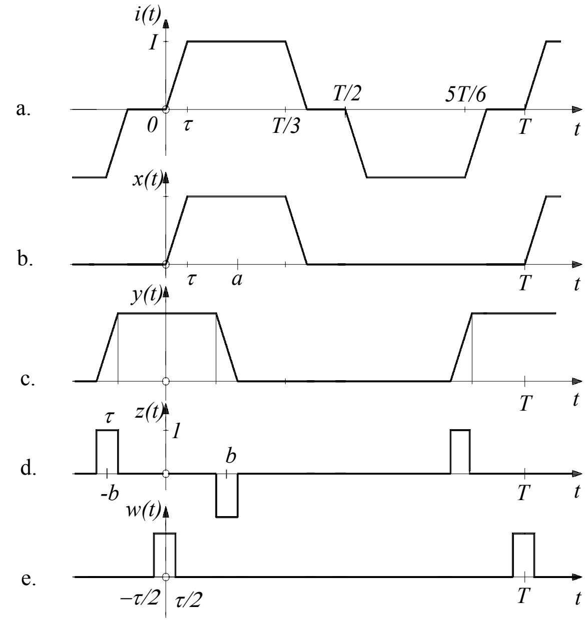

Figure 6. (a) Trapezoidal approximation of the supply current $i(t)$ of a three-phase ac-dc converter with an inductive filter; (b) its positive component $x(t)$; (c) shifted positive component $y(t)$; (d) its derivative $z(t)$ and (e) shifted positive component $w(t)$ of derivative $z(t)$.

The current can be considered as a linear form of two components $x(t)$ and $x(t-T/2)$, where $x(t)$ is shown in Fig. 6(b), thus $i(t) = x(t) - x(t-T/2)$, hence

$$\boldsymbol{I}_n = (1 - e^{-jn\omega_1\frac{T}{2}})\,\boldsymbol{X}_n = (1 - e^{-jn\pi})\,\boldsymbol{X}_n = \begin{cases} 0, & \text{for } n = 2k \\ 2\boldsymbol{X}_n, & \text{for } n = 2k+1 \end{cases}$$

Since it is easier to calculate the crms values for symmetrical quantities than for quantities without any symmetries, we can treat quantity $x(t)$ as the quantity $y(t)$ shown in Fig. 6(c), shifted by $a = (T/3 + \tau)/2$, that means, $x(t) = y(t-a)$, hence

$$\boldsymbol{X}_n = e^{-jn\omega_1 a}\,\boldsymbol{Y}_n = e^{-jn(\frac{\pi}{3}+\frac{\mu}{2})}\,\boldsymbol{Y}_n$$

The quantity $y(t)$ is an integral of the rectangular pulses $z(t)$, shown in Fig. 6(d), namely

$$y(t) = \frac{I}{\tau}\int z(t)\,dt$$

hence

$$\boldsymbol{Y}_n = \frac{I}{\tau}\frac{1}{jn\omega_1}\boldsymbol{Z}_n = \frac{I}{jn\mu}\boldsymbol{Z}_n$$

The quantity $z(t)$ is a linear form of the pulse $w(t)$ shown in Fig. 6(e), shifted by $\pm b$, where $b = a - \tau/2 = T/6$, namely,

$$z(t) = w(t+b) - w(t-b)$$

Thus

$$\boldsymbol{Z}_n = (e^{jn\omega_1 b} - e^{-jn\omega_1 b})\,\boldsymbol{W}_n = 2j\sin(n\frac{\pi}{3})\,\boldsymbol{W}_n$$

where the crms values of harmonics of the pulses $w(t)$ are equal to

$$\boldsymbol{W}_n = \frac{\sqrt{2}}{T}\int_{-T/2}^{T/2} w(t)\, e^{-jn\omega_1 t} dt = \frac{\sqrt{2}}{T}\int_{-\tau/2}^{\tau/2} e^{-jn\omega_1 t} dt = \frac{\sqrt{2}}{n\pi}\sin(n\frac{\mu}{2})$$

Finaly, the supply current harmonics have the crms values

$$\boldsymbol{I}_n = \frac{2}{3}\sqrt{2}\,\frac{\sin(n\,\pi/3)}{n\,\pi/3}\,\frac{\sin(n\,\pi/2)}{n\,\pi/2}\, I\, e^{-jn(\frac{\pi}{3}+\frac{\mu}{2})}$$

For I = 100 A and commutation angle $\mu = 10^0$, the crms values of the current harmonics up to the 17^{th}-order are equal to

$\boldsymbol{I}_1 = 77.87\, e^{-j65^0}$ A , $\boldsymbol{I}_5 = 15.10\, e^{-j145^0}$ A $\boldsymbol{I}_7 = 10.45\, e^{-j95^0}$ A

$\boldsymbol{I}_{11} = 6.05\, e^{-j175^0}$ A , $\boldsymbol{I}_{13} = 4.79\, e^{-j125^0}$ A , $\boldsymbol{I}_{17} = 3.08\, e^{j155^0}$ A

The sum of these six harmonics is shown in Fig. 7.

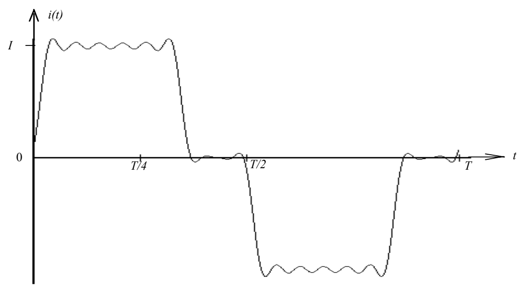

Figure 7. Sum of six harmonics of the trapezoidal approximation of the supply current of a three-phase ac-dc converter.

HARMONICS IN LINEAR CIRCUITS WITH LUMPED RLC PARAMETERS

Linear circuits with lumped RLC parameters are described in terms of Kirchoff's voltage and current laws and the voltage-current relations for the circuit RLC parameters.

1. *Kirchoff's current law* (KCL) for a node of K branches,

$$\sum_{k=1}^{K} i_k(t) = 0 \tag{34}$$

is a linear form of the branch currents,

$$i_k(t) = I_{k0} + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty}\boldsymbol{I}_{kn}\, e^{jn\omega_1 t} \tag{35}$$

Due to linearity of crms values, the KCL is satisfied for a node if for each harmonic

$$\sum_{k=1}^{K}\boldsymbol{I}_{kn} = 0 \tag{36}$$

2. *Kirchoff's voltage law* (KVL) for a closed path with M voltages,

$$\sum_{m=1}^{M} u_m(t) = 0 \tag{37}$$

is a linear form of voltages,

$$u_m(t) = U_{m0} + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty}\boldsymbol{U}_{mn}\, e^{jn\omega_1 t} \tag{38}$$

KVL is satisfied for the closed path if for each harmonic

$$\sum_{m=1}^{M}\boldsymbol{U}_{mn} = 0 \tag{39}$$

3. *The voltage-current relations* can be written in one of the following forms:

$$i(t) = Y_0 U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty}\boldsymbol{Y}_n\boldsymbol{U}_n\, \mathrm{e}^{jn\omega_1 t} \tag{40}$$

or

$$u(t) = Z_0 I_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty}\boldsymbol{Z}_n\boldsymbol{I}_n\, \mathrm{e}^{jn\omega_1 t} \tag{41}$$

with

$$\boldsymbol{Z}_n = Z_n\, e^{j\varphi_n} = R_n + jX_n = \frac{1}{\boldsymbol{Y}_n} \tag{42}$$

referred to as an *impedance for the n th-order harmonic*. The *s*ymbol

$$\boldsymbol{Y}_n = Y_n\, e^{-j\varphi_n} = G_n + jB_n \tag{43}$$

denotes an *admittance* for that harmonic. The impedance $\boldsymbol{Z}_n$ for a series RLC branch, shown in Fig. 8(a), is equal to

$$\boldsymbol{Z}_n = R + jn\omega_1 L + \frac{1}{jn\omega_1 C} \tag{44}$$

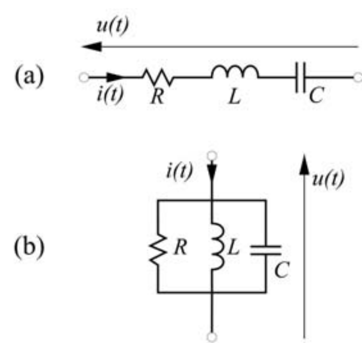

Figure 8. (a) Series RLC branch and (b) parallel RLC branch.

For a parallel RLC branch, shown in Fig. 8(b), the impedance $\boldsymbol{Z}_n$ is calculated as

$$\boldsymbol{Z}_n = \frac{1}{G + jn\omega_1 C + \frac{1}{jn\omega_1 L}}\,,\ \text{with } G = \frac{1}{R} \tag{45}$$

These voltage-current relations along with Kirchoff's laws provide fundamentals for all methods of steady-state analysis of linear circuits in the presence of voltage and current harmonics. The harmonic approach enables us to describe linear circuits in terms of a set of algebraic equations with complex coefficients, separately for each harmonic.

HARMONICS OF SYMMETRICAL THREE-PHASE QUANTITIES

When ac electric energy is conveyed to large customers, three-phase, three-wire systems are usually used. The lines and termi-nals of three-phase devices have to be ordered and tagged, for example as terminals R, S, and T, as shown in Fig. 9. Three-phase systems are built to achieve the symmetry of voltages and currents, as far as possible.

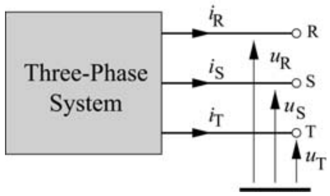

Figure 9. Three-phase, three-wire system, its terminals, line currents and line-to-ground voltages.

The voltage provided to a customer is of the *positive sequence*. This means that a particular phase of the voltage–for example, a maximum or a zero crossing, after–it is observed at terminal R, is observed after one third cycle at terminal S, and next, after another one third cycle, at terminal T. Thus, a symmetrical and positive sequence three-phase quantity, for example a voltage, satisfies the relationship

$$u_S(t) = u_R(t-\frac{T}{3}), \quad u_T(t) = u_R(t-2\frac{T}{3}) = u_R(t+\frac{T}{3}) \tag{46}$$

Such a symmetrical, positive sequence three-phase voltage is shown in Fig. 10.

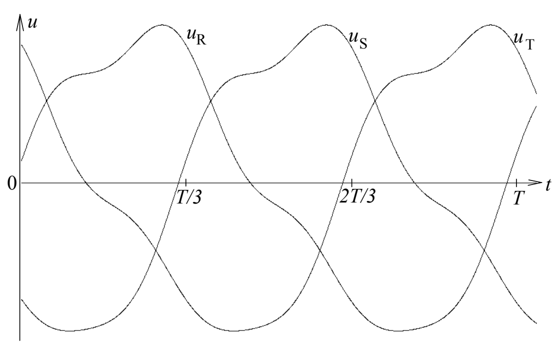

Figure 10. Example of three-phase nonsinusoidal symmetrical voltages of the positive sequence composed of harmonics of the second, third, and fourth order.

When the voltage at terminal R contains the n^{th}-order harmonic, namely

$$u_{Rn}(t) = \sqrt{2}\, U_{Rn} \cos(n\omega_1 t - \alpha_{Rn}) \tag{47}$$

then this harmonic at terminal S and T are equal to

$$u_{Sn}(t) = \sqrt{2}\, U_{Rn} \cos[n\omega_1(t-\frac{T}{3}) - \alpha_{Rn}] = \sqrt{2}\, U_{Rn} \cos(n\omega_1 t - \alpha_{Rn} - n\frac{2\pi}{3}) \tag{48}$$

$$u_{Tn}(t) = \sqrt{2}\, U_{Rn} \cos[n\omega_1(t+\frac{T}{3}) - \alpha_{Rn}] = \sqrt{2}\, U_{Rn} \cos(n\omega_1 t - \alpha_{Rn} + n\frac{2\pi}{3}) \tag{49}$$

When a voltage harmonic is of the order $n = 3k$, then $3k$ x $2\pi/3$ = k x 2π. Harmonics of such an order when observed at terminals R, S and T are in phase, i.e.,

$$u_{Rn}(t) = u_{Sn}(t) = u_{Tn}(t) \tag{50}$$

Such harmonics are referred to as the *zero sequence harmonics*. They are not able to propagate in three-wire systems. Such sys-tems behave as open circuits for zero sequence harmonics.

When a voltage harmonic is of the order $n = 3k+1$, then $(3k+1)$ x $2\pi/3 = k$ x $2\pi + 2\pi/3$. This means that the same phase of such a harmonic is observed in the same sequence as the three-phase quantity, namely

$$u_{Sn}(t) = u_{Rn}(t-T/3n), \quad u_{Tn}(t) = u_{Rn}(t+T/3n) \tag{51}$$

Such harmonics are referred to as the *positive sequence harmonics*.

When a voltage harmonic is of the order of $n = 3k - 1$, then $(3k-1)$ x $2\pi/3 = k$ x $2\pi - 2\pi/3$. It means that the same phase of such a harmonic is observed in the opposite sequence than the sequence of the three-phase quantity, namely

$$u_{Sn}(t) = u_{Rn}(t+T/3n), \quad u_{Tn}(t) = u_{Rn}(t-T/3n) \tag{52}$$

Such harmonics are referred to as *negative sequence harmonics*. There is no difference in the propagation of the positive and the negative sequence harmonics. However, they create magnetic fields rotating in opposite directions in electric motors. An example of voltage harmonics of the second, third and fourth order, that is, the negative, zero and positive sequence, are shown in Fig. 11. Just these harmonics, with amplitude 25 % of the fundamental, result in the voltage distortion shown in Fig. 10.

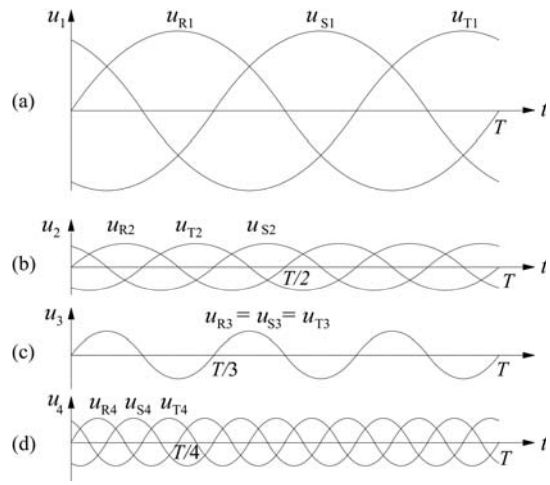

Figure 11. Harmonics of the voltages shown in Fig. 10; (a) the fundamental harmonic; (b) the second-order harmonic which is of the negative sequence; (c) the third-order harmonic which is of the zero sequence; and (d) the fourth-order harmonic which is of the positive sequence.

INSTANTENEOUS POWER IN SINGLE-PHASE CIRCUITS

The *instantaneous power* $p(t)$ at a cross-section of a circuit where the voltage $u(t)$ and the current $i(t)$ are observed is defined as the rate of electric energy $W(t)$ flow from the supply source to the load, namely

$$p(t) = \frac{d}{dt} W(t) = u(t)\, i(t) \tag{53}$$

As the rate of energy flow, the instantaneous power has a clear physical interpretation. When this rate is negative, energy flows back from the load to the supply source. The instantaneous power of a passive resistive load, $p(t) = R\, [i(t)]^2$, is non-negative and it is commonly assumed that, in spite of fluctuation of the instantaneous power, there is no energy oscillation between the supply source and resistive loads. A phase-shift between the voltage and current could be a cause of energy oscillation. The change of energy stored in electric or magnetic fields of inductors and/or capacitors is the only cause of such a phase shift in linear, time-invariant circuits. When a circuit is time-variant (in particular, with periodic switching), then a phase shift between voltage and current harmonics may occur even without energy storage capability. A light dimmer, where a semiconductor device known as a *triac* is used as a periodic switch to control the rms value of the current of an incandesent bulb, is a common example of such a circuit. The fundamental harmonic of the supply current is shifted with respect to the supply voltage in such a circuit without energy storage. In spite of this phase shift and the presence of *reactive power*, there is no reciprocating energy oscillation between the supply and the load, since the instantaneous power $p(t)$ is non-negative in such a circuit.

When a circuit has, including all sources and loads, K components with voltages $u_k(t)$ and currents $i_k(t)$ then

$$\sum_{k=1}^{K} u_k(t)\, i_k(t) = \sum_{k=1}^{K} p_k(t) = 0 \tag{54}$$

This is a conclusion from Tellegan's law and is referred to as the *balance principle for the instantaneous power*. This means that instantaneous power of all sources and loads is balanced at each instant of time.

The formula in Eq. (53) for the instantaneous power, when the voltage and current are expressed as a sum of harmonics,

$$p(t) = u(t)\, i(t) = \sum_{n=0}^{\infty} u_n(t) \sum_{n=0}^{\infty} i_n(t) \tag{55}$$

can be a source of a substantial misconception. This is because the product of two Fourier series contains an infinite number of oscillating components which is interpreted by some authors (45) as a proof that energy oscillates between the supply source and the load. In fact, such oscillating terms in the instantaneous power may exist even if there is no energy flow between the source and the load at all. This is illustrated (50) with the circuit shown in Fig. 12.

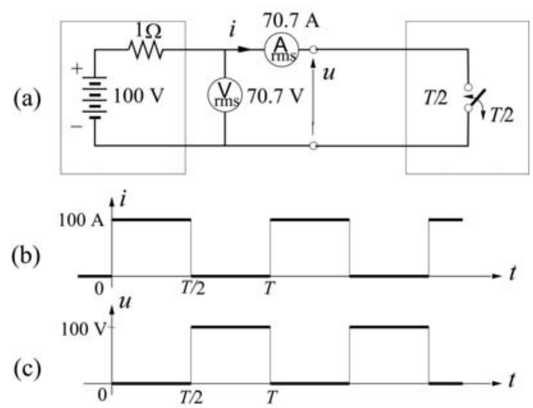

Figure 12. (a) Dc source loadad with a periodic switch, (b) load voltage and (c) load current. The instantaneous power at the load terminals, calculated as the product of the voltage and current harmonics, contains an infinite number of oscillating components, but there is no energy oscillations, since the instantaneous power is equal to zero. This circuit demonstrates that the apparent power may have a nonzero value without energy oscillations.

The load, supplied from a dc source, consists only of a periodic switch. The product of the load voltage and current Fourier series contains an infinite number of oscillating components, while at the same time there is no energy flow to such a load. In spite of the presence of the oscillating components of the instantaneous power $p(t)$, energy does not flow to the load when the switch is open or closed. There is only a very small amount of energy in the load associated with the stray capacitance and inductance. The product $u(t)\ i(t)$ is equal to zero over the whole period of time, apart from instants where the voltage and cur-rent have discontinuities.

ACTIVE POWER IN SINGLE-PHASE CIRCUITS

The *active power* is the average value of the instantaneous power $p(t)$ over a single period of the voltage, namely

$$P = \overline{p(t)} = \frac{1}{T}\int_0^T u(t)\, i(t)\, dt = (u,i) \tag{56}$$

The active power in systems with sinusoidal voltages and currents is a synonymous with the *useful power*. This may not be true in the presence of harmonics, since the active power associated with harmonics may not contribute to useful work, but to various harmful effects.

When a circuit has, including all sources and loads, K components with voltages $u_k(t)$ and currents $i_k(t)$, then the mean value of the instantaneous power of all components, that means their active power, fulfills the relationship,

$$\overline{\sum_{k=1}^{K} u_k(t)\, i_k(t)} = \sum_{k=1}^{K} \overline{p_k(t)} = \sum_{k=1}^{K} P_k = 0 \tag{57}$$

referred to as the *balance principle for the active power*.

The active power is the scalar product of the voltage and current, then if the crms values of the voltage and current harmonics are equal to $\boldsymbol{U}_n = U_n\, e^{j\alpha_n}$ and $\boldsymbol{I}_n = I_n\, e^{j\beta_n}$, respectively, then the active power can be expressed, according to Eq. (25), as

$$P = \mathrm{Re}\sum_{n=0}^{\infty} \boldsymbol{U}_n \boldsymbol{I}_n^{*} = \sum_{n=0}^{\infty} U_n I_n \cos\varphi_n, \quad \text{with } \varphi_n = \alpha_n - \beta_n \tag{58}$$

Since $\boldsymbol{I}_n = \boldsymbol{Y}_n \boldsymbol{U}_n = (G_n + jB_n)\boldsymbol{U}_n$, then the active power can be expressed as

$$P = \sum_{n=0}^{\infty} G_n U_n^2 \tag{59}$$

Similarly, since $\boldsymbol{U}_n = \boldsymbol{Z}_n \boldsymbol{I}_n = (R_n + jX_n)\boldsymbol{I}_n$, the active power can be expressed as

$$P = \sum_{n=0}^{\infty} R_n I_n^2 \tag{60}$$

The term, which can alternatively be expressed as

$$P_n = \mathrm{Re}\{\boldsymbol{U}_n \boldsymbol{I}_n^{*}\} = U_n I_n \cos\varphi_n = G_n U_n^2 = R_n I_n^2 \tag{61}$$

is called the *harmonic active power* of the n^{th}-order harmonic. Equation (57), which describes the balance principle for the active power in a circuit with harmonic orders $n \in N$, can be written in the form

$$\sum_{k=1}^{K} P_k = \sum_{k=1}^{K} \Big(\sum_{n\in N} P_{kn}\Big) = \sum_{n\in N} \Big(\sum_{k=1}^{K} P_{kn}\Big) = 0 \tag{62}$$

This equation is fulfilled for any set N, only if

$$\sum_{k=1}^{K} P_{kn} = 0 \tag{63}$$

for all harmonic orders n. This means that the sum of harmonic active power of all components of an electric circuit has to be equal to zero for each harmonic separately. This is a *balance principle for harmonic active power*.

APPARENT POWER IN SINGLE-PHASE CIRCUITS

When the load has the active power specified with Eq. (56), then the rms value of the supply source voltage is, according to Eq. (23), equal to

$$||u|| = \sqrt{\sum_{n=0}^{\infty} U_n^2} \tag{64}$$

and the rms value of the source current is equal to

$$||i|| = \sqrt{\sum_{n=0}^{\infty} I_n^2} \tag{65}$$

The supply source has to provide the voltage and current of the rms values $||u||$ and $||i||$ independently of the load active power P, and these two rms values affect the power ratings of the supply source and the active power loss inside of the source indepen-dently of each other. Therefore, the power rating of supply sources is characterized by the product of the voltage and current rms values $||u||$ and $||i||$ they are able to provide, referred to as an *apparent power*, namely

$$||u||\, ||i|| = S \tag{66}$$

This is not a physical quantity, however, but a conventional one. The adjective *apparent* emphasizes the fictitious nature of this power. There is no physical phenomenon related to the apparent power. For example, the apparent power S in the circuit shown in Fig. 12 is equal to $S = ||i||\, ||u|| = 70.7 \text{ x } 70.7 = 5000$ VA, and this power is not related to any power phenol-menon in the load, since there is no energy flow to the load (the switch), and in particular, this power is not related to any reciprocating oscillation of energy between the load and the source.

Because the apparent power is a conventional quantity, other conventions are also possible. Nonetheless, definition in Eq. (66) is commonly used in the power theory of single-phase electrical circuits.

BUDEANU'S REACTIVE AND DISTORTION POWERS

Apart from powers discussed above, a *reactive power* Q is defined for circuits with sinusoidal voltages and currents.

When the load voltage has only a single harmonic of the n^{th} -order, which means it has the waveform $u_n(t) = \sqrt{2}\, U_n \cos n\omega_1 t$, the current is equal to $i_n(t) = \sqrt{2}\, I_n \cos(n\omega_1 t - \varphi_n)$, and the instantaneous power $p(t)$ can be decomposed into the two following components

$$p(t) = P_n(1 + \cos 2n\omega_1 t) + Q_n \sin 2n\omega_1 t \tag{67}$$

with

$$P_n = U_n I_n \cos\varphi_n\,, \quad Q_n = U_n I_n \sin\varphi_n \tag{68}$$

The second term in Eq. (67) is an oscillating component of the instantaneous power. Its amplitude Q_n is referred to as the *reactive power of the n^th^-order harmonic*. It is denoted by Q in sinusoidal systems, where this power is a component of the power equation

$$P^2 + Q^2 = S^2 \tag{69}$$

The reactive power for single-phase circuits with nonsinusoidal waveforms was defined by C.I. Budeanu (49) in 1927. To distinguish this definition from other definitions of the reactive power, it is denoted by here Q_B, namely

$$Q_B = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n = \sum_{n=1}^{\infty} Q_n \tag{70}$$

With such a definition of the reactive power, we have

$$P^2 + Q_B^2 \le S^2 \tag{71}$$

Therefore, Budeanu introduced a new power quantity

$$D = \sqrt{S^2 - (P^2 + Q_B^2)} \tag{72}$$

and called it the *distortion power*. Budeanu's definitions of the reactive and distortion powers are supported by the *IEEE Standard Dictionary of Electrical and Electronics Terms* (5), and they are widely disseminated in electrical engineering.

By an analogy to its interpretation in circuits with sinusoidal waveforms, the reactive power Q_B is interpreted as a measure of the increase in apparent power S due to energy oscillation between the source and the load. The distortion power D is interpreted as a measure of the increase in apparent power due to waveform distortion. Unfortunately, these two interpretations are erroneous (33). According to Eq. (70), the reactive power is defined by Budeanu as a sum of amplitudes Q_n of the oscil-lating components of the instantaneous power with different frequencies, $2n\omega_1$. These amplitudes, according to Eq. (68), can be positive or negative. Therefore, oscillation of energy between the source and the load may exists even if the sum of these amplitudes is equal to zero. This is illustrated with the load shown in Fig. 13. The load is supplied with the voltage $u(t) = \sqrt{2}\,(100\sin\omega_1 t + 25\sin 3\omega_1 t)$ V . The parameters of the load were chosen such that the reactive power Q_B is equal to zero.

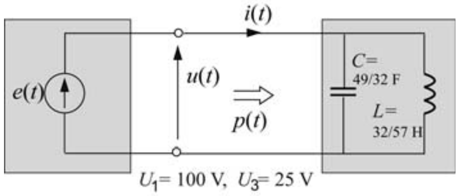

Figure 13. Circuit with Budeanu's reactive power Q_B equal to zero but with energy oscillation between the source and the load.

As shown in Figure 14, there are intervals of time when the instantaneous power $p(t)$ is negative, so that energy flows back to the source, thus there is an oscillation of energy between the load and the source. The reactive power Q_B is no measure of the effect of this oscillation on the apparent power.

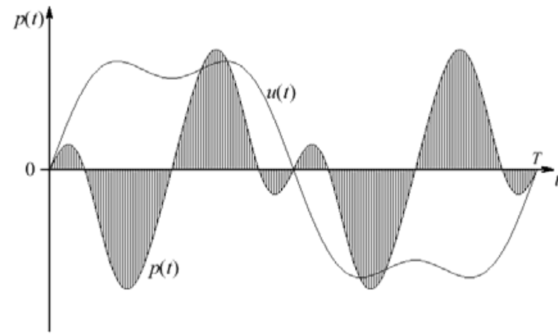

Figure 14. The supply voltage $u(t)$ and the instantaneous power $p(t)$ in the circuit shown in Fig. 13. Change of the sign of the instantaneous power $p(t)$ means that there is oscillation of energy in this circuit between the source and the load.

Also the distortion power D is interpreted erroneously. When the supply voltage has harmonics of orders from a set N, the formula (72) can be written in the form

$$D = \sqrt{\frac{1}{2}\sum_{r\in N}\sum_{s\in N} U_r^2 U_s^2 |\mathbf{Y}_r - \mathbf{Y}_s|^2} \tag{73}$$

This means, that distortion power D is equal to zero only if the load admittances for all harmonics of the order $n \in N$,

$$\mathbf{Y}_n = \text{const.} \tag{74}$$

However, in order to meet this condition the load current must be distorted with respect to the load voltage. This current is not distorted but only shifted, i.e., $i(t) = Y\,u(t-\tau)$, on the condition that

$$\mathbf{Y}_n = Y\,e^{-jn\omega_1\tau} \tag{75}$$

Thus, apart from resistive loads, the condition (74) for the zero distortion power D and the condition (75) for the lack of waveform distortion are mutually exclusive.

The situation where the distortion power D is equal to zero in spite of the distortion of the load current with respect to the supply voltage is illustrated with the load shown in Fig. 15. It is assumed that the supply voltage contains the fundamental and the third-order harmonics. The load parameters were chosen such that the admittance for these harmonics are mutually equal, namely, $\mathbf{Y}_1 = \mathbf{Y}_3 = 1e^{-j\frac{\pi}{2}}$ S , thus according to Eq. (73) the distortion power D is equal to zero.

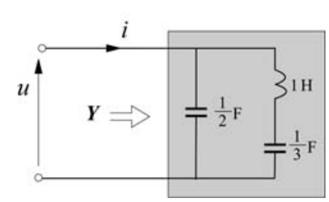

Figure 15. A circuit that has the same load admittance for the fundamental and the third-order harmonic, and consequently zero distortion power *D* in spite of current distortion.

However, the waveform of the load current at the supply voltage $u(t) = \sqrt{2}\,(100\sin\omega_1 t + 50\sin 3\omega_1 t)$ V, plotted in Fig. 16, shows that the voltage and the current waveforms are mutually distorted.

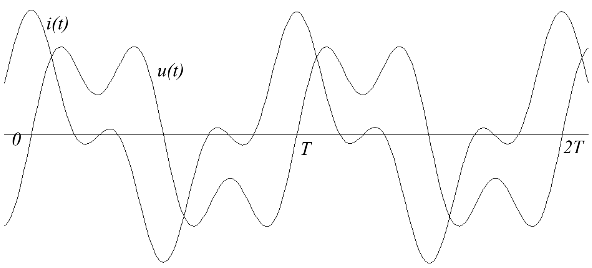

Figure 16. The load voltage and current waveform in the circuit shown in Fig. 15. They are mutually distorted dispite of zero distortion power *D*.

The reactive power defined with Eq. (70) proves also to be useless for improvement of the power factor in the presence of harmonics. In systems with sinusoidal waveforms, the value of the reactive power *Q* enables the design of a compensator that improves the power factor to unity. Unfortunately, all attempts to do the same in nonsinusoidal systems using the value of Budeanu's reactive power Q_B have failed. The reasons for this were explained in Ref. (33). A current harmonic can be decomposed as follows

$$i_n(t) = \sqrt{2}\, I_n \cos(n\omega_1 t - \varphi_n) = \sqrt{2}\,\left(\frac{P_n}{U_n}\cos n\omega_1 t + \frac{Q_n}{U_n}\sin n\omega_1 t\right) \quad (76)$$

Hence, the rms value of the supply current can be expressed as

$$\|i\| = \sqrt{\sum_{n\in N}\|i_n\|^2} = \sqrt{\sum_{n\in N}\left(\frac{P_n}{U_n}\right)^2 + \sum_{n\in N}\left(\frac{Q_n}{U_n}\right)^2} \quad (77)$$

This formula shows that at unchanged harmonic active powers P_n, the supply current has a minimum rms value, not when

$$\sum_{n\in N} Q_n = Q_B = 0, \text{ but when } \sum_{n\in N}\left(\frac{Q_n}{U_n}\right)^2 = 0$$

This has nothing to do with Budeanu's reactive power, Q_B.

CURRENT'S PHYSICAL COMPONENTS POWER THEORY OF LINEAR, TIME-VARIANT LOADS

When a load, shown in Fig. 17(a), with harmonic admittances $\boldsymbol{Y}_n$, supplied with the voltage $u(t)$, has the active power *P*, then a resistive load, shown in Fig. 17(b), is equivalent to that load with respect to the active power if its conductance is equal to

$$G_e = \frac{P}{\|u\|^2} \quad (78)$$

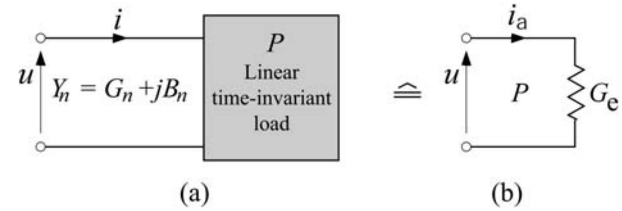

Figure 17. (a) Linear time-invariant load and (b) equivalent load with respect to the active power *P* at the same voltage *u*. The equivalent load is a resistive load, which draws the active current i_a from the supply source.

Such a load draws the current

$$i_a(t) = G_e u(t) \quad (79)$$

referred to as the *active current*. It can be considered as the main component of the load current associated with the load active power. The concept of the active current was introduced by S. Fryze in 1931 (50). The remaining current of the load

$$i(t) - i_a(t) = (G_0 - G_e)U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty}(G_n + jB_n - G_e)\boldsymbol{U}_n e^{jn\omega_1 t} \quad (80)$$

can be decomposed into two components. The current

$$i_r(t) = \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n \boldsymbol{U}_n e^{jn\omega_1 t} \quad (81)$$

is referred to as a *reactive current*. Its concept was introduced in 1972 by W. Shepherd and P. Zakikhani (28). The second current

$$i_s(t) = (G_0 - G_e)U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty}(G_n - G_e)\,\boldsymbol{U}_n e^{jn\omega_1 t} \quad (82)$$

is referred to as a *scattered current*. Its presence in the load current was revealed by L.S. Czarnecki in Ref. (32). Thus, the load current can be decomposed into three components associa-ted with three different phenomena, with permanent energy delivery to the load; with the phase shift between the voltage and current harmonics; and with the change of the load conduc-tance with harmonic order, namely This compnonents are refer-red to as *current's physical components (CPC).*

$$i(t) = i_a(t) + i_s(t) + i_r(t) \quad (83)$$

The decomposition of the load current into the physical components, $i_a(t)$, $i_r(t)$ and $i_s(t)$ forms the fundamentals of the *CPC Power Theory* (67).

The scalar products of the CPCs, (i_a, i_s), (i_a, i_r) and (i_s, i_r), are equal to zero, (32). Thus, they are mutually orthogonal, and therefore, the rms value of the load current can be expressed in terms of the rms values of the current's physical components, namely

$$\|i\|^2 = \|i_a\|^2 + \|i_s\|^2 + \|i_r\|^2 \quad (84)$$

where

$$\|i_{\mathrm{a}}\| = \frac{P}{\|u\|} \tag{85}$$

$$\|i_{\mathrm{s}}\| = \sqrt{\sum_{n=0}^{\infty}(G_n - G_{\mathrm{e}})^2 U_n^2} \tag{86}$$

$$\|i_{\mathrm{r}}\| = \sqrt{\sum_{n=1}^{\infty}(B_n U_n)^2} = \sqrt{\sum_{n=1}^{\infty}\left(\frac{Q_n}{U_n}\right)^2} \tag{87}$$

The relation between the rms value of the current physical components, i_{a}, i_{s} and i_{r} and the rms value of the supply current is the same as the relation between the length of the sides of a rectangular box, shown in Fig. 18, and its diagonal.

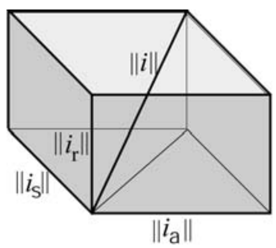

Figure 18. Geometrical illustration of the relationship between the rms values of the active, scattered and reactive currents. When the sides of the rectangular box are proportional to the rms values $\|i_{\mathrm{a}}\|$, $\|i_{\mathrm{s}}\|$ and $\|i_{\mathrm{r}}\|$, the diagonal is proportional to the supply current rms value $\|i\|$.

Decomposition of the load current into physical components enables the development of the power equation of the load. Multiplying Eq. (84) by the square of the voltage rms value $\|u\|^2$, we obtain the power equation

$$S^2 = P^2 + D_{\mathrm{s}}^2 + Q^2 \tag{88}$$

where

$$D_{\mathrm{s}} = \|u\|\,\|i_{\mathrm{s}}\| \tag{89}$$

is referred to as a *scattered power*, and

$$Q = \|u\|\,\|i_{\mathrm{r}}\| \tag{90}$$

is a *reactive power*. The reactive power Q is used to be associated in common interpretations with energy oscillation between the supply source and the load. It was proven in (71) that this is a major misinterpretation of power phenomena in electrical circuits. The reactive power occurs as an effect of the phase shift between the load voltage and its current or their harmonics.

The scattered current $i_{\mathrm{s}}(t)$ and the scattered power D_{s} occur in a circuit when the load conductance G_n changes with harmonic order n around a constant equivalent conductance G_{e}. This circuit phenomenon contributes to an increase in the rms value of the load current and the apparent power. The reactive current $i_{\mathrm{r}}(t)$ and the reactive power Q occur when there is a phase shift between the voltage and current harmonics, i.e., when there is at least one non zero Q_n value. Unlike the reactive power defined by Budeanu, the reactive power Q defined with Eq. (90) is a measure of the apparent power increase (32, 40) due to a phase shift between the voltage and current harmo-nics

The *power factor* of the supply source, which is a measure of the supply source utilization, is the ratio of the active and apparent power of the source. It can be expressed in terms of the rms value of CPCs, i_{a}, i_{s}, and i_{r}:

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + D_{\mathrm{s}}^2 + Q^2}} = \frac{\|i_{\mathrm{a}}\|}{\sqrt{\|i_{\mathrm{a}}\|^2 + \|i_{\mathrm{s}}\|^2 + \|i_{\mathrm{r}}\|^2}} \tag{91}$$

Both the scattered and the reactive currents contribute to degradation of the power factor.

Numerical illustration. The load shown in Fig. 19 is supplied with the voltage

$$u(t) = U_0 + \sqrt{2}\,\mathrm{Re}\{\boldsymbol{U}_1\,\mathrm{e}^{j\omega_1 t} + \boldsymbol{U}_5\,\mathrm{e}^{j5\omega_1 t}\} =$$

$$= 20 + \sqrt{2}\,\mathrm{Re}\{100\,\mathrm{e}^{j\omega_1 t} + 5\,\mathrm{e}^{j5\omega_1 t}\}\ \ \mathrm{V}$$

of the rms value

$$\|u\| = \sqrt{\sum_{n=0,1,5} U_n^2} = \sqrt{20^2 + 100^2 + 5^2} = 102.1\ \mathrm{V}$$

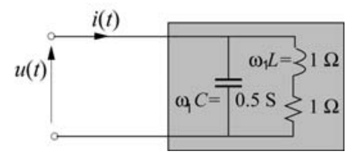

Figure 19. Example of a linear, time-invariant load.

The load admittance

$$\boldsymbol{Y}_n = G_n + jB_n = \frac{1}{R + jn\omega_1 L} + jn\omega_1 C =$$

$$= \frac{R}{R^2 + (n\omega_1 L)^2} + j\left(n\omega_1 C - \frac{n\omega_1 L}{R^2 + (n\omega_1 L)^2}\right)$$

for harmonic orders n = 0, 1, 5 has the values

$$Y_0 = 1\ \mathrm{S},\quad \boldsymbol{Y}_1 = 0.5\ \mathrm{S},\quad \boldsymbol{Y}_5 = 2.31\,e^{j89^{\circ}} = (0.038 + j2.31)\ \mathrm{S}$$

Thus the load current

$$i(t) = Y_0 U_0 + \sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}_1\boldsymbol{U}_1\,\mathrm{e}^{j\omega_1 t} + \boldsymbol{Y}_5\boldsymbol{U}_5\,\mathrm{e}^{j5\omega_1 t}\} =$$

$$= 20 + \sqrt{2}\,\mathrm{Re}\{50\,\mathrm{e}^{j\omega_1 t} + 11.55\,\mathrm{e}^{j89^{\circ}}\mathrm{e}^{j5\omega_1 t}\}\ \mathrm{A}$$

has the rms value

$$\|i\| = \sqrt{\sum_{n=0,1,5} I_n^2} = \sqrt{20^2 + 50^2 + 11.55^2} = 55.07\ \ \mathrm{A}$$

The load active power is equal to

$$P = \sum_{n=0,1,5} G_n U_n^2 = 1\cdot 20^2 + 0.5\cdot 100^2 + 0.038\cdot 5^2 = 5401 \text{ W}$$

thus, the load equivalent conductance is equal to

$$G_e = \frac{P}{||u||^2} = \frac{5401}{102.1^2} = 0.51808 \text{ S}$$

The active current, which is the current's physical component needed for the load active power, has the waveform

$$i_a(t) = G_e u(t) = 10.36 + \sqrt{2}\,\text{Re}\{51.81\,e^{j\omega_1 t} + 2.59\,e^{j5\omega_1 t}\} \text{ A}$$

and the rms value

$$||i_a|| = \frac{P}{||u||} = \frac{5401}{102.1} = 52.9 \text{ A}$$

The scattered current, that is, the current's physical component caused by the load conductance variation, has the waveform

$$i_s(t) = (G_0 - G_e)U_0 + \sqrt{2}\,\text{Re}\sum_{n=1,5}(G_n - G_e)\,\boldsymbol{U}_n\,e^{jn\omega_1 t} =$$
$$= 9.64 + \sqrt{2}\,\text{Re}\{1.81\,e^{j\omega_1 t} + 2.4\,e^{j5\omega_1 t}\} \text{ A}$$

and the rms value

$$||i_s|| = \sqrt{\sum_{n=0,1,5}(G_n - G_e)^2 U_n^2} = \sqrt{9.64^2 + 1.81^2 + 2.4^2} = 10.1 \text{ A}$$

The reative current, that is, the current's physical component caused by the phase shift between the voltage and current harmonics as the waveform

$$i_r(t) = \sqrt{2}\,\text{Re}\sum_{n=1,5} jB_n\,\boldsymbol{U}_n\,e^{jn\omega_1 t} = \sqrt{2}\,\text{Re}\{j11.5\,e^{j5\omega_1 t}\} \text{ A}$$

and the rms value, $||i_r|| = 11.5$ A . One can verify that the root of squares of the rms value of the current's physical components is equal to the rms value of the load current. Indeed,

$$\sqrt{||i_a||^2 + ||i_s||^2 + ||i_r||^2} = \sqrt{52.9^2 + 10.1^2 + 11.5^2} = 55.07 \text{ A} = ||i||$$

Although the scattered and reactive currents are both useless power currents, they are associated with different power phe-nomena. Also, they are affected in a different way by shunt reactive compensators connected as shown in Fig. 20.

An ideal compensator (i.e., a compensator that has no active power loss) does not change either the load conductances for

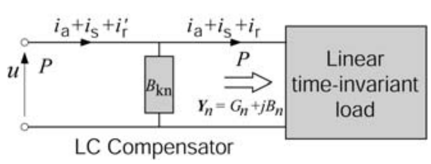

Figure 20. Circuit with reactive LC compensator. When the supply voltage is not affected by the compensator, it only affects the reactive current leaving the scattered current unchanged.

harmonic frequencies G_n or the equivalent conductance G_e. Hence, if the supply voltage is not affected by such a compen-sator, meaning if the supply source impedance can be neglect-ted, then it does not affect the scattered current. On the other hand, such a compensator affects the reactive current. When the compensator has susceptance B_{cn}, then it changes the rms value of the reactive component of the supply current i' to the value

$$||i'_r|| = \sqrt{\sum_{n=1}^{\infty}(B_{cn} + B_n)^2 U_n^2} \tag{92}$$

In particular, if the susceptance of the compensator, B_{cn}, satisfies the condition

$$B_{cn} = -B_n \tag{93}$$

for each harmonic of the supply voltage, then the reactive cur-rent of the load is totally compensated. This improves the *power factor* λ of the supply source to the maximum value

$$\lambda_{max} = \frac{P}{S_{min}} = \frac{||i_a||}{||i'||_{min}} = \frac{||i_a||}{\sqrt{||i_a||^2 + ||i_s||^2}} \tag{94}$$

The power factor can be improved to unity with a reactive com-pensator that compensates also the scattered current. However, it has to have not only a shunt but also a line branch (36).

CURRENT'S PHYSICAL COMPONENTS AND POWERS OF SINGLE-PHASE HARMONIC GENERATING LOADS

Current harmonics can be generated in passive loads due to a periodic change of their parameters, mining when the load is time-variant. Also, current harmonics are generated in passive nonlinear loads. Harmonics can also occur in circuits with active loads that contain sources of voltage or current harmo-nics. Such loads, referred to as *harmonic generating loads* (HGLs), cannot be described in terms of powers defined for linear, time-invariant (LTI) loads. This is illustrated with the following circuit.

Numerical illustration. The circuit shown in Fig. 21 is com-posed of a voltage source of the fundamental harmonic equal to $e(t) = 100\sqrt{2}\sin\omega_1 t$ V, with an internal resistance, and a resis-tive load with a current source of the second-order harmonic, equal to $j(t) = 50\sqrt{2}\sin 2\omega_1 t$ A. Thus, the load can be consi-dered as an active HGL. At the junction x-x, where the energy flow is observed, the voltage, current and their rms values are equal to:

$$u(t) = \sqrt{2}(80\sin\omega_1 t + 40\sin 2\omega_1 t) \text{ V}, \quad ||u|| = 89.44 \text{ V}$$
$$i(t) = \sqrt{2}(20\sin\omega_1 t - 40\sin 2\omega_1 t) \text{ A}, \quad ||i|| = 44.72 \text{ A}$$

The apparent power $S = ||u||\,||i|| = 4000$ VA. There is no active power P in this junction, however, since

$$P = \frac{1}{T}\int_0^T u(t)\, i(t)\, dt = P_1 + P_2 = 1600 - 1600 = 0$$

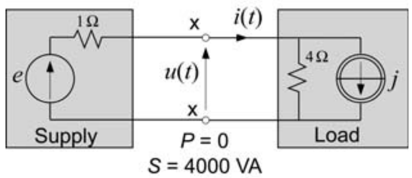

Figure 21. Circuit with harmonic generating load (HGL). The supply source provides a sinusoidal voltage of the fundamental frequency, while the load generates the second-order harmonic. The load active power P has a value of zero at nonzero apparent power S. The non-zero value of the apparent power S in this circuit cannot be explained in terms of the active, reactive and scattered powers.

The active power P is equal to zero because the active power P_2 of the second-order harmonic is negative and equal to the active power of the fundamental harmonic, P_1. Moreover, there is no reactive power Q in this junction, since the fundamental harmonics of the voltage and current are in phase, while the second-order harmonics are shifted by 180^0. Also, there is no scattered power D_{S}, since the conductance of the load does not change with the harmonic order. Thus, the presence of a non-zero apparent power S cannot be explained in terms of any of the known powers. We are not able to write the power equation for the voltage and current observed at the cross section x-x. The presence of current harmonics originating on the right side of the cross section observed is the main obstacle to writing the current and power equations in the known form.

When a current harmonic is observed in a cross section x-x, and the load is not linear and time-invariant, then there is insufficient information to conclude whether or not this current harmonic occurred because of the supply voltage harmonic or it was generated in the HGL. When the load is supplied from an ideal voltage source, then the presence of a harmonic in the current along with its lack in the voltage means that it is generated in the load. However, in real circuits, because of the voltage drop, the set of the current harmonic orders is identical with the set of the voltage harmonic orders. The *sign of the harmonic active power* P_n may indicate (35) where the dominating source of harmonic active power is, in the supply source or in the load.

Assume that a load (A) and its supply source (B) in the circuit shown in Fig. 22 are unknown and the voltage and current observed at the cross section x-x have harmonics from a set N.

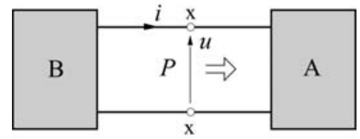

Figure 22. General structure of single-phase loads. The side A is assumeed to be a load, the side B is assumed to be the supply source.

Active powers Pn can be calculated individually for each harmonic. When $P_n \geq 0$ it means this power is dissipated in the load. When $P_n < 0$ it means that it is dissipated in the supply source. The sign of P_n enables us to decompose the set N into to sub-sets, N_{A} and N_{B}, as well as to define the voltage, current and active power components as follows:

If $P_n \geq 0,$ then $n \in N_{\mathrm{A}}$ and

$$\sum_{n\in N_{\mathrm{A}}} i_n = i_{\mathrm{A}}, \qquad \sum_{n\in N_{\mathrm{A}}} u_n = u_{\mathrm{A}}, \qquad \sum_{n\in N_{\mathrm{A}}} P_n = P_{\mathrm{A}} \tag{95}$$

If $P_n < 0$ then $n \in N_{\mathrm{B}}$ and

$$\sum_{n\in N_{\mathrm{B}}} i_n = i_{\mathrm{B}}, \qquad \sum_{n\in N_{\mathrm{B}}} u_n = -u_{\mathrm{B}}, \qquad \sum_{n\in N_{\mathrm{B}}} P_n = -P_{\mathrm{B}} \tag{96}$$

Thus, the load current, voltage and power can be expressed as

$$i = \sum_{n\in N} i_n = i_{\mathrm{A}} + i_{\mathrm{B}}, \quad u = \sum_{n\in N} u_n = u_{\mathrm{A}} - u_{\mathrm{B}}, \quad P = \sum_{n\in N} P_n = P_{\mathrm{A}} - P_{\mathrm{B}} \tag{97}$$

Equations (97) can be interpreted (35) as follows. The current i at terminals x-x in a circuit with HGLs contains a *supply originated current* i_{A} and a *load generated current* i_{B}. Similarly, the terminal voltage u contains a *supply originated voltage* u_{A} and a *load generated voltage* u_{B}. Moreover, the active power P at the cross-section observed is composed of a *supply originated active power* P_{A} and a *load generated active power* P_{B}.

The currents i_{A} and i_{B} have no common harmonics; thus their scalar product $(i_{\mathrm{A}}, i_{\mathrm{B}}) = 0$, so that they are mutually orthogonal. Hence the current rms value fulfills the relation

$$||i||^2 = ||i_{\mathrm{A}}||^2 + ||i_{\mathrm{B}}||^2 \tag{98}$$

For harmonic orders $n \in N_{\mathrm{A}}$ the load can be considered as a passive load of admittance

$$\boldsymbol{Y}_{n\mathrm{A}} = G_{n\mathrm{A}} + jB_{n\mathrm{A}} = \frac{\boldsymbol{S}_n^*}{U_n^2} \tag{99}$$

where $\boldsymbol{S}_n$ denotes the *harmonic complex power*,

$$\boldsymbol{S}_n = P_n + jQ_n = \boldsymbol{U}_n \boldsymbol{I}_n^* \tag{100}$$

and for the remaining harmonics the load can be considered as a current source of the current $j_{\mathrm{A}}(t) = i_{\mathrm{B}}(t)$, connected as shown in Fig. 23.

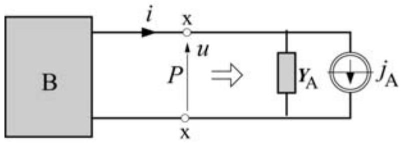

Figure 23. Equivalent circuit of a harmonic generating load (A). For harmonics of the order from the set N_A the load is equivalent to a passive linear load of admittance $\boldsymbol{Y}_A$. For harmonics of order from the set N_B (negative P_n) the load is equivalent to the current source $j_A = i_B$

With respect to the active power P_A at voltage u_A, the load is equivalent to a resistive load of the conductance

$$G_{eA} = \frac{P_A}{||u_A||^2} \tag{101}$$

which draws the active current

$$i_{aA} = G_{eA}\, u_A \tag{102}$$

The remaining part of the current i_A can be decomposed into the scattered and reactive components

$$i_{sA} = \sqrt{2}\mathrm{Re}\sum_{n\in N_A} (G_{nA} - G_{eA})\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{103}$$

$$i_{rA} = \sqrt{2}\mathrm{Re}\sum_{n\in N_A} jB_{nA}\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{104}$$

The formula for the scattered current was written assuming that the voltage u_A does not contain dc component. If the voltage u_A contains this component, formula (103) can be modified to include it. Thus, taking into account Eq. (97), the load current can be decomposed into four physical components (CPC)

$$i = i_{aA} + i_{sA} + i_{rA} + i_B \tag{105}$$

They are mutually orthogonal (35); hence their rms values fulfill the relationship

$$||i||^2 = ||i_{aA}||^2 + ||i_{sA}||^2 + ||i_{rA}||^2 + ||i_B||^2 \tag{106}$$

This relation can be visualized with the help of the polygon shown in Fig. 24, with sides whose length are proportional to the CPC rms value. It can be drawn, of course, with any sequence of the sides.

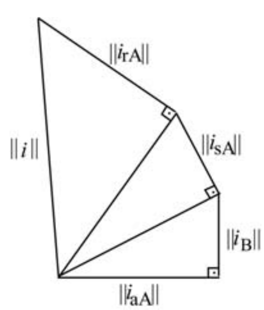

Figure 24. Geometrical illustration of the relationship between the rms values of the CPC in single-phase circuits with harmonic generating loads.

Four different power phenomena are responsible for the rms value of the load current. The interpretation of the active, scattered and reactive currents is similar to that for linear, time-invariant loads. However, these power currents in circuits with HGL are associated with only a part of the voltage observed at the cross-section x-x, namely, with the supply-originated voltage u_A, and therefore with the supply source harmonics. The load generated current, i_B, not only increases the current rms value but also reduces the active power at the load terminals, since the active power associated with this current, P_B, is negative. It dissipates in the supply source resistance.

When the load is linear and time-invariant, then the load current contains only the supply-originated current (i.e., $i = i_A$), and the load active power $P = P_A$. The presence of the load-generated current i_B increases the active power loss in the supply source; thus it increases fuel consumption by the electric energy producer and the needed power ratings of the equip-ment. At the same time, the load-generated power P_B reduces the active power P and the bill for energy delivered to the customer, which is proportional to the integral of the active power, P. Thus, energy producers are loosing part of their revenue (53) when they serve harmonics generating loads.

The voltages u_A and u_B are orthogonal; thus the rms value of the supply voltage is equal to

$$||u||^2 = ||u_A||^2 + ||u_B||^2 \tag{107}$$

and the apparent power at the cross-section x-x can be expressed as

$$\begin{aligned} S = ||u||\cdot||i|| &= \sqrt{||u_A||^2 + ||u_B||^2}\sqrt{||i_A||^2 + ||i_B||^2} = \\ &= \sqrt{S_A^2 + S_B^2 + S_E^2} \end{aligned} \tag{108}$$

with

$$S_A \triangleq ||u_A||\cdot||i_A|| = \sqrt{P_A^2 + D_{sA}^2 + Q_A^2} \tag{109}$$

$$S_B \triangleq ||u_B||\cdot||i_B||, \qquad S_E \triangleq \sqrt{||u_A||^2||i_B||^2 + ||u_B||^2||i_A||^2} \tag{110}$$

The apparent power denoted by S_A is the *supply-originated* apparent power, while that denoted by S_B is the *load-generated* apparent power. The load-generated apparent power only occurs when there is a voltage response u_B to the load-generated current i_B, that is, when the supply source has an internal impedance. The last component of the apparent power, S_E, occurs even in an ideal circuit, when the supply voltage source is connected with a current source of harmonic orders different than the supply voltage harmonics. The voltage source of voltage u_A and current i_A has to withstand the extorted current i_B. When this voltage source has an impedance then the voltage u_B is extorted as well. Therefore, the power S_E is referred to as an *extorted apparent power*.

The power equation for the circuit considered in the *illustration* at the beginning of this Section can be written just in terms of apparent powers S, P_A, P_B, S_B and S_E. All other powers are equal to zero.

The power factor λ of a source with a HGL can be expressed in the form

$$\lambda = \frac{P}{S} = \frac{P_A - P_B}{\sqrt{P_A^2 + D_{As}^2 + Q_A^2 + S_B^2 + S_E^2}} \tag{111}$$

which shows all causes of its degradation. Very often the internal voltage of the supply source can be considered as purely sinusoidal of the fundamental frequency, which means $u_A = u_1$. In such a case $P_A = P_1$, $D_{As} = 0$, $Q_A = Q_1$. Moreover, a *displacement power factor* λ_1 (43), equal to the cosine of the phase shift of the voltage and current fundamental harmonics, can be separated in such a case, namely

$$\lambda = \frac{P}{S} = \frac{U_1 I_1 \cos\varphi_1}{U_1 \|i\|} = \frac{I_1}{\sqrt{I_1^2 + \sum_{n=2} I_n^2}} \cos\varphi_1 = \frac{1}{\sqrt{1+\delta_i^2}}\,\lambda_1 \tag{112}$$

where

$$\delta_i = \frac{\|i_h\|}{I_1} = \frac{1}{I_1}\sqrt{\sum_{n=2}^{\infty} I_n^2} \tag{113}$$

is a *current distortion factor*.

THREE-PHASE SYSTEMS - DOUBTS WITH RESPECT TO APPARENT POWER DEFINITIONS

The basic structure of a three-phase, three-wire systems is shown in Fig. 25.

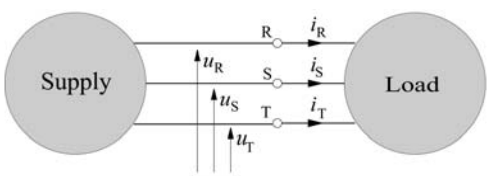

Figure 25. Structure of three-phase, three-wire system

There were numerous attempts to explain and describe power properties of three-phase systems with nonsinusoidal voltages and currents using Budeanu's or the Fryze's approach to power definitions. Unfortunately, even apparently successful results of such an extension convey all misconceptions and deficiencies of these two approaches with such an extension. Also, an extension from sinusoidal to nonsinusoidal condition requires that power properties of three-phase systems in sinusoidal conditions are described properly. Unfortunately, the commonly used power equation of three-phase systems

$$S^2 = P^2 + Q^2 \tag{114}$$

provides a true value of the apparent power and power factor only if the load is balanced. Some misconceptions with respect to definition of the apparent power are demonstrated below.

The active and reactive powers in three-phase, three-wire systems, shown in Fig. 25, with sinusoidal supply voltage and sinusoidal line currents are defined as follows

$$P = \frac{1}{T}\int_0^T (u_R i_R + u_S i_S + u_T i_T)\,dt = \sum_{f=R,S,T} U_f I_f \cos\varphi_f \tag{115}$$

$$Q = \sum_{f=R,S,T} U_f I_f \sin\varphi_f \tag{116}$$

The apparent power in such systems is defined according to the conclusion (68) of the joint committee of AIEE and NELE (presently, Edison Institute) in 1920. According to (68), the apparent power is defined as

$$S = \sqrt{P^2 + Q^2} = S_G \tag{117}$$

This quantity is known as the *geometric apparent power*. It can also be defined as

$$S = U_R I_R + U_S I_S + U_T I_T = S_A \tag{118}$$

It is known as the *arithmetic apparent power*. These definitions are provided by the *IEEE Standard Dictionary of Electrical and Electronics Terms* (5). There is a third definition of the apparent power, suggested by Buchholz (51) in 1922:

$$S = \sqrt{U_R^2 + U_S^2 + U_T^2}\;\sqrt{I_R^2 + I_S^2 + I_T^2} = S_B \tag{119}$$

but not referred to in Standard (5). These three definitions result in the same value of apparent power S, only if the line currents are symmetrical. Otherwise these values are different. This is demonstrated with the following illustration.

Numerical illustration. Let us consider a single-phase resistive load supplied from a three-phase circuit as shown in Fig. 26.

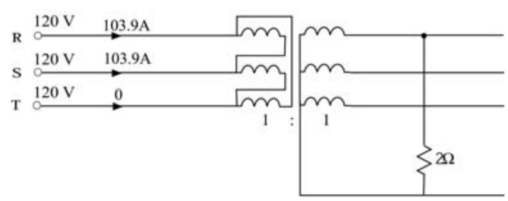

Figure 26. Example of three-phase circuit

Assuming that the line-to-ground voltage RMS value is 120 V, transformer turn ratio is 1:1, the active power at the supply terminals is P = 21.6 kW, while the apparent power, depending on the definition, is

S_A = 24.9 VA, S_G = 21.6 kVA, S_B = 30.5 kVA

Consequently, power factor depends on the selected definition of the apparent power and is equal to, respectively,

λ_A = 0.86, λ_G = 1, λ_B = 0.71

The reactive power in the system considered is Q = 0, thus, power equation (114) is satisfied only for the geometric definition of the apparent power. However, the question arises: *is the power factor of such an unbalanced load equal to* $\lambda = 1$?

The apparent power is not a physical, but a conventional quantity. Various objectives could be taken into account when a convention for the apparent power definition is selected. One of them, and probably particularly important, is such a definition that results in such a value of power factor that characterizes correctly the power loss at energy delivery. In such a case, the issue of selection of the apparent power definition is equivalent to the question: *which value* $\lambda_{\rm A}$, λ_G *or* $\lambda_{\rm B}$ *characterizes power loss on energy delivery?*

The answer to this question was based on the following reasoning presented in (70) in 1999. At first, a circuit with a balanced resistive load was found, a circuit that at the load active power P = 100 kW has the power loss, $\Delta P_{\rm S}$ = 5 kW, on delivery. Parameters of such a circuit are shown in Fig. 27.

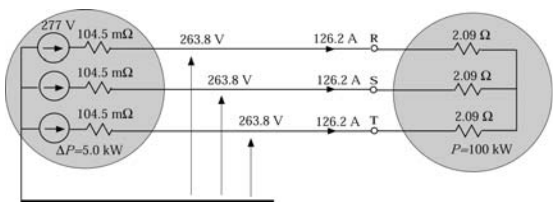

Figure 27. Circuit with balanced resistive load

In the next step, the same source supplies an unbalanced resistive load, shown in Fig. 28, with the same active power P = 100 kW.

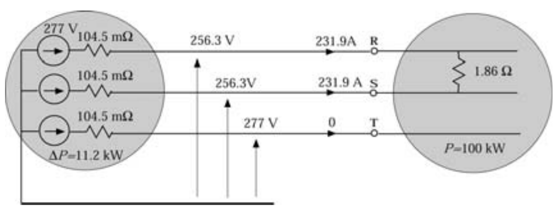

Figure 28. Circuit with unbalanced load

Depending on definition of the apparent power, it is equal to

$$S_{\rm A} = 119 \text{ kVA}, \quad S_{\rm G} = 100 \text{ kVA}, \quad S_{\rm B} = 149 \text{ kVA}$$

and the power factor is equal to, respectively,

$$\lambda_{\rm A} = 0.84, \qquad \lambda_{\rm G} = 1, \qquad \lambda_{\rm B} = 0.67$$

Observe, that in spite of the same load active power, the power loss on energy delivery has increased in the circuit with the unbalanced load from $\Delta P_{\rm s}$ = 5.0 kW to $\Delta P_{\rm s}$ = 11.2 kW. It means that the load shown in Fig. 28 is not a load with unity power factor. This conclusion disqualifies geometric definition (117) of the apparent power. However, still we do not know whether $\lambda_{\rm A}$ or $\lambda_{\rm B}$ provides the true value of the power factor. To answer this question, let us find the power factor of a balanced RL load that supplied from the same source will have the same active power, P = 100 kW, and will causes the same power loss, $\Delta P_{\rm s}$ = 11.2 kW. Such an RL balanced load has parameters shown in Fig. 29.

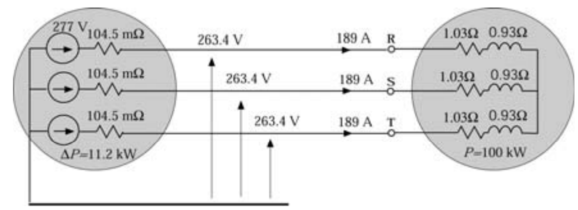

Figure 29. Balanced load equivalent to unbalanced load in Fig. 28 with respect to power loss in the source

The load in this circuit is balanced thus, the apparent power does not depend on the selected definition of the apparent power and $S_{\rm A} = S_{\rm B}$ = 149 kVA. Consequently, the power factor is $\lambda_{\rm B} = \lambda$ = 0.67. It means that the power factor has a true value only if the apparent power S is calculated according to definition (119). Arithmetic and geometric definitions of the apparent power result in an erroneous value of the power factor. However, when the apparent power S is calculated according to definition (119), power equation (114) is not fulfilled. Thus, this power equation is erroneous even for sinusoidal voltages and currents. It is true only for balanced loads supplied with a symmetrical voltage. However, power properties of such sys-tems are trivial and could be described phase by phase as properties of single-phase systems.

CURRENTS' PHYSICAL COMPONENTS OF THREE-PHASE, LINEAR, TIME-INVARIANT (LTI) LOADS

To describe a three-phase system as a whole, not only as a connection of three separate phases, it is convenient to arrange the phase voltages and currents observed at terminals R, S and T of a three-phase circuit, as shown in Fig. 25, into *three-phase vectors* (34), namely

$$\begin{bmatrix} u_{\rm R}(t) \\ u_{\rm S}(t) \\ u_{\rm T}(t) \end{bmatrix} = \boldsymbol{u}(t) \; = \; \boldsymbol{u}\,, \qquad \begin{bmatrix} i_{\rm R}(t) \\ i_{\rm S}(t) \\ i_{\rm T}(t) \end{bmatrix} = \boldsymbol{i}(t) \; = \; \boldsymbol{i} \tag{120}$$

Let us consider a three-phase device shown in Fig. 30.

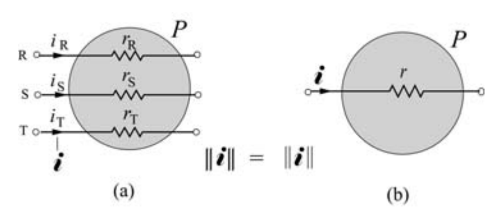

Figure 30. (a) Three-phase, three-wire symmetrical device and (b) single-phase device with current rms value $||i||$ equivalent to the three-phase current rms value $||\boldsymbol{i}||$ with respect to the active power loss.

When the current $\boldsymbol{i}$ flows through a three-phase device shown in Fig. 30(a), then the active power of this device is equal to

$$P = r_{\rm R}\,||i_{\rm R}||^2 + r_{\rm S}\,||i_{\rm S}||^2 + r_{\rm T}\,||i_{\rm T}||^2 \tag{121}$$

Three-phase equipment is built so that it is as symmetrical as possible, thus, it can be assumed that $r_R = r_S = r_T = r$. In such a case

$$P = r(\|i_R\|^2 + \|i_S\|^2 + \|i_T\|^2) = r\frac{1}{T}\int_0^T \boldsymbol{i}^T(t)\,\boldsymbol{i}(t)\,dt \triangleq r\|\boldsymbol{i}\|^2 \qquad (122)$$

where superscript T denotes a matrix transposition. The quantity

$$\|\boldsymbol{i}\| \triangleq \sqrt{\frac{1}{T}\int_0^T \boldsymbol{i}^T(t)\,\boldsymbol{i}(t)\,dt} = \sqrt{\|i_R\|^2 + \|i_S\|^2 + \|i_T\|^2} \qquad (123)$$

is referred to (34) as the *rms value of a three-phase current*. This is the rms value of the single-phase current in a circuit shown in Fig. 30(b), which is equivalent to the three-phase current with respect to the active power in a symmetrical three-phase, three-wire device. Similarly,

$$\|\boldsymbol{u}\| \simeq \sqrt{\frac{1}{T}\int_0^T \boldsymbol{u}^T(t)\,\boldsymbol{u}(t)\,dt} = \sqrt{\|u_R\|^2 + \|u_S\|^2 + \|u_T\|^2} \qquad (124)$$

is referred to as the *rms value of a three-phase voltage.*

The instantaneous power $p(t)$ of a three-phase load is defined as the rate of energy $W(t)$ flow to the load, namely

$$p(t) = \frac{d}{dt}W(t) = u_R(t)\,i_R(t) + u_S(t)\,i_S(t) + u_T(t)\,i_T(t) = \boldsymbol{u}^T\,\boldsymbol{i} \qquad (125)$$

The instantaneous power $p(t)$ in three-phase systems with a symmetrical and sinusoidal supply voltage $\boldsymbol{u}$ and a balanced load is constant, equal to the load active power P, independently of the load reactive power Q. This means that reactive power may occur in such a circuit without any reciprocating oscillation of energy between the supply source and the load. On the other hand, if the supply voltage $\boldsymbol{u}$ in such a system contains harmonics, than the instantaneous power $p(t)$ contains oscillating components even if the load is purely resistive and consequently the power factor is equal to one. Though some authors (46) claim that all nonactive powers occur only due to energy oscillation, there is no relation between such a reciprocating oscillation of energy between the load and the supply source and reactive power Q in three-phase systems. Also, it is important to observe that a single term, for example, $u_R(t)\,i_R(t)$, cannot be interpreted (47) as an instantaneous power $p_R(t)$ of a single phase of the three-phase system, since generally it is not possible to separate energy delivered to a three-phase load by a single phase. Moreover, any point in a three-phase system could be chosen as a reference point without affecting power phenomena, whereas single-phase products like $u_R(t)\,i_R(t)$ may change with the change of the reference point. When, for example, terminal R is chosen as a reference, then $u_R(t) \equiv 0$ and $p_R(t) \equiv 0$. Thus, there is no relation between such single-phase voltage and current products and power phenomena in three-phase circuits.

The active power of a three-phase load is defined as the mean value of the instantaneous power

$$P = \overline{p(t)} = \frac{1}{T}\int_0^T \boldsymbol{u}^T(t)\,\boldsymbol{i}(t)\,dt = (\boldsymbol{u},\boldsymbol{i}) \qquad (126)$$

The symbol $(\boldsymbol{u},\boldsymbol{i})$ denotes the scalar product of a three-phase load voltage and current. Generally, the scalar product of three-phase vectors, $\boldsymbol{x}$ and $\boldsymbol{y}$, is defined as

$$(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^T(t)\,\boldsymbol{y}(t)\,dt \qquad (127)$$

These quantities are *orthogonal* when their scalar product is equal to zero. The rms values of three-phase orthogonal quantities fulfill the relationship

$$\|\boldsymbol{z}\|^2 = \|\boldsymbol{x} + \boldsymbol{y}\|^2 = \|\boldsymbol{x}\|^2 + \|\boldsymbol{y}\|^2 \qquad (128)$$

When the entries of a three-phase vector $\boldsymbol{x}$ have harmonics from a set N, the vector can be expressed in the form

$$\boldsymbol{x}(t) = \sum_{n\in N}\boldsymbol{x}_n(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}\begin{bmatrix}\boldsymbol{X}_{Rn}\\ \boldsymbol{X}_{Sn}\\ \boldsymbol{X}_{Tn}\end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}\boldsymbol{X}_n\, e^{jn\omega_1 t} \qquad (129)$$

The scalar product defined with Eq. (127) in the time-domain, can be calculated in the frequency domain as

$$(\boldsymbol{x},\boldsymbol{y}) = \mathrm{Re}\sum_{n\in N}\boldsymbol{X}_n^T\,\boldsymbol{Y}_n^* \qquad (130)$$

In particular, the active power of the load can be expressed in the frequency-domain as

$$P = (\boldsymbol{u},\boldsymbol{i}) = \mathrm{Re}\sum_{n\in N}\boldsymbol{U}_n^T\,\boldsymbol{I}_n^* \qquad (131)$$

The rms value $\|\boldsymbol{x}\|$ of the vector $\boldsymbol{x}$ can be expressed as

$$\|\boldsymbol{x}\| = \sqrt{(\boldsymbol{x},\boldsymbol{x})} = \sqrt{\sum_{n\in N}\boldsymbol{X}_n^T\boldsymbol{X}_n^*} = \sqrt{\sum_{n\in N}(X_{Rn}^2 + X_{Sn}^2 + X_{Tn}^2)} \qquad (132)$$

A three-phase load has the active power P on the condition that the supply source provides the voltage of the rms value $\|\boldsymbol{u}\|$ and is capable of providing current of the rms value $\|\boldsymbol{i}\|$. By an analogy to the apparent power of single-phase sources, the product of these two rms values can be considered (34, 39, 51) as the apparent power of three-phase sources, namely

$$\|\boldsymbol{u}\|\cdot\|\boldsymbol{i}\| = S \qquad (133)$$

When voltages and currents are sinusoidal then this definition is equivalent to the definition given by Eq. (119). The last defini-tion is valid for systems with no sinusoidal voltages and current, but with the following restriction.

The supply source may produce voltage harmonics of the zero sequence, in particular, the third order. Such harmonics contribute to the voltage rms value $\|\boldsymbol{u}\|$ increase in the distribution equipment. They cause no current in three-wire systems, however, and do not deliver energy to loads. Therefore, when the load power factor λ is of concern, the zero sequence harmonics should be neglected when the voltage rms value $||\boldsymbol{u}||$ is calculated. Otherwise, even an ideal resistive balanced load would not have a unity power factor, and the customer cannot be blamed for that. Consequently, when power properties of customers' loads are analyzed, it is assumed that the load voltage $\boldsymbol{u}$ contains no zero-sequence harmonics.

Let us assume that a load is supplied symmetrically with a single voltage harmonic of the n^{th}-order

$$\boldsymbol{u}_n(t) = \sqrt{2}\ \text{Re}\begin{bmatrix} \boldsymbol{U}_{\text{R}n} \\ \boldsymbol{U}_{\text{S}n} \\ \boldsymbol{U}_{\text{T}n} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\ \text{Re}\ \boldsymbol{U}_n e^{jn\omega_1 t} \tag{134}$$

and the load current is equal to

$$\boldsymbol{i}_n(t) = \sqrt{2}\ \text{Re}\begin{bmatrix} \boldsymbol{I}_{\text{R}n} \\ \boldsymbol{I}_{\text{S}n} \\ \boldsymbol{I}_{\text{T}n} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\ \text{Re}\ \boldsymbol{I}_n e^{jn\omega_1 t} \tag{135}$$

The complex power of the load for the n^{th}-order harmonic has the value

$$\boldsymbol{S}_n = P_n + jQ_n = \boldsymbol{U}_n^{\text{T}} \boldsymbol{I}_n^* \tag{136}$$

A *symmetrical resistive* load having a star structure as shown in Fig. 31(b) is equivalent to the load shown in Fig. 31(a) with respect to the active power P_n at voltage $\boldsymbol{u}_n$, when its phase conductance is equal to

$$G_{\text{e}n} = \frac{P_n}{3U_{\text{R}n}^2} = \frac{P_n}{||\boldsymbol{u}_n||^2} \tag{137}$$

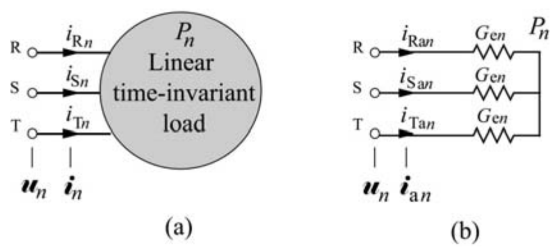

Figure 31. (a) Linear, time-invariant load and (b) symmetrical resistive load equivalent to the load in (a) with respect to the active power P_n of the n^{th}-order harmonic. It draws symmetrical active current $\boldsymbol{i}_{an}$.

This conductance can be referred to as *equivalent conductance* of the load for the n^{th}-order harmonic. A load of such a conductance draws a symmetrical current

$$\boldsymbol{i}_{\text{a}n}(t) = G_{\text{e}n}\, \boldsymbol{u}_n(t) = \sqrt{2}\ \text{Re}\, G_{\text{e}n}\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{138}$$

which can be referred to as an *active current of the n^{th}-order harmonic*. Its rms value is equal to

$$||\boldsymbol{i}_{\text{a}n}|| = G_{\text{e}n}\, ||\boldsymbol{u}_n|| \tag{139}$$

A *symmetrical reactive* load having a star structure as shown in Fig. 32(b) is equivalent (34, 39) to the load shown in Fig. 32(a) with respect to the reactive power Q_n at voltage $\boldsymbol{u}_n$, when its phase susceptance is equal to

$$B_{\text{e}n} = -\frac{Q_n}{3U_{\text{R}n}^2} = -\frac{Q_n}{||\boldsymbol{u}_n||^2} \tag{140}$$

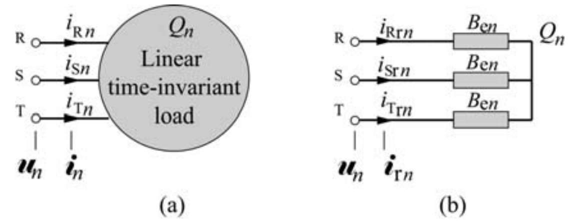

Figure 32. (a) Linear, time-invariant load and (b) symmetrical reactive load, equivalent to the load (a) with respect to the reactive power Q_n of the n^{th}-order harmonic. It draws reactive current $\boldsymbol{i}_{\text{r}n}$.

This susceptance can be referred to as *equivalent susceptance* of the load for the n^{th}-order harmonic. A load of such a susceptance draws a symmetrical current

$$\boldsymbol{i}_{\text{r}n}(t) = B_{\text{e}n} \frac{d}{d\,(n\omega_1 t)} \boldsymbol{u}_n(t) = \sqrt{2}\,\text{Re}\{jB_{\text{e}n}\, \boldsymbol{U}_n\, e^{jn\omega_1 t}\} \tag{141}$$

which can be referred to as a *reactive current of the n^{th}-order harmonic*. Its rms value is equal to

$$||\boldsymbol{i}_{\text{r}n}|| = |B_{\text{e}n}|\, ||\boldsymbol{u}_n|| \tag{142}$$

The equivalent conductance $G_{\text{e}n}$ and susceptance $B_{\text{e}n}$ are the real and imaginary parts of an *equivalent admittance* (34, 39) for the n^{th}-order harmonic.

$$\boldsymbol{Y}_{\text{e}n} \triangleq G_{\text{e}n} + jB_{\text{e}n} = \frac{\boldsymbol{S}_n^*}{||\boldsymbol{u}_n||^2} \tag{143}$$

Each three-phase load has a delta equivalent load shown in Fig. 33(b).

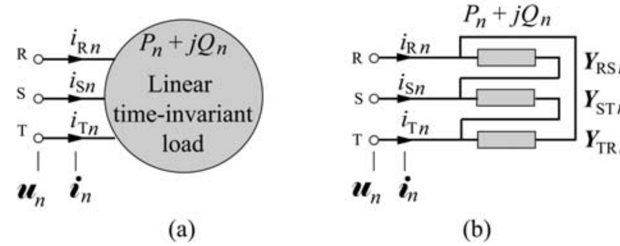

Figure 33. (a) Linear, time-invariant load; (b) load having a delta structue and equivalent to load (a) with respect to the line current $\boldsymbol{i}_n$.

The complex apparent power $\boldsymbol{S}_n$ of such a load supplied with a symmetrical voltage can be calculated as

$$\boldsymbol{S}_n = (\boldsymbol{Y}^*_{\mathrm{RS}\,n} + \boldsymbol{Y}^*_{\mathrm{ST}\,n} + \boldsymbol{Y}^*_{\mathrm{TR}\,n})\,||\boldsymbol{u}_n||^2 \tag{144}$$

thus, the equivalent admittance $\boldsymbol{Y}_{\mathrm{e}n}$ of the load for the n^{th}-order harmonic is equal to

$$\boldsymbol{Y}_{\mathrm{e}n} = \boldsymbol{Y}_{\mathrm{RS}n} + \boldsymbol{Y}_{\mathrm{ST}n} + \boldsymbol{Y}_{\mathrm{TR}n} \quad (145$$

When the load is unbalanced, its current is asymmetrical, while the active and reactive currents are symmetrical. Therefore, the load current contains a component

$$\boldsymbol{i}_{\mathrm{u}n} = \boldsymbol{i}_n - (\boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n}) \tag{146}$$

referred to (34) as an *unbalanced current* of the n^{th}-order harmonic. The crms value of this current in phase R is equal to

$$\begin{aligned}\boldsymbol{I}_{\mathrm{Ru}n} &= \boldsymbol{I}_{\mathrm{R}\,n} - (\boldsymbol{I}_{\mathrm{Ra}n} + \boldsymbol{I}_{\mathrm{Rr}n}) = \\ &= (\boldsymbol{Y}_{\mathrm{RS}n}\boldsymbol{U}_{\mathrm{RS}n} - \boldsymbol{Y}_{\mathrm{TR}\,n}\boldsymbol{U}_{\mathrm{TR}\,n}) - \boldsymbol{Y}_{\mathrm{e}\,n}\boldsymbol{U}_{\mathrm{R}\,n} = \\ &\triangleq \boldsymbol{A}_{\mathrm{R}\,n}\,\boldsymbol{U}_{\mathrm{R}\,n}\end{aligned} \tag{147}$$

where

$$\boldsymbol{A}_{\mathrm{R}n} \triangleq \boldsymbol{A}_n \triangleq -(\boldsymbol{Y}_{\mathrm{ST}n} + \beta\boldsymbol{Y}_{\mathrm{TR}n} + \beta^*\,\boldsymbol{Y}_{\mathrm{RS}n}) \tag{148}$$

is the *unbalanced admittance* of the load for the n^{th}-order harmonic. The symbol β denotes a complex *turn coefficient* dependent on the harmonic sequence, namely

$$\beta \triangleq 1e^{js\frac{2\pi}{3}}, \quad \begin{cases} s = 1, \text{ for positive sequence harmonics} \\ s = -1, \text{ for negative sequence harmonics}\end{cases} \tag{149}$$

The unbalanced current in lines S and T is equal to

$$\begin{aligned}\boldsymbol{I}_{\mathrm{Su}n} &= \beta^*\boldsymbol{A}_n\boldsymbol{U}_{\mathrm{S}\,n} = \beta\,\boldsymbol{I}_{\mathrm{Ru}n}\\ \boldsymbol{I}_{\mathrm{Tu}n} &= \beta\,\boldsymbol{A}_n\boldsymbol{U}_{\mathrm{T}\,n} = \beta^*\,\boldsymbol{I}_{\mathrm{Ru}n}\end{aligned} \tag{150}$$

which means that the unbalanced current $\boldsymbol{i}_{\mathrm{u}n}$ has a sequence which is the opposite to the voltage harmonic $\boldsymbol{u}_n$. This means that it is also opposite to the sequence of the active and reactive currents, $\boldsymbol{i}_{\mathrm{a}n}$ and $\boldsymbol{i}_{\mathrm{r}n}$. The unbalanced current $\boldsymbol{i}_{\mathrm{u}n}$ can be expressed in a compact form as follows

$$\begin{aligned}\boldsymbol{i}_{\mathrm{u}n}(t) &= \sqrt{2}\,\mathrm{Re}\begin{bmatrix}1 & 0 & 0\\ 0 & \beta^* & 0\\ 0 & 0 & \beta\end{bmatrix}\boldsymbol{A}_n\begin{bmatrix}\boldsymbol{U}_{\mathrm{R}\,n}\\ \boldsymbol{U}_{\mathrm{S}\,n}\\ \boldsymbol{U}_{\mathrm{T}\,n}\end{bmatrix}e^{jn\omega_1 t} = \\ &= \sqrt{2}\,\mathrm{Re}\{\boldsymbol{b}_n\boldsymbol{A}_n\boldsymbol{U}_n\,e^{jn\omega_1 t}\}\end{aligned} \tag{151}$$

Its rms value is equal to

$$||\boldsymbol{i}_{\mathrm{u}n}|| = A_n\,||\boldsymbol{u}_n|| \tag{152}$$

In such a way, the n^{th}-order load current harmonic has been decomposed into three physical components:

$$\boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n} \tag{153}$$

Their scalar products $(\boldsymbol{i}_{\mathrm{a}n}, \boldsymbol{i}_{\mathrm{r}n}) = (\boldsymbol{i}_{\mathrm{a}n}, \boldsymbol{i}_{\mathrm{u}n}) = (\boldsymbol{i}_{\mathrm{r}n}, \boldsymbol{i}_{\mathrm{u}n}) = 0$, thus they are orthogonal (39), and hence rms values of the physical components of the current harmonics fulfill the relationship

$$||\boldsymbol{i}_n||^2 = ||\boldsymbol{i}_{\mathrm{a}\,n}||^2 + ||\boldsymbol{i}_{\mathrm{r}\,n}||^2 + ||\boldsymbol{i}_{\mathrm{u}\,n}||^2 \tag{154}$$

In three-phase, three-wire systems with sinusoidal waveforms, there is no need to keep the index n, and relation (154) represents (39) the final decomposition of the load current into physical components (CPC), that means the active, reactive and unbalanced currents,

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}} \tag{155}$$

It is important to observe that the supply current in three-phase, three-wire systems is composed, in general, not only the active and reactive current, but also an unbalanced current. Their rms values satisfy the relationship

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2 \tag{156}$$

Thus, the unbalanced current contributes to the supply current rms value in the same way as the active and reactive currents. Observe however, that this equation is developed under the assumption that the supply voltage is symmetrical. The current decomposition and the power equation of loads at asymmetrical supply was developed in (72).

Equation (156) after multiplying both sides by voltage rms value $||\boldsymbol{u}||$, results in the power equation

$$S^2 = P^2 + Q^2 + D_{\mathrm{u}}^2 \tag{157}$$

with

$$D_{\mathrm{u}} \triangleq ||\boldsymbol{u}||\cdot||\boldsymbol{i}_{\mathrm{u}}|| \tag{158}$$

Power equation (157) is the valid equation of three-phase, three-wire systems with sinusoidal voltages and currents. It reveals a new, unknown earlier power referred to as the *unbalanced power* D_{u}. The commonly used power equation in the form (114) is erroneous. It is valid only if the voltages and currents are symmetrical.

The power factor λ can be related to the current physical components and load parameters as follows

$$\lambda \triangleq \frac{P}{S} = \frac{||\boldsymbol{i}_{\mathrm{a}}||}{\sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2}} = \frac{G_{\mathrm{e}}}{\sqrt{G_{\mathrm{e}}^2 + B_{\mathrm{e}}^2 + A^2}} \tag{159}$$

and this formula shows that the unbalanced current affects the power factor in a manner similar to the way that the reactive current does.

It is important to observe (39) that not only the reactive current but also the unbalanced current can be compensated by a passive reactive compensator. Thus the power factor can be improved to unity, independently of the load imbalance.

When the supply voltage contains harmonics of the orders from a set N, then the load current can be expressed as

$$\boldsymbol{i} = \sum_{n\in N}\boldsymbol{i}_n = \sum_{n\in N}(\boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n}) \tag{160}$$

The three-phase load at the distorted voltage $\boldsymbol{u}$ has the same active power P as a resistive symmetrical load if its conductance is equal to

$$G_{\rm e} = \frac{P}{||\boldsymbol{u}||^2} \tag{161}$$

and such a load draws only the active current from the source, namely

$$\boldsymbol{i}_{\rm a}(t) = G_{\rm e}\, \boldsymbol{u}(t) = \sqrt{2}\,{\rm Re}\sum_{n\in N} G_{\rm e} \boldsymbol{U}_{\rm n}\, e^{jn\omega_1 t} \tag{162}$$

When the load equivalent conductance G_{en} changes with the harmonic order, a difference

$$\boldsymbol{i}_{\rm s}(t) \triangleq \sum_{n\in N} \boldsymbol{i}_{{\rm a}n}(t) - \boldsymbol{i}_{\rm a}(t) = \sqrt{2}\,{\rm Re}\sum_{n\in N} (G_{{\rm e}n} - G_{\rm e})\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{163}$$

occurs in the load current. This is a scattered current. The reactive and unbalanced currents are the sum of the reactive and unbalanced harmonic currents, namely

$$\boldsymbol{i}_{\rm r}(t) \triangleq \sum_{n\in N} \boldsymbol{i}_{{\rm r}\,n} = \sqrt{2}\,{\rm Re}\sum_{n\in N} jB_{{\rm e}n}\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{164}$$

$$\boldsymbol{i}_{\rm u}(t) \triangleq \sum_{n\in N} \boldsymbol{i}_{{\rm u}\,n} = \sqrt{2}\,{\rm Re}\sum_{n\in N} \boldsymbol{b}_n\, \boldsymbol{A}_n\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{165}$$

Thus, the load current of a three-phase linear and time-invariant load supplied with a symmetrical nonsinusoidal voltage has four components

$$\boldsymbol{i} = \boldsymbol{i}_{\rm a} + \boldsymbol{i}_{\rm s} + \boldsymbol{i}_{\rm r} + \boldsymbol{i}_{\rm u} \tag{166}$$

Each of these four currents is associated with a different power phenomenon, namely, the active power of the load, a change of its conductance with harmonic order, a phase-shift between the voltage and current harmonics and the load imbalance. Therefore, similarly as in single-phase systems, they are referred to as currents' physical components, (CPCs).

The scalar products of all these power currents are equal to zero (34), thus they are mutually orthogonal, so that their rms values fulfill the relationship

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\rm a}||^2 + ||\boldsymbol{i}_{\rm s}||^2 + ||\boldsymbol{i}_{\rm r}||^2 + ||\boldsymbol{i}_{\rm u}||^2 \tag{167}$$

which means, that these four power phenomena contribute to the increase of the load current rms value independently of each other. This relationship can be illustrated with the polygon shown in Fig. 34.

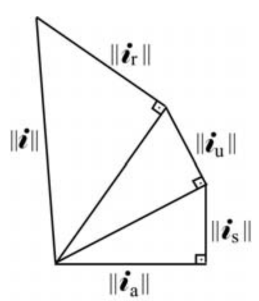

Figure 34. Geometrical illustration of the relationship between the rms values $||\boldsymbol{i}_{\rm a}||$, $||\boldsymbol{i}_{\rm r}||$ and $||\boldsymbol{i}_{\rm u}||$ of the currents physical components and the supply current rms value $||\boldsymbol{i}||$ of three-phase LTI loads.

Multiplying Eq. (167) by the square of the load voltage rms value $||\boldsymbol{u}||^2$ results in the power equation

$$S^2 = P^2 + D_{\rm s}^2 + Q^2 + D_{\rm u}^2 \tag{168}$$

with the *scattered power* of three-phase loads, defined as

$$D_{\rm s} \triangleq ||\boldsymbol{u}||\cdot||\boldsymbol{i}_{\rm s}|| \tag{169}$$

CURRENTS' PHYSICAL COMPONENTS AND POWERS OF THREE-PHASE HARMONIC GENERATING LOADS

The previously presented approach to analysis of power phenomena in single-phase circuits with HGLs can be applied to three-phase, three-wire circuits, shown in Fig. 35, with HGLs.

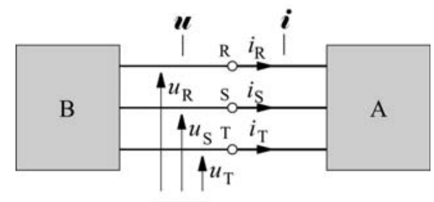

Figure 35. General structure of three-phase, three-wire circuits, with sub-circuit A assumed to be a load and sub-circuit B assumed to be a supply system.

The sign of the harmonic active power P_n enables us to con-clude where the source of this power is located, namely, in the supply source or in the HGL, and to decompose the set of har-monic orders N into to subsets, $N_{\rm A}$, $N_{\rm B}$.

If $P_n \geq 0$, then $n \in N_{\rm A}$ and

$$\sum_{n\in N_{\rm A}} \boldsymbol{i}_n \triangleq \boldsymbol{i}_{\rm A}, \quad \sum_{n\in N_{\rm A}} \boldsymbol{u}_n \triangleq \boldsymbol{u}_{\rm A}, \quad \sum_{n\in N_{\rm A}} P_n \triangleq P_{\rm A} \tag{170}$$

If $P_n < 0$ then $n \in N_{\rm B}$ and

$$\sum_{n\in N_{\rm B}} \boldsymbol{i}_n \triangleq \boldsymbol{i}_{\rm B}, \quad \sum_{n\in N_{\rm B}} \boldsymbol{u}_n \triangleq -\boldsymbol{u}_{\rm B}, \quad \sum_{n\in N_{\rm B}} P_n \triangleq -P_{\rm B} \tag{171}$$

Thus, similarly as in single-phase circuits, the load current, vol-tage and power can be expressed as

$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \boldsymbol{i}_{\rm A} + \boldsymbol{i}_{\rm B}$$

$$\boldsymbol{u} = \sum_{n\in N} \boldsymbol{u}_n = \boldsymbol{u}_{\rm A} - \boldsymbol{u}_{\rm B} \tag{172}$$

$$P = \sum_{n\in N} P_n = P_{\rm A} - P_{\rm B}$$

Interpretation of this decomposition is exactly the same (37) as interpretation of such a decomposition in single-phase circuits, except that the voltages and currents are superseded by three-phase vectors of line voltages and currents. The current vectors $\boldsymbol{i}_{\rm A}$ and $\boldsymbol{i}_{\rm B}$ are mutually orthogonal; hence

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\rm A}||^2 + ||\boldsymbol{i}_{\rm B}||^2 \tag{173}$$

For harmonic orders $n \in N_A$ the three-phase load can be considered as a passive load of having equivalent delta structure as shown in Figure 36

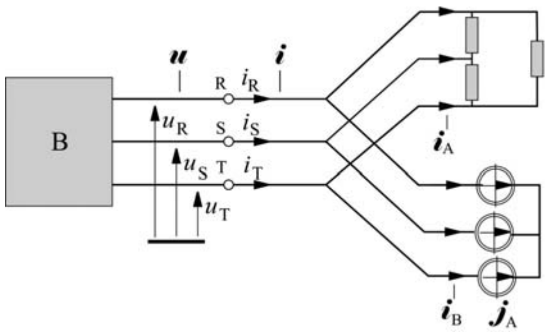

Figure 36. Equivalent circuit of a three-phase harmonic-generating load (A). For harmonics of the order from the set N_A, (nonnegative P_n) the load is equivalent to a passive linear load. For harmonics of the order from the set N_B, (negative P_n) the load is equivalent to the current source with $\boldsymbol{j}_A = \boldsymbol{i}_B$.

and equivalent admittance

$$Y_{enA} \triangleq G_{enA} + jB_{enA} = \frac{\boldsymbol{S}_n^*}{||\boldsymbol{u}_n||^2} \tag{174}$$

where the harmonic complex power $\boldsymbol{S}_n$ of a three-phase load can be calculated as

$$\boldsymbol{S}_n \triangleq P_n + jQ_n = \boldsymbol{U}_n^T \boldsymbol{I}_n^* \tag{175}$$

The remaining harmonics, i.e., of the order $n \in N_B$, are considered to be harmonics of a current source located in the subcircuit A, meaning harmonic generating load, $\boldsymbol{j}_A(t) = \boldsymbol{i}_B(t)$.

With respect to the active power P_A at voltage $\boldsymbol{u}_A$, the load is equivalent to a resistive symmetrical load having a star configuration and the conductance per phase equal to

$$G_{eA} \triangleq \frac{P_A}{||\boldsymbol{u}_A||^2} \tag{176}$$

Such a load draws the active current

$$\boldsymbol{i}_{aA} \triangleq G_{eA}\, \boldsymbol{u}_A \tag{177}$$

The remaining part of the current $\boldsymbol{i}_A$ can be decomposed into the scattered, reactive and unbalanced currents

$$\boldsymbol{i}_{sA} \triangleq \sqrt{2}\mathrm{Re} \sum_{n\in N_A} (G_{enA} - G_{eA})\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{178}$$

$$\boldsymbol{i}_{rA} \triangleq \sqrt{2}\ \mathrm{Re} \sum_{n\in N_A} jB_{enA}\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{179}$$

$$\boldsymbol{i}_{uA} \triangleq \sqrt{2}\ \mathrm{Re} \sum_{n\in N_A} \boldsymbol{b}_n \boldsymbol{A}_n\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{180}$$

Thus, taking into account eqn. (164), the load current can be decomposed into five physical components, namely

$$\boldsymbol{i} = \boldsymbol{i}_{aA} + \boldsymbol{i}_{sA} + \boldsymbol{i}_{rA} + \boldsymbol{i}_{uA} + \boldsymbol{i}_B. \tag{181}$$

They are mutually orthogonal (38); hence their rms values fulfill the relationship

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{aA}||^2 + ||\boldsymbol{i}_{sA}||^2 + ||\boldsymbol{i}_{rA}||^2 + ||\boldsymbol{i}_{uA}||^2 + ||\boldsymbol{i}_B||^2 \tag{182}$$

This relation can be visualized with the help of the polygon shown in Fig. 37. Five different power phenomena are responsible for the load current rms value. The active, scattered, reactive, unbalanced and the load generated current are associated with these phenomena.

The voltage vectors $\boldsymbol{u}_A$ and $\boldsymbol{u}_B$ orthogonal, thus the rms value of the supply voltage is equal to

$$||\boldsymbol{u}||^2 = ||\boldsymbol{u}_A||^2 + ||\boldsymbol{u}_B||^2 \tag{183}$$

and the source apparent power can be expressed as

$$S \triangleq ||\boldsymbol{u}||\cdot||\boldsymbol{i}|| = \sqrt{||\boldsymbol{u}_A||^2 + ||\boldsymbol{u}_B||^2}\sqrt{||\boldsymbol{i}_A||^2 + ||\boldsymbol{i}_B||^2} = \sqrt{S_A^2 + S_B^2 + S_E^2} \tag{184}$$

with

$$S_A \triangleq ||\boldsymbol{u}_A||\cdot||\boldsymbol{i}_A|| = \sqrt{P_A^2 + D_{sA}^2 + Q_A^2 + D_{uA}^2} \tag{185}$$

$$S_B \triangleq ||\boldsymbol{u}_B||\cdot||\boldsymbol{i}_B||, \quad S_E \triangleq \sqrt{||\boldsymbol{u}_A||^2||\boldsymbol{i}_B||^2 + ||\boldsymbol{u}_B||^2||\boldsymbol{i}_A||^2} \tag{186}$$

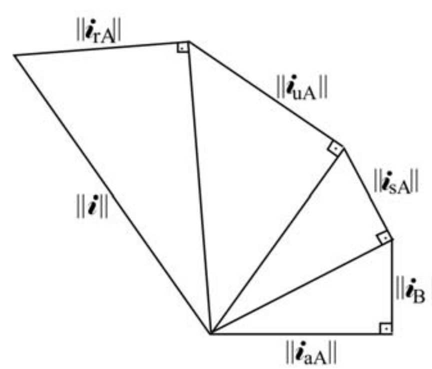

Figure 37. Geometrical illustration of the relationship between the rms value of the currents' physical components and the supply current rms value $||\boldsymbol{i}||$ in three-phase circuits with harmonic-generating loads.

Although the extorted apparent power S_E and the HGL-originated apparent power S_B look very similar, there is a substantial difference between them. There is no power phenomenon behind the extorted power S_E. It occurs only because the voltages $\boldsymbol{u}_A$ and $\boldsymbol{u}_B$ as well as currents $\boldsymbol{i}_A$ and $\boldsymbol{i}_B$ have rms values the supply source has to withstand. In the case of the apparent S_B it can be decomposed not only into the active power P_B, but also into a scattered, reactive and unbalanced powers, dependent on the power phenomena inside of the supply source.

The power factor λ of a three-phase unbalanced HGL can be expressed in the form

$$\lambda \triangleq \frac{P}{S} = \frac{P_{\rm A} - P_{\rm B}}{\sqrt{P_{\rm A}^2 + D_{\rm sA}^2 + Q_{\rm A}^2 + D_{\rm uA}^2 + S_{\rm B}^2 + S_{\rm E}^2}} \tag{187}$$

This formula reveals all power components that contribute to deterioration of the power factor in three-phase circuits with harmonic generating loads.

INSTANTANEOUS REACTIVE POWER p-q THEORY

Since the main approaches to power theory, as suggested by Budeanu and Fryze, were not capable of describing power properties and providing fundamentals for compensation of single-phase systems, a new concept, known as the *Instantaneous Reactive Power (IRP) p-q Theory*, has been developed by Akagi, Kanazawa and Nabae (59) in 1984. It was to provide mathematical fundamentals for the control of Pulse Width Modulated (PWM) inverter-based switching compensators, commonly known as "active power filters". According to Authors (73), the development of the IRP p-q Theory was a response to "*...the demand to instantaneously compensate the reactive power"* and the adjective *"instantaneous"* suggested that this theory could instantaneously provide information needed for a compensator control. Moreover, harmonic analysis is not needed for that purpose. Consequently, the IRP p-q Theory gained very high popularity (74–79).

Unfortunately, it was proven in (80, 81) that the IRP p-q Theory misinterprets power phenomena in electrical circuits. There is no physical phenomenon that is characterized by the instantaneous reactive power q. It can occur even in purely resistive circuits. Moreover, the instantaneous powers p and q do not enable for instantaneous identification of power properties of the load. They have to be observed over a whole period T for that purpose. Moreover, there is no direct relation of these two powers to power phenomena. This is because even in a sinusoidal situation there are three different phenomena that determine power properties of three-phase loads. These are: (1) the permanent flow of energy to the load characterized by the load active power, P; (2) phase shift between the voltage and current characterized by the load reactive power Q; and (3) the load current asymmetry and consequently, the load unbalanced power D. Three different phenomena cannot be identified with only two power quantities, p and q, the Instantaneous Reactive Power p-q Theory is based upon.

ADVANCED TOPICS THAT HAVE NOT BEEN DISCUSSED

The Currents' Physical Components (CPC) Power Theory is currently the most powerful tool for explanation and description of power phenomena in electrical systems with sinusoidal and nonsinusoidal voltages and currents. It applies not only to single-phase, but also to three-phase, three-wire systems with linear, time-invariant loads as well as with harmonic generating loads. The CPC Power Theory has proved its effectiveness in revealing major misconceptions in power theories developed by Budeanu, Fryze, Shepherd and Kusters, as well as in the Instan-taneous Reactive Power p-q Theory.

The CPC Power Theory is also a major theoretical tool for design and control of compensator for improving the power factor and reducing harmonic distortion. It provides fundamentals for design of reactive compensators both of the reactive current (32, 82), scattered current (36) and unbalanced currents (42, 83, 84). It can also be used, instead of the IRP p-q Theory, as a fundamental for switching compensator control algorithm (62, 85, 86, 94). As it was demonstrated in (86), the CPC-based control algorithm is more universal than the IRP p-q Theory-based algorithm and enables external controllability and adaptive operation of the compensator.

The progress in the development of the power theory of nonsinusoidal systems, obtained mainly due to the CPC concept, now enables (87) an extension of this theory beyond its traditional scope, meaning power phenomena and compensation in systems with periodic voltages and currents.

Due to fast varying, or in particular, pulsing loads, voltages and currents are loosing periodicity and consequently, the harmonic approach and the CPC power theory in its classical form cannot be applied. However, voltages and currents in electrical systems with time-varying or pulsing loads can be considered as *semi-periodic*. This concept is explained in (88). It was demon-strated in (87) that the CPC-based power theory can be exten-ded to systems with semi-periodic quantities and conesequ-ently, it enables description of power phenomena in such systems in power terms and provides a control algorithm for switching compensators (89).

The Reader should be aware that this article does not cover all issues on harmonics and powers. Due to an increase in number and in power of loads that cause current distortion, there is a lot of research on systems with nonsinusoidal voltages and currents. The scope of this research is very wide. There is still research related to mathematical fundamentals of analysis of such systems (90-92) and attempts aimed at standardization (7) of power quantities, based on an intuition rather than on a rigorous analysis of power phenomena. A lot of research is focused on compensation. This includes research on control algorithms for individual switching compensators (85, 86, 93, 94, 95) and on optimization and compensation of the whole system (96, 97).

BIBLIOGRAPHY

1. W. Shepherd and P. Zand, *Energy Flow and Power Factor in Nonsinusoidal Circuits*, Cambridge Univ. Press, Cambridge, 1978.
2. J. Arrillaga, D. Bradley, P. Bogdar, *Power System Harmonics,* John Wiley and Sons, Chichester, UK, 1985.
3. A. Kloss, *Harmonics* (in German), Technische Akademie Wuppertal, vde-verlag, 1989.

4. T. Heydt, *Electric Power Quality,* Stars in Circle Publication, 1991.
5. *The New IEEE Standard Dictionary of Electrical and Electronics Terms*, 1992.
6. *IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems*, IEEE Std. 519-1992.
7. *IEEE Trial-Use Standard. Definitions for the measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced, or unbalanced conditions,* IEEE Std. 1459-2000.
8. CIGRE Working Group 36.05, Harmonics, characteristic parameters, methods of study, estimates of existing values in the network, *Electra*, No. 77, 35-54, 1985.
9. A.C. Liew, Excessive neutral currents in three-phase fluoresent lighting circuits, CH2499-2/87/000-1364, IEEE, 1987.
10. R. Dwyer, A.K. Khan, M. McGranaghan, T. Houy, Le Tang, R.K. McCluskey, Evaluation of harmonic impacts from compact fluorescent lights on distribution system, 95 WM 105-7 PWRS, 1995.
11. N. Mohan, T.M. Undeland, W.P. Robbins, *Power Electronics*, John Willey & Sons, 1995.
12. M.H. Rashid, A.I. Maswood, A noval method of harmonic assessement generated by three-phase ac-dc converters under unbalanced supply conditions, *IEEE Trans. on Ind. Appl.,* Vol. 24, No. 4, pp.590-597, July/August 1998.
13. W. Xu, J.Marti, H.W. Dommel, A multiphase harmonic load flow solution technique, *IEEE Trans. on Power Systems*, Vol. PS-6, 174-182, Feb. 1991.
14. H.W. Dommel, Electromagnetic Transients Program Reference Manual (EMTP Theory Book), Univ. of British Columbia, 1986.
15. D. Xia, G.T. Heydt, Harmonic power flow studies, *IEEE Trans. on PAS*, Vol. PAS-101, 1257-1270, 1982.
16. T.J. Densen, P.S. Bodger, J. Arrillaga, Three-phase transmission system modeling for harmonic penetration studies, *IEEE Trans. on PAS*, Vol. PAS-103, 256-263, 1984.
17. J. Arrillaga, C.D. Callagham, Double-iterative algorithm for the analysis of power and harmonic flow at ac/dc convertor terminals, *IEE Proc.*, Pt. C, Vol. 136, No. 6, 319-324, Nov. 1989.
18. J.R. Linders, Electric wave distortions: their hidden cost and containment, *IEEE Trans. on Ind. Appl.*, Vol. IA-15, No. 5, 381-190, Sept./Oct. 1979.
19. M.T. Bishop, J.F. Baranowski, D. Heath, S.J. Benna, Evaluating harmonic-induced transformer heating, *IEEE Trans. on Power Delivery*, Vol. 11, No. 1, 305-311, January 1996.
20. E.F. Fuchs, R. Fei, A new computer aided method for the efficiency measurement of low-loss transformers and inductors under nonsinusoidal condotions, *IEEE Trans. on Power Delivery*, Vol. 11, No. 1, 292-304, January 1996.
21. IEEE Task Force on Harmonics Impact, Effects of harmonics on equipment, *IEEE Trans. on Power Delivery*, Vol. 8, No. 2, 672-670, April 1993.
22. G. Massey, Estimation method for power system harmonic effect on power distribution transformers, *IEEE Trans. on Ind. Appl.*, Vol. 30, No. 2, 485-489, March/April 1994.
23. A. Carullo and M. Parvis, Power meter for highly distorted three-phase systems, *IEEE Trans. Instr. Measur.*, Vol. IM-46, No. 6, 1262-1267, April 1997.
24. A.A. Girgis, M.C. Clapp, E.B. Makram, J.Qiu, J.G. Dalton, R.C. Catoe, Measurement and characterization of harmonics and high frequency distortion for large industrial loads, *IEEE Trans. on Power Delivery*, PRWD-5, No. 1, 427-434, January 1990.
25. A.A. Girgis, W.B. Cang, E.B. Makram, A digital recursive measurement scheme for on-line tracking of power system harmonics, *IEEE Trans. on Power Del.,* Vol. 6, No. 3, 1153-1160, July 1991.
26. R. Arsenau, P. Filipski, Application of a three-phase nonsinusoidal calibration system for testing energy and demand meters under nonsinusoidal field conditins, *IEEE Trans. on Power Delivery*, Vol. 3, No. 3, 874-879, January 1996.
27. L.S. Czarnecki, Z. Staroszczyk, Dynamic on-line measurement of equivalent parameters of three-phase systems for harmonic frequencies, *Europ. Trans. on Electr. Pow., ETEP*, Vol. 6, No.5, 329-336, Sept./Oct. 1996.
28. W. Shepherd and P. Zakikhani, Suggested definition of reactive power for nonsinusoidal systems, *Proc. IEE,* Vol. 119, No. 9, 1361-1362, Sept. 1972.
29. D. Sharon, Reactive power definitions and power factor improvement in nonlinear systems, *Proc. IEE*, Vol. 120, No. 6, 704-706, June 1973.
30. M. Depenbrock, Active and reactive powers (in German), *ETG-Fachtagung "Blindleistung"*, Aachen, October 1979.
31. M. Depenbrock, The FDB-method, a generally applicable tool for analysing power relations, *IEEE Trans. on Power Systems*, PS-8, No. 2, 381-386, 1993.
32. L. S. Czarnecki, Considerations on the reactive power in nonsinusoidal situations," *IEEE Trans. on Instr. Measur.,* Vol. IM-34, 399-404, March 1984.
33. L.S. Czarnecki, What is wrong with the Budeanu concept of reactive and distortion powers and why it should be abandoned, *IEEE Trans. on Instr. Measur.,* Vol. IM-36, No. 3, 834-837, Sept. 1987.
34. L.S. Czarnecki, Orthogonal decomposition of the current in a three-phase nonlinear asymmetrical circuit with nonsinusoidal voltage, *IEEE Trans. on Instr. Measur.,* Vol. IM-37, No. 1, 30-34, 1988.
35. L.S. Czarnecki and T. Swietlicki, Powers in nonsinusoidal networks, their analysis, interpretation and measurement, *IEEE Trans. on Instr. Measur.*, Vol. IM-39, No. 2, 340-345, April 1990.
36. L.S. Czarnecki, Scattered and reactive current, voltage, and power in circuits with nonsinusoidal waveforms and their compensation, *IEEE Trans. on I M*, Vol. 40, No. 3, 563-567, June 1991.
37. L.S.Czarnecki, Current and power equations at bidirectional flow of harmonic active power in circuits with rotating machines, *Europ. Trans. on Electrical Power Systems, ETEP,* Vol. 3, No.1, 45-52, Jan./Febr. 1993.
38. L.S. Czarnecki, Dynamic, power quality oriented approach to theory and compensation of asymmetrical systems under nonsinusoidal conditions, *Europ. Trans. on Electrical Power, ETEP*, Vol. 5, 347-358, Sept./Oct. 1994.
39. L.S. Czarnecki, Power related phenomena in three-phase unbalanced systems, *IEEE Trans. on Power Delivery,* Vol. 10, No. 3, 1168-1176, 1995.
40. S.Q. Sun, H. Kiyokawa, Decomposition of voltage, current and power, *Proc. IEE, Part B,* Vol. 138, No.1, 35-39, January 1991.
41. J. Willems, Instantaneous sinusoidal and harmonic active and deactive currents in three-phase systems, *Europ. Trans. on Electrical Power, ETEP*, Vol. 5, Sept./Oct. 335-346, 1994.
42. L.S. Czarnecki, Reactive and unbalanced currents compensation in three-phase circuits under nonsinusoidal conditions, *IEEE Trans. Instr. Measur.,* Vol. IM-38, No. 3, pp. 754-459, June 1989.

43. D. Sharon, Power factor definition and power transfer quality in nonsinusoidal systems, *IEEE Trans. on Instrum. and Measur*. Vol. 45, No. 3, 728-733, June1996.

44. IEEE Working Group on Nonsinusoidal Situations, Practical definitions for powers in systems with nonsinusoidal waveforms and unbalanced loads, *IEEE Trans. on Power Delivery,* Vol. 11, No. 1, 79-100, January 1996.

45. A.E. Emanuel, Powers in nonsinusoidal situations. A review of definitions and physical meaning. IEEE Trans. on Power Delivery, Vol. 5, No. 3, 1377-1389, 1990.

46. A.E. Emanuel, Apparent and reactive powers in polyphase and single-phase systems, *Europ. Trans. on Electrical Power, ETEP*, Vol. 3, No. 1, 7-14, Jan./Feb. 1993.

47. L.S. Czarnecki, Misinterpretation of some power properties of electric circuits, IEEE Trans. on Power Delivery, Vol. 9, No. 4, 1760-1769, October 1994.

48. P.Ch. Steinmetz, Is a phase-shift in the current of an electric arc? (in German), *Elektrotechnische Zeitschrift,* Heft 42, 567-568, 1892

49. C.I. Budeanu, Reactive and apparent powers (in French), *Institut Romain de l'Energie*, Bucharest 1927.

50. S. Fryze, Active, reactive and apparent powers in systems with distorted waveforms (in Polish), Przeglad Elektrotechniczny, Z. 7, 193-203, 1931; Z. 8, 225-234, Z. 22, 673-676. 1932.

51. F. Buchholz, Apparent power of a three-phase unbalanced load (in German), *Licht und Kraft*, No. 2, 8-11, 1922.

52. A. McEachern, G.T. Heydt, M. McGranaghan, W. Moncrief, W. Grady, Revenue and harmonics: an evaluation of some proposed rate structures, *IEEE Trans. on PD,* Vol. 10, No. 1, January 1995.

53 L.S. Czarnecki, Comments on active power flow and energy accounts in electrical systems with nonsinusoidal voltage and asymmetry, *IEEE Trans. on Power Delivery,* Vol. 11, No. 3, 1244-1250, July.1996.

54. D.A. Steeper, R.P. Stratford, Reactive power compensation and harmonic suppression for industrial power systems using thyristor converers, *IEEE Trans. on Ind. Appl.*, Vol. IA-12, No. 3, 232-254, May/June 1976.

55. D.A. Gonzales, J.C. McCall, Design of filters to reduce harmonic distortionmin industrial power systems, *Proc.of 1980 IEEE Annual Meeting,* Toronto, Canada, 361-365.

56. L.S. Czarnecki, Effect of minor harmonics on the performance of resonant harmonic filters in distribution systems, *IEE Proc., Electr. Pow. Appl.*, Vol. 144, No.5, 349-356, Sept. 1997.

57. L. Gyugyi, E. Strycula, Active ac power filters, *IEEE Trans. on Ind. Appl.*, Vol. IA-12, No. 3, 529-535, May/June 1976.

58. M. Depenbrock, Kompensation schnell verandliche Blindstrome, *ETZ-A,* Bd. 98, 408-411, 1977.

59. H. Akagi, Y. Kanazawa, A. Nabae, Instantaneous reactive power compensators comprising switching devices without energy storage components, *IEEE Trans. on IA*, Vol. IA-20, 625-630, 1984.

60. E. Makram, S. Varadan, Analysis of reactive power and power factor correction in the presence of harmonics and distortion, *Electric Power Systems Research*, 26, 211-218, 1993.

61. L. Rossetto, P. Tenti, Using ac-fed PWM converters as instantaneous reactive power compensators, *IEEE Trans. on Power Electronics,* Vol. 7, No. 1, 224-230 January 1992.

62. L. Czarnecki, Application of running quantities for a control of an adaptive hybrid compensator, *Europ. Trans. on Electrical Power Systems, ETEP,* Vol. 6, No.5, 337-344, Sept./Oct. 1996.

63. F.A. Farret, L.L. Freris, Minimisation of uncharacteristic harmonics in HVDC convrters through firing angle modulation, *IEE Proc.*, Vol. 137, Pt. C, No. 1, 45-52, January 1990.

64. M.J. Willers, M.G. Egan, J.M.D. Murphy, S. Daly, An ac-dc converter with low input distortion and near unity power factor, *Proc. of the EPE* 1993, Brighton.

65. C. Licitra, L. Malesani, G. Spiazzi, P. Tenti, A. Testa, Single-ended soft switching electronic ballast with unity power factor, *IEEE Trans. on Ind. Appl.*, Vol. 29, No. 2, 382-388, 1993.

66. H. Dym, H.P. McKean, *Fourier Series and Integrals,* Academic Press, Inc., London, 1972

67. L.S. Czarnecki, Currents' Physical Components (CPC) in circuits with nonsinusoidal voltages and currents. Part 1: Single-phase linear circuits, *Electrical Power Quality and Utilization Journal,* Vol. XI, No. 2, pp. 3-14, 2005.

68. Joint Committee of AIEE and NELE, Apparent power in three-phase systems, Trans. AIEE, 39, 1450, 1920.

69. L.S. Czarnecki, Currents' Physical Components (CPC) in circuits with nonsinusoidal voltages and currents. Part 2: Three-phase linear circuits, *Electrical Power Quality and Utilization Journal,* Vol. XII, No. 1, pp. 3-14, 2006.

70. L.S. Czarnecki, Energy flow and power phenomena in electrical circuits: illusions and reality, *Archiv fur Elektrotechnik*, (82), No. 4, pp. 10-15, 1999.

71. L.S. Czarnecki, Physical interpretation of the reactive power in terms of the CPC power theory, *7th Int. Workshop on Power Definitions and Meas. under Non-Sinusoidal Condit.,* Cagliari, 2006.

72. L.S. Czarnecki, Powers of asymmetrically supplied loads in terms of the CPC power theory, *7th Int. Workshop on Power Definitions and Meas. under Non-Sinusoidal Conditions*, Cagliari, 2006.

73. H. Akagi and A. Nabae, The p-q theory in three-phase systems under non-sinusoidal conditions, *Europ. Trans. on Electrical Power, ETEP,* Vol. 3, No. 1, pp. 27-31, 1993.

74. P. Salmeron, J.C. Montano, Instantaneous power components in polyphase systems under nonsinusoidal conditions, *IEE Proc. on Science, Meas. and Techn.,* Vol. 143, No. 2, pp. 239-297, 1996.

75. J.M. Morendaz-Eguilaz, J. Peracaula, Understanding AC power using the generalized instantaneous reactive power theory: considerations for instrumentation of three-phase electronic converters, *Proc. of the IEEE Int. Symp., Ind. Electr., ISIE '99*, Vol. 3, pp. 1273-1277, 1999.

76. J.M. Aller, at al, Advantages of the instantaneous reactive power definitions in three-phase system measurement, *IEEE Power Eng. Review*, Vol. 19, No. 6, pp. 54-56, June 1999.

77. F.Z. Peng, J.-S. Lai, Generalized instantaneous reactive power theory for three-phase power systems, *IEEE Trans. on Instr. Meas*., Vol. 45, No. 1, pp. 293-297, Feb. 1996.

78. K. Kim, F. Blaabjerg, B. Bak Jensen, Spectral analysis of instanttaneous powers in single-phase and three-phase systems with use of p-q-r theory, *IEEE Trans. on Power Electronics*, Vol. 17, No. 5, pp. 711-720, Sept. 2002.

79. M. Depenbrock, D.A. Marshal and J.D. van Wyk, Formulating requirements for universally applicable power theory as control algorithm in power compensators, *Europ. Trans. on Electrical Power, ETEP,* Vol. 4, No. 6, pp. 445-456, Nov./Dec. 1994.

80. L.S. Czarnecki, Comparison of the Instantaneous Reactive Power, p-q, Theory with Theory of Current's Physical Components, *Archiv fur Elektrotechnik*, Vol. 85, No. 1, pp. 21-28, Feb. 2004.

81. L.S. Czarnecki, On some misinterpretations of the Instantaneous Reactive Power p-q Theory, *IEEE Trans. on Power Electronics,* Vol. 19, No.3, pp. 828-836, 2004.

82. L.S. Czarnecki, Minimization of reactive power in nonsinusoidal situation, *IEEE Trans. Instr. Measur.,* Vol. IM-36, No. 1, pp. 18-22, March 1987.

83. L.S. Czarnecki and M.S. Hsu, Thyristor controlled susceptances for balancing compensators operated under nonsinusoidal conditions, *Proc. IEE, B, EPA*., Vol. 141, No. 4, pp. 177-185, 1994.

84. L.S. Czarnecki, S.M. Hsu, G. Chen, Adaptive balancing compensator, *IEEE Trans. on Power Delivery,* Vol. 10, No. 3, pp. 1663-1669, July 1995.

85. A. Firlit, Current's physical components theory and p-q power theory in the control of the three-phase shunt active power filter, *7th Int. Workshop on Power Definitions and Measurement under Non-Sinusoidal Conditions,* Cagliari, Italy, 2006.

86. H.L. Ginn III, A hybrid reference signal generator for active compensators, *7th Int. Workshop on Power Definitions and Measurement under Non-Sinusoidal Conditions*, Cagliari, Italy, 2006.

87. L.S. Czarnecki, Power theory of electrical circuits with quasi-periodic waveforms of voltages and currents, *Europ. Trans. on Elect. Power, ETEP*, Vol. 6, No. 5, pp. 321-328, Sept./Oct. 1996.

88. L.S. Czarnecki, Circuits with semi-periodic currents: main features and power properties, *Europ. Trans. on Electrical Power, ETEP,* Vol. 12, No. 1, pp. 41-46, Jan./Feb. 2002.

89. L.S. Czarnecki, E.A. Mendrela, H.L. Ginn, Decomposition of pulsed load current for hybrid compensation, *L'Energia Elettrica*, Vol. 81, pp. 150-156, 2004.

90. H. Lev-Ari, A.M. Stanković, Hilbert space techniques for modeling and compensation of reactive power in energy processing systems, *IEEE Trans. Circuits & Systems*, Vol. 50, No. 4, pp. 540-556, Apr. 2003.

91. H. Lev-Ari, A.M. Stanković, Fundamental principles in orthogonal decomposition of current and apparent power, *7th Int. Workshop on Power Definitions and Measurement under Non-Sinusoidal Conditions*, Cagliari, Italy, 2006.

92. P. Tenti, P. Mattavelli, A time-domain approach to power terms definitions under non-sinusoidal conditions, *6th Int. Workshop on Power Definitions and Measurement under Non-Sinusoidal Conditions*, Milan, Italy, 2003.

93. P. Mattavelli, A closed-loop selective harmonic compensation for active filters, *IEEE Trans. on Industy Appl.,* Vol. 37, No. 1, pp. 81-89, Jan/Feb 2001.

94. A. Firlit, Comperative analysis of control algorithms based on the p-q and the current's physical components(L.S.Czarnecki) for hybrid compensator, *7th Int. Conf. on Electric Power Quality and Utilization*, Krakow, Poland, 2003.

95. J.M. Maza Ortega, et al., Reference current computation methods for active power filters: accuracy assessment in the frequency domain, *IEEE Trans. PE*, Vol.20, No. 2, pp. 446-456, 2005.

96. H. Lev-Ari, A.M. Stanković, Optimizing power flows in lossy polyphase systems: effects of source impedance, *7th Int. Workshop on Power Definitions and Measurement under Non-Sinusoidal Conditions*, Cagliari, Italy, 2006.

97. P. Tenti, P. Mattavelli, E. Tedeschi, Compensation techniques based on reactive power conservation, *7th Int. Workshop on Power Definitions and Measurement under Non-Sinusoidal Conditions,* Cagliari, Italy, 2006.