

Currents' Physical Components (CPC) In Circuits with Nonsinusoidal Voltages and Currents

Part 1: Single-Phase Linear Circuits

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Abstract: - The paper summarizes the present state of discussions on power phenomena, power definitions and compensation in single-phase linear circuits. The main concepts developed over several decades on how various non-active powers in circuits with nonsinusoidal voltages and currents should be defined and compensated are presented along with a detailed discussion of these concepts. The paper presents also the concept of the single-phase load current decomposition into Currents' Physical Components (CPC) and its application for power definitions and compensation in such circuits. The paper is not addressed to scientists and experts on powers, but rather to common electrical engineers that sometimes face power related problems in nonsinusoidal systems.

Index Terms: - nonsinusoidal voltages and currents, harmonics, power definitions, power phenomena, apparent, reactive, scattered, unbalanced currents or powers.

1. INTRODUCTION

Power properties of circuits with nonsinusoidal voltages and currents have been investigated and debated for the entire XXth century. Some scientists have devoted their entire scientific lives to this purpose. A number of "schools" of various interpretations of power properties and power definitions were established, taught at universities for decades and widespread through academic textbooks. Followers of these schools adhere to them often with a sort of religious zeal and discussions are sometimes very emotional.

At such a level of controversy, a common electrical engineer could be confused. According to author's observations, usually they do not know how to write a power equation of a load when the supply voltage is nonsinusoidal. Therefore, this paper is not addressed to experts on powers, but to common engineers, not particularly involved in power issues, with a main message that could surprise a Reader: power properties of systems with nonsinusoidal voltages and currents are not so complex as this controversy suggests.

The paper starts with a discussion of the most widespread traditional approach to power definitions, developed by Budeanu [1] It had the strongest impact on our interpretation of power properties. Although it was demonstrated in 1987 [2] that Budeanu's definitions are erroneous, they are still supported by the IEEE Standard Dictionary [3]. However, a Reader not interested in the historical background could omit the first Sections and go directly to Sec. V, where the Currents' Physical Components (CPC) approach is discussed. It is only recommended that the Reader would acquaint himself with symbols used in this paper.

The process of energy delivery, as well as the equipment rating in the electrical system is described in terms of powers. Beginnings of studies on powers and power theory could be traced down to the moment at the end of the XIXth century, when it was observed that the product of the supply voltage and the load current RMS values, known as the apparent power S , could be higher than the load active power P . This difference is interpreted in various ways and these interpretations, as it will be demonstrated in this paper, affect tech-

nical methods of reduction of this difference with a compensator. Particular interpretations and related definitions of powers are referred to as Budeanu's, Fryze's or other power theories.

Power theory can be located on a junction of mathematics, physics and technology. Therefore, it is not sufficient for a power theory to be mathematically correct. This is a trivial expectation. Identification of physical phenomena that affect the apparent power and description of these phenomena in terms of powers seem to be much more difficult. Even worse usually is with technical value of power theories. Most of them have failed to provide solutions for practical problems and first of all, for compensation

Power theory, being founded in mathematics, should provide strict results. As long as the conditions for which powers were defined are fulfilled, results of power theory cannot depend on the level of waveform distortion or selection of structure and circuit parameters. This feature of power theory is used extensively in this paper. Very simplified circuits with high distortion are used to provide clear and vivid proofs that some concepts of power theory can lead to absurd results.

Electric power of a device, in a physical sense, is the rate of electric energy flow to this device,

$$p(t) = \frac{dW}{dt}. \quad (1)$$

Being a fundamental physical quantity, this power does not however have technical applications, because usually it is a time-varying function. Electrical equipment cannot be specified in terms of this power. It is mainly used for theoretical studies on energy flow. Other powers have to be defined.

II. POWERS IN SYSTEMS WITH SINUSOIDAL CURRENTS

Powers defined for single-phase systems with sinusoidal voltages and currents form a natural reference for power definitions in more complex situations, in particular, in systems under nonsinusoidal conditions and in three-phase systems.

Power properties of single-phase circuits shown in Fig. 1 with sinusoidal supply voltage and current

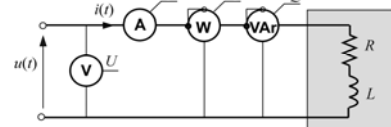


Fig. 1. Single-phase load with meters

$$u(t) = \sqrt{2}U \sin \omega t,$$

$$i(t) = \sqrt{2}I \sin(\omega t - \varphi),$$

are described in terms of three powers. The active or real power,

$$P = \frac{1}{T} \int_0^T u(t) i(t) dt = UI \cos \varphi, \quad (2)$$

is a mean value, over period T , of the rate of energy flow from the source to the load. The reactive power

$$Q = \frac{1}{T} \int_0^T u(t) i(t - \frac{T}{4}) dt = UI \sin \varphi, \quad (3)$$

is equal to the amplitude of energy oscillation between the supply and the load. The apparent power

$$S = \sqrt{\frac{1}{T} \int_0^T u^2(t) dt} \sqrt{\frac{1}{T} \int_0^T i^2(t) dt} = UI, \quad (4)$$

is a product of supply voltage and current RMS values.

These three powers are measured by meters connected as shown in Fig 1 and satisfy power equation

$$S^2 = P^2 + Q^2. \quad (5)$$

Electric energy conversion into other forms of energy needed for the energy consumer is determined by the active power, P , while equipment for energy delivery has to be rated with respect to the voltage and current RMS values, i.e., with respect to the apparent power, S . The ratio of these two powers

$$\lambda = \frac{P}{S}, \quad (6)$$

called power factor, specifies effectiveness of utilization of energy delivery equipment.

III. SYMBOLS AND FUNDAMENTALS

Harmonics used for description and analysis of nonsinusoidal voltages and currents could cause an increase in complexity of various expressions to such a degree that they could become illegible, or at least, difficult to comprehend. Therefore, effective mathematical tools and compact symbols are crucial for a clear presentation of power properties of such systems.

Periodic voltages and currents, denoted generally by $x(t) = x$, of the same period T , will not be presented in this paper in a common form of the Fourier series,

$$x = X_0 + \sum_{n \in N} a_n \cos n\omega_1 t + \sum_{n \in N} b_n \sin n\omega_1 t, \quad (7)$$

but in its complex form, namely, as

$$x = \sum_{n \in N_0} x_n(t) = X_0 + \sqrt{2} \operatorname{Re} \sum_{n \in N} X_n e^{jn\omega_1 t}, \quad (8)$$

where $x_n(t) = x_n$ is the n^{th} order harmonic of x , X_0 is its mean value, N_0 is the set of all harmonic orders, n , along with $n = 0$, and

$$X_n = X_n e^{j\psi_n} = \frac{a_n - jb_n}{\sqrt{2}} = \frac{\sqrt{2}}{T} \int_0^T x(t) e^{-jn\omega_1 t} dt, \quad (9)$$

is the complex RMS (CRMS) value of the n^{th} order harmonic.

A scalar product of two periodic quantities $x(t)$ and $y(t)$ of the same period T is defined as

$$(x, y) = \frac{1}{T} \int_0^T x(t) y(t) dt. \quad (10)$$

Calculation of the scalar product with formula (10) involves integration of the product of time functions, therefore this formula specifies scalar product in a *time-domain*. Description of quantity $x(t)$ in terms of its harmonic CRMS values X_n is referred to as its presentation in a *frequency-domain*. The scalar product in this domain is equal to

$$(x, y) = \operatorname{Re} \sum_{n \in N_0} X_n Y_n^*, \quad (11)$$

where Y^* is a complex conjugate number. It means, integration in the time-domain is replaced by summation in the frequency-domain.

The RMS value of periodic quantity $x(t)$ is defined as

$$\|x\| = \sqrt{(x, x)} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt} = \sqrt{\sum_{n \in N_0} X_n^2}, \quad (12)$$

and can be calculated in the time- or in the frequency-domain. The RMS value of a sum of quantities $x(t)$ and $y(t)$ is equal to

$$\|x+y\| = \sqrt{\|x\|^2 + 2(x, y) + \|y\|^2}. \quad (13)$$

This RMS value is related to only $\|x\|$ and $\|y\|$ values

$$\|x+y\| = \sqrt{\|x\|^2 + \|y\|^2}, \quad (14)$$

only if $(x, y) = 0$. Quantities that fulfill this condition are referred to as *orthogonal quantities*. The RMS value of their sum can be calculated according to (14), i.e., without calculating their scalar product.

Quantities $x(t)$ and $y(t)$ are orthogonal in three cases, namely if

$$(i) \quad x(t) y(t) \equiv 0, \quad (15)$$

i.e., if one of them is non-zero, the second has to have zero value.

$$(ii) \quad x = \sqrt{2} X \sin(\omega t - \psi), \quad y = \sqrt{2} Y \sin(\omega t - \psi \pm \pi/2), \quad (16)$$

i.e., if they are sinusoidal functions shifted by 90 degrees.

$$(ii) \quad x = \sqrt{2} X_r \sin(r\omega_1 t - \psi), \quad y = \sqrt{2} Y_s \sin(s\omega_1 t - \phi), \quad r \neq s, \quad (17)$$

i.e., if they are harmonics of different orders.

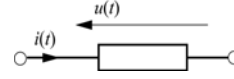


Fig. 2. Two terminal device

A voltage-current relation of a linear, time-invariant two-terminal device shown in Fig. 2 can be expressed in terms of the device admittance Y_n , specified by its conductance G_n and susceptance B_n for the n^{th} order harmonic or impedance Z_n , specified by its resistance R_n and reactance X_n , namely

$$Y_n = Y_n e^{-j\varphi_n} = G_n + jB_n, \quad Z_n = Z_n e^{j\varphi_n} = R_n + jX_n. \quad (18)$$

If the device voltage has the waveform

$$u = U_0 + \sqrt{2} \operatorname{Re} \sum_{n \in N} U_n e^{jn\omega_1 t}, \quad (19)$$

then, its current is equal to

$$i = I_0 + \sqrt{2} \operatorname{Re} \sum_{n \in N} I_n e^{jn\omega_1 t} = G_0 U_0 + \sqrt{2} \operatorname{Re} \sum_{n \in N} Y_n U_n e^{jn\omega_1 t}. \quad (20)$$

Observe, that the definition of the active power, given by eqn. (2) is identical to the definition of the scalar product (10) thus, the active power of the device is equal to

$$P = (u, i) = \operatorname{Re} \sum_{n \in N_0} U_n I_n^* = \sum_{n \in N_0} G_n U_n^2 = \sum_{n \in N_0} R_n I_n^2. \quad (21)$$

The apparent power of such a device is defined as

$$S = \|u\| \cdot \|i\| = \sqrt{\sum_{n \in N_0} U_n^2} \cdot \sqrt{\sum_{n \in N_0} I_n^2}. \quad (22)$$

Definitions of other powers and the power equation of single-phase systems will be discussed in next sections.

IV. OVERVIEW OF POWER DEFINITIONS

Discussions on definition of powers for nonsinusoidal systems started in 1892 when it was observed that power equation (5) in such systems could not be satisfied. Budeanu, and Fryze have contributed to the most remarkable and widespread approaches to power definitions for systems with nonsinusoidal voltages and currents and only these approaches will be drafted and their flaws will be discussed in this paper.

Budeanu introduced in 1927 the following definition of the reactive power, supported by the IEEE Standard Dictionary [3],

$$Q = \sum_{n \in N} U_n I_n \sin \varphi_n = \sum_{n \in N} Q_n = Q_B, \quad (23)$$

interpreted, like the reactive power in single-phase systems with sinusoidal voltages and currents, as a measure of the apparent power increase due to energy oscillations between the supply and the load. He introduced also *the distortion power*, defined as

$$D = \sqrt{S^2 - P^2 - Q^2}, \quad (24)$$

also supported by Ref. [3] and interpreted as a measure of apparent power increase due to the waveform distortion. Unfortunately, these two interpretations are erroneous. It is a conclusion from the following reasoning.

The reactive power of the n^{th} order harmonic, $Q_n = U_n I_n \sin \varphi_n$ is indeed the amplitude of the oscillating component of the instantaneous power. However, these amplitudes for different harmonics could be of the opposite sign, so that they could cancel mutually. Consequently, energy oscillations between the supply source and the load could exist even at zero Budeanu's reactive power Q .

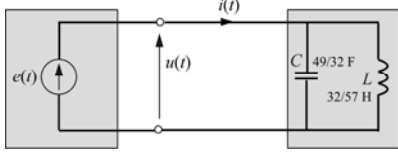


Fig. 3. Circuit with zero Budeanu's reactive power Q , but with energy oscillation

harmonic $Y_3 = j4$ S. If the supply voltage is

$$u(t) = \sqrt{2}(100 \sin \omega_1 t + 25 \sin 3\omega_1 t) \text{ V}, \quad \omega_1 = 1 \text{ rd/s},$$

then the load current is equal to

$$i(t) = \sqrt{2} [25 \sin(\omega_1 t - 90^\circ) + 100 \sin(3\omega_1 t + 90^\circ)] \text{ A}.$$

The reactive power calculated according to Budeanu's definition is

$$Q = Q_1 + Q_3 = 2500 - 2500 = 0.$$

However, in spite of zero reactive power Q , there is energy oscillation in this circuit, because instantaneous power $p(t)$, shown in Figure 4, changes its sign. When it is positive, energy flows to the load, when it is negative it flows back to the supply source.

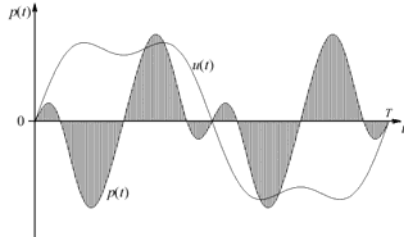


Fig. 4. Instantaneous power of the load shown in Fig. 3

Definition (24)

of distortion power

could be rearranged to the form

$$D = \sqrt{S^2 - P^2 - Q^2} = \sqrt{\frac{1}{2} \sum_{r \in N} \sum_{s \in N} U_r^2 U_s^2 / |Y_r - Y_s|^2}. \quad (25)$$

The expression under the root is a sum of terms that are equal to zero for $r = s$. If harmonics of different order r^{th} and s^{th} are present in the supply voltage, it means RMS values U_r and U_s are not equal to zero, but the load admittances are different, i.e., $Y_r \neq Y_s$, for these harmonics, then these terms have a positive value. Such terms cannot cancel mutually thus distortion power is equal to zero only if for each harmonic

$$Y_r = Y_s, \quad (26)$$

i.e., if the load admittance Y_n does not change with harmonic order. Unfortunately, this condition differs from the condition for the lack of the voltage and current mutual distortion. Indeed, the load current is not distorted with respect to the supply voltage, but only shifted, if

$$i(t) = a u(t - \tau). \quad (27)$$

If the supply voltage is periodic, then the CRMS values of the voltage and current harmonics have to satisfy relation

$$I_n = a U_n e^{-jn\tau} = Y_n U_n. \quad (28)$$

It means, a load does not cause current distortion only if the load admittance

$$Y_n = a e^{-jn\tau} = a e^{-j\varphi_n}, \quad (29)$$

has a constant magnitude, but the phase $\varphi_n = n\tau$, that is proportional to the harmonic order, n . Thus, condition for zero distortion power, (26), and condition for the lack of distortion, (29), could be fulfilled at the same time only if for a purely resistive load. Otherwise, these conditions contradict each other. It means that distortion power D has nothing in common with the waveform distortion.

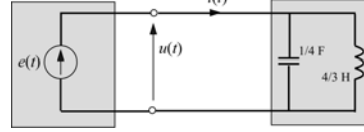


Fig. 5. Circuit with current distortion, but with non-zero distortion power D

the 1st and the 3rd order harmonics are

$$Y_1 = j\frac{1}{4} - j\frac{3}{4} = \frac{1}{2} e^{-j\frac{\pi}{2}} \text{ S},$$

$$Y_3 = j\frac{3}{4} - j\frac{1}{4} = \frac{1}{2} e^{j\frac{\pi}{2}} = \frac{1}{2} e^{-j\frac{3\pi}{2}} \text{ S}.$$

Thus, the load admittance satisfies the condition for the lack of the voltage and current mutual distortion (29) and indeed, the load current has the waveform

$$\begin{aligned} i(t) &= \sqrt{2} [50 \sin(\omega_1 t - \frac{\pi}{2}) + 15 \sin(3\omega_1 t - \frac{3\pi}{2})] = \\ &= \sqrt{2} [50 \sin \omega_1 (t - \frac{T}{4}) + 15 \sin 3\omega_1 (t - \frac{T}{4})] = \frac{1}{2} u(t - \frac{T}{4}), \end{aligned}$$

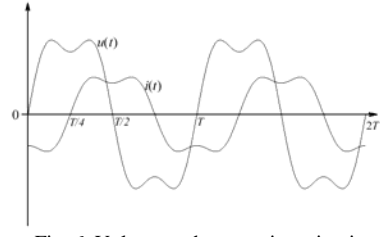


Fig. 6. Voltage and current in a circuit with non-zero distortion power, D

shown in Fig. 6, thus, it is shifted, but not distorted with respect to the voltage waveform. Distortion power D is not equal to zero, however. It could be calculated from eqn. (24) or (25). Equation (25) for only two voltage harmonics simplifies to one term

and the distortion power is equal to

$$D = U_1 U_3 / |Y_1 - Y_3| = 100 \cdot 25 \cdot / \frac{1}{2} e^{-j\frac{\pi}{2}} - \frac{1}{2} e^{j\frac{\pi}{2}} = 2.5 \text{ kVA}.$$

These two illustrations demonstrate that Budeanu's power theory provides an erroneous interpretation of power properties of single-phase circuits. Reactive and distortion powers are not associated as suggested with energy oscillation and current distortion.

The Budeanu's power theory is also useless for practical applications, in particular, for the power factor improvement. The reactive power Q in systems with sinusoidal voltages and currents is a basic quantity needed for power factor calculation and for compensator design. The reactive power Q defined by Budeanu cannot be used for such purposes. Also, no method of compensation of Budeanu's distortion power D was developed.

Illustration 3. Observe that Budeanu's reactive power Q of the load shown in Fig. 3 is equal to zero while the load current RMS value is

$$\|i\| = \sqrt{I_1^2 + I_3^2} = \sqrt{100^2 + 25^2} = 103.1 \text{ A}.$$

However, in spite of zero Budeanu's reactive power Q , the supply current could be compensated entirely by a shunt compensator shown in Fig. 7. Its admittance for the 1st and the 3rd order harmonics is $Y_{c1} = j1/4$ S and $Y_{c3} = -j4$ S. Consequently, the load with the compensator has zero admittance for these two harmonics and behaves as an open circuit.

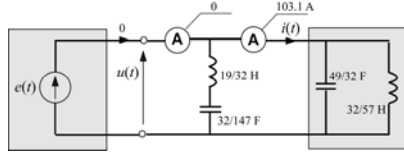


Fig. 7. Example of compensation of a load with zero Budeanu reactive power Q

Fryze introduced [4] definition of the reactive power based on the load current decomposition into the active and reactive currents in a time-domain, without any use of a harmonic concept,

$$i(t) = i_a(t) + i_{rF}(t), \quad (30)$$

with the active current defined as a current component that is proportional to the supply voltage

$$i_a(t) = G_e u(t), \quad G_e = \frac{P}{\|u\|^2}, \quad (31)$$

and of minimum RMS value needed to provide the active power P .

The active and reactive currents are orthogonal mutually, thus, their RMS values satisfy the relation

$$\|i\|^2 = \|i_a\|^2 + \|i_{rF}\|^2, \quad (32)$$

which after multiplication by the square of the voltage RMS value results in the power equation

$$S^2 = P^2 + Q_F^2, \quad (33)$$

with Fryze's definition of the reactive power

$$Q_F = \|u\| \|i_{rF}\|. \quad (34)$$

It is a very simple definition, easy for instrumentation and for evaluating the value of the useless component of the apparent power. Unfortunately, it also has major deficiencies. There is no explicit relation of this current to load properties and parameters and consequently, it is not clear how to shape the load properties to reduce this power. It is because all power phenomena in the load, other than permanent energy conversion, contribute together to this power. Thus, it has no cognitive merits.

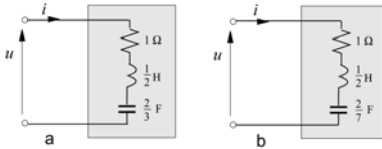


Fig. 8. Two loads that cannot be distinguished in terms of Fryze's active and reactive powers

The Fryze's power theory also does not provide fundamentals for improving power factor and power quality. The knowledge of the Fryze's reactive power Q_F does not enable us to design a compensator for reducing this power. Moreover, it is even unclear whether compensation of this power is possible or not. As illustrated below, this conclusion applies even to linear loads.

Illustration 4. Let us consider two loads shown in Fig. 8 supplied with the same distorted voltage

$$u(t) = 100\sqrt{2}(\sin\omega_1 t + \sin 3\omega_1 t) \text{ V}, \quad \omega_1 = 1 \text{ rd/s}.$$

The load in Fig. 8a has admittance for the 1st and the 3rd harmonics equal to,

$$Y_1 = 0.5 + j0.5 = 0.5\sqrt{2}e^{j45^\circ} \text{ S}, \quad Y_3 = 0.5 - j0.5 = 0.5\sqrt{2}e^{-j45^\circ} \text{ S}.$$

Thus

$$I_1 = Y_1 U_1 = 70.1 \text{ A}, \quad I_3 = Y_3 U_3 = 70.1 \text{ A},$$

and the load current RMS value is

$$\|i\| = \sqrt{I_1^2 + I_3^2} = \sqrt{70.1^2 + 70.1^2} = 100 \text{ A}.$$

Since

$$\|u\| = \sqrt{U_1^2 + U_3^2} = \sqrt{100^2 + 100^2} = 140.2 \text{ V}.$$

Thus, apparent power is equal to

$$S = \|u\| \|i\| = 140.2 \times 100 = 14.0 \text{ kVA},$$

while the active power

$$P = \sum_{n=1,3} G_n U_n^2 = 0.5 \times 100^2 + 0.5 \times 100^2 = 10 \text{ kW},$$

and the Fryze's reactive power is equal to

$$Q_F = \sqrt{S^2 - P^2} = \sqrt{14.0^2 - 10^2} = 10 \text{ kVAr}.$$

Power factor of the load is $\lambda = P/S = 0.71$. Similar calculations for the load shown in Fig. 8b result in

$$Y_1 = 0.1 + j0.3 = 0.316e^{j72^\circ} \text{ S}, \quad Y_3 = 0.9 - j0.3 = 0.95e^{-j18.4^\circ} \text{ S}.$$

$$I_1 = Y_1 U_1 = 31.6 \text{ A}, \quad I_3 = Y_3 U_3 = 95 \text{ A}, \quad \|i\| = \sqrt{I_1^2 + I_3^2} = 100 \text{ A},$$

and consequently

$$P = \sum_{n=1,3} G_n U_n^2 = 0.1 \times 100^2 + 0.9 \times 100^2 = 10 \text{ kW},$$

$$S = 14.0 \text{ kVA}, \quad Q_F = 10 \text{ kVAr}, \quad \lambda = 0.71.$$

It means that loads in Figs. 8a and 8b cannot be distinguished in terms of Fryze's powers and the power factor.

Admittance of both loads has a non-zero imaginary part, thus it can be compensated by a shunt reactive compensator. The load in Fig. 8a can be compensated for the 1st and the 3rd order harmonics, if the compensator has admittance $Y_{c1} = -j0.5$ S and $Y_{c3} = j0.5$ S. Such a compensator has a structure and parameters shown in Fig. 9a. It changes the admittance as seen from the supply source to a real value equal to $Y_1' = Y_3' = 0.5$ S. Thus, it reduces the RMS value of the supply current harmonics to

$$I_1' = Y_1' U_1 = 50 \text{ A}, \quad I_3' = Y_3' U_3 = 50 \text{ A},$$

and the supply current RMS value to

$$\|i'\| = \sqrt{(I_1')^2 + (I_3')^2} = \sqrt{50^2 + 50^2} = 70.1 \text{ A}.$$

Consequently, the apparent power is reduced to

$$S' = \|u\| \|i'\| = 140.2 \times 70.1 = 10 \text{ kVA}.$$

Thus, the Fryze's reactive power after compensation $Q_F = 0$ and power factor is improved to $\lambda = 1$.

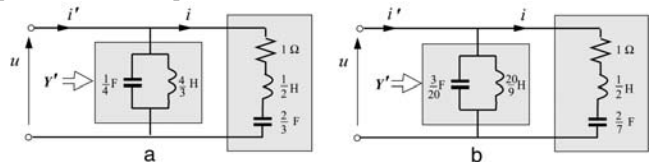


Fig. 9. Loads shown in Fig. 8a and b with compensators of reactive power

Susceptance of the load in Fig. 8b can be compensated entirely, if the compensator has admittance for harmonics $Y_{c1} = -j0.3$ S and $Y_{c3} = j0.3$ S. Parameters of such a compensator are shown in Fig. 9b. It changes admittance for harmonics to $Y_1' = 0.1$ S and $Y_3' = 0.9$ S and the RMS value of the current harmonics to

$$I_1' = Y_1' U_1 = 10 \text{ A}, \quad I_3' = Y_3' U_3 = 90 \text{ A}.$$

Hence, $\|i'\| = 90.5 \text{ A}$. Thus, after compensation

$$S' = 12.7 \text{ kVA}, \quad Q_F = \sqrt{S^2 - P^2} = \sqrt{12.7^2 - 10^2} = 8 \text{ kVAr}.$$

It means, that the reactive power Q_F cannot be compensated entirely by such a compensator. The power factor is improved only to 0.78.

It is interesting to observe that admittance for harmonics of the compensated load, Y' , in Fig. 9b is a real number, thus there is no phase shift between the voltage and current harmonics. In spite of

this, there is still a non-zero Fryze's reactive power and power factor lower than unity. The loads in Fig. 8a and b do not differ in terms of Fryze powers, but only one of them could be compensated to unity power factor. Moreover, compensators shown in Fig. 9 cannot be designed based on the Fryze's power theory because it is formulated in the time-domain, while frequency-domain is needed for a reactive compensator design.

Illustration 4 demonstrates that the Fryze's approach does not provide fundamentals for power factor improvement with reactive compensators, such as those shown in Fig. 9. However, there are opinions that this theory provides fundamentals for improving power factor with switching compensators, known under a common name of active filters. These compensators are power electronics devices, connected as shown in Fig. 10, and controlled in such a way

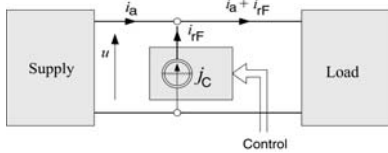


Fig. 10. Circuit with reactive current compensator

that can compensate the Fryze's reactive current, i_{rF} , thus, leaving only the useful, active current i_a in the supply current. Symbol j_c in Fig. 10 denotes controlled current source that injects the reactive current into the system.

Although there are opinions that reduction of the supply current to its active component is the utmost goal of compensation, there are some doubts. Let us suppose that the supply voltage is distorted and an RL load is passive and linear. Impedance of such loads increases with frequency thus the load current is less distorted than the supply voltage. The supply current would be more distorted after such compensation, because active current is proportional to the supply voltage. It could not be acceptable from the point of view of power quality improvement. More doubts occur when the load is nonlinear and generates current harmonics, as shown in the following Illustration.

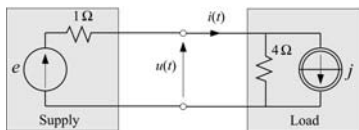


Fig. 11. Circuit with harmonic generating load

Illustration 5. Let us consider a circuit shown in Fig. 11, with sinusoidal supply voltage

$$e = 100\sqrt{2} \sin \omega_1 t \text{ V,}$$

and nonlinear loads, that generates the 3rd order current harmonic

$$j = 50\sqrt{2} \sin 3\omega_1 t \text{ A.}$$

The voltage, current and the active power at the load terminals are

$$u = 80\sqrt{2} \sin \omega_1 t - 40\sqrt{2} \sin 3\omega_1 t \text{ V,}$$

$$i = 20\sqrt{2} \sin \omega_1 t + 40\sqrt{2} \sin 3\omega_1 t \text{ A.}$$

$$P = \frac{1}{T} \int_0^T u i dt = \sum_{n=1,3} U_n I_n \cos \varphi_n = 1600 - 1600 = 0,$$

and consequently, the active current

$$i_a = \frac{P}{\|u\|^2} u \equiv 0.$$

It means, that only the Fryze's reactive current,

$$i_{rF} = i = 20\sqrt{2} \sin \omega_1 t + 40\sqrt{2} \sin 3\omega_1 t = i_1 + i_3,$$

is present in the load current. However, it would be a wrong conclusion that just this current should be compensated. Power factor in this circuit declines because the 3rd order current harmonic generated in the load j reduces active power P and energy delivered to the load. This harmonic can be considered as a source of energy flow

from the load back to the supply source. The compensator should inject the generated current j into the system, but not the Fryze's reactive current i_{rF} , to eliminate this phenomenon and to improve power factor.

The phenomenon of energy flow from a load back to the supply at harmonic frequencies is not, of course, so visible in real systems as in the circuit shown in Fig. 11. Parameters in this circuit were purposely selected to obtain zero active power and a clear picture of this phenomenon. Nonetheless, current harmonics generated in the load due to its non-linearity contribute usually to reduction of the active power at load terminals.

The Fryze's approach to identification of power properties could be summarized as follows. The active current, i_a , i.e., the minimum current that has to be present in the supply current at a specified active power P of the load, is an important and useful concept. However, Fryze's reactive current, i_{rF} , and reactive power, Q_F , cannot be interpreted in terms of any phenomenon in the load. Moreover, a design of reactive compensators is not possible with this approach, while active compensation, based on injection of the Fryze's reactive current, remains at least controversial.

V. CURRENT'S PHYSICAL COMPONENTS

The Currents' Physical Components (CPC) based power theory explains power properties of single- and three-phase systems with linear, time-invariant loads, i.e., loads that do not generate current harmonics, and with harmonics generating loads (HGL). Unfortunately, the whole theory cannot be presented in a single paper, even written in two parts.



Fig. 12. Load and its equivalent with respect to active power P

The CPC theory has been developed, from the original concept presented in [5], step by step, starting with the same circuits that were the main object of the Budeanu and Fryze's attempts, i.e., single-phase circuits with linear, time-invariant loads and nonsinusoidal supply voltage, as shown in Fig. 12a. Association of current components with physical phenomena in the load was the main imperative at the development of the CPC based power theory.

It starts from Fryze's separation of the active current and this current is preserved in the CPC theory. However, unlike Fryze's theory, the CPC theory recognizes the importance of harmonics and frequency properties of a circuit for power properties of electrical loads and power definitions. Therefore, the supply voltage and the load current should be expressed in terms of harmonics

$$u(t) = U_0 + \sqrt{2} \operatorname{Re} \sum_{n \in N} U_n e^{jn\omega_1 t}, \quad (35)$$

$$i(t) = Y_0 U_0 + \sqrt{2} \operatorname{Re} \sum_{n \in N} Y_n U_n e^{jn\omega_1 t}. \quad (36)$$

Hence, the active current, $i_a(t)$, i.e., the current of a resistive equivalent load, shown in Fig. 12b, that at the same voltage $u(t)$ has the same active power P as the original load, can be expressed as

$$i_a(t) = G_e u(t) = \sqrt{2} \operatorname{Re} \sum_{n \in N_0} G_e U_n e^{jn\omega_1 t}. \quad (37)$$

The remaining component of the load current, after subtracting the active current, is equal to

$$\begin{aligned} i(t) - i_a(t) &= (Y_0 - G_e)U_0 + \sqrt{2} \operatorname{Re} \sum_{n \in N} (Y_n - G_e)U_n e^{jn\omega_1 t} = \\ &= (Y_0 - G_e)U_0 + \sqrt{2} \operatorname{Re} \sum_{n \in N} (G_n + jB_n - G_e)U_n e^{jn\omega_1 t}. \end{aligned} \quad (38)$$

The active current $i_a(t)$ is a minimum current of a load that at voltage $u(t)$ has active power P , therefore, this remaining current component does not contribute to permanent energy transmission. It can be decomposed into the following components

$$(G_0 - G_e)U_0 + \sqrt{2} \operatorname{Re} \sum_{n \in N} (G_n - G_e)U_n e^{jn\omega_1 t} = i_s(t), \quad (39)$$

$$\sqrt{2} \operatorname{Re} \sum_{n \in N} jB_n U_n e^{jn\omega_1 t} = i_r(t). \quad (40)$$

The first component, $i_s(t)$, occurs in the load current only if the load conductance G_n changes with harmonic order n . In such a case conductance G_n is smaller for some harmonics than equivalent conductance G_e and it is higher for others. In general, this conductance is scattered around G_e value. Therefore, current $i_s(t)$ is referred to as a *scattered current*. Such a change of the conductance is not an “exotic” property, but a common property of linear, passive, time-invariant loads.

Illustration 6. Let us calculate conductance G_n for a few harmonic orders of a common RL load shown in Fig. 13, assuming that the load has parameters $R = 1\Omega$ and $\omega_1 L = 1\Omega$. The conductance of such a load for harmonic frequencies is equal to

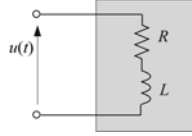


Fig. 13. RL load

$$G_n = \operatorname{Im}\{Y_n\} = \operatorname{Im} \frac{1}{Z_n} = \operatorname{Im} \frac{1}{R + jn\omega_1 L} = \frac{R}{R^2 + (n\omega_1 L)^2}.$$

This conductance for parameters given in the illustration is equal to:

$$G_0 = 1 \text{ S}, \quad G_1 = 0.5 \text{ S}, \quad G_2 = 0.2 \text{ S}, \quad G_3 = 0.1 \text{ S}, \quad G_4 = 0.06 \text{ S},$$

The component, $i_r(t)$, specified with formula (40), occurs in the load current only if the load susceptance B_n for at least one harmonic is not equal to zero, i.e., if at least one current harmonic is shifted with respect to the supply voltage harmonic. A reactive power

$$Q_n = U_n I_n \sin \varphi_n = -B_n U_n^2, \quad (41)$$

is associated with such a harmonic. Therefore, current $i_r(t)$ is referred to as a *reactive current*. The formula (38), with definitions (39) and (40), can be written in the form

$$i(t) = i_a(t) + i_s(t) + i_r(t). \quad (42)$$

It means that the load current can be decomposed into three components associated with three distinctive physical phenomena in the load: (i) permanent energy conversion; (ii) change of the load conductance G_n with harmonic order n and (iii) phase shift between the voltage and current harmonics. Therefore, these currents are called *current's physical components*. Their RMS value is equal to

$$\|i_a\| = G_a \|u\| = \frac{P}{\|u\|}, \quad (43)$$

$$\|i_s\| = \sqrt{\sum_{n \in N_0} (G_n - G_e)^2 U_n^2}, \quad (44)$$

$$\|i_r\| = \sqrt{\sum_{n \in N} B_n^2 U_n^2} = \sqrt{\sum_{n \in N} \left(\frac{Q_n}{U_n}\right)^2}. \quad (45)$$

The possibility of calculating the load current RMS value having RMS values of currents i_a , i_s and i_r depends on orthogonality of these currents. According to formula (40) harmonics of the reactive current are shifted by 90° with respect to harmonics of the active and the scattered currents thus, due to property (16), these currents are mutually orthogonal, i.e., their scalar product

$$(i_r, i_a) = 0, \text{ and } (i_r, i_s) = 0. \quad (46)$$

However, orthogonality of the scattered and active currents is not obvious. Let us calculate with formula (11) their scalar product:

$$\begin{aligned} (i_s, i_a) &= \operatorname{Re} \sum_{n \in N_0} (G_n - G_e)U_n \cdot G_e U_n^* = G_e \sum_{n \in N_0} (G_n - G_e)U_n^2 = \\ &= G_e \left(\sum_{n \in N_0} G_n U_n^2 - G_e \sum_{n \in N_0} U_n^2 \right) = G_e (P - G_e \|u\|^2) = 0. \end{aligned} \quad (47)$$

Thus, all current's physical components are mutually orthogonal and consequently

$$\|i\|^2 = \|i_a\|^2 + \|i_s\|^2 + \|i_r\|^2. \quad (48)$$

This relationship could be visualized geometrically with a rectangular box shown in Fig. 14. If the length of edges is proportional to RMS values of current's physical components, then the box diagonal is proportional to the load current RMS value, $\|i\|$.

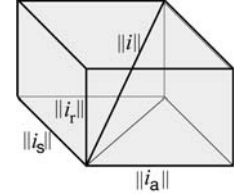


Fig. 14. Rectangular box of RMS values of current's physical components

Multiplying eqn. (48) by the square of the voltage RMS value, the power equation is obtained

$$S^2 = P^2 + D_s^2 + Q^2, \quad (49)$$

where

$$D_s = \|u\| \|i_s\| \quad \text{and} \quad Q = \|u\| \|i_r\|, \quad (50)$$

are a *scattered power* and a *reactive power*, respectively.

Current decomposition (42) reveals an earlier unknown phenomenon of increase of the current RMS value and apparent power S due to a change of the load conductance G_n with harmonic order.

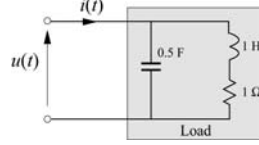


Fig. 15. Example of a load

Illustration 7. Let us calculate the RMS value of current's physical components of the load shown in Fig. 15 if the supply voltage is

$$u = 50 + \sqrt{2} \operatorname{Re}\{100 e^{j\omega_1 t} + 20 e^{j5\omega_1 t}\} \text{ V}, \quad \omega_1 = 1 \text{ rd/s}, \quad \|u\| = 113.58 \text{ V}.$$

The set of voltage harmonics is $N_0 = \{0, 1, 5\}$ and the load admittance for harmonic orders from this set is equal to

$$Y_0 = 1 \text{ S}, \quad Y_1 = 0.5 \text{ S}, \quad Y_5 = 0.04 + j2.31 \text{ S},$$

and consequently, the load current is

$$i = 50 + \sqrt{2} \operatorname{Re}\{50 e^{j\omega_1 t} + 46.2 e^{j89^\circ} e^{j5\omega_1 t}\} \text{ A},$$

and its RMS value is equal to

$$\|i\| = \sqrt{I_0^2 + I_1^2 + I_5^2} = \sqrt{50^2 + 50^2 + 46.2^2} = 84.47 \text{ A}.$$

To decompose the current into physical components and calculate their RMS value, the active power and equivalent conductance of the load have to be calculated. The active power is

$$P = \sum_{n \in 0,1,5} G_n U_n^2 = 7.516 \text{ kW},$$

so that, the equivalent conductance of the load has the value

$$G_e = \frac{P}{\|u\|^2} = \frac{7516}{113.58^2} = 0.5826 \text{ S}.$$

The RMS value of the load current physical components is equal to

$$\|i_a\| = G_a \|u\| = 66.17 \text{ A},$$

$$\|i_s\| = \sqrt{\sum_{n \in 0,1,5} (G_n - G_e)^2 U_n^2} = 24.93 \text{ A},$$

$$\|i_r\| = \sqrt{\sum_{n \in 1,5} B_n^2 U_n^2} = 46.2 \text{ A}.$$

It is easy to verify that calculated RMS values satisfy relationship (48) and indeed

$$\|i\| = \sqrt{\|i_a\|^2 + \|i_s\|^2 + \|i_r\|^2} = \sqrt{66.17^2 + 24.93^2 + 46.2^2} = 84.47 \text{ A}.$$

Thus, decomposition (42) is strictly satisfied even for very high distortion. This conclusion is trivial to some degree, because decomposition (42) was not founded on any approximations.

The scattered and reactive powers of the load shown in Fig 15 are equal to

$$D_s = 2.83 \text{ kVA} \quad \text{and} \quad Q = 5.25 \text{ kVAr}.$$

The scattered power D_s at a distortion level more realistic than that assumed in Illustration 7, is usually much smaller than the active power. However, the power equation of loads with nonsinusoidal supply voltage cannot be fulfilled without the scattered power.

VI. POWER FACTOR IMPROVEMENT

Compensation in systems with sinusoidal voltages and currents has a clear meaning. Reduction of energy loss at its delivery (*i*) is the major objective of such compensation. At the same time, it reduces the voltage drop and investment cost of equipment for energy delivery. The objective of compensation in systems with non-sinusoidal voltages and currents could be different. It could be the same as previously (*i*), or it could be focused on preventing customer (*ii*) or power utility (*iii*) equipment from disturbances caused by harmonics, by reduction of waveform distortion. These different objectives may require different compensating or filtering equipment or some sort of trade off between different compensation goals. Objectives (*ii*) and (*iii*) are mainly a subject of studies on power quality and go beyond the subject of this paper, focused on power factor rather than on power quality improvement.

Power factor, i.e., the ratio of the active and apparent powers, can be expressed as

$$\lambda = \frac{P}{S} = \frac{\|i_a\|}{\sqrt{\|i_a\|^2 + \|i_s\|^2 + \|i_r\|^2}}, \quad (51)$$

thus, both scattered and reactive currents contribute to power factor degradation and could be improved by reduction of these currents.

There are two kinds of compensators. The most common are reactive devices, such as capacitor banks or filters. Less common are switching compensators, known as active power filters. Some hybrid structures composed of reactive and switching compensators are possible as well. Reactive compensators are usually shunt devices, i.e., connected as shown in Fig. 16.

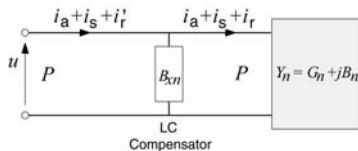


Fig.16. Shunt reactive compensator

An ideal, lossless reactive compensator has zero conductance for all harmonic frequencies. Moreover, if the load is supplied with a fixed voltage, then the active power and equivalent conductance G_e are not affected by the compensator. It means, such a compensator is not capable of compensating the scattered current. It modifies only the reactive current RMS to the value

$$\|i_r\| = \sqrt{\sum_{n \in N} (B_n + B_{cn})^2 U_n^2}. \quad (52)$$

In particular, if for each harmonic of the order from set N ,

$$B_{cn} = -B_n, \quad (53)$$

then the reactive current is entirely compensated and power factor reaches its maximum, possible at shunt reactive compensation:

$$\lambda_{\max} = \frac{\|i_a\|}{\sqrt{\|i_a\|^2 + \|i_s\|^2}}. \quad (54)$$

Examples of such compensation were given in Illustrations 3 and 4. Compensators in these examples satisfy condition (53). Relation (54) also explains why it was not possible to compensate the load in Fig. 8b to unity power factor while this was possible for the load in Fig. 8a. The load in Fig. 8a has the same conductance for the fundamental and for the 3rd order harmonic, thus, it has zero scattered current. This conductance changes for the load in Fig 8b, thus, it has a non-zero scattered current which cannot be compensated by any shunt reactive compensator.

Scattered and active currents and powers are not affected by a shunt reactive compensator only if such a compensator does not change the load voltage, but this is possible only when the supply bus has an infinite power. Otherwise they change with the voltage harmonics RMS value, U_n , change due to a voltage drop on the supply source impedance. This change could be particularly high when a resonance between the compensator and the supply source occurs in the system. Usually it is the resonance between capacitance of the compensator and inductance of the supply source. It could lead to a disastrous increase of waveform distortion in systems with capacitor banks, designed under the assumption that voltages and currents are sinusoidal, while there were some distribution voltage harmonics or/and current harmonics generated by the load, but neglected during the compensator design.

If a capacitor compensates the reactive power Q of the load entirely, its capacitance has to be equal to

$$C = \frac{Q}{\omega_1 U^2} = \frac{P}{\omega_1 U^2} \tan \varphi, \quad (55)$$

and this capacitor could resonate with the supply inductance L_s .

The impedance of RL loads increases with frequency, therefore, a circuit with such a load supplied from an inductive source

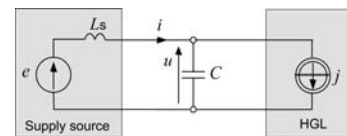


Fig.17. Equivalent circuit of compensated load for $\omega \gg \omega_1$

and compensated by a capacitor, behaves in the range of frequency $\omega \gg \omega_1$, as the circuit shown in Fig. 17. If the load generates harmonics, they occur in this equivalent circuit as a current source, j .

Dependence of the load voltage on frequency in the circuit with a voltage resonance can be expressed by the Load-to-Distribution Voltage (L/DV) transmittance,

$$A(j\omega) = \frac{U(j\omega)}{E(j\omega)} = \frac{1}{1 - \omega^2 L_s C} = \frac{1}{1 - (\omega/\omega_r)^2}, \quad (56)$$

where

$$\omega_r = 1/\sqrt{L_s C}, \quad (57)$$

is the resonant frequency. Inductance L_s depends on the short circuit power of the bus where the capacitor bank is installed. If the supply source reactance is much higher than the source resistance, then the short circuit power S_{sc} is approximately equal to

$$S_{sc} \approx \frac{E^2}{\omega_1 L_s}. \quad (58)$$

Hence, the resonant frequency could be expressed as

$$\omega_r = \frac{U}{E} \sqrt{\frac{S_{sc}}{P \tan \varphi}} \omega_1 \approx \sqrt{\frac{S_{sc}}{P \tan \varphi}} \omega_1, \quad (59)$$

thus, the resonant frequency ω_r is determined entirely by the short circuit power, load active power and its power factor, $\lambda = \cos \varphi$.

Observe that the magnitude of the L/DV transmittance $A(j\omega)$ is higher than one for frequency ω such $(\omega/\omega_r)^2 < 2$. It means that all distribution voltage harmonics of the frequency below $\sqrt{2} \omega_r$ are amplified by a compensator. This amplification increases to infinity

at resonant frequency. It is limited in real systems by resistance of the supply source and the load, in particular, purely resistive ones.

Illustration 8. The plot of the magnitude of the L/DV transmittance for a system with the ratio $S_{sc}/P = 50$, the reactance to resistance of the supply $X_s/R_s = 5$, and power factor $\lambda = 0.71$ is shown in Fig. 18.

All harmonics of the distribution voltage up to 10th order are amplified in such a situation. The highest amplification, approximately twenty times, is for the 7th order harmonic.

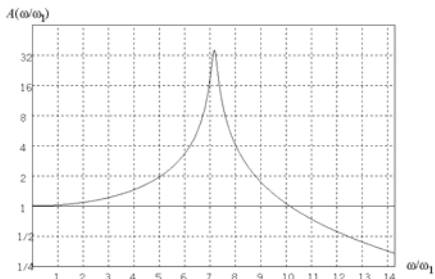


Fig. 18. Plot of L/DV transmittance versus relative frequency ω/ω_1

is fulfilled for each voltage harmonic, usually requires compensators built of a high number of reactive components. In such a situation it could be reasonable to give up a whole compensation of the reactive current for its minimization by a compensator with complexity limited to two reactive components. Such a compensator should have a structure and parameters that make resonance for harmonic frequency impossible. Such requirements could be fulfilled by the LC compensator shown in Fig. 19.

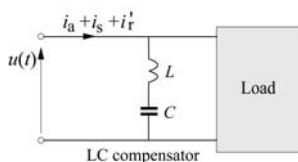


Fig. 19. Series LC compensator

If series inductance L is selected such that frequency range of amplification is below the 2nd order harmonic, i.e.,

$$\frac{1}{\sqrt{(L_s + L)C}} < \sqrt{2} \omega_1, \quad (60)$$

then the compensator does not increase waveform distortion and it can be assumed that voltage harmonics are not affected by the compensator. Since the LC branch susceptance is equal to

$$B_{xn} = \text{Im}\{Y_{xn}\} = \frac{n\omega_1 C}{1 - n^2 \omega_1^2 LC}, \quad (61)$$

the compensator changes the RMS value of the reactive current, according to formula (52), to

$$\|i_r'\| = \sqrt{\sum_{n \in N} \left(B_n + \frac{n\omega_1 C}{1 - n^2 \omega_1^2 LC} \right)^2 U_n^2}. \quad (62)$$

It has minimum when a derivative of this RMS value with respect to capacitance C is equal to zero, i.e., $d\|i_r'\|/dC = 0$. It results in formula

$$\omega_1 C \sum_{n \in N} \frac{n^2 B_n U_n^2}{(1 - n^2 \omega_1^2 LC)^3} + \sum_{n \in N} \frac{n B_n U_n^2}{(1 - n^2 \omega_1^2 LC)^2} = 0. \quad (63)$$

Unfortunately, capacitance C is not in an explicit form in this formula. Therefore, numerical methods are needed for capacitance calculation. In particular, it could be found using iterative formula

$$C_{k+1} = - \frac{\sum_{n \in N} \frac{n B_n U_n^2}{(1 - n^2 \omega_1^2 LC_k)^2}}{\omega_1 \sum_{n \in N} \frac{n^2 B_n U_n^2}{(1 - n^2 \omega_1^2 LC_k)^3}} \rightarrow C. \quad (64)$$

This iteration can start from the capacitance value given by (55), while any inductance that satisfies condition (60) could be selected. After capacitance C is calculated, condition (60) should be verified again. If it is not satisfied, a different inductance should be selected.

VII. CONCLUSIONS

The paper demonstrates that it is possible to associate power related phenomena in single-phase linear loads under nonsinusoidal conditions with distinctive components of the load current and define non-active powers associated with these components. Such powers characterize power properties of the load much better than the reactive and distortion powers introduced by Budeanu and supported by the IEEE Standard Dictionary of Electrical and Electronics Terms [3]. The same is the reactive power definition suggested by Fryze. Moreover, both Fryze's reactive power, as well as powers supported by [3], do not provide any fundamentals for compensator design while the CPC based power definitions provide a clear base for the power factor improvement in single-phase circuits with nonsinusoidal supply voltage.

The paper demonstrates that power phenomena in single-phase linear loads with nonsinusoidal supply voltage are not complex. It seems, that some widespread but erroneous definitions of powers and their interpretations are the main obstacle for understanding power properties of such loads.

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BIOGRAPHY

Leszek S. Czarnecki, Alfredo M. Lopez Distinguished Professor, received the M.Sc. and Ph.D. degrees in electrical engineering and Habil. Ph.D. degree from the Silesian University of Technology, Poland, in 1963, 1969 and 1984, respectively, where he was employed as an Assistant Professor. Beginning in 1984 he worked for two years at the Power Engineering Section, Division of Electrical Engineering, National Research Council of Canada as a Research Officer. In 1987 he joined the Electrical Engineering Dept. at Zielona Gora University of Technology. In 1989 Dr. Czarnecki

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Dr. Czarnecki climbed the main ridge of Ruwenzori, Kilimanjaro and Mt. Kenya in Africa, Lhotse in Himalaya to 8350m, Cordillera Huayashi in Andes and he climbed solo McKinley in Alaska.

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