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Decomposition of Pulsed Loads Current for Hybrid Compensation

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Summary: ***Pulsed loads take energy from a distribution system in short intervals of time, i.e., their supply current amplitude changes in a wide range. Thus, by affecting the supply voltage amplitude, such loads could disturb the supply conditions of other loads. When the level of these disturbances is unacceptable, a compensator of the power variation might be needed. The supply current of pulsed loads could also be strongly distorted, thus, compensation of the power variation should be accompanied by compensation of the current distortion. These two different goals impose different requirements with respect to energy storage of a compensator and the speed of its operation. These goals could be achieved by two different compensators connected in a hybrid structure.***

The paper reports results of an investigation on decomposition of the current of a pulsed load into physical components and their prediction for control of a hybrid compensator. Considerations are illustrated using as an example of the decomposition of the supply current of a power pump that provides supply for an electromagnetic linear accelerator.

I. INTRODUCTION

Pulsed loads in electrical systems are loads that take energy from a distribution system only in short intervals of time, separated by pauses with much lower energy consumption. Thus, the supply current over a few or several cycles of the supply voltage could have much higher amplitude than its average value. Spot welders are well known traditional examples of such pulsed loads. Devices for high power flashes of ultraviolet light or X-ray beams for food anti-contamination are other examples of such loads. An electric gun is probably the most impressive example of a pulsed load. Such a gun or a linear accelerator takes energy, needed for projectile acceleration, from a distribution system, over a few seconds and accumulates it in a capacitor. In the case of electromagnetic guns, this energy is next released, in approximately 2 ms time, in the form of kinetic energy of a projectile. The projectile is accelerated by the Lorenz force in the magnetic field created in the gun barrel by the capacitor discharge current on the order of tens or even hundreds of kA [1]. A simplified structure of this type of load, with only the main functional components, is shown in Fig 1.

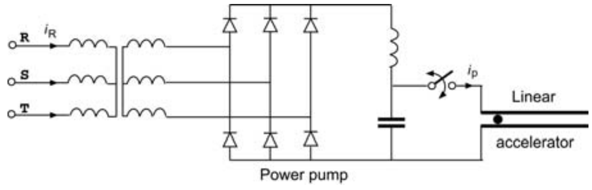

Fig. 1. Example of a pulsed load

Even if real circuits may differ substantially from the one, shown in the Figure, such loads when considered from the distribution system point of view, have some common characteristics. They are built of one or a few charging circuits [2] that operate as *energy or power pumps.* Such a pump, gradually, over a few or several cycles of the supply voltage, elevates the energy of a storage capacitor and, when this energy is released in a short interval of time, a high power pulse is created. Therefore, investigation of power properties of pulsed loads will be illustrated and verified in this paper using as an example the power pump shown in Fig. 1. For some set of circuit parameters, that are irrelevant for study in this paper, the variation of the supply current, i_R, between output current pulses, i_p, is shown in Fig. 2.

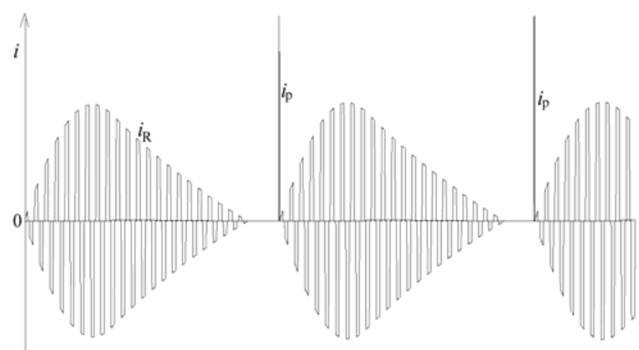

Fig. 2. Example of pulsed load supply and output currents

The active power variation and consequently, variation of the amplitude of the supply current of pulsed load in a wide range can affect the bus voltage RMS value. If a pulsed load is a major load on the distribution system, this voltage variation could reach an unacceptable level. It could be reduced only by an increase of the distribution system power or by compensation. When the distribution power cannot be elevated, compensation remains the only option for reducing the effects of pulsed loads on the distribution system voltage.

Energy storage in pulsed loads supplied from an AC system requires that the current is rectified, as in the case of the power pump shown in Fig. 1, and rectification usually causes supply current distortion, that changes with the change of active power of the pulsed load. Due to the time scale used in Fig. 2, this distortion is not clearly visible. It becomes visible when only a few cycles of that waveform, as shown in Fig. 3, are observed. Due to commutation, a delay of the supply current with respect to the supply voltage also occurs in such a circuit and consequently, a pulse of reactive power will accompany the active power pulse.

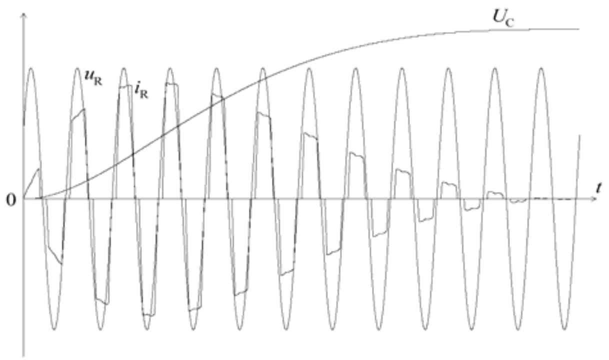

Fig. 3. Supply voltage and current of a pulsed load and the capacitor voltage.

The investigation on power properties of pulsed loads reported in this paper attempts to answer the two following questions.

(i) Can a pulsed load be described in power terms?

(ii) How can control signals be generated for a compensator that could reduce harmful components of the pulsed load supply current?

II. CURRENT DECOMPOSITION

The supply current of pulsed loads is non-periodic and consequently, it cannot be expressed in terms of Fourier series. Such a load cannot be described in terms of the active and reactive powers, P and Q, defined for systems with periodic voltages and currents. Also, the RMS value is defined for periodic quantities, hence the apparent power S of pulsed loads cannot be specified.

For static application, the supply current of a pulsed load can be decomposed into orthogonal components and a reactive compensator could be designed using the Fourier Transform [3]. Such an approach does not enable us for generating control signals for an adaptive switching compensator.

However, properties of the supply current of the pulsed load, at least that considered in this paper, fit properties of a subset of non-periodic currents that were called ***semi-periodic*** currents in Ref. [4]. These are currents that lost periodicity due to time variance of the load parameters, although the permanent energy transfer in systems with such currents is associated only with their semi-sinusoidal fundamental component that is in phase with the supply voltage. Because the energy transfer is associated only with the fundamental, the period T of the supply voltage has a particular importance for semi-periodic currents. It could be considered as a time-frame for analysis of power properties of systems with semi-periodic currents.

Since the variation of active and reactive powers and the supply current distortion are the dominating features at the pulsed load terminals, features of secondary importance such as the voltage distortion, its asymmetry or the load imbalance are neglected in the following analysis. Thus, it could be simplified to analysis of power properties of a single-phase load.

When the load is observed in the instant t, then the mean value of the instantaneous power $p(t)$, per phase, calculated over the preceding interval $< t-T,\ t >$ can be considered as the active power per phase, in that instant t.

$$\tilde{P}(t) = \frac{1}{T}\int_{t-T}^{t} u(t)\ i(t)\,dt\ , \tag{1}$$

When the current is not periodic, then this mean value is, of course, not a constant but a function of time and is referred to [5] as a running active power.

Let the voltage and current be specified by a sequence of uniformly distributed samples, N samples per period T, i.e., taken every interval $\Delta\tau = T/N$, and let us denote these samples by $u_n = u(n\Delta\tau)$ and $i_n = i(n\Delta\tau)$. At a sufficiently high number of samples per period, N, the running active power at the instant $t_k = k\Delta\tau$ can be calculated as

$$\tilde{P}(t_k) = \tilde{P}(k\Delta\tau) = \tilde{P}_k = \frac{1}{N}\sum_{n=k-N+1}^{n=k} u_n\, i_n \tag{2}$$

Its variation for the voltage and current plotted in Fig. 3 is shown in Fig. 4.

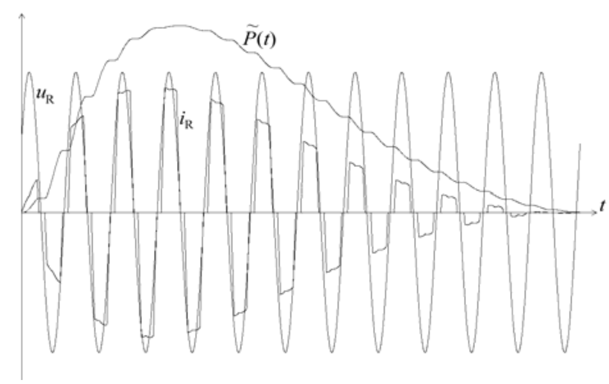

Fig. 4. Variation of the running active power

A segment of the semi-periodic current observed over the window of duration T can be considered as one period of a *periodic extension* of what is observed in the window thus, this segment can be described in terms of Fourier series.

It was assumed that the supply voltage is sinusoidal and the load is balanced, thus, the supply current in the observation window can be decomposed into the active and reactive components of the fundamental harmonic and a distorted current

$$i = i_1 + i_d = i_{1a} + i_{1r} + i_d \tag{3}$$

referred to as *physical components* of the supply current. This decomposition can be performed when the last sample in the observation window, $i_k = i(k\Delta\tau)$, is provided by the acquisition system. It is enough to calculate the complex RMS (CRMS) value of the fundamental harmonic of the current in the observation window for this purpose.

The Discrete Fourier Transform (DFT) provides a discrete value of the fundamental harmonic at $t_k = k\Delta\tau$,

$$\tilde{\boldsymbol{I}}_{1k} = \frac{\sqrt{2}}{N}\sum_{n=k-N+1}^{n=k} i_n\, e^{-j\frac{2\pi}{N}n} = \tilde{I}_{1k}\, e^{-j\tilde{\varphi}_k}\ . \tag{4}$$

Thus, the fundamental harmonic of the supply current at the instant t_k is equal to

$$i_{1k} = \sqrt{2}\,\mathrm{Re}\{\tilde{\boldsymbol{I}}_{1k}\, e^{j\omega_1 k\Delta\tau}\}\,, \tag{5}$$

and the distorted current

$$i_{\mathrm{d}k} = i_{\mathrm{k}} - i_{1k}. \tag{6}$$

Let us assume that the supply voltage of the reference phase changes as a cosine function,

$$u(t) = U\sqrt{2}\cos\omega_1 t\,, \tag{7}$$

thus, its CRMS value is $\boldsymbol{U} = U$. Having the supply voltage as a reference, the value of the fundamental harmonic at the instant t_k can be decomposed into two components, one in phase and another in quadrature with the supply voltage.

$$\begin{aligned} i_{1k} &= \sqrt{2}\,\mathrm{Re}\{\tilde{\boldsymbol{I}}_{1k}\, e^{j\omega_1 k\Delta\tau}\} = \tilde{I}_{1k}\sqrt{2}\,\cos(\omega_1 k\Delta\tau - \tilde{\varphi}_k) = \\ &= \tilde{I}_{1k}\sqrt{2}\cos\tilde{\varphi}_k\cos(\omega_1 k\Delta\tau) + \tilde{I}_{1k}\sqrt{2}\sin\tilde{\varphi}_k\sin(\omega_1 k\Delta\tau) = \\ &= i_{1\mathrm{a}\,k} + i_{1\mathrm{r}\,k}\,, \end{aligned} \tag{8}$$

where

$$i_{1\mathrm{a}\,k} = \tilde{I}_{1k}\sqrt{2}\cos\tilde{\varphi}_k\cos(\omega_1 k\Delta\tau)\,, \tag{9}$$

$$i_{1\mathrm{r}\,k} = \tilde{I}_{1k}\sqrt{2}\sin\tilde{\varphi}_k\sin(\omega_1 k\Delta\tau)\,. \tag{10}$$

The running active power, per phase, at the instant t_k is equal to

$$\tilde{P}_k = U\,\tilde{I}_{1k}\cos\tilde{\varphi}_k\,, \tag{11}$$

thus, the in-phase current component can be expressed as

$$\begin{aligned} i_{1\mathrm{a}\,k} &= \tilde{I}_{1k}\sqrt{2}\cos\tilde{\varphi}_k\cos(\omega_1 k\Delta\tau) = \\ &= \frac{\tilde{P}_k}{U^2}\sqrt{2}\,U\cos(\omega_1 k\Delta\tau) = \tilde{G}_{\mathrm{e}k}\,u(k\Delta\tau). \end{aligned} \tag{12}$$

Thus, the in-phase component can be considered as an active current. The coefficient

$$\tilde{G}_{\mathrm{e}k} = \frac{\tilde{P}_k}{U^2}\,, \tag{13}$$

is the equivalent conductance of the load at $t_k = k\Delta\tau$.

Due to a phase-shift of the fundamental current harmonic, a reactive power occurs at the load terminals. Its value, per phase, at the instant t_k is

$$\tilde{Q}_k = U\,\tilde{I}_{1k}\sin\tilde{\varphi}_k\,, \tag{14}$$

Thus, the quadrature current component can be expressed as

$$i_{1\mathrm{r}k} = \frac{\tilde{Q}_k}{U^2}\sqrt{2}\,U\sin(\omega_1 k\Delta\tau) = \tilde{B}_{\mathrm{e}k}\left.\frac{du(t)}{d(\omega_1 t)}\right|_{t_k}\,, \tag{15}$$

so that, it is the reactive current of the load and

$$\tilde{B}_{\mathrm{e}k} = -\frac{\tilde{Q}_k}{U^2}\,, \tag{16}$$

is the equivalent susceptance of the load at the instant t_k.

With a change of the observation window, both the fundamental harmonic and distortion of the periodic extension of the segment observed in the window change. Consequently, the CRMS value of the fundamental harmonic is not a constant, but a function of time thus, the current i_1 is not a sinusoidal current. It is referred to as a fundamental semi-harmonic of the supply current. Also, the active and reactive currents are no longer sinusoidal currents. The values specified with formulae (12) and (15) are discrete values of the active and reactive current, $i_{1\mathrm{a}}(t)$ and $i_{1\mathrm{r}}(t)$. Their waveforms for a few periods of the supply voltage are shown in Fig. 5.

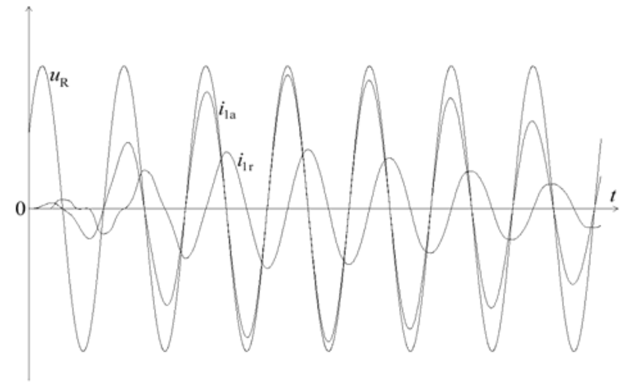

Fig. 5. Active and reactive current waveforms

Subtracting the fundamental semi-harmonic i_1 from the supply current separates the distorted component, i_{d}, i.e.,

$$i_{\mathrm{d}}(t) = i(t) - i_1(t). \tag{17}$$

Its waveform is shown in Fig. 6.

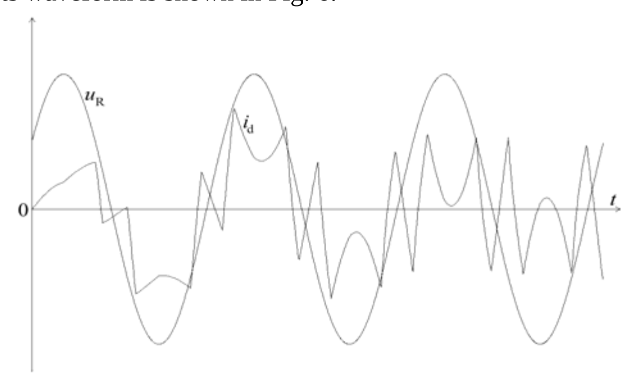

Fig. 6. Distorted current waveform

This decomposition into the active, reactive and distorted currents, done at the end of the observation window, at the instant t_k is an orthogonal decomposition, since we can interpret the segment of the waveform observed in this window as one period of a hypothetical periodic extension of what was observed. Thus, if the scalar product at the instant $t_k = k\Delta\tau$ is defined as

$$(x,y)_k = (x,y)_{t=k\Delta\tau} = \frac{1}{T}\int_{t-T}^{t=k\Delta\tau} x(t)\,y(t)\,dt\,, \tag{18}$$

then

$$(i_{1\mathrm{a}}, i_{1\mathrm{r}})_k = 0, \quad (i_{1\mathrm{a}}, i_{\mathrm{d}})_k = 0, \quad (i_{1\mathrm{r}}, i_{\mathrm{d}})_k = 0\,. \tag{19}$$

Consequently, if the RMS value at the instant t_k is defined as

$$\|\tilde{x}\|_k = \|\tilde{x}\|_{t=k\Delta\tau} = \sqrt{(x,x)_k} = \sqrt{\frac{1}{T}\int_{t-T}^{t=k\Delta\tau} x^2(t)\,dt}\,, \tag{20}$$

hence

$$\|\tilde{i}\|_k^{\,2} = \|\tilde{i}_{1a}\|_k^{\,2} + \|\tilde{i}_{1r}\|_k^{\,2} + \|\tilde{i}_{d}\|_k^{\,2}\,. \tag{21}$$

Multiplying this equation by the square of the voltage RMS value, U, the following power equation, per phase, is obtained

$$\tilde{S}_k^2 = \tilde{P}_k^2 + \tilde{Q}_k^2 + \tilde{D}_k^2\,, \tag{22}$$

with

$$\begin{aligned} \tilde{S}_k &= U\|\tilde{i}\|_k\,, \\ \tilde{P}_k &= U\|\tilde{i}_{1a}\|_k\,, \\ \tilde{Q}_k &= U\|\tilde{i}_{1r}\|_k\,, \\ \tilde{D}_k &= U\|\tilde{i}_{d}\|_k \end{aligned} \tag{23}$$

Observe that these powers and the power equation were obtained without any approximations. As long as the conditions for which they were developed are fulfilled, these are the exact values of the active, reactive and distorted powers at the instant $t_k = k\Delta\tau$.

The power factor of the load is equal to

$$\tilde{\lambda}_k = \frac{\tilde{P}_k}{\tilde{S}_k} = \frac{\|\tilde{i}_{1a}\|_k}{\sqrt{\|\tilde{i}_{1a}\|_k^{\,2} + \|\tilde{i}_{1r}\|_k^{\,2} + \|\tilde{i}_{d}\|_k^{\,2}}} = \frac{1}{\sqrt{1+\tilde{\delta}_{ik}^2}}\,\tilde{\lambda}_{1k}\,, \tag{24}$$

where

$$\tilde{\lambda}_{1k} = \frac{\|\tilde{i}_{1a}\|_k}{\|\tilde{i}_{1}\|_k} = \frac{\|\tilde{i}_{1a}\|_k}{\sqrt{\|\tilde{i}_{1a}\|_k^{\,2} + \|\tilde{i}_{1r}\|_k^{\,2}}} \tag{25}$$

denotes power factor of the fundamental semi-harmonic and

$$\tilde{\delta}_{ik} = \frac{\|\tilde{i}_{d}\|_k}{\|\tilde{i}_{1}\|_k} = \frac{\|\tilde{i}_{d}\|_k}{\sqrt{\|\tilde{i}_{1a}\|_k^{\,2} + \|\tilde{i}_{1r}\|_k^{\,2}}} \tag{26}$$

is the coefficient of the supply current harmonic distortion.

These results fit well into the concept of the Currents Physical Components (CPC) power theory [6], in the sense that is possible to associate specific currents and powers with distinctively different power phenomena in a circuit with semi-periodic currents. Furthermore, this association is accurate as well as all power quantities can be measured. The only difference is that when the current is semi-periodic, all of these powers are functions of time.

III. IMPACT ON THE SUPPLY

Power factor, λ, and the coefficient of the current harmonic distortion, δ_i, are the major measures of the loading quality in the case of conventional harmonic generating loads, i.e., with periodic currents. This is due to the following reasons.

Let us assume that the load power, the short-circuit power S_{sc} and the reactance-to-resistance ratio, $\xi_s = X_s/R_s$, at the bus, where a conventional load is installed, are known. Then the knowledge of λ and δ_i values, as well as the knowledge of the current harmonic spectrum, enable us to evaluate the increase of the active power loss in the supply, the voltage drop and the coefficient of the bus voltage harmonic distortion, δ_u, defined as

$$\delta_u = \frac{\|u_d\|}{U_1} \tag{27}$$

where u_d is distorted component of the supply voltage.

Unfortunately, the value λ and δ_i are not sufficient for characterization of pulsed loads, even if their time variance is known. The power pulse itself is not yet characterized. An approach to its characterization is, of course, a matter of convention.

A pulsed load could be characterized by such parameters as its maximum power, pulse duration, power time-variance, power factor and the supply current distortion. However, it seems that the best perspective could be obtained by evaluation of the impact of such a load on the supply as compared to the impact of a conventional load.

The distribution system can be loaded by a conventional load to a level specified by standards or by an agreement between customers and utilities. The acceptable loading power, $S_{AL,}$ is the primary loading parameter. The required level of the power factor, λ, and the limit for the supply current harmonic distortion, δ_i, could be specified as well.

As long as the apparent power of a pulsed load and other loads does not exceed the acceptable loading power, S_{AL}, and distortion is kept within required limits, there is no reason for concern. The presence of a pulsed load can cause that for a short interval of time the acceptable loading power, S_{AL}, is surpassed.

Assuming that the pulsed load is a major load on the supply bus and its maximum apparent power is denoted by S_{Pmax}, then the ratio

$$\vartheta = \frac{S_{Pmax}}{S_{AL}}\,, \tag{28}$$

could serve as a measure of pulse loading. The time interval, τ, when the pulsed load apparent power, S_P, is higher than the acceptable loading power, S_{AL}, provides another important piece of information about such loading.

There are two major effects of pulsed loads on the distribution system. When the coefficient ϑ is larger than one, then a notch in the supply voltage RMS value that exceeds an accepted value, of duration τ is expected. Moreover, it is probable, that the voltage notch will be accompanied by an increase in the voltage distortion, δ_u.

Deep notches in the supply voltage, accompanied by a strong waveform distortion, have a dangerous potential to disturb the system. In particular, digital and computer-like equipment, instrumentation and control systems could be affected.

The effect of pulsed loads on the distribution system could be reduced by an increase of the short-circuit power at the bus where such loads are installed or by compensators. The first option requires an increase in the power rating of the distribution or even generating equipment. In order to reduce the harmful effects by half, the short-circuit power at the supply bus has to be approximately doubled which would require double the power rating of the bus supply transformer. Thus, this is a very expansive option. Installation of a compensator seems to be more justified.

IV. COMPENSATORS OF PULSED LOADS

A compensator of a pulsed load has to not only reduce effects of the active and reactive power variation but also the supply current distortion. These two tasks are substantially different from the point of view of the required capability of energy storage of the compensator and its speed of response to waveform changes. The net energy over a voltage period transmitted by harmonics to a compensator of the waveform distortion is equal to energy dissipated in the device. Thus, it needs relatively low capability of energy storage. It only need to provide a DC voltage supply for a PWM inverter which is the core component of such a compensator. Conversely, the variation of the load active power can be compensated only if the compensator has a substantial capability of energy storage. At the same time, the rate of active and reactive powers change of pulsed loads usually remains much lower than the rate of change of the current instantaneous values. Thus, a compensator of the variation of active and reactive powers could be much slower than a compensator of the current distortion. Consequently, it would be reasonable to build compensators of pulsed loads as hybrid compensators, composed of a compensator of power variations (PQ variab. comp.) and a compensator of the current distortion (Dist. comp.). Its structure is shown in Fig. 7.

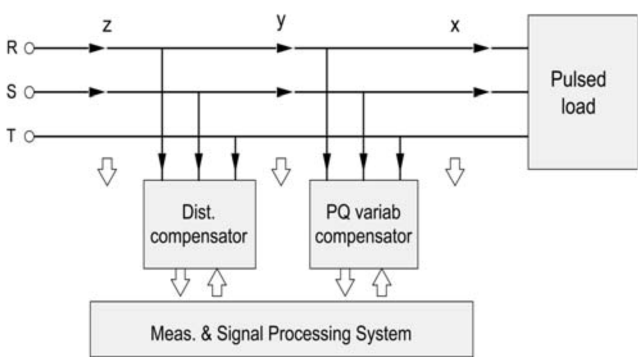

Fig. 7. Structure of a hybrid compensator of pulsed loads

A conventional switching compensator, commonly known under the misleading name of "active power filter" can be used as a compensator of the current distortion. However, there are a variety of options with respect to the compensator of the load power variability, because the amount and the method of energy storage is a key factor for design of such a compensator. Energy could be stored in a capacitor, in a battery or in a fly-wheel. Even a super-conducting storage device could be considered for very high power applications. The structure of the compensator depends, of course, on the selection of a method of energy storage. However, independently of the storage method selection, the compensator of the power variability has to be able to provide a pulse of energy, i.e., to provide a current of the opposite sign to the sum of active and reactive currents, $i_{1a} + i_{1r}$, shown in Fig. 5, i.e., a semi-sinusoidal current.

A measurement and digital signal processing system has to provide information on the required currents of both compensators. Information on instantaneous values of the distorted current is needed for the control of the distortion compensator while information on the amplitude and phase of the semi-fundamental current is needed for the control of the *PQ* variability compensator. Observe that different kinds of information are needed for control of each of these two compensators. One is controlled in the time-domain, the second in the frequency-domain.

The system voltages and currents needed for generating the control signals, could be measured in one of cross-sections, x, y or z. Depending on the selection, both compensators are operated with a closed loop feedback control, with an open loop control, or one of them would be operated with an open loop control while the other with a closed loop. Although the selection is crucial for the control accuracy and the compensator stability, this is beyond the scope of this paper. Instead, the focus is on such a decomposition of the load current that the control of the hybrid compensator would be possible.

A question crucial for the compensator control is: *how to the generate control signals?* Such a signal generation requires extrapolation of the known information about the load, acquired in the observation window, into the future to predict the expect-ed control signals.

V. PREDICTION OF CONTROL SIGNALS

Control signals should be generated after a sequential set of data about the load is acquired by a measurement system, with as little delay possible. Therefore, various time consuming methods of non-periodic signal analysis and estimation, such as Fourier Transform or wavelets seem to have a limited potential as prospective tools for generation of a control signal for the compensator.

This paper reports results of a study on whether it is possible to predict the control signals by a simple extrapolation of the current physical component values, calculated after at the instant $t_k = k\Delta\tau$, when the last samples of voltages and current were acquired, to the instant, $t_{k+1} = (k+1)\Delta\tau$, that means to the nearest future beyond the observation window.

To evaluate the extrapolation error, the power pump, selected previously as an example of pulsed loads, was analyzed and results over several cycles of the supply voltage were stored for comparison. These results of circuit analysis were used previously to simulate measurement over a moving observation window and for the current decomposition into its physical components. Now they will be used to evaluate the error of the predicted control signal.

The equivalent conductance of the load, updated according to formula (13) at each instant of time t_k, is like the active power, a slowly varying function of time. Its value, extrapolated to the instant of time $t_{k+1} = (k+1)\Delta\tau$, is equal to

$$\tilde{G}_{ek+1} = \tilde{G}_{ek} + \Delta\tilde{G}_e = \tilde{G}_{ek} + (\tilde{G}_{ek} - \tilde{G}_{ek-1}) = 2\,\tilde{G}_{ek} - \tilde{G}_{ek-1}\,. \quad (29)$$

Now, this value can be used to predict the value of the active current at the instant t_{k+1}, namely

$$i_{1a\,k+1} = \tilde{G}_{ek+1}\, u[(k+1)\Delta\tau] = \tilde{G}_{ek+1}\sqrt{2}\,U\cos[\omega_1(k+1)\Delta\tau]\,, \quad (30)$$

where $u[(k+1)\Delta\tau]$ is a known value of the supply voltage. Exactly in the same way, the reactive current can be predicted, namely

$$i_{1rk+1} = \tilde{B}_{ek+1}\left.\frac{du(t)}{d(\omega_1 t)}\right|_{t_{k+1}} = -\tilde{B}_{ek+1}\sqrt{2}\,U\sin[\omega_1(k+1)\Delta\tau]\,, \quad (31)$$

where $\tilde{B}_{ek+1}$ is the extrapolated value of the load susceptance, equal to

$$\tilde{B}_{ek+1} = \tilde{B}_{ek} + \Delta\tilde{B}_e = \tilde{B}_{ek} + (\tilde{B}_{ek} - \tilde{B}_{ek-1}) = 2\,\tilde{B}_{ek} - \tilde{B}_{ek-1}\,. \quad (32)$$

Plots of predicted values of the active and reactive currents are not drawn here, because they cannot be distinguished visually from actual values of these currents, shown in Fig. 5. The accuracy of prediction can be found calculating the prediction error

$$\varepsilon_{k+1} = 1 - \frac{i_{x\,k+1}}{(i_{x\,k+1})_{act}}\,, \quad (33)$$

where x = 1a, or x = 1r, respectively and $(i_{xk+1})_{act}$ denotes the value of i_{xk+1} obtained from the circuit analysis. This error for the load considered in this paper does not exceed 1.3% for the active current prediction and 2.0% for the reactive current prediction.

The predicted value of distorted current is equal to

$$i_{d\,k+1} = i_{k+1} - (i_{1a\,k+1} + i_{1r\,k+1})\,, \quad (34)$$

where i_{k+1} is the extrapolated value of the supply current equal to

$$i_{k+1} = i_k + \Delta i = i_k + (i_k - i_{k-1}) = 2\,i_k - i_{k-1}\,. \quad (35)$$

However, unlike the equivalent conductance and susceptance, the supply current is not a slowly varying quantity. Thus, the supply current value can be extrapolated with much lower accuracy as compared to the equivalent conductance or susceptance values. Therefore, the predicted distorted current can contain an error much higher than the predicted active or reactive currents. The predicted value of the distorted current at the sampling rate of 128 samples per period T is shown in Fig. 8.

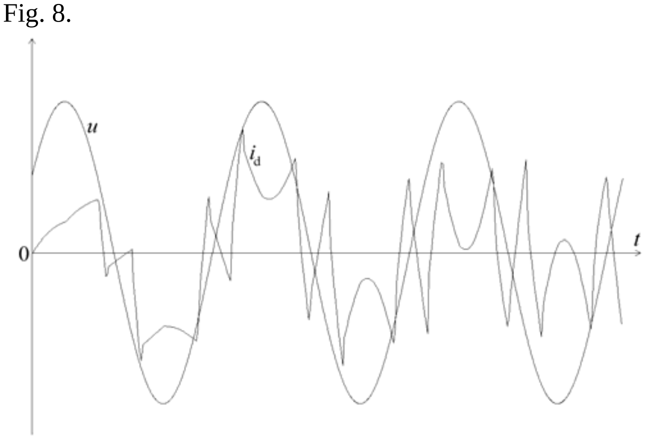

Fig. 8. Predicted distorted current waveform

Since the difference between the predicted and the actual waveform of distorted current is not well visible by comparison of Figs. 8 and 6, this difference is shown in Fig. 9. It shows that the prediction accuracy declines at the beginning and the end of diodes' commutation. This is because of the increased rate of change of the current waveform slope when the diode commutation begins and ends.

The prediction accuracy could be elevated by a more sophisticated method of measured data extrapolation than used in this section or by an increase in the sampling rate. With availability of simultaneous sampling A/D four channels converters that can have a sampling frequency on the order of 1Mhz, the acceptable sampling rate is practically not bounded by A/D converters but by computation speed and by amount of computation that has to be performed between two neighboring samples to generate control signals.

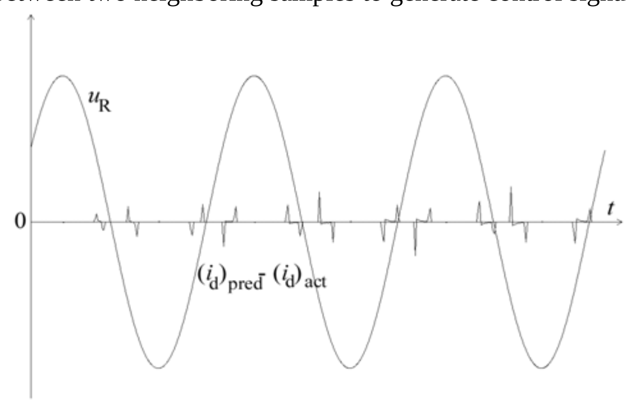

Fig. 9. Difference between predicted and actual distorted current at sampling rate of 128 samples per period T

Therefore, the accuracy of the control signal prediction can be elevated by reduction of the amount of calculation needed for generation of this signal.

VI. COMPUTATIONAL ASPECTS

The number of multiplications needed for the prediction of current components i_{1a}, i_{1r} and i_d is the major computational burden of the method discussed. Assuming that values of $\sqrt{2}\,U\cos(\omega_1 k\Delta\tau)$ and $\sqrt{2}\,U\sin(\omega_1 k\Delta\tau)$ are not calculated but stored in a look up table, and equivalent parameters G_{ek} and B_{ek} are known, then only two multiplications are needed to predict the values of components i_{1a}, i_{1r} and i_d. Some error can occur, of course, due to possible variation of the supply voltage RMS value U.

Unfortunately, the FFT on N samples with respect to the supply current fundamental semi-harmonic has to be calculated at the instant t_k, according to formula (4), to calculate the CRMS value $\tilde{\boldsymbol{I}}_1$ and equivalent parameters G_{ek} and B_{ek}.

Assuming that N complex values

$$\frac{\sqrt{2}}{N}\, e^{-j\frac{2\pi}{N}n}\,, \quad (36)$$

are stored in a look up table, this calculation requires N multiplications of a real number by a complex number, thus, $2N$ multiplications of real numbers. However, this number of multiplications can be reduced in an iterative procedure.

The formula (4) can be rearranged as follows

$$\tilde{\boldsymbol{I}}_{1k} = \tilde{I}_{1k}\, e^{-j\tilde{\varphi}_k} = \frac{\sqrt{2}}{N}\sum_{n=k\text{-}N+1}^{n=k} i_n\, e^{-j\frac{2\pi}{N}n} =$$

$$= \frac{\sqrt{2}}{N}\sum_{n=k\text{-}N}^{n=k-1} i_n\, e^{-j\frac{2\pi}{N}n} + (i_k - i_{k\text{-}N})\frac{\sqrt{2}}{N} e^{-j\frac{2\pi}{N}\mathrm{k}} =$$

$$= \tilde{\boldsymbol{I}}_{1k-1} + (i_k - i_{k\text{-}N})\frac{\sqrt{2}}{N} e^{-j\frac{2\pi}{N}\mathrm{k}} \;. \tag{37}$$

It means that only two multiplications are needed to update the CRMS value calculated at the instant $t_{\mathrm{k-1}}$ to the new value at the instant t_{k}. Two additional multiplications are needed for calculating equivalent conductance and susceptance, since

$$\tilde{\boldsymbol{Y}}_{\mathrm{e}k} = \tilde{G}_{\mathrm{e}k} + jB_{\mathrm{e}k} = \frac{\tilde{\boldsymbol{I}}_{1k}}{U} = \tilde{I}_{1k}\,\frac{\cos\tilde{\varphi}_k}{U} + j\tilde{I}_{1k}\,\frac{\sin\tilde{\varphi}_k}{U}\,. \tag{38}$$

Thus again, at the cost of some inaccuracy caused possible variation of by the supply voltage RMS value U, with a help of a look up table, only two multiplications are needed for calculating the load equivalent conductance and susceptance.

It means that decomposition of the semi-periodic supply current of a pulsed load into physical components and prediction of their value for the compensator control require only six multiplications of real numbers.

VI. CONCLUSIONS

This paper demonstrates using an example of a pulsed load that when supplied from a sinusoidal voltage source has a semi-periodic and distorted supply current, that it is possible to decompose such a current into its physical components and describe such a load in terms of active, reactive and distortion powers that are functions of time. It is also possible to predict the values of these currents for control of a hybrid compensator. Moreover, it was shown that such decomposition and prediction are not computationally toilsome. In fact, only six multiplications are needed.

VII. REFERENCES

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