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An Optimization Based Method for Selection of Resonant Harmonic Filter Branch Parameters

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Abstract—**Distribution voltage harmonics and load current harmonics other than harmonics to which a resonant harmonic filter (RHF) is tuned, deteriorate the filter efficiency in reducing harmonic distortion. To reduce this effect an optimization based design method is developed for the conventional RHF. It takes into consideration the interaction of the filter with the distribution system and provides filter parameters which give the maximum effectiveness with respect to harmonic suppression. The results for optimized filters, applied in a typical case, are given.**

Index terms--**passive harmonic filters, harmonic distortion, harmonic filter design, shunt tuned harmonic filters**

I. INTRODUCTION

Due to the presence of harmonic generating loads (HGLs) in distribution systems, resonant harmonic filters very often operate in the presence of distribution voltage harmonics as well as the load current harmonics other than those to which the filter is tuned. Some of the voltage and current harmonics could be amplified by the filter resonance with the distribution system inductance, L_s, as seen from the bus where the RHF is installed. Moreover, the filter as seen from the distribution system has very low impedance at tuned frequencies. Consequently, with the increase of distortion of the distribution voltage and the amount of non-characteristic harmonics in the load current, the efficiency of the filter in reducing distortion of the bus voltage and the supply current declines.

Harmonic amplification caused by the filter resonance with the distribution system inductance depends on frequencies of this resonance and can be reduced by a selection of the filter parameters. Harmful effects of the filter's low impedance at tuning frequencies can be reduced [2-6, 10, 11] by detuning the filter from frequencies of characteristic harmonics. Unfortunately, this detuning reduces the attenuation of the load current harmonics. Thus, to improve the filter efficiency, a trade off between attenuation and amplification of particular harmonics is needed. This trade off can be achieved by a "trail and error" approach or by an optimization procedure.

Unfortunately, the complex interaction of the filter with the distribution system and the number of filter design parameters makes the best selection of parameters by trial and error methods very difficult. In order to solve complex problems where the best selection of parameter values is not readily apparent, optimization techniques may be employed. A method of applying optimization techniques to the design of RHFs will be explored and presented in this paper along with some performance data for the optimized filters. Such an optimization based design method consists of several phases. First, a filter prototype is designed which satisfies some basic design requirements. Next, the prototype is analyzed to obtain frequency characteristics using the transmittance approach [7, 17] and to obtain performance measures. The optimization is then performed and analysis is done to determine performance. However, as with most optimization routines, the minimum value of distortion obtained may only be a local minimum. Because of this, the optimization and analysis should be repeated using different starting filter prototypes in order to find several local minima. The best final design can then be chosen out of the set obtained. In order to ensure that all local minima that meet the constraint requirements are found, the behavior of the filter cost function is investigated.

The studies discussed in this paper are confined to RHFs applied to reduction of harmonic distortion caused by six-pulse ac/dc converters and rectifiers as well as by other minor nonlinear loads supplied from the same bus. Characteristic harmonics of such converters and rectifiers are of the order $n = 6k \pm 1$, while their asymmetry, asymmetry of the thyristors' firing angle and other loads contribute [2-4, 10, 12] to the presence of other current harmonics, referred to as non-characteristic. The study is limited to filters composed of four resonant LC branches that provided a low impedance path for the 5^{th}, 7^{th}, 11^{th} and 13^{th} order harmonics. Filters with high-pass branches were beyond the scope of this study.

Amplification of some harmonics by a resonance of such filters with the distribution system inductance is one of the main concerns [2-4, 7, 12]. This resonance cannot be avoided, therefore, RHFs are often superseded by switching compensators, commonly known as "active harmonic filters"(AHFs). Such devices have a number of advantages over RHFs. However, the power rating of AHFs is limited by their transistors' switching power. Moreover, high frequency switching, necessary for operation of these devices, is a source of electromagnetic interference. Therefore, RHFs, might still be an important alternative, in particular, if an optimization procedure could elevate their efficiency.

II. TRADITIONAL DESIGN OF RHFs

Resonant harmonic filters are designed traditionally by calculating the capacitance C_k and inductance L_k, in such a way, that each branch has a resonance at a frequency equal to or in a vicinity of harmonic frequency, $\omega_k = \zeta_k \omega_1$. Furthermore, each filter branch compensates reactive power

$$Q_k = d_k Q, \tag{1}$$

where Q is the load reactive power per-phase and d_k is the coefficient of the reactive power allocation to particular branches of the filter.

The opinions with respect to the reactive power allocation to particular branches are divided. According to Ref. [1], this allocation is irrelevant for the filter properties. Consequently, it could be assumed that each branch compensates the same reactive power, i.e., allocation coefficients have the same value. The reactive power allocation for a two branch filter of the 5th and 7th order harmonics assumed in Ref [9] is in proportion of $Q_5/Q_7 = 2{:}1$, while in Ref. [10] this proportion is $Q_5/Q_7 = 8{:}3$. According to Ref. [8], the reactive power allocation should be "*...proportional to total harmonic current each filter will carry*".

In the presence of distribution voltage harmonics, the filter branches are typically tuned to a frequency below the harmonic frequency. It increases the branch reactance at the harmonic frequency and keeps it inductive, even if the capacitance of capacitor bank declines with aging. However, there are substantial differences in opinions on how much the branches should be detuned. Reference [9] assumes that filters are detuned by 5% below harmonic frequencies, while Ref. [2] suggests that detuning should be in the range of 3 to 10% below these frequencies. Indeed, detuning assumed in Ref. [10] amounts to 8% for all branches, i.e., the relative detuning is the same for all branches. Thus, there is the lack of a clear recommendation with respect to the filter detuning.

When a harmonic filter is under design, the attenuation of dominating, characteristic harmonics is the subject of main concern. However, harmonics other than the characteristic harmonics are always present. Their level is reported in numerous papers [2-6 10, 12]. The traditional approach to filter design essentially neglects the presence of non-characteristic harmonics in the load current and the distribution voltage harmonics in the filter design process, considering them as kind of "minor" [7] harmonics.

III. RESONANT FREQUENCY LOCATIONS

In order to adjust filter parameters for the purpose of avoiding resonance at harmonic frequencies, the relation between reactive power allocation and resonant frequency locations is needed. The quality factor of filter inductors is usually very high for RHFs, furthermore, supply and load inductance dominate the supply and load impedance at harmonic frequencies. Therefore, to find the resonant frequencies we may consider a reactive equivalent circuit. The equivalent network as seen by the supply for such a circuit having a filter with K branches is shown below in Figure 1.

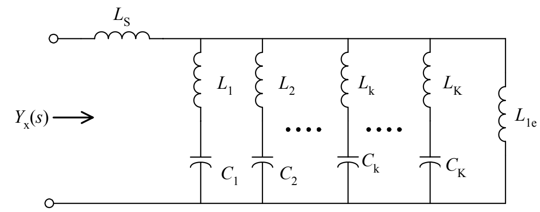

Figure 1. Equivalent one-port network viewed from the supply terminals.

The lumped impedance of the filter branches and the load equivalent inductance L_{1e} are connected in series with the equivalent supply inductance L_s. This means that there will be series resonance as seen by the supply that give high values of admittance. The admittance $Y_x(s)$ is given by

$$Y_x(s) = \frac{1}{s\,L_s + 1/Y_a(s)} \tag{2}$$

where

$$Y_a(s) = \frac{s\,C_1}{s^2 L_1 C_1 + 1} + \frac{s\,C_2}{s^2 L_2 C_2 + 1} + \ldots + \frac{s\,C_K}{s^2 L_K C_K + 1} + \frac{1}{sL_{1e}} \tag{3}$$

The impedance $Y_a(s)$ can be expressed in terms of the reactive power allocation coefficients, d_k, as

$$Y_a(s) = \frac{1}{s\,L_{1e}} + \sum_{k=1}^{K} \frac{s\,a_k}{\dfrac{s^2}{(\zeta_k \omega_1)^2} + 1} \tag{4}$$

where

$$a_k = \frac{B_1 d_k}{\omega_1}\left(1 - \frac{1}{\zeta_k^2}\right). \tag{5}$$

For higher values of ζ_k, $(1 - 1/\zeta_k^2) \approx 1$, and therefore, with the fundamental frequency normalized to $\omega_1 = 1$ the admittance $Y_a(s)$ can be approximated as

$$Y_a(s) \approx \frac{1}{s\,L_{1e}} + \sum_{k=1}^{K} \frac{s\,a_k}{\dfrac{s^2}{z_k^2} + 1}, \quad \text{where } a_k = d_k B_1. \tag{6}$$

Finally, the driving point admittance $Y_x(s)$ can be expressed as

$$Y_x(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{\sum_{k=0}^{K} y_k s^{2K}} \tag{7}$$

where the zeros of the polynomial $D(s)$ are the resonant frequency locations. Since the filter tuning frequencies should be selected prior to the reactive power allocation, the filter's

zeros, z_k, are fixed. Therefore, values of the coefficients y_k are determined only by the reactive power allocation.
For a two branch RHF

$$D(s) = s^4 + \frac{y_1}{y_2}s^2 + \frac{y_0}{y_2} \tag{8}$$

and

$$D(\omega) = \omega^4 - \frac{y_1}{y_2}\omega^2 + \frac{y_0}{y_2} \tag{9}$$

where

$$\begin{aligned}
y_2 &= L_{1e}L_s\left[\left(\frac{1}{L_{1e}} + \frac{1}{L_s}\right) + a_1 z_1^2 + a_2 z_2^2\right] \\
y_1 &= L_{1e}L_s\left[\left(\frac{1}{L_{1e}} + \frac{1}{L_s}\right)\left(z_1^2 + z_2^2\right) + \left(z_1 z_2\right)^2\left(a_1 + a_2\right)\right] \\
y_0 &= L_{1e}L_s\left[\left(\frac{1}{L_{1e}} + \frac{1}{L_s}\right)\left(z_1 z_2\right)^2\right]
\end{aligned} \tag{10}$$

so that the resonant frequencies ω_r can be obtained from the formula

$$\omega_r^2 = \frac{1}{2}\left(\frac{y_1}{y_2} \pm \sqrt{(\frac{y_1}{y_2})^2 - 4\frac{y_0}{y_2}}\right). \tag{11}$$

Because $a_1 + a_2 = B_1$, changing the reactive power allocation only effects the coefficient y_2. Also, $z_1 < z_2$ and, consequently, as a_2 increases and a_1 declines, the lower frequency pole p_1 will increase in value and the separation between the poles will decrease.
As shown by equation (7), the pole locations of three and four branch RHFs are given by the zeros of cubic and quartic polynomials respectively. Although there are formulas for the solution of cubic and quartic polynomials, the complexity is such that it is not possible to draw conclusions about the effect of the reactive power allocation on the resonant frequency locations. This adds another level of complexity to the trail and error method of design described in the previous section when more than two branches are needed.

IV. OPTIMIZATION OF FILTER EFFICIENCY

The filter efficiency might be improved if the fixed rules with respect to the reactive power allocation, i.e., selection of allocation coefficients, a_k, to the filter branches and their tuned frequencies, ω_k, are abandoned for a selection of parameters that minimizes the voltage and current distortion.

There are many different possibilities with respect to optimization techniques that could be used for the optimization of filter effectiveness in reduction of distortion. Some optimization methods were tested and found to give poor performance due to the complex behavior of the cost function. It was not always possible to reach a minimum point of the function without being relatively close to it. The Polak-Ribiere variation of the Fletcher-Reeves method was applied using outside penalty methods to approximate a constrained cost function. Although some good results were obtained using this method it still exhibited difficulty in reaching a local minimum in some cases due to the problem of ill-conditioning. Finally, to overcome the ill-conditioning problem the method of multipliers [15] was implemented. The method works well for this application and in all testing it was able to reach a constrained local minimum of the cost function even if the starting point was far away from that minimum.

For optimization procedures it is convenient to have a single measure of the performance of a filter with respect to attenuation of harmonics in the supply current and harmonics in the bus voltage. Such a measure can be constructed as follows. The distorted component of the supply current before a filter is installed, denoted i_{d0}, can be compared to the distorted component of the supply current after the installation of a harmonic filter. Such a performance coefficient with respect to a filter's effect on the supply current is referred to as the *effectiveness in reduction of current distortion*, defined in percent as

$$\varepsilon_i = (1 - \frac{\| i_d \|}{\| i_{d0} \|}) \times 100\,. \tag{12}$$

The maximum effectiveness that a filter can achieve is 100% which means that the distorted component of the supply current rms value, $||i_d||$, is reduced by the filter to zero.

The *effectiveness in reduction of voltage distortion* is a performance coefficient with respect to a filter's effect on the bus voltage distortion. It is defined in percent as

$$\varepsilon_u = (1 - \frac{\| u_d \|}{\| u_{d0} \|}) \times 100\,, \tag{13}$$

where $||u_d||$ is the distorted component of the bus voltage rms value after the filter is installed, and $||u_{d0}||$ is the distorted component of the bus voltage rms value before the filter is installed.

In order to utilize optimization methods to maximize filter effectiveness with respect to harmonic suppression, ε_i and ε_u should be maximized. This can be accomplished by the minimization of $||i_d||$ and $||u_d||$. Unfortunately, minimizing the voltage distortion at the bus and minimizing the supply current distortion are not equivalent tasks [17]. There has to be a tradeoff based on the requirements of a particular filter application. The rms values of the distorted component of the supply current and bus voltage can be combined into a linear form where each one is multiplied by a weighting coefficient. Such a linear form is expressed as

$$f(\mathbf{x}) = W_i \frac{\|i_d\|}{\|i_{d0}\|} + W_u \frac{\|u_d\|}{\|u_{d0}\|} . \quad (14)$$

where W_i is the weighting coefficient of the supply current distortion and W_u is the weighting coefficient of the bus voltage distortion. How the weighting is set determines whether the minimization technique effects the current or voltage distortion more strongly. Adjustment of filter parameters by an optimization routine may lead to change of the load reactive power compensation that is provided by the filter. However, in most cases it may not be reasonable to allow the compensation of the load reactive power to be reassigned to any value which minimizes $f(\mathbf{x})$. Therefore, a method of constrained optimization must be applied. Finally, a form that is more suitable to optimization algorithms [16] and that is equivalent with respect to the location of the minimum is

$$f_c(\mathbf{x}) = W_i \frac{\|i_d\|^2}{\|i_{d0}\|^2} + W_u \frac{\|u_d\|^2}{\|u_{d0}\|^2} . \quad (15)$$

To implement the method of multipliers an augmented Lagrangian was formed using the cost function (15) and the reactive power constraints. For the case of unity load reactive compensation the augmented Lagrangian is equal to

$$\begin{aligned} L_c(\mathbf{x}, \lambda, \boldsymbol{\mu}) &= f_c(\mathbf{x}) + \lambda h(\mathbf{x}) + \frac{1}{2} w |h(\mathbf{x})|^2 \\ &+ \frac{1}{2w} \sum_{i=1}^{K} \left\{ \left[\max\{0, \mu_i + w g_i(\mathbf{x})\} \right]^2 - \mu_i^2 \right\} \end{aligned} \quad (16)$$

The equality constraint $h(\mathbf{x})$ is

$$h(\mathbf{x}) = B_{f1} U_1^2 - Q_L = 0 \quad (17)$$

where B_{f1} is the filter susceptance at the fundamental frequency, which is a function of the variables $\mathbf{x}$, and Q_L is the load reactive power. The inequality constraints require that the n filter circuit elements be positive values. However, it is not necessary to constrain each filter circuit element separately. The susceptance of each k filter branch is capacitive at the fundamental frequency and can be expressed as

$$B_{k1} = \frac{\omega_1 C_k}{1 - \left(\frac{\omega_1}{\zeta_k} \right)^2} \quad (18)$$

and the inductance is related to the capacitance as

$$L_k = \frac{1}{(\zeta_k \omega_1)^2 C_k} \quad (19)$$

where $\zeta_k \omega_1$ is the branch tuned frequency and is always positive. If the susceptance is negative then by (18) the branch capacitance is negative which in turn yields a negative value of the branch inductance by (19). Thus, for a filter with K branches, there are K inequality constraints, and they can be expressed as

$$g_i(\mathbf{x}) = -B_{i1} \le 0 \quad (20)$$

For the case where there is a range of over or under-compensation of load reactive power no equality constraints are needed and the augmented Lagrangian becomes

$$L_c(\mathbf{x}, \boldsymbol{\mu}) = f_c(\mathbf{x}) + \frac{1}{2w} \sum_{i=1}^{K+2} \left\{ \left[\max\{0, \mu_i + w g_i(\mathbf{x})\} \right]^2 - \mu_i^2 \right\} . \quad (21)$$

The constraint $g_1(\mathbf{x})$ specifies the upper limit of the over-compensation and is equal to

$$g_1(\mathbf{x}) = -B_{f1} U_1^2 + c_1 Q_L \le 0 \quad (22)$$

where a multiplied by the reactive power of the load, Q_L, specifies the upper limit of the overcompensation. If the reactive power of the filter exceeds aQ_L then $g_1(\mathbf{x})$ will increase. The lower limit of under-compensation is specified by $g_2(\mathbf{x})$ and is equal to

$$g_2(\mathbf{x}) = B_{f1} U_1^2 - c_2 Q_L \le 0 . \quad (23)$$

where bQ_L specifies the lower limit. If the reactive power of the filter is lower than bQ_L, then $g_2(\mathbf{x})$ will increase. As previously the other K constraints are simply to ensure that filter circuit elements are positive, and they are also given by (20).

The method of multipliers requires the adjustment of the penalty weighting factor, w, similarly as for the penalty method. The weighting factor was updated according to

$$w_{k+1} = \gamma w_k \quad (24)$$

However, in this case there was no need to dynamically adjust the weighting as in the case of a penalty method. A constant value for the weight increase of γ=1.2 was used to obtain the results presented. The method converged to a constrained local minimum of $f_c(\mathbf{x})$ from any valid starting point.

V. COST FUNCTION BEHAVIOR

The cost function $f_c(\mathbf{x})$ has multiple local minima and maxima. When filter parameters are selected such that a resonant frequency approaches a harmonic frequency a sharp increase in the cost function value may occur. Consequently, local minima occur in a number of regions due to the presence of several resonant frequencies.

The cost function may be observed using a surface or a contour plot. Unfortunately, it is not possible to visualize more than three dimensions, therefore, consider a simplified example of a two branch conventional RHF. Assume that the filter zeros are fixed so that the cost function, $f_c(\mathbf{x})$, of the two branch filter is a function of two variables, the reactive power allocation of each branch, d_1 and d_2. The two-branch filter with branches tuned to the 5^{th}, 7^{th} order harmonics, is connected to a bus having a short circuit power 25 times higher than the load active power. The power factor for the fundamental frequency is λ_1=0.707. All inductors' q-factors are equal to 50 at the tuned frequency, and the reactance to resistance ratio of the supply is 10. The load generated current harmonics in percent of the fundamental are J_2 = 0.1%, J_3 = 5%, J_4 = 0.2%, J_5 = 17%, J_6 = 0.2%, J_7 = 11%, J_8 = 0.2%, J_9 = 5%. Distribution voltage distortion contains a uniform harmonic noise on the level of E_n = 0.1% of the fundamental up to n = 9. Minimization of the current distortion and bus voltage distortion are considered equally important, therefore, the weighting factors of equation (15), W_i and W_u, are equal to 0.5. The cost function scaled by a factor of 10 for convenience in plotting is

$$f_c(\mathbf{x}) = f(d_1, d_2) = 5\frac{\|i_d\|^2}{\|i_{d0}\|^2} + 5\frac{\|u_d\|^2}{\|u_{d0}\|^2}\ . \qquad (25)$$

Figure 2 shows the surface plot of the cost function, $f_c(\mathbf{x})$, drawn as a function of the reactive power allocation of each branch, d_1 and d_2, in percent of the total filter reactive power. Four local minima are visible in the plot within the range 0.1-0.9 of d_1 and d_2.

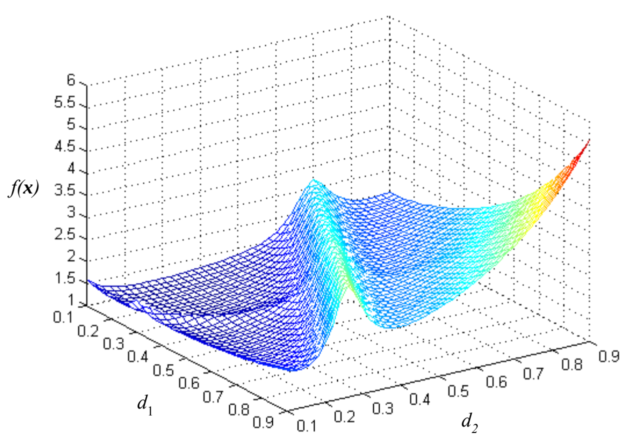

Figure 2. Surface plot of the cost function.

A contour plot of the cost function is shown in Figure 3. The plot shows the multiple local minima that are separated by the ridges formed when the resonant frequencies and harmonic frequencies coincide. The ridge that runs down the center of the plot is formed at the values of d_1 and d_2 for which a resonance is located at the 4^{th} order harmonic as shown in Fig. 4 (a).

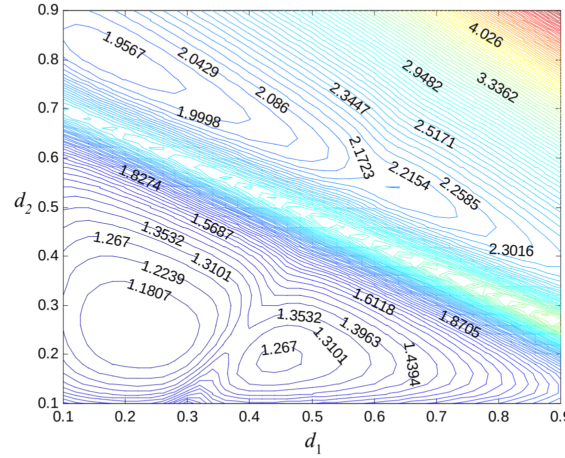

Figure 3. Contour plot of the cost function $f_c(\mathbf{x})$.

Figure 4 (b) shows the resonant frequency locations for values of d_1 and d_2 that correspond to a local minimum of $f_c(\mathbf{x})$. At this local minimum the resonant frequencies are much further from harmonic frequencies.

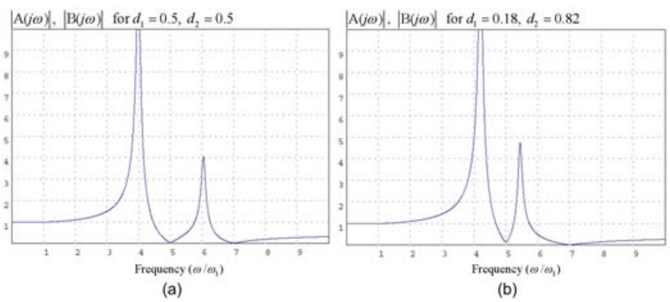

Figure 4. Resonant bands of amplification for values of d_1 and d_2 on a ridge (a) and near a minimum (b) of $f_c(\mathbf{x})$.

Although there are four local minima for the cost function, $f_c(\mathbf{x})$, this function is unconstrained. If full load reactive power compensation is required and the reactive power allocated to a particular branch cannot be lower than 10% of the filter reactive power, then the constrained problem is

$$\begin{aligned} &\text{minimize } f_c(\mathbf{x}) \\ &\text{subject to } g_1(\mathbf{x}) = d_1 + d_2 - 1 = 0 \\ &\qquad\qquad\quad g_2(\mathbf{x}) = d_1 - 0.1 \ge 0 \\ &\qquad\qquad\quad g_3(\mathbf{x}) = d_2 - 0.1 \ge 0 \end{aligned} \qquad (26)$$

The constrained cost function will yield a minimum which is confined to a line drawn from the top left corner to the lower right of the contour plot. The boundary of the plot shown is specified by all three constraints. In this case the minimum would be chosen from the best of those local minima that the constraint line intersects.

The presence of multiple minima indicates that a method of global optimization is needed. Since the mechanism that causes multiple local minima in the case of this cost function is known, it is possible to use the optimization techniques described above. These can be employed by repeated use of the routines at starting points near each local minimum. The local minima can then be compared and the global minimum identified.

It should be mentioned that although a particular local minimum may be the best with respect to the cost function value, it may not be acceptable from the viewpoint of sensitivity. A matrix sensitivity measure could be developed based on the behavior of the Hessian matrix for the region around the optimal point.

VI. PERFORMACE OF OPTIMIZED FILTERS

A distribution system that supplies a six-pulse controlled converter is used as a test system. The converter is supplied from a 60 Hz symmetrical three-phase distribution system with a short circuit power of 21.2 pu and with a reactance to resistance ratio at the fundamental frequency, X_s/R_s equal to 10. The distorted component of the load generated current, j, is composed of the characteristic harmonics of a six pulse converter with the RMS value J_5 = 18%, J_7 = 13%, J_{11} = 8%, and J_{13} = 7% of the fundamental. The load current also contains minor harmonics caused by the thyristor firing control asymmetry. The distribution of the minor harmonics in the load current is random, it varies from converter to converter and also with changes in the firing angle. It is assumed for this test system that the minor harmonics in the current have a uniform value up to the 12th order harmonic. This assumption is further justified by [14] which provides the current spectrum of a typical converter and shows minor harmonics which are of approximately the same magnitude. Harmonics above the 13th order are neglected since they cannot be amplified by filter resonance. It is assumed that minor harmonics comprise a distorted component of the load current, denoted as δ_{jm}, equal to 1.5% of the fundamental, i.e., of the value J_n=0.53% of the fundamental. The IEEE recommended limit for the distortion of the distribution voltage e, given in Table 2.2, is δ_e = 5% of the fundamental. Therefore, various levels of voltage distortion up to δ_e= 5% are used to evaluate filters for the range of allowed voltage distortion. The magnitude of the voltage harmonics are assumed to decline as $1/n$ and the even order harmonics have a magnitude which is 25% of the odd order harmonics. Effectiveness of filters designed according to three strategies for this test system is shown in Figure 5.

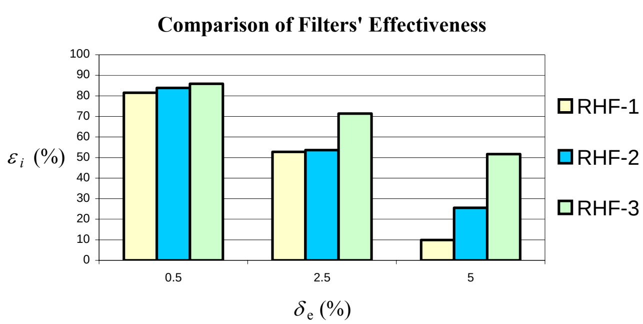

Figure 5. Comparison of effectiveness ε_i for the various RHF design strategies.

The first filter set, designated RHF-1, is designed according to Ref. [1]. The load reactive power compensation is shared equally by the four branches. The second filter set, designated RHF-2, represents the group of filters designed using a trial and error approach which is based on simulation. Unfortunately, results of such a strategy would vary according to the experience and design philosophies of the filter designer. Therefore, optimization with additional restrictions could be performed to simulate the best-case design using the trial and error approach. Literature that promotes this approach only suggests a slight de-tuning of the filter branches from the characteristic harmonic frequencies of the load current while all adjustments are performed on the branch reactive power allocation. Therefore, the set RHF-2 is designed by applying optimization techniques with the additional restriction that the filter's tuning frequencies may be only slightly de-tuned. The third filter set, designated RHF-3, represents the group of filters designed using the optimization approach described in this paper. Both the tuned frequencies and the branch reactive power allocation are selected by the optimization algorithm.

VII. CONCLUSIONS

Resonant harmonic filters designed according to traditional methods may have unacceptably low effectiveness when installed in systems with a dense harmonic spectrum. In order to cope with the complexity of the interaction of the filter with the system, optimization based design is needed. Results show that optimization based design substantially increases the effectiveness of filters in the presence of minor harmonics. Finally, although traditional optimization theory can be applied successfully to this application care must be taken due to the large number of local extrema and the generally ill-behaved nature of the cost function.

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