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Instantaneous Reactive Power p-q Theory and Power Properties of Three-Phase Systems

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Abstract - **The paper investigates how power phenomena and properties of three-phase systems are described and interpreted by the Instantaneous Reactive Power (IRP) p-q Theory.**

The paper demonstrates that this theory misinterprets power properties of electrical systems or provides some results that at least defy a common sense or meaning of some notions in electrical engineering. For example, it suggests the presence of an instantaneous reactive current in supply lines of purely resistive loads and the presence of an instantaneous active current in supply lines of purely reactive loads. Moreover, it suggests that line currents of linear loads with sinusoidal supply voltage contain a nonsinusoidal component.

The paper shows moreover that the IRP p-q Theory is not capable to identify power properties of three-phase loads instantaneously. A pair of instantaneous values of *p* and *q* powers does not allow us to conclude whether the load is resistive, reactive, balanced or unbalanced. It is known that a load imbalance reduces power factor. However, the IRP p-q Theory does not identify the load imbalance as the cause of power factor degradation.

Key words – **Powers, p-q theory; currents' physical components, CPC, switching compensators, active power filters, unbalanced systems, power theory.**

I. INTRODUCTION

The Instantaneous Reactive Power (IRP) p-q Theory is based on the Clarke Transform of voltages and currents in three-phase systems into α and β orthogonal coordinates. Its development was a response [3] to "*...the demand to instantaneously compensate the reactive power...*" Originally, this theory was formulated by Akagi, Kanazawa and Nabae [1, 2] for the active power filter control.

Power properties of three-phase systems are described by the IRP p-q Theory in two orthogonal α and β coordinates in terms of two, *p* and *q* instantaneous powers. They are referred to [3] as the instantaneous real and the imaginary powers or more commonly [4-6], as the instantaneous active and reactive powers. According to Authors' of Ref. [3] claim: "*…the instantaneous imaginary* (reactive) *power q was introduced on the same basis as the conventional real power p in three-phase circuits and then the instantaneous reactive power in each phase was defined with the focus on the physical meaning and the reason for naming…*" Because of it, the IRP p-q Theory has become a very attractive theoretical tool not only for the active power filter control [3-6], but also for analysis and identification [7-12, 18] of power properties of three-phase systems with nonsinusoidal voltages and currents.

As long as the IRP p-q Theory is considered only as a control algorithm then, the interpretation of power phenomena as suggested by this theory is irrelevant. It is sufficient for an algorithm to be acceptable that it enables us to reach the control objectives. However, to be considered as a power theory, the IRP p-q Theory should also satisfy other expectations. First, it should provide a credible interpretation of power properties and phenomena in power systems.

Power properties of three-phase systems are expressed by the IRP p-q Theory in terms of only two, active and reactive, *p* and *q*, powers, while power properties of such systems, even without any harmonic distortion, depend on three independent phenomena. These are: (i) permanent energy transmission, (ii) the voltage and current phase shift and (iii) line current asymmetry due to the load imbalance. These phenomena are characterized by the active, reactive, and unbalanced powers, *P*, *Q* and *D*. These powers specify [14] the apparent power:

$$S = \sqrt{P^2 + Q^2 + D^2}\ , \qquad (1)$$

and consequently, power factor, $\lambda = P/S$. The number of power quantities in the IRP p-q Theory can make one suspicious that this theory does not characterize power phenomena in three-phase unbalanced systems correctly. One might suggest that this major deficiency of the IRP p-q Theory could be overcome by combining the *Q* and *D* powers into a single power quantity. Unfortunately, this would be only an apparent solution. The reactive power *Q* is a measure of the effect of the current phase shift on the apparent power *S*, while the unbalanced power *D* is a measure of the effect of the load imbalance on this power. Thus, these two powers are associated with quite different phenomena in electrical loads.

The IRP p-q Theory has been developed for three-phase systems with nonsinusoidal voltages and currents. However, if a theory is true at distorted waveforms, it has to provide credible results also when applied to systems with sinusoidal voltages and currents, because such systems form a sub-set of systems with distorted waveforms. Therefore, this paper investigates only how does the IRP p-q Theory describe power properties of linear, three-phase, three-wire systems with sinusoidal voltages and currents. Results obtained using the IRP p-q Theory are compared with results obtained using the Currents' Physical Components (CPC) Power Theory, developed [13-15] for three-phase systems under nonsinusoidal conditions.

II. OUTLINE OF THE IRP p-q THEORY

To avoid any confusion between symbols used for the active current as defined in Ref. [16] and denoted by i_a, and the line current of phase "a", as well to be in accordance with symbols used in Refs. [13-15], the phase indices R, S and T are used in this paper, instead of a, b, and c, it means, voltages and currents are denoted as shown in Fig. 1.

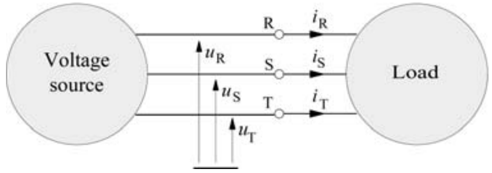

Fig. 1. Three-phase, three-wire system

The Clarke's Transform of phase voltages to the α and β coordinates has the form:

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix}. \quad (2)$$

It is assumed that line voltages (u_R, u_S, u_T) are referenced to an artificial zero, i.e., $u_R + u_S + u_T = 0$. At such a condition, the Clarke's Transform of phase voltages can be simplified to the form

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_R \\ u_S \end{bmatrix} = \boldsymbol{C} \begin{bmatrix} u_R \\ u_S \end{bmatrix}. \quad (3)$$

Similarly, in three-wire systems $i_R + i_S + i_T = 0$, thus, the Clarke's Transform of the line currents has the form

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_R \\ i_S \end{bmatrix} = \boldsymbol{C} \begin{bmatrix} i_R \\ i_S \end{bmatrix}. \quad (4)$$

With voltages and currents transformed to the α and β coordinates, the instantaneous active (real) power is defined, according to Ref. [3], as

$$p = u_\alpha i_\alpha + u_\beta i_\beta, \quad (5)$$

and the instantaneous reactive (imaginary) power as

$$q = u_\alpha i_\beta - u_\beta i_\alpha. \quad (6)$$

With these two, p and q, instantaneous powers, instantaneous active, and reactive currents are defined. The instantaneous active current, i_p, is defined in the α and β coordinates as

$$i_{\alpha p} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2} p, \qquad i_{\beta p} = \frac{u_\beta}{u_\alpha^2 + u_\beta^2} p, \quad (7)$$

and the instantaneous reactive current, i_q, in the α and β coordinates is defined as

$$i_{\alpha q} = \frac{-u_\beta}{u_\alpha^2 + u_\beta^2} q, \qquad i_{\beta q} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2} q. \quad (8)$$

Line currents can be obtained from currents in the α and β coordinates with the inverse Clarke's Transform:

$$\begin{bmatrix} i_R \\ i_S \end{bmatrix} = \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \boldsymbol{C}^{-1} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}, \quad (9)$$

thus, active and reactive currents in supply lines are equal to

$$\begin{bmatrix} i_{Rp} \\ i_{Sp} \end{bmatrix} = \boldsymbol{C}^{-1} \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix}, \qquad \begin{bmatrix} i_{Rq} \\ i_{Sq} \end{bmatrix} = \boldsymbol{C}^{-1} \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix}. \quad (10)$$

Illustration 1a. Let us apply the IRP p-q Theory to a circuit with a resistive load as shown in Fig. 2, assuming that the load is supplied from a symmetrical source of a sinusoidal, positive sequence voltage, with

$$u_R = \sqrt{2}\, U \cos \omega_1 t, \qquad U = 277 \text{ V}, \quad (11)$$

assuming that the Δ/Y transformer is a loss-less, ideal transformer with the turn ratio 1:1.

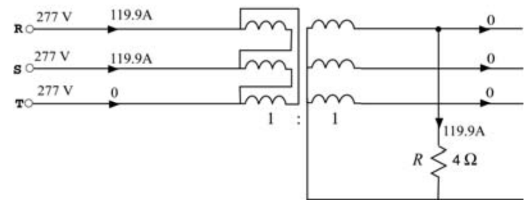

Fig. 2. Example of circuit with resistive load

With such assumptions, the Clarke's Transform of phase voltages results in

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \boldsymbol{C} \begin{bmatrix} \sqrt{2}\, U \cos\omega_1 t \\ \sqrt{2}\, U \cos(\omega_1 t - 120^o) \end{bmatrix} = \begin{bmatrix} \sqrt{3} U \cos\omega_1 t \\ \sqrt{3} U \sin\omega_1 t \end{bmatrix}. \quad (12)$$

The line currents,

$$i_R = \sqrt{2}\, I \cos(\omega_1 t + 30^0) = -\, i_S, \qquad I = 119.9 \text{ A}, \qquad i_T = 0,$$

could be transformed to

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \boldsymbol{C} \begin{bmatrix} i_R \\ -\, i_R \end{bmatrix} = \begin{bmatrix} \sqrt{3}\, I \cos(\omega_1 t + 30^o) \\ -I \cos(\omega_1 t + 30^o) \end{bmatrix}. \quad (13)$$

Thus, the instantaneous real power of such a load is equal to

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3} U I [1 + \cos 2(\omega_1 t + 30^0)], \quad (14)$$

and the instantaneous reactive power,

$$q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3} U I \sin 2(\omega_1 t + 30^0). \quad (15)$$

Since instantaneous reactive power, q, of the load is not equal to zero hence, according to the IRP p-q Theory, a reactive current occurs in supply lines. Its value in the α and β coordinates is equal to

$$i_{\alpha q} = \frac{-u_\beta}{u_\alpha^2 + u_\beta^2} q = I \sin 2(\omega_1 t + 30^0) \sin\omega_1 t, \quad (16)$$

$$i_{\beta q} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2} q = -I \sin 2(\omega_1 t + 30^0) \cos\omega_1 t. \quad (17)$$

Having these values, the inverse Clarke's Transform results in the reactive current in supply lines, namely

$$\begin{bmatrix} i_{Rq} \\ i_{Sq} \end{bmatrix} = I \sin 2(\omega_1 t + 30^0)\, \boldsymbol{C}^{-1} \begin{bmatrix} \sin\omega_1 t \\ -\cos\omega_1 t \end{bmatrix} = I \sqrt{\frac{2}{3}} \sin 2(\omega_1 t + 30^0) \begin{bmatrix} \sin\omega_1 t \\ \sin(\omega_1 t - 120^0) \end{bmatrix}. \quad (18)$$

One could expect that only an active current could occur in a purely resistive, linear circuit. However, according to the IRP p-q Theory, in spite of the lack of any reactive elements in the load and consequently, zero reactive power, Q, there is a reactive current in supply lines. It means that the reactive current in the IRP p-q Theory cannot be associated with the reactive power, Q, of the load. It occurs in supply lines even of purely resistive loads. It means that results of the IRP p-q Theory deny a common meaning of a reactive current. Also, observe that all currents in linear circuits with sinusoidal supply voltage are sinusoidal. However, when the IRP p-q Theory is applied for calculating the instantaneous active current of the load in the α and β coordinates, then

$$i_{\alpha p} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2}\, p \;=\; I\,[1 + \cos 2(\omega_1 t + 30^0)]\cos\omega_1 t\,, \qquad (19)$$

$$i_{\beta p} = \frac{u_\beta}{u_\alpha^2 + u_\beta^2}\, p \;=\; I\,[1 + \cos 2(\omega_1 t + 30^0)]\sin\omega_1 t\,. \qquad (20)$$

Having the active current in the α and β coordinates, its value in the load supply lines can be calculated, namely

$$\begin{bmatrix} i_{Rp} \\ i_{Sp} \end{bmatrix} = \boldsymbol{C}^{-1}\begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} = I\sqrt{\frac{2}{3}}\,[1+\cos 2(\omega_1 t+30^0)]\begin{bmatrix} \cos\omega_1 t \\ \cos(\omega_1 t - 120^0) \end{bmatrix}. \qquad (21)$$

This active current is not sinusoidal, however. Its waveform in line R can be expressed in the form

$$i_{Rp} = \frac{I}{\sqrt{6}}[2\cos\omega_1 t + \cos(\omega_1 t + 60^0) + \cos(3\omega_1 t + 60^0)]\,, \qquad (22)$$

thus, it contains the third order harmonic. Thus, the IRP p-q Theory suggests a phenomenon that does not exist in the circuit. This conclusion obtained from the IRP p-q Theory, also observed by Willems [17], is in an evident contradiction to a common meaning of the active current [16, 19] that was introduced to electrical engineering by Fryze in 1932. The active current is the current component proportional to the supply voltage and of the value indispensable for providing the load active power *P*. The active current is defined as

$$i_a = \frac{P}{||u||^2}\,u\,, \qquad (23)$$

where $||u||$ denotes the supply voltage rms value. Fryze's definition of the active current was generalized in Ref. [14] for three-phase systems as follows,

$$\begin{bmatrix} i_{Ra} \\ i_{Sa} \\ i_{Ta} \end{bmatrix} = \boldsymbol{i}_a = \frac{P}{||\boldsymbol{u}||^2}\boldsymbol{u} = \frac{P}{||\boldsymbol{u}||^2}\begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix}, \qquad (24)$$

where $||\boldsymbol{u}||$ denotes the rms value of the supply three-phase voltage, that means,

$$||\boldsymbol{u}|| = \sqrt{||u_R||^2 + ||u_S||^2 + ||u_T||^2}\,. \qquad (25)$$

Thus, the active current in three-phase systems with a sinusoidal supply voltage is sinusoidal.

The main properties of the active current, for which the concept of this current is so important for power theory and compensation, are summarized [17] by Willems: "*The active current is hence a current which yields the same average* (i.e. active) *power as the load current. It is the current with the smallest rms value having this property, and hence it realizes the lowest line losses and the largest power factor.*" The instantaneous active current that results from the IRP p-q Theory does not have any of these properties. Unlike the active current defined by Fryze, it is not the current that should remain in supply lines after total compensation of the load, that means to unity power factor. Indeed, the active current in the IRP p-q Theory has nothing in common with the meaning of the active current as it has been known in electrical engineering since 1932 when this concept was introduced by Fryze. It is a quite different quantity, but its name, i.e., "*the instantaneous active current*" may cause a confusion, especially since formula (24) also specifies the instantaneous values of the active current.

Illustration 2a. Let us apply the IRP p-q Theory to the circuit shown in Fig. 3 with a purely reactive load, assuming that it is supplied like the load in Illustration 1a, thus, with the voltage

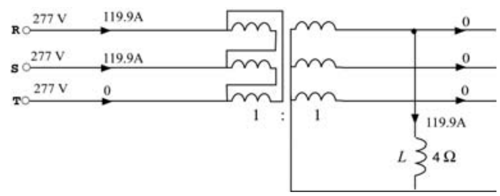

Fig. 3. Example of three-phase circuit with purely reactive load

in the α and β coordinates specified by formula (12). The line currents in such a circuit are equal to

$$i_R = \sqrt{2}\,I\cos(\omega_1 t - 60^0), \quad i_S = -\,i_R, \;\; i_T = 0, \;\; I = 119.2 \text{ A},$$

and could be transformed to

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \boldsymbol{C}\begin{bmatrix} \sqrt{2}\,I\cos(\omega_1 t - 60^o) \\ -\sqrt{2}\,I\cos(\omega_1 t - 60^o) \end{bmatrix} = \begin{bmatrix} \sqrt{3}\,I\cos(\omega_1 t - 60^o) \\ -\,I\cos(\omega_1 t - 60^o) \end{bmatrix}. \qquad (26)$$

Thus, the instantaneous real and imaginary powers are

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}\,U I\cos(2\omega_1 t - 30^0)\,. \qquad (27)$$

$$q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}\,U I\,[1 + \sin(2\omega_1 t - 30^0)]\,. \qquad (28)$$

In spite of zero active power, there is a non-zero active current in the circuit. Its value in the α and β coordinates is equal to

$$i_{\alpha p} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2}\,p = \frac{I}{2}[\cos(\omega_1 t - 30^0) + \cos(3\omega_1 t - 30^0)]\,, \qquad (29)$$

$$i_{\beta p} = \frac{u_\beta}{u_\alpha^2 + u_\beta^2}\,p = \frac{-I}{2}[\sin(\omega_1 t - 30^0) - \sin(3\omega_1 t - 30^0)]\,, \qquad (30)$$

and in the phase coordinates this current is equal to

$$\begin{bmatrix} i_{Rp} \\ i_{Sp} \end{bmatrix} = \boldsymbol{C}^{-1}\begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} = \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix} \times \begin{bmatrix} \frac{I}{2}[\cos(\omega_1 t - 30^0) + \cos(3\omega_1 t - 30^0)] \\ -\frac{I}{2}[\sin(\omega_1 t - 30^0) - \sin(3\omega_1 t - 30^0)] \end{bmatrix}. \qquad (31)$$

In particular, this current in line R is

$$i_{Rp} = \frac{I}{\sqrt{6}}[\cos(\omega_1 t - 30^0) + \cos(3\omega_1 t - 30^0)]\,. \qquad (32)$$

One could expect that only a reactive current could occur in a purely reactive, linear circuit. However, according to the IRP p-q Theory, in spite of the lack of any resistive elements in the load and consequently, zero active power, *P*, there is an active current in supply lines. It means that results of the IRP p-q Theory deny the common meaning of both the active and reactive currents. Moreover, these currents are nonsinusoidal even if there is no source of harmonics in the supply source and the load.

Let us compare these results obtained from the IRP p-q Theory with results of analysis of power properties of circuits considered in Illustrations 1a and 2a using the Currents' Physical Components (CPC) Power Theory.

III. OUTLINE OF THE CPC POWER THEORY

The Currents' Physical Components (CPC) Power Theory, developed in Ref. [13] for three-phase, three-wire systems under nonsinusoidal conditions, is used in this paper as a reference for analysis of properties of the IRP p-q Theory, when it is applied to systems under sinusoidal conditions. Therefore, only these elements of the CPC Theory that are needed for describing three-phase, three-wire systems under sinusoidal conditions are outlined in this Section.

A linear load as shown in Fig. 4, supplied with a symmetrical sinusoidal voltage, with line-to-line admittances $\boldsymbol{Y}_{\mathrm{RS}}$, $\boldsymbol{Y}_{\mathrm{ST}}$

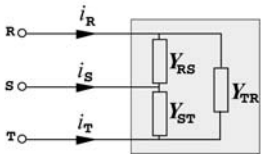

Fig. 4. Three-phase load

and $\boldsymbol{Y}_{\mathrm{TR}}$ can be characterized by equivalent admittance

$$\boldsymbol{Y}_{\mathrm{e}} = G_{\mathrm{e}} + jB_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}}\,, \tag{33}$$

and unbalanced admittance

$$\boldsymbol{A} = -\,(\boldsymbol{Y}_{\mathrm{ST}} + \alpha \boldsymbol{Y}_{\mathrm{TR}} + \alpha^{*}\boldsymbol{Y}_{\mathrm{TR}}), \qquad \alpha = 1e^{j120^{0}}\,. \tag{34}$$

If the complex rms (crms) values of symmetrical phase voltages of the supply are arranged into vectors

$$\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{U}_{\mathrm{S}} \\ \boldsymbol{U}_{\mathrm{T}} \end{bmatrix} = \boldsymbol{U}, \qquad \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{U}_{\mathrm{T}} \\ \boldsymbol{U}_{\mathrm{S}} \end{bmatrix} = \boldsymbol{U}^{\#}\,, \tag{35}$$

then, the line currents of the load, arranged into a vector

$$\boldsymbol{i} = \begin{bmatrix} i_{\mathrm{R}} \\ i_{\mathrm{S}} \\ i_{\mathrm{T}} \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{I}_{\mathrm{R}} \\ \boldsymbol{I}_{\mathrm{S}} \\ \boldsymbol{I}_{\mathrm{T}} \end{bmatrix} e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\,\boldsymbol{I}\,e^{j\omega_1 t}\,, \tag{36}$$

can be decomposed into three components, namely

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}\,, \tag{37}$$

where

$$\boldsymbol{i}_{\mathrm{a}} = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}}\,\boldsymbol{U}\,e^{j\omega_1 t}\}\,, \tag{38}$$

is the active current,

$$\boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re}\{\,jB_{\mathrm{e}}\,\boldsymbol{U}\,e^{j\omega_1 t}\,\}\,, \tag{39}$$

is the reactive current and

$$\boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\,\boldsymbol{U}^{\#}e^{j\omega_1 t}\,\}\,, \tag{40}$$

is the unbalanced current. These three components of the load line currents are associated with distinctive power related physical phenomena. Therefore, these currents are referred to as ***currents' physical components.***

The active, reactive and unbalanced current, $\boldsymbol{i}_{\mathrm{a}}$, $\boldsymbol{i}_{\mathrm{r}}$ and $\boldsymbol{i}_{\mathrm{u}}$, are mutually orthogonal [13] thus, rms values of these three currents fulfill the relationship

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2\,. \tag{41}$$

Multiplying this equation by the square of the supply voltage rms value, $||\boldsymbol{u}||$, the power equation

$$S^2 = P^2 + Q^2 + D^2\,, \tag{42}$$

can be obtained, where

$$Q = ||\boldsymbol{u}||\cdot||\boldsymbol{i}_{\mathrm{r}}||\,, \qquad D = ||\boldsymbol{u}||\cdot||\boldsymbol{i}_{\mathrm{u}}||\,, \tag{43}$$

are the reactive and unbalanced powers, respectively.

Let us apply the CPC Theory to circuits analyzed previously in Illustrations 1a and 2a using the IRP p-q Theory.

Illustration 1b. Since line-to-line admittances of the load are: $\boldsymbol{Y}_{\mathrm{RS}}$ = 0.25 S; $\boldsymbol{Y}_{\mathrm{ST}} = \boldsymbol{Y}_{\mathrm{TR}} = 0$, formula (33) results in the equivalent admittance,

$$\boldsymbol{Y}_{\mathrm{e}} = G_{\mathrm{e}} + jB_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}} = 0.25\ \mathrm{S}\,, \tag{44}$$

and formula (34) results in the unbalanced admittance

$$\boldsymbol{A} = -\,(\boldsymbol{Y}_{\mathrm{ST}} + \alpha\boldsymbol{Y}_{\mathrm{TR}} + \alpha^{*}\boldsymbol{Y}_{\mathrm{RS}}) = -\,\alpha^{*}\boldsymbol{Y}_{\mathrm{RS}} = 0.25\,e^{j60^{0}}\ \mathrm{S}\,, \tag{45}$$

thus, the active current of the load is equal to

$$\boldsymbol{i}_{\mathrm{a}} = \begin{bmatrix} i_{\mathrm{Ra}} \\ i_{\mathrm{Sa}} \\ i_{\mathrm{Ta}} \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}}\,\boldsymbol{U}\,e^{j\omega_1 t}\} = 69.2\,\sqrt{2}\begin{bmatrix} \cos\omega_1 t \\ \cos(\omega_1 t - 120^0) \\ \cos(\omega_1 t + 120^0) \end{bmatrix} \mathrm{A}. \tag{46}$$

Since equivalent susceptance B_{e} of the load considered is equal to zero, the reactive current does not occur in the load supply lines. The unbalanced current is equal to

$$\boldsymbol{i}_{\mathrm{u}} = \begin{bmatrix} i_{\mathrm{Ru}} \\ i_{\mathrm{Su}} \\ i_{\mathrm{Tu}} \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\,\boldsymbol{U}^{\#}e^{j\omega_1 t}\} = 69.2\sqrt{2}\begin{bmatrix} \cos(\omega_1 t + 60^0) \\ \cos(\omega_1 t + 180^0) \\ \cos(\omega_1 t - 60^0) \end{bmatrix} \mathrm{A}. \tag{47}$$

Indeed, the sum of the active and unbalanced currents is equal to the line current of the load, namely

$$\boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}} = 119.9\sqrt{2}\begin{bmatrix} \cos(\omega_1 t - 60^0) \\ \cos(\omega_1 t + 120^0) \\ 0 \end{bmatrix} \mathrm{A} = \boldsymbol{i}\,. \tag{48}$$

Thus, the supply current cannot contain any component other than the active and unbalanced currents. The rms value of three-phase active and unbalanced currents is equal to, respectively

$$||\boldsymbol{i}_{\mathrm{a}}|| = 69.2\sqrt{3} = 119.9\ \mathrm{A}\,, \quad ||\boldsymbol{i}_{\mathrm{u}}|| = 69.2\sqrt{3} = 119.9\mathrm{A}\,, \tag{49}$$

while the rms value of the three-phase supply current is

$$||\boldsymbol{i}|| = \sqrt{I_{\mathrm{R}}^2 + I_{\mathrm{S}}^2} = \sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2} = 169.6\ \mathrm{A}\,. \tag{50}$$

Since $||\boldsymbol{u}|| = 277\sqrt{3}$ V, the load powers are equal to

$$P = ||\boldsymbol{u}||\,||\boldsymbol{i}_a|| = 57.5 \text{ kW},$$
$$Q = ||\boldsymbol{u}||\,||\boldsymbol{i}_r|| = 0, \tag{51}$$
$$D = ||\boldsymbol{u}||\,||\boldsymbol{i}_u|| = 57.5 \text{ kVA},$$
$$S = ||\boldsymbol{u}||\,||\boldsymbol{i}|| = 81.3 \text{ kVA}.$$

These results are in an evident accordance with the fact that the load in Illustration 1 does not contain any reactive component and it is unbalanced. Unfortunately, the IRP p-q Theory does not have any power quantity associated with the load imbalance, although it contributes to the apparent power increase. Instead, the load imbalance causes, according to the IRP p-q Theory, that a reactive current occurs apparently in the supply current, even if the load has zero reactive power.

Illustration 2b. Since line-to-line admittances of the load are: $\boldsymbol{Y}_{RS} = - j¼$ S, $\boldsymbol{Y}_{ST} = \boldsymbol{Y}_{TR} = 0$, formula (33) results in the equivalent admittance,

$$\boldsymbol{Y}_e = G_e + jB_e = \boldsymbol{Y}_{RS} + \boldsymbol{Y}_{ST} + \boldsymbol{Y}_{TR} = -j\,0.25 \text{ S}, \tag{52}$$

and formula (34) results in the unbalanced admittance

$$\boldsymbol{A} = -(\boldsymbol{Y}_{ST} + \alpha\boldsymbol{Y}_{TR} + \alpha^*\boldsymbol{Y}_{RS}) = -\alpha^*\boldsymbol{Y}_{RS} = 0.25\,e^{-j30^0} \text{ S}. \tag{53}$$

Because equivalent conductance G_e is equal to zero, hence

$$\boldsymbol{i}_a = \begin{bmatrix} i_{Ra} \\ i_{Sa} \\ i_{Ta} \end{bmatrix} = \sqrt{2}\,\text{Re}\{G_e\,\boldsymbol{U}\,e^{j\omega_1 t}\} = 0, \tag{54}$$

thus, the active current does not occur in the supply current. The reactive current is

$$\boldsymbol{i}_r = \begin{bmatrix} i_{Rr} \\ i_{Sr} \\ i_{Tr} \end{bmatrix} = \sqrt{2}\,\text{Re}\{jB_e\,\boldsymbol{U}\,e^{j\omega_1 t}\} =$$
$$= 69.2\sqrt{2}\begin{bmatrix} \cos(\omega_1 t - 90^0) \\ \cos(\omega_1 t + 150^0) \\ \cos(\omega_1 t + 30^0) \end{bmatrix} \text{A}. \tag{55}$$

The unbalanced current is equal to

$$\boldsymbol{i}_u = \begin{bmatrix} i_{Ru} \\ i_{Su} \\ i_{Tu} \end{bmatrix} = \sqrt{2}\,\text{Re}\{\boldsymbol{A}\,\boldsymbol{U}^{\#}\,e^{j\omega_1 t}\} =$$
$$= 69.2\sqrt{2}\begin{bmatrix} \cos(\omega_1 t - 30^0) \\ \cos(\omega_1 t + 120^0) \\ \cos(\omega_1 t - 150^0) \end{bmatrix} \text{A}. \tag{56}$$

Indeed, the sum of the reactive and unbalanced currents is equal to the line current of the load, namely

$$\boldsymbol{i}_r + \boldsymbol{i}_u = 119.9\sqrt{2}\begin{bmatrix} \cos(\omega_1 t - 60^0) \\ \cos(\omega_1 t + 120^0) \\ 0 \end{bmatrix} \text{A} = \boldsymbol{i}. \tag{57}$$

Thus, the supply current cannot contain any component other than the active and unbalanced currents. The rms value of three-phase reactive and unbalanced currents is equal to, respectively,

$$||\boldsymbol{i}_r|| = 69.2\sqrt{3} = 119.9\text{A}, \quad ||\boldsymbol{i}_u|| = 69.2\sqrt{3} = 119.9\text{A}, \tag{58}$$

while the rms value of the three-phase supply current is

$$||\boldsymbol{i}|| = \sqrt{I_R^2 + I_S^2} = \sqrt{||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2} = 169.6 \text{ A}. \tag{59}$$

Since $||\boldsymbol{u}|| = 277\sqrt{3}$ V, the load powers are equal to

$$P = ||\boldsymbol{u}||\,||\boldsymbol{i}_a|| = 0,$$
$$Q = ||\boldsymbol{u}||\,||\boldsymbol{i}_r|| = 57.5 \text{ kVAr}, \tag{60}$$
$$D = ||\boldsymbol{u}||\,||\boldsymbol{i}_u|| = 57.5 \text{ kVA},$$
$$S = ||\boldsymbol{u}||\,||\boldsymbol{i}|| = 81.3 \text{ kVA}.$$

These results are in a full accordance with the fact that the load in Illustration 2 is purely reactive and unbalanced. However, according to the IRP p-q Theory, the load imbalance causes that an active current apparently occurs in the supply current.

IV. INSTANTANEOUS IDENTIFICATION OF POWER PROPERTIES

The attractiveness of the IRP p-q Theory in electrical engineering community is due to the opinion, expressed by authors of this Theory [3], that it enables an instantaneous identification of power properties of the load and consequently, instantaneous compensation of the reactive power. Let us check whether such an instantaneous identification is possible or not.

Indeed, the p and q powers could be known only with a delay needed for calculation, after samples of voltages and currents are provided by a measuring system. Does it mean, however, that power properties of the load are identified the same moment when these two powers are calculated?

To answer this question, let us return to results of Illustration 1a. According to formulae (14) and (15), instantaneous values of powers p and q taken at the instant of time such that $2(\omega_1 t_k + 30^0) = 90^0$, are mutually equal, with only the opposite sign, i.e., $p_k = -q_k$. According to formulae (27) and (28), such a situation that $p_k = -q_k$ can also occur in the circuit considered in Illustration 2a, namely, at such an instant of time that $2\omega_1 t_k - 30^0 = 0$. It means that the same pair of samples of the p and q powers can occur in a purely resistive and purely reactive loads. Thus, power properties of the load cannot be specified instantaneously in terms of the instantaneous active and reactive, p and q, powers. A sequence of measurements of the load voltages and currents, usually over period T, is needed to specify power properties of a load.

V. CONCLUSIONS

The paper demonstrates that the IRP p-q Theory, when applied to three-phase systems with sinusoidal voltages and currents, provides results that defy common comprehension of power properties of such systems. It is because the IRP p-q Theory suggests that

(i) An instantaneous reactive current can occur in supply lines of purely resistive loads.

(ii) An instantaneous active current can occur in supply lines of purely reactive loads.

(iii) A distorted current component can exist in supply currents of a linear load supplied with sinusoidal voltages.

Moreover, a pair of p and q powers calculated at some instant of time does not provide information on power properties of three-phase loads. No conclusions can be drawn with respect to the load power properties. The knowledge of these two powers does not allow us to answer the question: *is this load active, reactive, balanced or unbalanced?*

The main deficiency of the IRP p-q Theory results from the fact that three independent power phenomena, i.e., permanent energy transmission associated with the active power, P, phase-shift associated with reactive power, Q, and supply current asymmetry associated with unbalanced power, D, are described by the IRP p-q Theory in terms of only two powers, p and q. Two powers cannot characterize three independent power phenomena.

Misinterpretations of power phenomena in three-phase systems, caused by the IRP p-q Theory, could be to some degree attributed to names of currents, referred to as the instantaneous active and reactive currents, that were earlier used [16] in electrical engineering for currents defined in a different meaning. However, even if these names are modified, the main deficiency of this Theory will remain. The IRP p-q Theory does not provide any information as to reasons of the apparent power S increase and the power factor decline. Does this power increase due to the presence of reactive components in the load or due to its imbalance? It is an intrinsic deficiency of this Theory.

The IRP p-q Theory was developed for three-phase systems under nonsinusoidal conditions. Unfortunately, it has a number of deficiencies, as compiled above, even when it is applied to a relatively simple, sinusoidal situation, where the power phenomena are known. These deficiencies could be considered as irrelevant when the IRP p-q Theory is used as the fundamental for a switching compensator control algorithm, but not when it is considered as a power theory, it means, as a theoretical tool that explains power phenomena and properties of electrical systems.

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