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Currents' Physical Components (CPC) In Circuits with Nonsinusoidal Voltages and Currents

Part 2: Three-Phase Three-Wire Linear Circuits

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Abstract: - **The paper summarizes the present state of discussions on power phenomena, power definitions and compensation in three-phase three-wire linear circuits with nonsinusoidal voltages and currents. In particular, definitions of the arithmetic and geometric apparent powers are discussed. It is shown that these two powers result in an erroneous value of power factor of unbalanced loads even if voltages and currents are sinusoidal. A reasoning that leads to a right selection of the apparent power definition is presented as well. The paper also presents the concept of the three-phase current decomposition into Currents' Physical Components (CPC) and its application for power definitions and compensation in three-phase circuits.**

Index Terms, - power definitions, power phenomena, apparent, reactive, scattered, unbalanced currents or powers.

1. INTRODUCTION

There were numerous attempts to explain and describe power properties of three-phase systems with nonsinusoidal voltages and currents using Budeanu's or the Fryze's approach to power definitions. Unfortunately, even apparently successful results of such an extension convey all misconceptions and deficiencies of these two approaches, discussed in Part 1, with such an extension. Also, an extension from sinusoidal to nonsinusoidal condition requires that power properties of three-phase systems in sinusoidal conditions are described properly. Unfortunately, the commonly used power equation of three-phase systems

$$S^2 = P^2 + Q^2, \tag{1}$$

provides a true value of the apparent power and power factor only if the load is balanced. Some misconceptions with respect to definition of the apparent power are demonstrated in the following Section.

II. DOUBTS WITH RESPECT TO APPARENT POWER DEFINITIONS

The active and reactive powers in three-phase, three-wire systems, shown in Fig. 1, with sinusoidal supply voltage and sinusoidal line currents are defined as follows

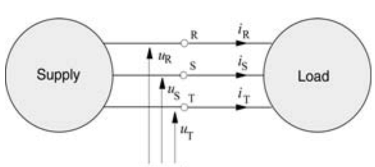

Fig. 1. Three-phase, three-wire system

$$P = \frac{1}{T}\int_0^T (u_R i_R + u_S i_S + u_T i_T)\,dt = \sum_{f=R,S,T} U_f I_f \cos\varphi_f \,, \tag{2}$$

$$Q = \sum_{f=R,S,T} U_f I_f \sin\varphi_f \,. \tag{3}$$

However, there are two different definitions of the apparent power in the IEEE Standard Dictionary of Electrical and Electronics Terms [1] namely

$$S = U_R I_R + U_S I_S + U_T I_T = S_A \,, \tag{4}$$

$$S = \sqrt{P^2 + Q^2} = S_G \,, \tag{5}$$

referred to as arithmetic and geometric apparent powers, respectively. There is a third definition, suggested in Ref. [2],

$$S = \sqrt{U_R^2 + U_S^2 + U_T^2}\ \sqrt{I_R^2 + I_S^2 + I_T^2} = S_B \,, \tag{6}$$

but not referred to in Standard [1]. These three definitions result in the same value of apparent power S, only if the line currents are symmetrical. Otherwise these values are different.

Illustration 1. An electric train loads only a sin-gle phase of a three-phase system. Let us sup-pose that such a load and its supply is simplified to the structure with a loss-less transformer and parameters shown in Fig. 2.

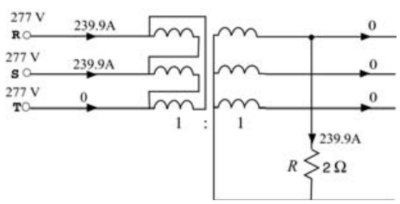

Fig. 2. Example of three-phase supply of a single-phase load

Assuming that the line-to-ground voltage RMS value is 277 V and transformer turn ratio is 1:1, the active power at the supply terminals is P = 115.1 kW, while the apparent power, depending on the definition, is

$$S_A = 132.9 \text{ kVA}, \quad S_G = 115.1 \text{ kVA}, \quad S_B = 162.8 \text{ kVA}.$$

Consequently, power factor depends on the selected definition of the apparent power and is equal to, respectively,

$$\lambda_A = 0.86, \qquad \lambda_G = 1, \qquad \lambda_B = 0.71.$$

Observe, that the reactive power in the system considered is $Q = 0$, thus, power equation (1) is satisfied only for the geometric definition of the apparent power. However, the question arises: *is the power factor at supply terminals indeed equal to* $\lambda = \lambda_G = 1$? It will be shown in Section V that this would be a wrong conclusion.

III. SYMBOLS AND FUNDAMENTALS

Symbols introduced in the first part of this paper for single-phase systems with nonsinusoidal voltages and currents could be generalized to three-phase, three-wire systems, shown in Fig. 1, as follows.

Three-phase, sinusoidal line-to-ground voltages u_R, u_S and u_T and line currents i_R, i_S and i_T, denoted generally by x_R, x_S and x_T, could be arranged into three-phase vectors:

$$\boldsymbol{x} = \begin{bmatrix} x_R \\ x_S \\ x_T \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{X}_R \\ \boldsymbol{X}_S \\ \boldsymbol{X}_T \end{bmatrix} e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\, X\, e^{j\omega_1 t} \,. \tag{7}$$

If these quantities are nonsinusoidal, but periodic of the period T, and three-phase vectors of harmonics are denoted by $\boldsymbol{x}_n$, then these quantities could be expressed as

$$\boldsymbol{x} = \sum_{n\in N} \boldsymbol{x}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} X_n\, e^{jn\omega_1 t} \ . \tag{8}$$

Observe, that it was assumed that the voltages and currents in the considered systems do not contain a DC component. If needed it could be included.

The scalar product of three-phase periodic quantities $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ of the same period T is defined as

$$(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\,\boldsymbol{y}(t)\,dt = \frac{1}{T}\int_0^T (x_{\mathrm{R}}\,y_{\mathrm{R}} + x_{\mathrm{S}}\,y_{\mathrm{S}} + x_{\mathrm{T}}\,y_{\mathrm{T}})\,dt\,, \tag{9}$$

where superscript T denotes a transposed matrix $\boldsymbol{x}(t)$. This scalar product can be calculated in the frequency-domain as

$$(\boldsymbol{x},\boldsymbol{y}) = \mathrm{Re}\sum_{n\in N} X_n^{\mathrm{T}}\, Y_n^{*} \ . \tag{10}$$

The asterisk denotes a conjugate number. Observe, that definition of the scalar product (10) is identical with the formula for calculating the active power in three-phase systems, specified with eqn. (2), i.e.,

$$P = (\boldsymbol{u},\boldsymbol{i}) = \mathrm{Re}\sum_{n\in N} U_n^{\mathrm{T}}\, I_n^{*} \ . \tag{11}$$

The RMS value of a three-phase vector is defined as

$$||\boldsymbol{x}|| = \sqrt{(\boldsymbol{x},\boldsymbol{x})} = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\cdot\boldsymbol{x}(t)\,dt} = \sqrt{\sum_{n\in N} X_n^{\mathrm{T}}\cdot X_n^{*}} \ . \tag{12}$$

The last formula could be rearranged to the form

$$||\boldsymbol{x}|| = \sqrt{\sum_{n\in N} X_n^{\mathrm{T}}\cdot X_n^{*}} = \sqrt{\sum_{n\in N}[\boldsymbol{X}_{\mathrm{R}n},\boldsymbol{X}_{\mathrm{S}n},\boldsymbol{X}_{\mathrm{T}n}]\cdot\begin{bmatrix}\boldsymbol{X}_{\mathrm{R}n}^{*}\\ \boldsymbol{X}_{\mathrm{S}n}^{*}\\ \boldsymbol{X}_{\mathrm{T}n}^{*}\end{bmatrix}} = $$
$$= \sqrt{\sum_{n\in N}(X_{\mathrm{R}n}^2 + X_{\mathrm{S}n}^2 + X_{\mathrm{T}n}^2)} = \sqrt{||x_{\mathrm{R}}||^2 + ||x_{\mathrm{S}}||^2 + ||x_{\mathrm{T}}||^2}\,, \tag{13}$$

thus, the RMS value, or a three-phase quantity, is equal to the root of the sum of squares of the RMS value of phase quantities.

Formulae (12) and (13) provide [4] a mathematical definition of the RMS value of a three-phase quantity. *A crucial question is: has such a definition any physical and technical meaning?* To answer this question, let us calculate the active power of a three-phase device, shown in Fig. 3a, with nonsinusoidal currents i_{R}, i_{S} and i_{T}.

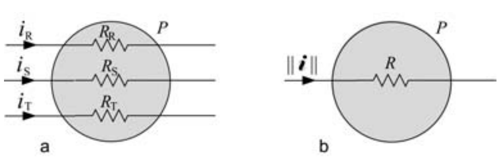

Fig. 3. Three-phase device (a) and its equivalent (b)

Assuming that line resistances R_{R}, R_{S} and R_{T}, do not change with harmonic frequency, the active power of such a device is equal to

$$P = R_{\mathrm{R}}\,||i_{\mathrm{R}}||^2 + R_{\mathrm{S}}\,||i_{\mathrm{S}}||^2 + R_{\mathrm{T}}\,||i_{\mathrm{T}}||^2 \ . \tag{14}$$

Transmitting equipment is usually built in such a way that the phase symmetry is preserved to a high degree. Therefore, it can be assumed that $R_{\mathrm{R}} = R_{\mathrm{S}} = R_{\mathrm{T}} = R$. In such a case

$$P = R\,(||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2) = R||\boldsymbol{i}||^2 \ . \tag{15}$$

Thus, any three-phase symmetrical device with phase resistance R and asymmetrical currents i_{R}, i_{S} and i_{T} is equivalent as to the active power P to a single-phase device, shown in Fig. 3b, with the same resistance and the current RMS value equal to

$$||\boldsymbol{i}|| = \sqrt{||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2} \ . \tag{16}$$

This reasoning provides a clear physical meaning for the RMS value of three-phase vectors. Moreover, such a three-phase device should be designed with respect to dissipation of the power that is proportional to the square of $||\boldsymbol{i}||$ value. Thus, this value has a clear technical meaning. Observe, that the RMS value $||\boldsymbol{u}||$ of three-phase voltage vector, $\boldsymbol{u}$, has the same physical and technical meanings.

IV. THE INSTANTANEOUS REACTIVE POWER p-q THEORY

Since the main approaches to power theory, as suggested by Budeanu and Fryze, were not capable, as demonstrated in Part 1, of describing power properties and providing fundamentals for compensation of single-phase systems, they are useless for the same purposes in three-phase systems.

In such a situation, a new concept, known as the Instantaneous Reactive Power (IRP) p-q Theory, has been developed by Akagi, Kanazawa and Nabae, [3]. It was to provide mathematical fundamentals for the control of PWM inverter based switching compensators, commonly known as "active power filters". According to Authors [3], the development of the Instantaneous Reactive Power p-q Theory was a response to "*...the demand to instantaneously compensate the reactive power"* and the adjective *"instantaneous"* suggested that this theory could instantaneously provide information needed for a compensator control.

The IRP p-q Theory is commonly used not only as a fundamental for compensator control, but also as a fundamental for description and interpretation of power properties of three-phase systems. Unfortunately, although the IRP p-q Theory seems to be useful for compensation, it misinterprets power properties of such systems. Detailed analysis of the IRP p-q Theory and conclusions from that analysis is presented in paper [5].

V. SELECTION OF APPARENT POWER DEFINITION

Before any attempt at clarifying power properties of three-phase systems with nonsinusoidal voltages and currents, an acceptable definition of the apparent power should be selected.

Similarly as in single-phase systems, the apparent power in three-phase systems is not a physical, but a conventional quantity. However, apparent power definitions compiled in Section II, when applied to unbalanced loads result, as it was demonstrated in Illustration 1, in different numerical values of the apparent power S and consequently, in different values of the power factor.

Various objectives could be taken into account when a convention for the apparent power definition is selected. One of them, and probably the most important, is such a definition that results in a right value of power factor, λ, i.e., the value that characterizes correctly the power loss at energy delivery. In such a case, the issue of selection of the apparent power definition boils down to the question: *which value λ_{A}, λ_{G} or λ_{B} provides the true value of the power factor of an unbalanced load, if this power factor is to characterize power loss on energy delivery?*

The answer to this question was based on the following reasoning. At first, a circuit with a balanced resistive load was found, a circuit that at the load active power P = 100 kW has 5% of the power loss, i.e., ΔP_{s} = 5 kW, on delivery. Parameters of such a circuit are shown in Fig. 4.

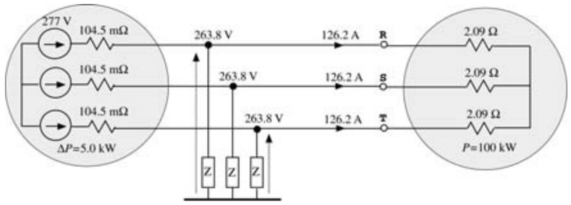

Fig. 4. Circuit with balanced resistive load

In the next step, the same source supplies an unbalanced resistive load, shown in Fig. 5, with the same active power P = 100 kW.

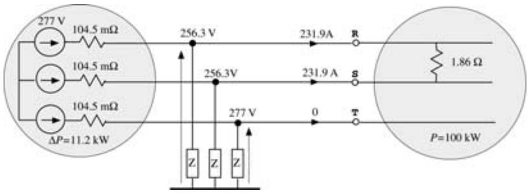

Fig. 5. Circuit with unbalanced load

Depending on definition of the apparent power, it is equal to

$$S_A = 119 \text{ kVA}, \quad S_G = 100 \text{ kVA}, \quad S_B = 149 \text{ kVA},$$

and the power factor is equal to, respectively,

$$\lambda_A = 0.84, \qquad \lambda_G = 1,. \qquad \lambda_B = 0.67.$$

Observe, that in spite of the same load active power, the power loss on energy delivery has increased in the circuit with the unbalanced load from ΔP_s = 5.0 kW to ΔP_s = 11.2 kW. It means, that the load shown in Fig. 5 is not a load with unity power factor. This conclusion disqualifies geometric definition (5) of the apparent power. However, still we do not know whether λ_A or λ_B provides the true value of the power factor. To answer this question, let us find the power factor of a balanced RL load with the same active power, P = 100 kW, that causes the same power loss, ΔP_s = 11.2 kW. Such an RL balanced load has parameters shown in Fig. 6.

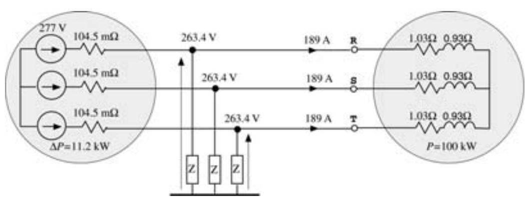

Fig. 6. Balanced load equivalent to unbalanced load in Fig. 6 with respect to power loss in the source

The load in this circuit is balanced thus, the apparent power does not depend on the selected definition of the apparent power and $S_A = S_B$ = 149 kVA. Consequently, the power factor is $\lambda_B = \lambda = 0.67$. It means, that the power factor has a true value only if the apparent power S is calculated according to definition (6). Arithmetic and geometric definitions of the apparent power result in an erroneous value of the power factor. However, when the apparent power S is calculated according to definition (6), power equation (1) is not fulfilled. For example, powers in Illustration 1 are equal to P = 115.1 kW, Q = 0, and S = 162.8 kVA. Equation (1) is not satisfied for such powers. Thus, it is erroneous even for sinusoidal voltages and currents. It is true only for balanced loads supplied with a symmetrical voltage. However, power properties of such systems are trivial and could be described phase by phase as properties of single-phase systems.

The same conclusion could be drawn from a different reasoning. The apparent power in single-phase systems is defined as the product of RMS values of the voltage and current at the source terminals, i.e., $S = ||u||\ ||i||$. Therefore, it seems to be reasonable to define the apparent power in three-phase systems in the same way, i.e., as the product of RMS value of three-phase voltage and current vectors, $\boldsymbol{u}$ and $\boldsymbol{i}$,

$$S = ||\boldsymbol{u}||\,||\boldsymbol{i}|| \,. \tag{17}$$

When voltages and currents are sinusoidal then this definition could be expressed in form (6).

Selection of the acceptable definition of the apparent power is the first step in a quest for a power equation of three-phase systems that would be valid, unlike equation (1), in systems with unbalanced loads as well. As demonstrated, such an equation is needed even for systems with sinusoidal voltages and currents.

Decomposition of the line currents of three-phase loads into components associated with distinctive physical phenomena in the circuit is of key importance in development of the power equation.

VI. CURRENTS' PHYSICAL COMPONENTS IN SYSTEMS UNDER SINUSOIDAL CONDITIONS

Considerations in this Section are confined to three-phase, three-wire circuits, shown in Fig. 7a, with linear, time-invariant loads supplied with a sinusoidal symmetrical voltage of positive sequence. For any such load there exists an equivalent resistive and balanced load, shown in Fig. 7b, that at the same voltage has the same active power, P, as the original load.

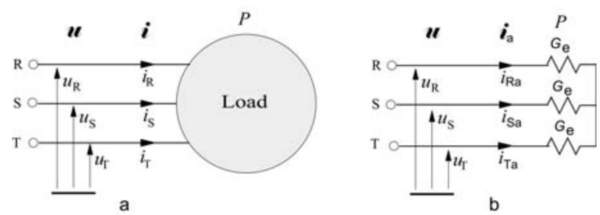

Fig. 7. (a) Three-phase load and (b) its equivalent load with respect to active power, P

The active power of the load in Fig. 7b is equal to

$$||u_R||^2 G_e + ||u_S||^2 G_e + ||u_T||^2 G_e = P \,, \tag{18}$$

thus, this load is equivalent to the original load with respect to the active power, if its phase conductance has the value

$$G_e = \frac{P}{||u_R||^2 + ||u_S||^2 + ||u_T||^2} = \frac{P}{||\boldsymbol{u}||^2} \,. \tag{19}$$

This conductance is referred to as the *equivalent conductance* of a three-phase load.

For any three-phase load supplied by a three-wire line, an equivalent load of Δ structure, shown in Fig. 8, can be found.

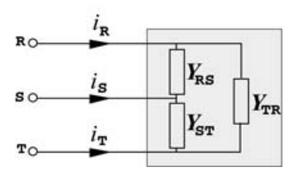

Fig. 8. Equivalent load in Δ structure

The active power of such a load is

$$P = \text{Re}\{\boldsymbol{Y}_{RS}||u_{RS}||^2 + \boldsymbol{Y}_{ST}||u_{ST}||^2 + \boldsymbol{Y}_{TR}||u_{TR}||^2\} \,. \tag{20}$$

The supply voltage is sinusoidal and symmetrical, thus

$$||u_{RS}|| = ||u_{ST}|| = ||u_{TR}|| = \sqrt{3}\,||u_R|| = ||\boldsymbol{u}|| \,, \tag{21}$$

hence

$$P = \text{Re}\{\boldsymbol{Y}_{RS} + \boldsymbol{Y}_{ST} + \boldsymbol{Y}_{TR}\}||\boldsymbol{u}||^2 = \text{Re}\,\boldsymbol{Y}_e\,||\boldsymbol{u}||^2 = G_e||\boldsymbol{u}||^2 \,. \tag{22}$$

The term

$$\boldsymbol{Y}_{RS} + \boldsymbol{Y}_{ST} + \boldsymbol{Y}_{TR} = \boldsymbol{Y}_e = G_e + jB_e \tag{23}$$

is called the *equivalent admittance* of three-phase loads. Its real part is equal to the load equivalent conductance G_e. Its imaginary part, B_e, is referred to as the *equivalent susceptance* of three-phase loads.

The line current of the equivalent resistive load is equal to

$$\boldsymbol{i}_a = \begin{bmatrix} i_{Ra}\\ i_{Sa}\\ i_{Ta}\end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix}G_e\boldsymbol{U}_R\\ G_e\boldsymbol{U}_S\\ G_e\boldsymbol{U}_T\end{bmatrix}e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\{G_e\,\mathcal{U}e^{j\omega_1 t}\} = G_e\,\boldsymbol{u}\,, \tag{24}$$

and is referred to as the *active current*. It is the smallest current needed for energy permanent conversion in the load with power P. The remaining part of the supply current, $\boldsymbol{i} - \boldsymbol{i}_a$, does not contribute to energy conversion. It is useless but it contributes to the supply current RMS value increase. Let us calculate this difference:

$$\boldsymbol{i} - \boldsymbol{i}_a = \sqrt{2}\,\mathrm{Re}\begin{bmatrix}\boldsymbol{I}_R - G_e\boldsymbol{U}_R\\ \boldsymbol{I}_S - G_e\boldsymbol{U}_S\\ \boldsymbol{I}_T - G_e\boldsymbol{U}_T\end{bmatrix}e^{j\omega_1 t}. \tag{25}$$

The complex RMS (CRMS) value of the line R current is

$$\boldsymbol{I}_R = (\boldsymbol{U}_R - \boldsymbol{U}_S)\boldsymbol{Y}_{RS} - (\boldsymbol{U}_T - \boldsymbol{U}_R)\boldsymbol{Y}_{TR} = \\ = (\boldsymbol{Y}_{RS} + \boldsymbol{Y}_{ST} + \boldsymbol{Y}_{TR})\boldsymbol{U}_R - (\boldsymbol{Y}_{ST}\boldsymbol{U}_R + \boldsymbol{Y}_{TR}\boldsymbol{U}_T + \boldsymbol{Y}_{RS}\boldsymbol{U}_S). \tag{26}$$

Since in systems with supply voltage of positive sequence

$$\boldsymbol{U}_T = \alpha\boldsymbol{U}_R, \quad \boldsymbol{U}_S = \alpha^*\boldsymbol{U}_R, \quad \alpha = 1e^{j2\pi/3}, \tag{27}$$

the CRMS value $\boldsymbol{I}_R$ can be expressed as follows,

$$\boldsymbol{I}_R = (\boldsymbol{Y}_{RS} + \boldsymbol{Y}_{ST} + \boldsymbol{Y}_{TR})\boldsymbol{U}_R - (\boldsymbol{Y}_{ST} + \alpha\boldsymbol{Y}_{TR} + \alpha^*\boldsymbol{Y}_{RS})\boldsymbol{U}_R = \boldsymbol{Y}_e\boldsymbol{U}_R + \boldsymbol{A}\boldsymbol{U}_R. \tag{28}$$

where

$$\boldsymbol{A} = -(\boldsymbol{Y}_{ST} + \alpha\boldsymbol{Y}_{TR} + \alpha^*\boldsymbol{Y}_{RS}) \tag{29}$$

will be referred to as an *unbalanced admittance*. Similarly, the CRMS value of the line S and T currents are equal to

$$\boldsymbol{I}_S = \boldsymbol{Y}_e\boldsymbol{U}_S + \boldsymbol{A}\boldsymbol{U}_T, \tag{30}$$

$$\boldsymbol{I}_T = \boldsymbol{Y}_e\boldsymbol{U}_T + \boldsymbol{A}\boldsymbol{U}_S. \tag{31}$$

When formulae (28), (30) and (31) are combined with (25), then the useless current can be expressed in the form

$$\boldsymbol{i} - \boldsymbol{i}_a = \sqrt{2}\,\mathrm{Re}\begin{bmatrix}\boldsymbol{Y}_e\boldsymbol{U}_R + \boldsymbol{A}\boldsymbol{U}_R - G_e\boldsymbol{U}_R\\ \boldsymbol{Y}_e\boldsymbol{U}_S + \boldsymbol{A}\boldsymbol{U}_T - G_e\boldsymbol{U}_S\\ \boldsymbol{Y}_e\boldsymbol{U}_T + \boldsymbol{A}\boldsymbol{U}_S - G_e\boldsymbol{U}_T\end{bmatrix}e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\,(jB_e\,\mathcal{U} + \boldsymbol{A}\,\mathcal{U}^{\#})\,e^{j\omega_1 t}. \tag{32}$$

where

$$\mathcal{U} = \begin{bmatrix}\boldsymbol{U}_R\\ \boldsymbol{U}_S\\ \boldsymbol{U}_T\end{bmatrix}, \qquad \mathcal{U}^{\#} = \begin{bmatrix}\boldsymbol{U}_R\\ \boldsymbol{U}_T\\ \boldsymbol{U}_S\end{bmatrix}. \tag{33}$$

This formula shows that the useless current in supply lines contains two components. The first of them

$$\sqrt{2}\,\mathrm{Re}\{jB_e\,\mathcal{U}e^{j\omega_1 t}\} = \boldsymbol{i}_r \tag{34}$$

occurs when the equivalent susceptance B_e of the load has a non-zero value. Since the reactive power

$$Q = -\mathrm{Im}\{\boldsymbol{Y}_{RS}\|u_{RS}\|^2 + \boldsymbol{Y}_{ST}\|u_{ST}\|^2 + \boldsymbol{Y}_{TR}\|u_{TR}\|^2\} = -B_e\,\|\boldsymbol{u}\|^2 \tag{35}$$

can occur only when this susceptance is non-zero, this current is associated with the reactive power, Q. Therefore, current $\boldsymbol{i}_r$ can be referred to as the *reactive current*. The second component

$$\sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\,\mathcal{U}^{\#}\,e^{j\omega_1 t}\} = \boldsymbol{i}_u \tag{36}$$

occurs in the supply current only when coefficient $\boldsymbol{A}$ is not equal to zero. This coefficient has zero value when

$$\boldsymbol{Y}_{RS} = \boldsymbol{Y}_{ST} = \boldsymbol{Y}_{TR}, \tag{37}$$

i.e., in balanced systems. Because the voltage vector $\mathcal{U}^{\#}$ has negative sequence, thus, current $\boldsymbol{i}_u$ is also of negative sequence and causes the supply current asymmetry. Since this current occurs in systems with unbalanced loads, it can be called an *unbalanced current*. Observe, however, that equality (37) is not a necessary condition to have zero unbalanced admittance. This observation will make compensation of unbalanced currents possible.

Formula (32) for the useless current when combined with formulae for the reactive and unbalanced currents leads to the following decomposition [4] of the supply current in three-phase systems

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_r + \boldsymbol{i}_u\,, \tag{38}$$

specified by three equivalent parameters of the load, G_e, B_e and $\boldsymbol{A}$.

These current components are associated, separately, with three distinctive physical phenomena in the circuit: (i) permanent energy conversion in the load due to its active power, (ii) current phase shift with respect to the supply voltage due to the load reactive power and (iii) supply current asymmetry due to the load imbalance. Therefore, these currents are referred to as *physical components* of the supply current. Their RMS are

$$\|\boldsymbol{i}_a\| = G_e\,\|\boldsymbol{u}\|\,, \tag{39}$$

$$\|\boldsymbol{i}_r\| = |B_e|\,\|\boldsymbol{u}\|\,, \tag{40}$$

$$\|\boldsymbol{i}_u\| = A\,\|\boldsymbol{u}\|\,. \tag{41}$$

The relation of the RMS value of the supply current $\|\boldsymbol{i}\|$ to the RMS value of its physical components depends on their mutual orthogonality.

The active and reactive currents are mutually orthogonal because these currents are shifted mutually by 90 degrees. However, their orthogonality to the unbalanced current is not evident. Let us calculate the scalar product of the active and unbalanced current,

$$(\boldsymbol{i}_a, \boldsymbol{i}_u) = \mathrm{Re}\{G_e\,\mathcal{U}^{T}(\boldsymbol{A}\,\mathcal{U}^{\#})^*\} = \mathrm{Re}\{G_e\,\boldsymbol{A}^*\}(1 + \alpha + \alpha^*)U_R^2 = 0. \tag{42}$$

Similarly, the scalar product of the reactive and unbalanced current

$$(\boldsymbol{i}_r, \boldsymbol{i}_u) = \mathrm{Re}\{jB_e\,\mathcal{U}^{T}(\boldsymbol{A}\,\mathcal{U}^{\#})^*\} = \mathrm{Re}\{jB_e\,\boldsymbol{A}^*\}(1 + \alpha + \alpha^*)U_R^2 = 0, \tag{43}$$

thus, all physical components are mutually orthogonal and hence, their RMS values have to satisfy the relationship

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_r\|^2 + \|\boldsymbol{i}_u\|^2\,. \tag{44}$$

This relation between RMS values of the current physical components could be visualized with the relation between the length of edges of a rectangular box and its diagonal, as it is shown in Fig. 9.

Multiplying equation (44) by the square of the supply voltage RMS value, $\|\boldsymbol{u}\|$, the power equation

$$S^2 = P^2 + Q^2 + D^2 \tag{45}$$

would be obtained, where

$$Q = \|\boldsymbol{u}\|\cdot\|\boldsymbol{i}_r\| = -B_e\,\|\boldsymbol{u}\|^2\,, \quad D = \|\boldsymbol{u}\|\cdot\|\boldsymbol{i}_u\| = A\,\|\boldsymbol{u}\|^2 \tag{46}$$

are the reactive and *unbalanced* powers, respectively. This power equation provides quantitative information on the effect of the power factor. Observe moreover, that this approach makes it possible to express the power factor in terms of not only the load powers but also the load equivalent parameters,

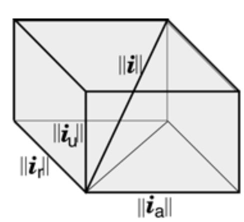

Fig. 9. Rectangular box of RMS values of current physical components

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2 + D^2}} = \frac{\|\boldsymbol{i}_a\|}{\sqrt{\|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_r\|^2 + \|\boldsymbol{i}_u\|^2}} = \frac{G_e}{\sqrt{G_e^2 + B_e^2 + A^2}}\,. \tag{47}$$

Illustration 3. Let us apply the CPC Theory to the circuit shown in Fig. 10, supplied with sinu-soidal voltage of the line-to-ground rms value U = 277 V, assuming that the load impedance is $\boldsymbol{Z}_{\rm R}$ = (3 + j1) Ω.

For such a load

$$\boldsymbol{Y}_{\rm RS} = \frac{1}{\boldsymbol{Z}_{\rm RS}} = \frac{1}{3+j1} = 0.30 - j\,0.10 = 0.316\,e^{-j18.4^{\circ}}\ \mathrm{S}\,,$$

hence, the equivalent and unbalance admittances are equal to

$$\boldsymbol{Y}_{\rm e} = G_{\rm e} + j\,B_{\rm e} = \boldsymbol{Y}_{\rm RS} = 0.30 - j0.10\ \mathrm{S},$$

$$\boldsymbol{A} = -\alpha^{*}\boldsymbol{Y}_{\rm RS} = 0.316\,e^{j41.6^{\circ}}\ \mathrm{S}\,.$$

The RMS value of the supply voltage vector $||\boldsymbol{u}||$ = 277 $\sqrt{3}$ = 480 V, thus, the RMS values of the supply current's physical components are

$$||\boldsymbol{i}_{\rm a}|| = G_{\rm e}\,||\boldsymbol{u}|| = 0.30 \times 480 = 144\ \mathrm{A},$$

$$||\boldsymbol{i}_{\rm r}|| = |B_{\rm e}|\,||\boldsymbol{u}|| = 0.10 \times 480 = 48\ \mathrm{A},$$

$$||\boldsymbol{i}_{\rm u}|| = A\,||\boldsymbol{u}|| = 0.316 \times 480 = 152\ \mathrm{A},$$

and the supply current RMS value is

$$||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_{\rm a}||^2 + ||\boldsymbol{i}_{\rm r}||^2 + ||\boldsymbol{i}_{\rm u}||^2} = \sqrt{144^2 + 48^2 + 152^2} = 215\ \mathrm{A}\,.$$

This result should be in accordance with the value calculated from the definition of the RMS value of the supply current vector. Indeed,

$$||\boldsymbol{i}|| = \sqrt{||i_{\rm R}||^2 + ||i_{\rm S}||^2 + ||i_{\rm T}||^2} = \sqrt{152^2 + 152^2} = 215\ \mathrm{A}\,.$$

The load powers are

$$P = G_{\rm e}\,||\boldsymbol{u}||^2 = 0.30 \times 480^2 = 69\ \mathrm{kW},$$

$$Q| = -B_{\rm e}\,||\boldsymbol{u}||^2 = 0.10 \times 480^2 = 23\ \mathrm{kVA},$$

$$D = A\,||\boldsymbol{u}||^2 = 0.316 \times 480^2 = 73\ \mathrm{kVA},$$

$$S = ||\boldsymbol{u}||\,||\boldsymbol{i}|| = 480 \times 215 = 103\ \mathrm{kVA},$$

and the power factor is

$$\lambda = \frac{P}{S} = \frac{69}{103} = 0.67\,.$$

Observe, that the power factor calculated with geometric definition of the apparent power is λ = 0.95.

VII. COMPENSATION OF REACTIVE AND UNBALANCED CURRENT

Power equation (45) shows that the unbalanced current contributes to the supply current RMS value and apparent power increase and consequently, the power factor decline in the same way as the reactive current. Thus, reduction of both currents contributes to power factor improvement. These currents can be reduced by a shunt *balancing compensator*. The load is compensated entirely when a vector of the line currents, $\boldsymbol{i}_{\rm c}$, of a balancing compensator is equal to the negative value of the sum of the load reactive and unbalanced currents, i.e., $\boldsymbol{i}_{\rm c} = -\boldsymbol{i}_{\rm r} - \boldsymbol{i}_{\rm u}$.

Balancing compensators can be built as reactive devices, composed of inductors and capacitors, or as fast switching devices, composed of a three-phase inverter with a measurement and a control system. In the case of reactive compensators, line currents are specified by structure and LC parameters of the compensator. In the case of switching compensators, line currents are shaped by fast switching of inverter's switches. Switching compensators are not discussed here because their operation depends mainly on the selected control algorithm, but this goes beyond the scope of this paper. Only one issue that differentiates compensation of the reactive and unbalanced currents with switching compensators is discussed here. It is the question: *does the energy have to be stored in the compensator during compensation?*

If the supply voltage at terminal R is $u_{\rm R} = \sqrt{2}\,U\cos(\omega t)$, then the instantaneous power of a reactive current compensator is equal to

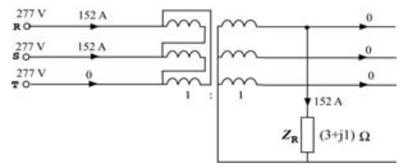

Fig. 10. Example of three-phase system with unbalanced load

$$p_{\rm r}(t) = \frac{dW}{dt} = -\boldsymbol{u}^{\rm T}\boldsymbol{i}_{\rm r} = -2 \quad \lfloor\cos(\omega t + 120^{0})\rfloor \ \lfloor\sin(\omega t + 120^{0})\rfloor$$

$$= -Q\,[\sin 2\omega t + \sin(2\omega t + 120^{0}) + \sin(2\omega t - 120^{0}) \equiv 0. \tag{48}$$

Thus, energy does not have to be stored in the compensator for the reactive power compensation, because there is no flow of energy between the compensator and the supply. Small energy storage is needed only to provide DC voltage or current for the inverter.

The instantaneous power of a compensator of the unbalanced current is equal to

$$p_{\rm u}(t) = -\boldsymbol{u}^{\rm T}\boldsymbol{i}_{\rm u} = -2AU^{2}\begin{bmatrix}\cos\omega t\\ \cos(\omega t - 120^{0})\\ \cos(\omega t + 120^{0})\end{bmatrix}^{\rm T}\begin{bmatrix}\cos(\omega t + \psi)\\ \cos(\omega t + 120^{0} + \psi)\\ \cos(\omega t - 120^{0} + \psi)\end{bmatrix} =$$

$$= -D\cos(2\omega t + \psi). \tag{49}$$

Thus, a compensator of unbalanced current cannot be built without a sufficient capability of energy storage. Since the frequency of energy oscillation is equal to 2ω, the compensator has to be able to store, at least, energy

$$W_{\rm s} = \int_{-\psi/2\omega + T/8}^{-\psi/2\omega + 3T/8} p_{\rm u}\,dt = D\int_{0}^{T/4}\sin 2\omega t\,dt = \frac{T}{2\pi}D\,. \tag{50}$$

A reactive compensator can have a structure shown in Fig. 11, i.e., it can be built of three reactance LC one-ports, connected in delta configuration and of branch susceptances $T_{\rm RS}$, $T_{\rm ST}$ and $T_{\rm TR}$. If these one-ports are ideal, lossless devices, then the compensator modifies only the reactive and unbalanced currents to

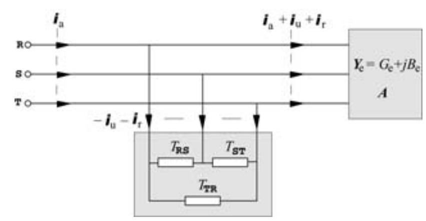

Fig. 11. Circuit with shunt compensator

$$\boldsymbol{i}'_{\rm r} = \sqrt{2}\,\mathrm{Re}\{j[\,B_{\rm e} + (T_{\rm ST} + T_{\rm TR} + T_{\rm RS})]\,\boldsymbol{U}e^{j\omega_1 t}\}\,, \tag{51}$$

$$\boldsymbol{i}'_{\rm u} = \sqrt{2}\,\mathrm{Re}\{[\boldsymbol{A} - j\,(T_{\rm ST} + \alpha T_{\rm TR} + \alpha^{*}T_{\rm RS})]\,\boldsymbol{U}^{\#}e^{j\omega_1 t}\}\,. \tag{52}$$

The reactive current is compensated entirely only if

$$B_{\rm e} + (T_{\rm ST} + T_{\rm TR} + T_{\rm RS}) = 0\,. \tag{53}$$

The unbalanced current is compensated entirely only if

$$\boldsymbol{A} - j\,(T_{\rm ST} + \alpha T_{\rm TR} + \alpha^{*}T_{\rm RS}) = 0\,. \tag{54}$$

The left side of the last equation is a complex number, thus, this equ-tion is satisfied only if it is satisfied for the real and for imaginary parts, i.e., if

$$\mathrm{Re}\{\boldsymbol{A} - j\,(T_{\rm ST} + \alpha T_{\rm TR} + \alpha^{*}T_{\rm RS})\} = 0\,. \tag{55}$$

$$\mathrm{Im}\{\boldsymbol{A} - j\,(T_{\rm ST} + \alpha T_{\rm TR} + \alpha^{*}T_{\rm RS})\} = 0\,. \tag{56}$$

Properties of the compensator are specified by three susceptances $T_{\rm RS}$, $T_{\rm ST}$ and $T_{\rm TR}$. These susceptances satisfy equations (53), (55) and (56), if they are equal to

$$T_{\mathrm{RS}} = (\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A} - \mathrm{Im}\,\boldsymbol{A} - B_{\mathrm{e}})/3\,,$$
$$T_{\mathrm{ST}} = (2\,\mathrm{Im}\,\boldsymbol{A} - B_{\mathrm{e}})/3\,, \tag{57}$$
$$T_{\mathrm{TR}} = (-\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A} - \mathrm{Im}\,\boldsymbol{A} - B_{\mathrm{e}})/3\,.$$

When a susceptance T_{XY} obtained from formulae (57) is positive, then a capacitor should be selected as the compensator branch. When this susceptance is negative, then an inductor should be selected. Their capacitance and inductance are, respectively, equal to

$$C_{\mathrm{XY}} = \frac{T_{\mathrm{XY}}}{\omega_1}\,, \qquad L_{\mathrm{XY}} = -\frac{1}{\omega_1 T_{\mathrm{XY}}}\,. \tag{58}$$

Illustration 4. Let us calculate parameters of a balancing compensator for entire reduction of the reactive and unbalanced currents in the system considered in Illustration 3. The equivalent susceptance of the load in Fig. 12 is $B_{\mathrm{e}} = -0.10$ S, while

$$\boldsymbol{A} = \mathrm{Re}\,\boldsymbol{A} + j\,\mathrm{Im}\,\boldsymbol{A} = 0.316\,e^{j41.6^0} = 0.236 + j0.210\ \ \mathrm{S}.$$

Hence, formulae (57) result in the following susceptances of the compensator: T_{RS}= 0.10 S, T_{ST}= 0.173 S, T_{TR} = – 0.173 S. Thus, capacitors should be connected between lines RS and lines ST. Inductor should be connected between lines TR. The structure and parameters of the compensator are shown in Fig. 12. Such a compensator reduces the RMS value of the supply current vector from $||\boldsymbol{i}||$ = 215 A to 144 A. It restores the supply current symmetry and improves power factor from $\lambda = 0.67$ to unity.

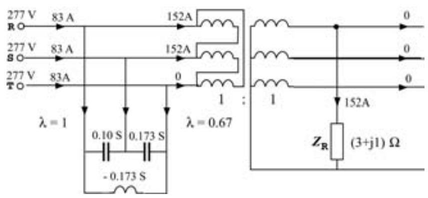

Fig. 12. Unbalanced load with balancing compensator

This very simple and transparent method of calculation of the compensator parameters is an example of benefits from the supply current decomposition according to CPC theory. It confirms the applicability of this approach to reactive compensator design, but first of all, this approach provides a new and credible power equation of three-phase systems under sinusoidal condition. Also, it provides a reliable starting point for identification of power properties of three-phase systems in more complex situations.

A generalization of the CPC power theory to systems considered in Section VI, but with nonsinusoidal supply voltage is the next step towards comprehension of power properties of three-phase systems with nonsinusoidal voltages and currents.

VIII. CURRENTS' PHYSICAL COMPONENTS AT DISTORTED SUPPLY VOLTAGE

Let us specify power properties and the power equation of three-phase systems considered in Section V. It is assumed in this Section that the supply voltage of a three-phase, three-wire system is nonsinusoidal, but symmetrical and of positive sequence, i.e.,

$$u_{\mathrm{S}}(t) = u_{\mathrm{R}}(t - T/3), \qquad u_{\mathrm{T}}(t) = u_{\mathrm{R}}(t + T/3), \tag{59}$$

and can be expressed as a sum of harmonics in the form

$$\boldsymbol{u} = \sum_{n\in N} \boldsymbol{u}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n\, e^{jn\omega_1 t}\,. \tag{60}$$

The supply current in three-wire systems cannot contain any harmonic of the zero order, i.e., $\boldsymbol{i}_{3k} = 0$, since such a circuit is open for zero order voltage harmonics. Therefore, it will be assumed that such harmonics do not exist in the supply voltage, i.e., $\boldsymbol{u}_{3k} = 0$. Such an assumption is equivalent to the assumption that the supply voltage is referenced to an artificial neutral point. Without such an assumption the concept of the RMS value of the voltage vector $||\boldsymbol{u}||$ is losing sense, because zero order harmonics of the voltage affect this value without affecting the active power of a three-phase, three-wire device.

The line current of a three-phase, three-wire load is

$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_n\, e^{jn\omega_1 t}\,. \tag{61}$$

When a linear, time-invariant load is supplied by a single voltage harmonic of the n^{th} order, $\boldsymbol{u}_n$, i.e., by a sinusoidal voltage, then the line current harmonic $\boldsymbol{i}_n$ can be decomposed, according to Section VI, into the active, reactive and unbalanced currents of that order n. It applies to each voltage harmonic, thus,

$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_n\, e^{jn\omega_1 t} = \sum_{n\in N} (\boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n}) =$$
$$= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} (G_{\mathrm{e}n}\boldsymbol{U}_n + jB_{\mathrm{e}n}\boldsymbol{U}_n + \boldsymbol{A}_n\boldsymbol{U}_n^{\#})\, e^{jn\omega_1 t}. \tag{62}$$

If the line-to-line admittances of the load for the n^{th} order harmonic are $\boldsymbol{Y}_{\mathrm{RS}n}$, $\boldsymbol{Y}_{\mathrm{ST}n}$, and $\boldsymbol{Y}_{\mathrm{TR}n}$, then its equivalent admittance for that harmonic is

$$\boldsymbol{Y}_{\mathrm{e}n} = G_{\mathrm{e}n} + jB_{\mathrm{e}n} = \boldsymbol{Y}_{\mathrm{RS}n} + \boldsymbol{Y}_{\mathrm{ST}n} + \boldsymbol{Y}_{\mathrm{TR}n}\,. \tag{63}$$

The equivalent conductance and susceptance for the n^{th} order harmonic could also be related to the active and reactive powers of that harmonic, i.e.,

$$G_{\mathrm{e}n} = \mathrm{Re}\{\boldsymbol{Y}_{\mathrm{e}n}\} = \frac{P_n}{||\boldsymbol{u}_n||^2}\,, \tag{64}$$

$$B_{\mathrm{e}n} = \mathrm{Im}\{\boldsymbol{Y}_{\mathrm{e}n}\} = -\frac{Q_n}{||\boldsymbol{u}_n||^2}\,. \tag{65}$$

The formula for the unbalanced admittance $\boldsymbol{A}_n$ calculation depends on the sequence of the voltage harmonic. For harmonics of the positive sequence, $n = 3k+1$, like the fundamental, $\boldsymbol{U}_{\mathrm{S}n} = \alpha^*\boldsymbol{U}_{\mathrm{R}n}$, and

$$\boldsymbol{A}_n = -(\boldsymbol{Y}_{\mathrm{ST}n} + \alpha\boldsymbol{Y}_{\mathrm{TR}n} + \alpha^*\boldsymbol{Y}_{\mathrm{RS}n})\,. \tag{66}$$

For harmonics of the negative sequence, $n = 3k - 1$, $\boldsymbol{U}_{\mathrm{S}n} = \alpha\boldsymbol{U}_{\mathrm{R}n}$, and

$$\boldsymbol{A}_n = -(\boldsymbol{Y}_{\mathrm{ST}n} + \alpha^*\boldsymbol{Y}_{\mathrm{TR}n} + \alpha\boldsymbol{Y}_{\mathrm{RS}n})\,. \tag{67}$$

With respect to the load active power P at voltage $\boldsymbol{u}$, the load is equivalent to a balanced resistive load, shown in Fig. 8b, of conductance G_{e} specified by formula (29). This formula is valid irrespective if the supply voltage is sinusoidal or not. Such an equivalent resistive load draws the active current from the supply source

$$\boldsymbol{i}_{\mathrm{a}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} G_{\mathrm{e}}\,\boldsymbol{U}_n\, e^{jn\omega_1 t}\,. \tag{68}$$

The active current is the smallest possible current of a load that at voltage $\boldsymbol{u}$ has active power P. The remaining component of the line current, $\boldsymbol{i} - \boldsymbol{i}_{\mathrm{a}}$, is useless and only increases the supply current RMS value $||\boldsymbol{i}||$. It is equal to

$$\boldsymbol{i} - \boldsymbol{i}_{\mathrm{a}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} (\boldsymbol{I}_n - G_{\mathrm{e}}\,\boldsymbol{U})\, e^{jn\omega_1 t}\,. \tag{69}$$

It means, that the useless current can be expressed as

$$\boldsymbol{i} - \boldsymbol{i}_{\mathrm{a}} = \sum_{n\in N} (\boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n}) - \boldsymbol{i}_{\mathrm{a}} =$$
$$= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} [(G_{\mathrm{e}n} - G_{\mathrm{e}})\,\boldsymbol{U}_n + jB_{\mathrm{e}n}\boldsymbol{U}_n + \boldsymbol{A}_n\boldsymbol{U}_n^{\#}]\, e^{jn\omega_1 t}. \tag{70}$$

Thus, the useless current has three components. The component

$$\sqrt{2}\,\mathrm{Re}\sum_{n\in N} (G_{\mathrm{e}n} - G_{\mathrm{e}})\,\boldsymbol{U}_n\, e^{jn\omega_1 t} = \boldsymbol{i}_{\mathrm{s}} \tag{71}$$

occurs in the load supply current when the load equivalent conductance G_{en} changes with harmonic order, i.e., when it is scattered around G_e value. Therefore, it is referred to as the *scattered current*. The component

$$\sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_{en}U_n e^{jn\omega_1 t} = \boldsymbol{i}_r \tag{72}$$

is the *reactive current* and the component

$$\sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{A}_n U_n^{\#} e^{jn\omega_1 t} = \boldsymbol{i}_u \tag{73}$$

is referred to as the *unbalanced current*. Like previously these currents are associated with distinctive physical phenolmena in the circuit. Consequently, the line currents of three-phase loads supplied with a nonsinusoidal voltage can be decomposed into four physical components,

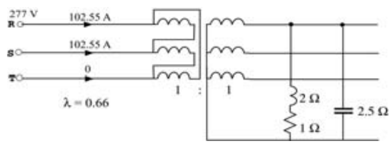

Fig. 13. Circuit with unbalanced load

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_s + \boldsymbol{i}_r + \boldsymbol{i}_u\,. \tag{74}$$

The RMS values of these components are, respectively,

$$||\boldsymbol{i}_a|| = G_e\,||\boldsymbol{u}||\,, \tag{75}$$

$$||\boldsymbol{i}_s|| = \sqrt{\sum_{n\in N}(G_{en} - G_e)^2||\boldsymbol{u}_n||^2}\,, \tag{76}$$

$$||\boldsymbol{i}_r|| = \sqrt{\sum_{n\in N}B_{en}^2||\boldsymbol{u}_n||^2}\,, \tag{77}$$

$$||\boldsymbol{i}_u|| = \sqrt{\sum_{n\in N}A_n^2||\boldsymbol{u}_n||^2}\,. \tag{78}$$

Orthogonality of these currents is not evident yet. Thus, it has to be verified.

Harmonics of different orders are orthogonal, hence the scalar product of three-phase vectors could be expressed in the form

$$(\boldsymbol{x},\boldsymbol{y}) = \Big(\sum_{n\in N}\boldsymbol{x}_n, \sum_{n\in N}\boldsymbol{y}_n\Big) = \sum_{n\in N}(\boldsymbol{x}_n, \boldsymbol{y}_n)\,, \tag{79}$$

thus, when harmonics of two different physical components are orthogonal, then these components are orthogonal as well. Orthogonality of the active, reactive and unbalanced current in sinusoidal conditions, i.e., for individual harmonics, was proven in Section VI. The reactive and unbalanced currents are also orthogonal to the scattered current. However, harmonics of the active and scattered currents are not mutually orthogonal, because in general,

$$(\boldsymbol{i}_{an}, \boldsymbol{i}_{sn}) = G_e(G_{en} - G_e)||\boldsymbol{u}_n||^2 \neq 0\,. \tag{80}$$

Nonetheless, the scalar product of entire currents,

$$\begin{aligned}(\boldsymbol{i}_a, \boldsymbol{i}_s) &= \sum_{n\in N}(\boldsymbol{i}_{an}, \boldsymbol{i}_{sn}) = \sum_{n\in N}G_e(G_{en} - G_e)||\boldsymbol{u}_n||^2 = \\ &= G_e\sum_{n\in N}(G_{en} - G_e)||\boldsymbol{u}_n||^2 = G_e(P - P) = 0.\end{aligned} \tag{81}$$

Thus, all physical components of the line current are orthogonal and consequently, its RMS value can be expressed as

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2\,, \tag{82}$$

while the power equation has the form

$$S^2 = P^2 + D_s^2 + Q^2 + D^2\,. \tag{83}$$

Illustration 5. Let the supply voltage of the phase R-to ground in the circuit shown in Fig. 13 be

$$u_R = \sqrt{2}\,\mathrm{Re}\{277\,e^{j\omega_1 t} + 11 e^{j5\omega_1 t}\}\ \mathrm{V}\,.$$

Since only admittance between phases R and S are not equal to zero and equal to

$$\boldsymbol{Y}_{RS1} = 0.2\ \mathrm{S},$$

$$\boldsymbol{Y}_{RS5} = 0.01 + j1.901 = 1.901 e^{j89.7^0}\ \mathrm{S}\,,$$

the line R current is

$$i_R = \sqrt{2}\,\mathrm{Re}\{95.95 e^{j\omega_1 t} + 36.22 e^{j89.7^0} e^{j5\omega_1 t}\}\ \mathrm{A}\,.$$

$i_S = -\,i_R$, and their RMS values are

$$||i_R|| = ||i_S|| = \sqrt{95.95^2 + 36.22^2} = 102.55\ \mathrm{A}\,.$$

The RMS value of the supply current vector is

$$||\boldsymbol{i}|| = \sqrt{||i_R||^2 + ||i_S||^2 + ||i_T||^2} = \sqrt{102.55^2 + 102.55^2} = 145.03\,\mathrm{A}\,.$$

Since the RMS value of the supply voltage vector is

$$||\boldsymbol{u}|| = \sqrt{||\boldsymbol{u}_1||^2 + ||\boldsymbol{u}_5||^2} = \sqrt{(277\sqrt{3})^2 + (11\sqrt{3})^2} = 480.156\ \mathrm{V}\,,$$

the apparent power of the load is equal to

$$S = ||\boldsymbol{u}||\,||\boldsymbol{i}|| = 69.6\ \mathrm{kVA}.$$

The load equivalent admittance for the 1st and 5th order harmonics are

$$\boldsymbol{Y}_{e1} = G_{e1} + jB_{e1} = \boldsymbol{Y}_{RS1},$$

$$\boldsymbol{Y}_{e5} = G_{e5} + jB_{e5} = \boldsymbol{Y}_{RS5}\,,$$

and the unbalanced admittance

$$\boldsymbol{A}_1 = \alpha^*\boldsymbol{Y}_{TR1} = 0.2 e^{-j120^0}\ \mathrm{S}, \qquad \boldsymbol{A}_5 = \alpha\boldsymbol{Y}_{TR5} = 1.901 e^{-j29.7^0}\ \mathrm{S}\,.$$

The load active power is

$$P = \sum_{n=1,5}G_{en}||\boldsymbol{u}_n||^2 = 0.2\,(277\sqrt{3})^2 + 0.01\,(11\sqrt{3})^2 = 46.04\ \mathrm{kW}\,,$$

and the power factor

$$\lambda = \frac{P}{S} = 0.66\,.$$

The equivalent conductance of the load is

$$G_e = \frac{P}{||\boldsymbol{u}||^2} = 0.1997\ \mathrm{S}\,,$$

and this enables us to calculate the RMS value of the supply current physical components

$$||\boldsymbol{i}_a|| = 95.88\ \mathrm{A},\quad ||\boldsymbol{i}_s|| = 3.79\ \mathrm{A},\quad ||\boldsymbol{i}_r|| = 36.22\ \mathrm{A},\quad ||\boldsymbol{i}_u|| = 102.55\ \mathrm{A}.$$

It could be verified that

$$\sqrt{||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2} = ||\boldsymbol{i}|| = 145.03\,\mathrm{A}\,.$$

Observe, that the RMS value of the scattered current is much smaller than the remaining ones. However, without it, current equations (74), (82) and the power equation (83) are not fulfilled.

IX. CONCLUSIONS

This paper demonstrates that the commonly used power equation (1) and arithmetic and geometric apparent powers, when applied to systems with unbalanced loads, provide an incorrect power factor and incorrect power rating for energy transmitting equipment.

At the same time, decomposition of the supply current into components associated with distinctive power phenomena in three-phase systems, according to the Currents' Physical Components power theory, under nonsinusoidal condition is possible. It enables us to define powers associated with distinctive power phenomena and provide fundamentals for a design of balancing compensators.

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BIOGRAPHY

Leszek S. Czarnecki, Alfredo M. Lopez Distinguished Professor, received the M.Sc. and Ph.D. degrees in electrical engineering and Habil. Ph.D. degree from the Silesian University of Technology, Poland, in 1963, 1969 and 1984, respectively, where he was employed as an Assistant Professor. Beginning in 1984 he worked for two years at the Power Engineering Section, Division of Electrical Engineering, National Research Council of Canada as a Research Officer. In 1987 he joined the Electrical Engineering Dept. at Zielona Gora University of Technology. In 1989 Dr. Czarnecki joined the Electrical and Computer Engineering Dept. of Louisiana State University, Baton Rouge, where he is a Professor of Electrical Engineering now. For developing a power theory of three-phase nonsinusoidal unbalanced systems and methods of compensation of such systems he was elected to the grade of Fellow IEEE.

Dr. Czarnecki climbed the main ridge of Ruwenzori, Kilimanjaro and Mt. Kenya in Africa, Lhotse in Himalaya to 8350m, Cordilliera Huayashi in Andas and he climbed solo McKinley in Alaska.

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