# Powers in Nonsinusoidal Networks: Their Interpretation, Analysis, and Measurement

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*Abstract*—**A nonlinear or periodically time-variant load sometimes has to be considered not as the receiver, but as the source of energy, at least for some harmonic frequencies. This can be explained in terms of its equivalent circuit, usually composed of passive elements and harmonic sources which make the power phenomena in such a circuit much more complex than in a linear circuit. The necessity of comprehension of these phenomena stems from the fact that they determine the efficiency of the power transmission and the possibility of power factor improvement. They also affect the energy accounts. A method is suggested for the apparent power decomposition into components related to current components of distinctively different physical interpretation. A digital analyzer for measuring these powers is also described.**

## I. Introduction

DUE TO a rapid progress in power electronics, more and more energy is transmitted now at nonsinusoidal voltage and current. It is quite obvious, however, that the instrumentation for controlling the flow of energy in such situations, due to a natural lag, remains almost the same as it is used for sinusoidal waveforms. There is a number of reasons for such a state of affairs. The border between sinusoidal and nonsinusoidal systems is vague; there is still a confusion [1], [2] on a theoretical level as to power properties of systems in the presence of harmonics, so, in the lack of clear theoretical justification, the efforts devoted for research on a new instrumentation are limited. Moreover, with a lack of evident economic reasons, the industry prefers to keep on using traditional instrumentation. This, however, does not provide information as to power properties of systems in a real nonsinusoidal situation, which requires the measurement of additional quantities, but only in a hypothetical, sinusoidal situation. Even these are burdened, however, with errors not known in the true sinusoidal situation. As a consequence of the lack of the proper instrumentation, even the research on the energy flow and verification of theoretical results are substantially limited. This, of course, affects both the efficiency optimization and the energy accounts. What perhaps is the most important is that there is the lack of economically bolstered incentives for reduction of harmonic "pollution" in the electrical "environment."

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Considering the present state of the measurement technology and its ever growing possibility, the main reason for the lack of power instrumentation for nonsinusoidal systems should not be seen as a technological nature. An essential disaccord as to the definition of power fundamental terms could be blamed as a main factor for it.

The intricacy of the power phenomena and a lasting confusion about power properties of nonsinusoidal circuits is the cause of this disaccord. It seems, therefore, that just comprehension of the physical phenomena that determine the energy flow in nonsinusoidal circuits is one of the main factors that condition the progress both in instrumentation and in research on nonsinusoidal systems. Such an interpretation for circuits with a nonsinosoidal voltage source applied to linear loads (Fig. 1) is provided in [3]. It is based on the current decomposition into the active $i_a$, scattered $i_s$, and reactive $i_r$ orthogonal currents, in a very plain physical interpretation. It requires that the complex rms values $\boldsymbol{U}_n$ and the load admittance $\boldsymbol{Y}_n$ for harmonic frequencies be known.

The terminals where the energy flow is observed often have to be considered, however, as the junction of two wholly unknown subsystems, (Fig. 2) where the active power can be transmitted in either direction, where this direction may change with harmonic frequency. Any power system with nonlinear loads or loads with periodically switched devices which are the sources of harmonic currents can be considered as just such a system.

The energy flow in such a system is much more complex, however, than in linear systems, so, the current decomposition, its physical interpretation, and the power equation suggested in [3] cannot be applied to it directly. The following examples illustrate some interpretational questions that occur in such circuits.

### A. Example 1

In the circuit shown in Fig. 3, the voltage and the current at terminals $x$-$x$ are equal to

$$i = \sqrt{2}(20 \sin \omega_1 t - 40 \sin 2\omega_1 t) \text{ A}$$

$$u = \sqrt{2}(80 \sin \omega_1 t + 40 \sin 2\omega_1 t) \text{ V}$$

hence, they have the rms values of

$$\|i\| = 44.72 \text{ A} \quad \text{and} \quad \|u\| = 89.44 \text{ V}$$

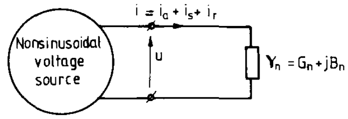


Fig. 1. Nonsinusoidal voltage source with linear load.

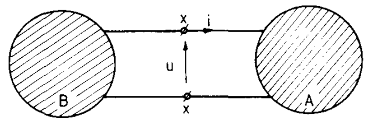


Fig. 2. Connection of not known subsystems.

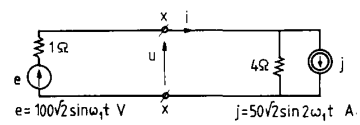


Fig. 3. Example of the circuit with $S \neq 0$, and $P = 0$.

so that, the apparent power $S$ measured at these terminals is equal to

$$S \triangleq \|u\| \, \|i\| = 4000 \text{ VA}.$$

There is not, however, any active power at terminals $x$-$x$, since

$$P \triangleq \frac{1}{T}\int_0^1 ui \, dt = 0.$$

Thus in terms of what kind of power can the difference between these two powers $S$ and $P$ be explained for this resistive circuit?

### *B. Example 2*

In the circuit shown in Fig. 4, the voltage and the current at terminals $x$-$x$ are equal to

$$i = \sqrt{2}(25 \sin \omega_1 t + 15 \cos \omega_1 t) \text{ A}$$

$$u = \sqrt{2}(75 \sin \omega_1 t - 15 \cos \omega_1 t) \text{ V}$$

and they have the complex rms values

$$\boldsymbol{I} = 15 - j25 = 29.15 \exp\{-j59.03°\} \text{ A}$$

$$\boldsymbol{U} = 15 - j75 = 76.48 \exp\{-j101.31°\} \text{ V}.$$

The complex apparent power is equal to

$$\boldsymbol{S} \triangleq \boldsymbol{U}\boldsymbol{I}^* = 2230 \exp\{-j42.27°\} \text{ VA}.$$

Thus despite the lack of reactance elements in the circuit, there is a reactive power:

$$Q \triangleq \text{Im}\{\boldsymbol{S}\} = -1500 \text{ VAr}$$

at terminals $x$-$x$. Elements that store energy in electric or magnetic fields are not necessary in circuits to have a reciprocating flow of energy, which results in a nonzero reactive power $Q$.

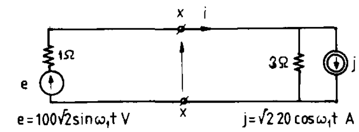


Fig. 4. Example of resistive circuit with nonzero reactive power.

## II. Orthogonal Decomposition of the Current

Let us consider a single-phase network shown in Fig. 2, composed of two subnetworks, $A$ and $B$, of a structure that is not known. We assume that only terminals $x$-$x$ are accessible for the voltage and current measurement. If voltage and current are periodic quantities of period $T$, then, they can be expressed with the Fourier series of the form:

$$u = U_0 + \sqrt{2}\,\text{Re}\sum_{n=1}^{\infty} \boldsymbol{U}_n \exp\{jn\omega_1 t\} \triangleq \sum_{n \in \mathfrak{N}} u_n \quad (1)$$

$$i = I_0 + \sqrt{2}\,\text{Re}\sum_{n=1}^{\infty} \boldsymbol{I}_n \exp\{jn\omega_1 t\} \triangleq \sum_{n \in \mathfrak{N}} i_n \quad (2)$$

where $\mathfrak{N}$ denotes the set of harmonic orders $n$, along with $n = 0$, of voltage and current harmonics with nonzero complex rms values:

$$\boldsymbol{U}_n \triangleq U_n \exp\{j\alpha_n\} \quad \text{and} \quad \boldsymbol{I}_n \triangleq I_n \exp\{j\beta_n\}. \quad (3)$$

The complex apparent power $\boldsymbol{S}_n$ of the $n$th-order harmonic at terminals $x$-$x$ is equal to

$$\boldsymbol{S}_n \triangleq \boldsymbol{U}_n \boldsymbol{I}_n^* \triangleq P_n + jQ_n. \quad (4)$$

$P_n$ is the active power transmitted from subnetwork $B$ to $A$ at frequency $n\omega_1$ and $Q_n$ is the reactive power of the $n$th-order harmonic, i.e., the generalized (positive or negative) amplitude of the reciprocating component of the instantaneous power $p_n \triangleq u_n i_n$. When the active power $P_n$ is not negative, i.e., $\text{Re}\{\boldsymbol{S}_n\} \geq 0$, then we can assume that subnetwork $B$ is the source of energy for the $n$th-order harmonic and subnetwork $A$ absorbs it as a passive load. Though the structure of subnetwork $A$ is not known and it may contain the harmonic sources of frequency $n\omega_1$, the network is equivalent as to power $\boldsymbol{S}_n$ at terminals $x$-$x$, to the network with subnetwork $A$ replaced by a passive element (Fig. 5) of admittance $\boldsymbol{Y}_{nA}$ equal to

$$\boldsymbol{Y}_{nA} \triangleq \boldsymbol{S}_n^*/U_n^2 \triangleq G_{nA} + jB_{nA}. \quad (5)$$

Subnetwork $B$ can be assumed to be a one-port of Norton structure, composed of a current source $j_{nB}$ and an admittance $\boldsymbol{Y}_{nB}$, though, neither $j_{nB}$ nor $\boldsymbol{Y}_{nB}$ are known.

Let $\mathfrak{N}_A$ denote the set of harmonic orders $n$ for which $P_n \geq 0$ and then denote

$$u_A \triangleq \sum_{n \in \mathfrak{N}_A} u_n, \quad i_A \triangleq \sum_{n \in \mathfrak{N}_A} i_n, \quad P_A \triangleq \sum_{n \in \mathfrak{N}_A} P_n. \quad (6)$$

When $P_n < 0$, we assume that subnetwork $A$ is the source of energy for frequency $n\omega_1$ and subnetwork $B$ absorbs it as a passive load. The original network in terms of power $\boldsymbol{S}_n$ at the terminals considered is equivalent to the network

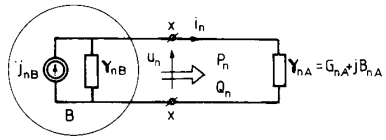

Fig. 5. Equivalent network for $P_n \geq 0$.

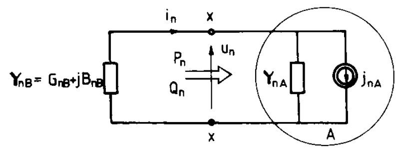

Fig. 6. Equivalent network for $P_n < 0$.

with subnetwork $\boldsymbol{B}$ replaced (Fig. 6) by a passive element of admittance $Y_{nB}$ equal to

$$Y_{nB} \triangleq -S_n^*/U_n^2 \triangleq G_{nB} + jB_{nB}. \tag{7}$$

Subnetwork $A$ can be assumed to be a one-port of Norton structure, composed of a source of current $j_{nA}$ and an admittance $Y_{nA}$, though neither $j_{nA}$ nor $Y_{nA}$ are known.

Let $\mathfrak{N}_B$ be the set of orders $n$ for which $P_n < 0$ and:

$$u_B \triangleq -\sum_{n\in\mathfrak{N}_B} u_n, \quad i_B \triangleq \sum_{n\in\mathfrak{N}_B} i_n, \quad P_B \triangleq -\sum_{n\in\mathfrak{N}_B} P_n. \tag{8}$$

In this way set $\mathfrak{N}$ is decomposed into two separable sets $\mathfrak{N}_A$ and $\mathfrak{N}_B$ of harmonic orders $n$, i.e., $\mathfrak{N}_A \cup \mathfrak{N}_B = \mathfrak{N}$ and $\mathfrak{N}_A \cap \mathfrak{N}_B = 0$, for which the active power is transmitted to subnetwork $A$ and to subnetwork $B$, respectively. This allows the following decomposition of the terminal quantities $i$, $u$, $P$:

$$i \triangleq \sum_{n\in\mathfrak{N}} i_n = \sum_{n\in\mathfrak{N}_A} i_n + \sum_{n\in\mathfrak{N}_B} i_n = i_A + i_B \tag{9}$$

$$u \triangleq \sum_{n\in\mathfrak{N}} u_n = \sum_{n\in\mathfrak{N}_A} u_n + \sum_{n\in\mathfrak{N}_B} u_n = u_A - u_B \tag{10}$$

$$P \triangleq \sum_{n\in\mathfrak{N}} P_n = \sum_{n\in\mathfrak{N}_A} P_n + \sum_{n\in\mathfrak{N}_B} P_n = P_A - P_B. \tag{11}$$

The network considered is equivalent, relative to the power at terminals $x$-$x$ to a "semi-identified" network shown in Fig. 7, such that

$$\text{for } n \in \mathfrak{N}_A: \begin{cases} j_{nA} \equiv 0 \\ Y_{nA} \triangleq S_n^*/U_n^2 \end{cases} \tag{12}$$

$$\text{for } n \in \mathfrak{N}_B: \begin{cases} j_{nB} \equiv 0 \\ Y_{nB} \triangleq -S_n^*/U_n^2 \end{cases} \tag{13}$$

but the remaining network parameters are not known.

Sets $\mathfrak{N}_A$ and $\mathfrak{N}_B$ are separable, thus currents $i_A$ and $i_B$ do not contain harmonics of the same order $n$, so they are mutually orthogonal, i.e., their scalar product $(i_A, i_B)$ defined as

$$(i_A, i_B) \triangleq \frac{1}{T}\int_0^T i_A i_B \, dt \tag{14}$$

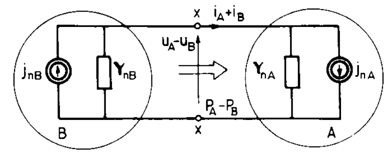

Fig. 7. Semi-identified equivalent network.

is equal to zero. Hence, the current's rms value $\|i\|$ at the terminals $x$-$x$ can be expressed as

$$\|i\|^2 = \|i_A\|^2 + \|i_B\|^2 \tag{15}$$

where

$$\|i_A\| = \sqrt{\sum_{n\in\mathfrak{N}_A} I_n^2} \qquad \|i_B\| = \sqrt{\sum_{n\in\mathfrak{N}_B} I_n^2}. \tag{16}$$

The same holds true, of course, for voltages $u_A$ and $u_B$, thus

$$\|u\|^2 = \|u_A\|^2 + \|u_B\|^2 \tag{17}$$

where

$$\|u_A\| = \sqrt{\sum_{n\in\mathfrak{N}_A} U_n^2} \qquad \|u_B\| = \sqrt{\sum_{n\in\mathfrak{N}_B} U_n^2}. \tag{18}$$

For voltage and current harmonics of order $n$ from set $\mathfrak{N}_A$ the subnetwork $A$ can be replaced by a passive load, thus according to [3], its current $i_A$ can be decomposed into the active, scattered, and reactive currents, $i_{aA}$, $i_{sA}$, $i_{rA}$, namely:

$$i_A = i_{aA} + i_{sA} + i_{rA}. \tag{19}$$

If

$$G_{eA} \triangleq P_A/\|u_A\|^2 \tag{20}$$

denotes the equivalent conductance of subnetwork $A$, then, these currents are defined as

$$i_{aA} \triangleq G_{eA} u_A \tag{21}$$

$$i_{sA} \triangleq \sqrt{2}\,\mathrm{Re} \sum_{n\in\mathfrak{N}_A} (G_{nA} - G_{eA}) U_{nA} \exp\{jn\omega_1 t\}. \tag{22}$$

If set $\mathfrak{N}_A$ contains $n = 0$, then the term $(G_{0A} - G_{eA}) U_{0A}$ should be added separately:

$$i_{rA} \triangleq \sqrt{2}\,\mathrm{Re} \sum_{n\in\mathfrak{N}_A} jB_{nA} U_{nA} \exp\{jn\omega_1 t\}. \tag{23}$$

All these currents are mutually orthogonal, i.e.,

$$(i_{aA}, i_{sA}) = 0, \quad (i_{sA}, i_{rA}) = 0, \quad (i_{aA}, i_{rA}) = 0 \tag{24}$$

therefore, their rms values fulfill relation

$$\|i_A\|^2 = \|i_{aA}\|^2 + \|i_{sA}\|^2 + \|i_{rA}\|^2 \tag{25}$$

where

$$\|i_{aA}\| = G_{eA}\|u_A\| \tag{26}$$

$$\|i_{sA}\| = \sqrt{\sum_{n\in\mathfrak{N}_A} (G_{nA} - G_{eA})^2 U_{nA}^2} \tag{27}$$

$$\|i_{rA}\| = \sqrt{\sum_{n\in\mathfrak{N}_A} B_{nA}^2 U_{nA}^2}. \tag{28}$$

The physical interpretation of these currents is the same as that presented in [3]. The active current $i_{aA}$ is the only current component that is indispensable for active power $P_A$ transmission. The remaining currents are useless and according to (26) they increase the current $i_A$ rms value.

The scattered current $i_{sA}$ occurs when the conductance $G_{nA} = P_{nA}/U_n^2$ changes with harmonic order, i.e., when the ability of harmonics for active power transmission varies with frequency.

The reactive current $i_{rA}$ occurs when, for any harmonic component of order $n \in \mathfrak{N}_A$, there is a reciprocating flow of energy at terminals x-x, i.e., when there is the harmonic reactive power $Q_n = -B_{nA}U_n^2$ at these terminals.

Everything that has been stated for subnetwork $A$ and harmonic order $n \in \mathfrak{N}_A$ holds true, of course, for subnetwork $B$ and $n \in \mathfrak{N}_B$. Its current $i_B$ can be decomposed as follows:

$$i_B = i_{aB} + i_{sB} + i_{rB}. \tag{29}$$

If

$$G_{eB} \triangleq P_B/\|u_B\|^2 \tag{30}$$

is the equivalent conductance of subnetwork $B$, then

$$i_{aB} \triangleq G_{eB}u_B \tag{31}$$

$$i_{sB} \triangleq \sqrt{2}\,\mathrm{Re} \sum_{n\in\mathfrak{N}_B} (G_{nB} - G_{eB})U_{nB} \exp\{jn\omega_1 t\}. \tag{32}$$

If $\mathfrak{N}_B$ contains $n = 0$, then the term $(G_{0B} - G_{nB})U_{0B}$ should be added separately:

$$i_{rB} \triangleq \sqrt{2}\,\mathrm{Re} \sum_{n\in\mathfrak{N}_B} jB_{nB}U_{nB} \exp\{jn\omega_1 t\}. \tag{33}$$

All these currents are mutually orthogonal, therefore, their rms values fulfill relation

$$\|i_B\|^2 = \|i_{aB}\|^2 + \|i_{sB}\|^2 + \|i_{rB}\|^2 \tag{34}$$

where

$$\|i_{aB}\| = G_{eB}\|u_B\| \tag{35}$$

$$\|i_{sB}\| = \sqrt{\sum_{n\in\mathfrak{N}_B} (G_{nB} - G_{eB})^2 U_{nB}^2} \tag{36}$$

$$\|i_{rB}\| = \sqrt{\sum_{n\in\mathfrak{N}_B} B_{nB}^2 U_{nB}^2}. \tag{37}$$

The physical interpretation of these currents is similar to that for subnetwork $A$. The active current $i_{aB}$ is the only current component that is indispensable for active power $P_B$ transmission. The remaining currents are useless and they increase the current $i_B$ rms value.

The scattered current $i_{sB}$ occurs when the conductance $G_{nB} = P_{nB}/U_n^2$ changes with harmonic order $n$, i.e., when the ability of the harmonics to cause active power transmission to subnetwork $B$ varies with frequency.

The reactive current $i_{rB}$ occurs when, for any harmonic component of order $n \in \mathfrak{N}_B$, there is a reciprocating flow of energy at terminals x-x, i.e., when there is a harmonic reactive power $Q_n = -B_{nB}U_n^2$ at these terminals.

Thus the current at terminals x-x can be considered as the sum of six components

$$i = (i_{aA} + i_{sA} + i_{rA}) + (i_{aB} + i_{sB} + i_{rB}) \tag{38}$$

of different physical interpretation and mutual orthogonality, hence:

$$\|i\|^2 = \|i_{aA}\|^2 + \|i_{sA}\|^2 + \|i_{rA}\|^2 + \|i_{aB}\|^2 + \|i_{sB}\|^2 + \|i_{rB}\|^2. \tag{39}$$

This relation is visualized as a polygon, shown in Fig. 8, built of five right-angled triangles with the length of the sides proportional to the rms value of the particular currents. It is important that this decomposition is obtained without any approximations or limits as to harmonic distortion level. It is valid even for a very high harmonic content. The decomposition presented above is illustrated in the example of the current decomposition in a network having known structure and parameters. This allows the verification of the results obtained with a circuit analysis program.

### A. Example 3

Let us consider the circuit shown in Fig. 9, with a source of distorted voltage and $\omega_1 = 1\ \mathrm{s}^{-1}$:

$$e = \sqrt{2}(100 \cos \omega_1 t + 50 \cos 3\omega_1 t + 10 \sin 4\omega_1 t)\ \mathrm{V}$$

and the source of distorted current:

$$j = \sqrt{2}(50 \cos 2\omega_1 t + 10 \sin 3\omega_1 t + 30 \cos 4\omega_1 t)\ \mathrm{A}.$$

A circuit analysis results in the following complex rms values of the voltage and the current at the x-x terminals

$$I_1 = 37.3 \exp\{-j27°\}\ \mathrm{A}$$
$$U_1 = 83.3 \exp\{-j0°\}\ \mathrm{V}$$
$$I_2 = 41.7 \exp\{-j0°\}\ \mathrm{A}$$
$$U_2 = 23.6 \exp\{-j135°\}\ \mathrm{V}$$
$$I_3 = 19.1 \exp\{-j70°\}\ \mathrm{A}$$
$$U_3 = 36.8 \exp\{-j5°\}\ \mathrm{V}$$
$$I_4 = 23.3 \exp\{-j2°\}\ \mathrm{A}$$
$$U_4 = 30.0 \exp\{-j109°\}\ \mathrm{V}$$

and hence, $\|i\| = 63.5$ A, $\|u\| = 98.8$ V.

The complex apparent powers $S_n \triangleq U_n I_n^*$ are equal to

$$S_1 = 2778\ \mathrm{W} + j1389\ \mathrm{VAr}$$
$$S_3 = 175\ \mathrm{W} + j679\ \mathrm{VAr}$$
$$S_2 = -694\ \mathrm{W} - j694\ \mathrm{VAr}$$
$$S_4 = -210\ \mathrm{W} - j669\ \mathrm{VAr}$$

thus $\mathfrak{N}_A = \{1; 3\}$, $\mathfrak{N}_B = \{2; 4\}$; so we can decompose the active power $P$ into $P_A$, $P_B$ components with values

$$P_B = 904\ \mathrm{W} \qquad P_A = 2953\ \mathrm{W}.$$

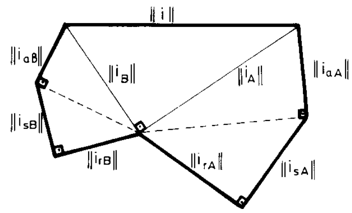


Fig. 8. Polygon of the current components rms values.

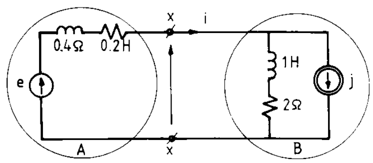


Fig. 9. Example of the circuit.

The rms values of the voltage and current components $i_A$, $i_B$, and $u_A$, $u_B$, calculated with (16) and (18) are equal to

$$\|i_B\| = 47.8 \text{ A}, \quad \|i_A\| = 41.9 \text{ A},$$

$$\|u_B\| = 38.2 \text{ V}, \quad \|u_A\| = 91.1 \text{ V},$$

so, that the equivalent conductances of the subnetworks are

$$G_{eB} = 0.620 \text{ S} \quad G_{eA} = 0.356 \text{ S}.$$

The equivalent admittances of both subnetworks for harmonics are equal to

$$Y_{2B} = (1.250 - j1.250) \text{ S}, \quad Y_{1A} = (0.400 - j0.200) \text{ S},$$

$$Y_{4B} = (0.233 - j0.741) \text{ S}, \quad Y_{3A} = (0.129 - j0.501) \text{ S},$$

and the rms values of particular currents can be calculated. For the active current with (26) and (35) we obtain

$$\|i_{aB}\| = 23.7 \text{ A} \quad \|i_{aA}\| = 32.4 \text{ A}.$$

For the scattered current with (27) and (36):

$$\|i_{sB}\| = 18.9 \text{ A} \quad \|i_{sA}\| = 9.1 \text{ A}$$

and for the reactive current, (28) and (37) result in

$$\|i_{rB}\| = 36.9 \text{ A} \quad \|i_{rA}\| = 24.9 \text{ A}.$$

It is easy to check that indeed the rms values of particular components fulfill (39).

## III. Components of the Apparent Power

The apparent power, i.e., the formal product of the voltage and current rms values, $S \triangleq \|i\| \, \|u\|$, has two interpretations. When it is used for rating equipment, the rms maximum values of the voltage and current are considered. It is also used for calculating the actual loading of the source with the actual rms values of the voltage and current. In this paper the apparent power is used in this last meaning, i.e., it is the product of actual rms values of the voltage and current in the cross section considered. Since usually only a part of the apparent power $S$ is related to the source loading with the active power $P$, the remaining part, related to the useless loading, is the subject of a great interest for theoretical, technical, and economic reasons. In systems with sinusoidal waveforms all these equations are solved in terms of reactive power $Q$. For nonsinusoidal systems, however, a similar quantity has not been found. The complexity of the power phenomena requires instead that a few different quantities be used. Even for linear loads, apart from the active power, two additional powers are required [3]. Similarly for the apparent power $S$, they are only formal products of the rms values of the voltage and current components, and their meaning and interpretation can only be explained in terms of these current components, that is, as an additional loading of the source caused by these currents. When the active power is transmitted in both directions, the situation becomes even more complex, however. With the decomposition suggested, the square of the terminal apparent power $S$ can be expressed as

$$S^2 \triangleq \|i\|^2 \|u\|^2 = (\|i_A\|^2 + \|i_B\|^2)\|u\|^2 \triangleq S_A^2 + S_B^2 \tag{40}$$

with

$$S_A^2 \triangleq \|i_A\|^2 \|u\|^2 = \|i_A\|^2 (\|u_A\|^2 + \|u_B\|^2) \triangleq S_{0A}^2 + S_{BA}^2 \tag{41}$$

$$S_B^2 \triangleq \|i_B\|^2 \|u\|^2 = \|i_B\|^2 (\|u_A\|^2 + \|u_B\|^2) \triangleq S_{0B}^2 + S_{AB}^2. \tag{42}$$

Terms $S_{0A}$ and $S_{0B}$ are apparent powers of subnetworks $A$ and $B$ considered as sourceless loads, and they can be decomposed [3] into the active, scattered, and reactive powers, as follows

$$S_{0A}^2 \triangleq \|i_A\|^2 \|u_A\|^2 = P_A^2 + D_{sA}^2 + Q_{rA}^2 \tag{43}$$

$$S_{0B}^2 \triangleq \|i_B\|^2 \|u_B\|^2 = P_B^2 + D_{sB}^2 + Q_{rB}^2 \tag{44}$$

where

$$D_{sA} \triangleq \|i_{sA}\| \, \|u_A\|, \quad Q_{rA} \triangleq \|i_{rA}\| \, \|u_A\| \tag{45}$$

$$D_{sB} \triangleq \|i_{sB}\| \, \|u_B\|, \quad Q_{rB} \triangleq \|i_{rB}\| \, \|u_B\|. \tag{46}$$

The terms $S_{BA} \triangleq \|u_B\| \, \|i_A\|$ is the apparent power of a voltage source of voltage $u_B$ with the forced current $i_A$ (Fig. 10(a)). Since this current does not contain any harmonics of the same order as the voltage $u_B$, the active, reactive, and the scattered powers are not associated with this current. Nonetheless, the source has to have a sufficient rating for such a current at this voltage. This also refers to the term $S_{AB} \triangleq \|u_A\| \, \|i_B\|$, (Fig. 10(b)). Since the presence of components $i_A$, $i_B$ in the current forces an additional apparent power, different from the $P$, $D_s$, $Q_r$, at the terminals observed, so that, the square of the total apparent power $S$ increases by the value

$$S_F^2 \triangleq S_{AB}^2 + S_{BA}^2 \tag{47}$$

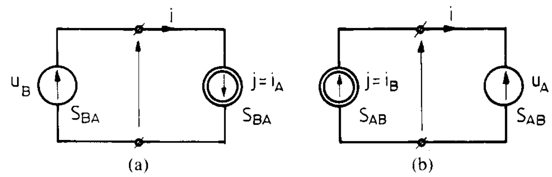


Fig. 10. (a) A source of voltage $u_B$ with the forced current $i_A$. (b) A source of voltage $u_A$ with the forced current $i_B$.

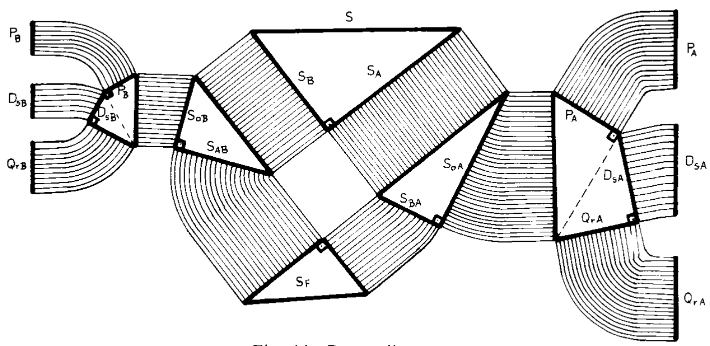


Fig. 11. Power diagram.

therefore, $S_F$ could be called a "forced" apparent power, and the terminal apparent power $S$ can be expressed as

$$S^2 = S_{0A}^2 + S_{0B}^2 + S_F^2. \tag{48}$$

Some relations between all these powers are visualized in the power diagram shown in Fig. 11. For the circuit analyzed in example 3 we obtain

$$D_{sB} = 720 \text{ VA}, \quad D_{sA} = 831 \text{ VA}, \quad P = 2049 \text{ W}$$

$$Q_{rB} = 1411 \text{ VAr}, \quad Q_{rA} = 2265 \text{ VAr}, \quad S_F = 4635 \text{ VA}$$

$$S_{0B} = 1824 \text{ VA}, \quad S_{0A} = 3813 \text{ VA}, \quad S = 6274 \text{ VA}.$$

## IV. Power Analyzer

The decomposition suggested is applied as the algorithm for a digital power analyzer. It is composed of a two-channel, 12-bit A/D conversion board built around a Z80 microprocessor, a sampling generator that is synchronized with the sampled waveforms $u(t)$, $i(t)$, and a personal microcomputer.

In the waveform period $T$ the conversion board connected at the terminals $x$-$x$ provides 64 voltage samples, interlaced with 64 current samples, to the microcomputer. Since the sampling rate is synchronized with the sampled frequency of period $T$, the voltage samples $u_k$ at points $t_k$ are uniformly spaced in time by $\tau = T/64$. The current samples $i_k$ are shifted with respect to the voltage samples $u_k$ by $\tau/2$. The samples, arranged in pairs $\langle u_k, i_k \rangle$ are processed with a 64-points FFT algorithm. This results in the complex rms values of 32 voltage harmonics, $\boldsymbol{U}_n$. Since the current samples are delayed by $\tau/2$, which is equivalent to sampling of the shifted current $i(t + \tau/2)$ at points $t_k$, the FFT procedure does not result in $\boldsymbol{I}_n$ values, but in $\boldsymbol{I}_n \exp\{jn\omega_1\tau/2\}$. Since $\omega_1\tau/2 = \pi/64$, the phase angles of the current harmonics can be corrected after the FFT procedure is completed.

When the values $\boldsymbol{U}_n \boldsymbol{I}_n$ are known, then, the complex apparent powers $\boldsymbol{S}_n = \boldsymbol{U}_n \boldsymbol{I}_n^*$ are calculated and with the aid of the sign of their real part, Re $\{\boldsymbol{S}_n\} = P_n$, sets $\mathfrak{N}_A$, $\mathfrak{N}_B$ are identified, and admittances $\boldsymbol{Y}_{nA} = \boldsymbol{S}_n^*/U_n^2$ and $\boldsymbol{Y}_{nB} = -\boldsymbol{S}_n^*/U_n^2$ are calculated. For sets $\mathfrak{N}_A$, $\mathfrak{N}_B$ the values of $P_A$, $P_B$, $\|u_A\|$, $\|u_B\|$ are calculated from (6), (8), and (18). This enables both the equivalent conductances $G_{eA} \triangleq P_A/\|u_A\|^2$ and $G_{eB} \triangleq P_B/\|u_B\|^2$ to be found and the rms values of the current orthogonal components $\|i_{aA}\|$, $\|i_{sA}\|$, $\|i_{rA}\|$ and $\|i_{aB}\|$, $\|i_{sB}\|$, $\|i_{rB}\|$ to be calculated with (26)–(28) and (35)–(37). Analysis is terminated with the calculation of the powers $D_{sA}$, $Q_{rA}$, $D_{sB}$, $Q_{rB}$, $S_F$ according to (45)–(47).

For the FFT and the power analysis procedure written in Borland Pascal 4.0, executed on a 12-MHz IBM/AT the power analysis is completed in 0.3 s.

## V. Conclusions

The decomposition suggested provides the basis for the physical interpretation of power phenomena in circuits with nonsinusoidal waveforms with any distribution of harmonic sources. Furthermore, it enables the identification of the power properties of the system at the specified terminals if only the instantaneous values of the voltage and current at these terminals are accessible for the measurement.

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