IEEE Transactions on Power Delivery, Vol. 21, No. 1, Jan. 2006, pp. 339-344

Could Power Properties of Three-Phase Systems be Described in Terms of the Poynting Vector?

Leszek S. Czarnecki, Fellow IEEE

Abstract: The Poynting Vector (PV) and Poynting Theorem (PT) are fundamental mathematical tools for calculating energy flow, its dissipation and storage in electromagnetic fields. Therefore, there are opinions that power properties of power systems should be described in terms of the PV and PT. The paper investigates whether the PV and PT provide an explanation of power properties and whether they enable calculation of powers in three-phase systems or not.

It is demonstrated in the paper that only the instantaneous power and active power can be expressed in terms of the PV, but not the reactive, apparent and unbalanced powers as well as the power factor. Consequently, the PV and PT do not provide information useful for physical interpretations of power properties of power systems. Also, they are useless for practical applications of power theory to power system problems, such as compensation.

Key words: Power properties, unbalanced power, energy flow

I. INTRODUCTION

Power properties of power systems are specified in terms of various powers. These powers provide information needed for system design, evaluation of its performance, control, energy accounts or compensator design. In the case of three-phase three-wire systems under sinusoidal conditions, the active, reactive and the apparent powers form a basic set of power quantities. The unbalanced power has to be added to this set [8] in the case of the load imbalance. These powers are next used for defining power features of a second level such as, for example, power factor, installed power, or demanded power. More powers are needed for describing power properties of systems under nonsinusoidal conditions; with supply voltage asymmetry or systems with fast time-variant loads.

The set of power definitions, their relationship, physical interpretations, fundamentals of energy accounts and compensation are considered as a power theory of electrical systems. Its development started when it was observed that the apparent power, S, i.e., the product of the voltage and current RMS values, could be higher than the load active power, P.

The subject and objectives of power theory of electrical systems are not well defined. However, an overview of numerous publications on power theory shows that the area of interest and the main objectives of this theory could be compiled as follows:

(i) Explanation of power properties of power systems and in particular, explanation of physical phenomena that cause an increase of the apparent power above the active power of the load.

(ii) Definition of powers and specification of power related data needed for selecting power ratings and design of equipment for energy transmission and distribution.

(iii) Providing data on the effectiveness of energy delivery to customers, i.e., data on the load power factor. It is needed for fair energy accounts between energy suppliers and customers.

(iv) Providing fundamentals for power factor improvement by compensation of useless or harmful components of the supply current.

The Poynting Vector (PV) is a fundamental concept of electromagnetic field theory with respect to energy flow. It is used extensively in such problems as energy radiation by antennas and other radiating high frequency devices with distributed parameters. Therefore, in recent discussions [1, 2] on definitions and interpretations of powers in systems with nonsinusoidal voltages and currents, i.e., in discussions on power theory development, the PV is increasingly often referred to as the very fundamental of this theory. There are even suggestions that the Poynting Vector should be applied as a basis for power quality evaluation.

Electromagnetic fields are specified in terms of space distribution of electric and magnetic field intensities. The Poynting Vector is defined as their vector product, namely

$$\vec{\boldsymbol{P}} = \vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}} \,, \qquad (1)$$

where $\vec{\boldsymbol{E}}$ denotes the electric field intensity, while $\vec{\boldsymbol{H}}$ denotes the magnetic field intensity. The PV is interpreted as the surface density of the rate of energy flow. Its direction is perpendicular to the surface specified by vectors of the electric and magnetic fields intensities.

Suggestions that the Poynting Vector concept should be considered as the very fundamental of the power theory [3] stem from the fact that the flux of this vector to volume V through its surface S is equal to the rate of energy W flow to this volume, i.e., to the instantaneous power, $p(t) = dW/dt$, of all devices confined by the surface S. Indeed, let us assume that electric energy is delivered to a load exclusively by its three-phase, three-wire supply lines, with the supply terminals denoted by R, S and T, line-to-ground voltages u_R, u_S and u_T and line currents i_R, i_S and i_T, as shown in Fig. 1.

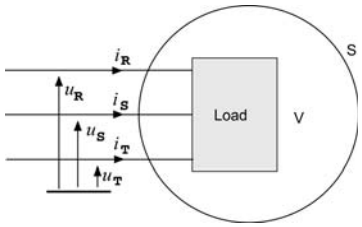

Fig.1. Three-phase load confined by surface S

The flux of the Poynting Vector through surface S, that crosses the supply lines only once, in such a case is equal to

$$\oiint_S \vec{\boldsymbol{P}} \bullet d\vec{\boldsymbol{S}} = u_R i_R + u_S i_S + u_T i_T = p(t)\ . \qquad (2)$$

Direct relation of the Poynting Vector to Maxwell equations, fundamental equations of electrical engineering, and to energy flow, its storage and dissipation in electromagnetic fields make the claim that power theory should be founded on the PV very appealing.

The objective of this paper is to investigate whether power theory could really be founded on the Poynting Vector or not. The answer to the question: *Could power properties of three-phase systems be interpreted and described in terms of the PV and the PT?* is of major importance for this investigation.

II. POWERS AND FLOW OF ENERGY

When power properties of electrical systems and power theory are discussed, the very first impression is that the energy flow is the main subject of this theory. Indeed, the instantaneous and the active powers, $p(t)$ and P, are clearly related to the flow of energy and these powers are the main quantities of power theory. However, a closer insight into issues of power theory shows that the main questions of this theory are not focused on the active power, but rather on the difference between this power and the apparent power.

One could say, that this difference, i.e., the reactive power in three-phase balanced systems, occurs because of oscillations of energy thus, it is related to the flow of energy. However, the instantaneous power, i.e., the rate of energy flow, $p(t) = dW/dt$, between a three-phase supply source and the load in three-phase balanced systems is constant, independently of the reactive power Q value. Thus, the presence of the reactive power in three-phase systems cannot be explained [4] in terms of energy oscillation between the supply source and the load. It could be even demonstrated [5] that also in single-phase circuits the reactive power can occur when loads are purely resistive, i.e., with no energy storage and no oscillation of energy between the supply and the load.

Consequently, the difference between the active and the apparent power, i.e., one of the major subjects of interest of power theory, cannot be explained in terms of energy flow. Moreover, there is no relationship between the apparent power S and energy flow. The apparent power S is a conventional quantity but not a physical quantity. This power in single-phase systems is defined as a product of voltage and current RMS values. There are a few different conventions with respect to the apparent power definition in three-phase systems. This means, that there is no physical phenomenon that is characterized by the apparent power. Long ago, in 1931, Fryze demonstrated [6] that the apparent power, S, can occur even in circuits with no flow of energy.

On the other hand, the PV and PT are focused entirely on the flow of energy, its storage and dissipation in electromagnetic fields. Therefore, the observation that the difference between the apparent and the active powers is not related to energy flow could make one suspicious about the suggestion that power theory could be founded on the Poynting Vector and the Poynting Theorem. The subject and objectives of power theory are different than only a description of energy flow in power systems.

III. CIRCUIT APPROACH AND FIELD APPROACH TO POWER THEORY

One could say that a field approach, i.e., the approach founded on the PV and PT, to power theory is more fundamental than that based on a circuit approach. Indeed, circuit analysis is only an approximation of electromagnetic field analysis, the approximation based on a concept of lumped parameters. Unfortunately, such a claim looks to be convincing only as long we do not have to use the field approach for detailed calculations.

Even now, in the age of computers, when powerful tools for modeling electric and magnetic fields are available, simplicity of calculations and analysis is still important. Computers do not supersede us in drawing intellectual conclusions. To support such a view, let us observe that the active power of a load, i.e., the mean value over period T of the instantaneous power, $p(t)$, can be found by calculation of the mean value of the Poynting Vector flux over the load boundary, S, namely

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T [\oiint_S \vec{\boldsymbol{P}} \bullet d\vec{\boldsymbol{S}}]\,dt\ , \qquad (3)$$

but first, the electric and magnetic field intensities on the surface S have to be calculated. However, in a case of three-phase loads the same result can be obtained much easier, having supply voltages and line currents of the load, when this power is calculated from formulae

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T (u_R i_R + u_S i_S + u_T i_T)\,dt\ . \qquad (4)$$

Using the field approach, the electric and magnetic field intensities on the surface S have to be calculated, while the flux of the PV does not even depend on the surface selected. Indeed, for common, not radiating loads, such as a motor supplied through a three-phase line as shown in Fig. 2, the flux of the PV has the same value for boundary S_1, S_2, or S_3, or for

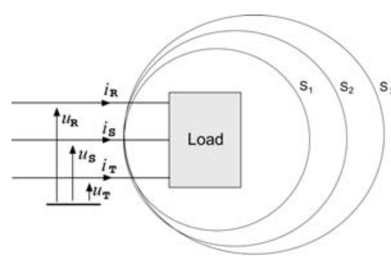

Fig.2. Three-phase loads and a few surfaces for the flux of the Poynting Vector calculation

any other boundary that confines the load. Only voltages and currents at the load terminals affect the instantaneous power, i.e., also the flux. Thus, there is no reasonable justification for a field approach to calculation of the active power. Only for a very simple geometry of the load and isotropic media of the space does the Poynting Vector flux provide the instantaneous power of the load with a reasonable amount of calculation. However, even in such a situation, calculating this flux could be considered only as an intellectual exercise for students rather than as a scientific approach to engineering problems. Geometry of power equipment is not simple and the medium is not isotropic and consequently, calculation of the active power using the Poynting Vector approach could be very toilsome without any visible benefits.

When a device or its element such as a screen, cover or a rotor is not supplied directly by conductors with distinctive voltages and currents, the Poynting Vector approach could provide a very useful, and perhaps even the only tool for energy flow, storage and dissipation analysis. The same is true for antennas and other energy radiating devices. However, when voltages and currents at the terminals of a device are known, and this device could be considered as a lumped RLC circuit, then the Poynting Vector approach seems to be entirely redundant from the computational point of view. Furthermore observe, that voltages and currents are the input quantities for power and energy meters.

IV. PHYSICAL INTERPRETATION

Even if a field approach to power theory would not be justified from computational point of view, the idea of founding this theory on the PV and PT could be acceptable, if the Poynting Vector and Theorem would be able to provide us with a physical interpretation of power phenomena in electrical systems. Therefore, let us focus our attention on the physical context of the PV and the PT and on their cognitive merits.

The Poynting Theorem has the mathematical form

$$\int_{\mathrm{V}} \vec{\boldsymbol{E}}\bullet\vec{\boldsymbol{j}}\,dV = -\int_{\mathrm{V}} (\vec{\boldsymbol{H}}\bullet\frac{\partial\vec{\boldsymbol{B}}}{\partial t} + \vec{\boldsymbol{E}}\bullet\frac{\partial\vec{\boldsymbol{D}}}{\partial t})dV - \oiint_{\mathrm{S}} (\vec{\boldsymbol{E}}\times\vec{\boldsymbol{H}})\bullet d\vec{\boldsymbol{S}}\,, \quad (5)$$

where $\vec{\boldsymbol{B}}$, $\vec{\boldsymbol{D}}$ and $\vec{\boldsymbol{j}}$ denote vectors of the magnetic and electric fields induction densities and the current surface density, respectively. This Theorem says that energy that enters volume V confined by surface S is dissipated over that volume and is stored in electric and magnetic fields. This is true, but this truth is now rather trivial. It has been a common knowledge in electrical engineering for a century. Thus, cognitive merits of the Poynting Theorem are not impressive.

This theorem also says how to calculate the rate of energy dissipation and storage in the volume V, but again, only when this volume is geometrically simple and the medium isotropic, are these calculations not very toilsome. For a common power system, which is geometrically very complex and medium is anisotropic, calculation of the rate of energy dissipation and storage is much more efficient when such a system is considered as a system with lumped RLC parameters.

One could say, as it is in the case of Ref. [1, 2], that the Poynting Vector provides us with an important physical interpretation relevant to energy flow, as a local surface density, *dp/dS*, of the instantaneous power *p*(*t*) flux, i.e., it informs us how energy enters volume V. However, such an interpretation could be questionable in some situations.

Consider for example a situation illustrated in Fig. 3, where the electric field is created by a charged capacitor while the magnetic field is created by a permanent magnet.

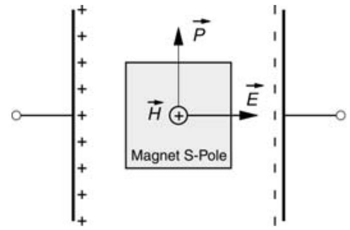

Fig. 3. Situation where there is no energy flow, while the Poynting Vector is not equal to zero

There is no, for sure, energy flow in such a situation, although the PV is not equal to zero. Thus, it seems that we cannot interpret the PV according to its common interpretation as a surface power density in such a situation. An expertise in electrodynamics is needed for clarification of doubts demonstrated by the above example. Could we expect that a power engineer that should understand power phenomena in power systems would have a sufficient expertise in electromagnetic fields and vector analysis needed for comprehension of these kinds of paradoxes?

The Poynting Vector describes an energetic aspect of the mutual dependence of electric and magnetic fields during propagation of electromagnetic waves specified by Maxwell equations. There is a common source of the electric and magnetic fields when an electromagnetic field propagates energy. However, there are numerous situations in power systems where sources of magnetic fields are independent of sources of electric fields. Motors with permanent magnets, DC motors, synchronous generators are just a few examples. It is up to experts on electrodynamics, not power engineers to interpret the relationship between the Poynting Vector and surface power density in such situations. However, even if this relationship could be interesting from the point of view of energy flow studies, it is irrelevant for power properties at the load terminals. These properties are specified in terms of powers. Therefore, the relationship between powers and the PV is crucial for answering the question: *could power properties of power systems be described in terms of the Poynting Vector and could power theory of such systems be founded on this Vector?*

V. CURRENTS' PHYSICAL COMPONENTS (CPC)

To demonstrate that power theory cannot be founded on the Poynting Vector, this theory does not have to be considered in its full complexity. It is enough to show that this is not possible for power theory of three-phase, three-wire systems as shown in Fig. 4, under sinusoidal conditions.

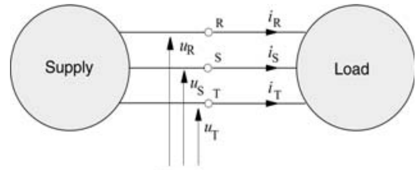

Fig. 4. Three-phase three-wire system

It is a common structure of a power system used for energy delivery and could be considered as a major subset of all three-phase systems. Any conclusion drawn with respect to the Poynting Vector approach and power theory of three-phase systems under more complex conditions, such as for example, under nonsinusoidal conditions, has to be true when such a conclusion is applied to this particular subset.

The power theory of three-phase, three-wire systems under nonsinusoidal conditions, referred to as the Currents' Physical Components (CPC) power theory, was presented in paper [7]. It could be drafted shortly for sinusoidal conditions [8] as follows.

Any three-phase load in the situation considered has the equivalent circuit shown in Fig. 5.

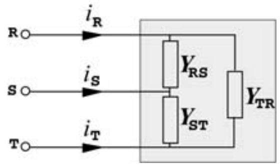

Fig. 5. Equivalent circuit of three-phase, static linear load

It can be characterized by two admittances. Namely, by the equivalent admittance

$$\boldsymbol{Y}_{\mathrm{e}} = G_{\mathrm{e}} + jB_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}}\ , \tag{6}$$

and the unbalanced admittance

$$\boldsymbol{A} = Ae^{j\psi} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha\boldsymbol{Y}_{\mathrm{TR}} + \alpha^{*}\boldsymbol{Y}_{\mathrm{RS}}),\quad \alpha = 1e^{j\frac{2\pi}{3}}\ . \tag{7}$$

Supply voltages and line currents in three-phase, three-wire systems can be arranged into three-phase vectors

$$\boldsymbol{u} = \begin{bmatrix} u_{\mathrm{R}} \\ u_{\mathrm{S}} \\ u_{\mathrm{T}} \end{bmatrix},\qquad \boldsymbol{i} = \begin{bmatrix} i_{\mathrm{R}} \\ i_{\mathrm{S}} \\ i_{\mathrm{T}} \end{bmatrix}, \tag{8}$$

and presented in a compact form

$$\boldsymbol{u} = \begin{bmatrix} u_{\mathrm{R}} \\ u_{\mathrm{S}} \\ u_{\mathrm{T}} \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{U}_{\mathrm{S}} \\ \boldsymbol{U}_{\mathrm{T}} \end{bmatrix} e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\,\boldsymbol{U}e^{j\omega_1 t}\ , \tag{9}$$

$$\boldsymbol{i} = \begin{bmatrix} i_{\mathrm{R}} \\ i_{\mathrm{S}} \\ i_{\mathrm{T}} \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{I}_{\mathrm{R}} \\ \boldsymbol{I}_{\mathrm{S}} \\ \boldsymbol{I}_{\mathrm{T}} \end{bmatrix} e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\,\boldsymbol{I}e^{j\omega_1 t}\ . \tag{10}$$

Symbols $\boldsymbol{U}$ and $\boldsymbol{I}$ in these formulae denote complex RMS (CRMS) values of phase voltages and currents. Three-phase vectors can be characterized by a *three-phase RMS value*. For quantity $\boldsymbol{x}(t)$ this three-phase RMS value is defined as

$$||\boldsymbol{x}|| = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\,\boldsymbol{x}(t)\,dt} = \sqrt{X_{\mathrm{R}}^2 + X_{\mathrm{S}}^2 + X_{\mathrm{T}}^2}\ , \tag{11}$$

where X denotes the RMS value of phase quantities.

Having admittances $\boldsymbol{Y}_{\mathrm{e}}$ and $\boldsymbol{A}$, the three-phase vector of the line currents can be decomposed into three components, namely

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}\ , \tag{12}$$

If the supply voltage of the load is sinusoidal, symmetrical of the positive sequence and the CRMS values of the supply voltage, $\boldsymbol{U}_{\mathrm{R}}$, $\boldsymbol{U}_{\mathrm{S}}$, and $\boldsymbol{U}_{\mathrm{T}}$, are arranged into three-phase vectors

$$\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{U}_{\mathrm{S}} \\ \boldsymbol{U}_{\mathrm{T}} \end{bmatrix} = \boldsymbol{U},\qquad \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{U}_{\mathrm{T}} \\ \boldsymbol{U}_{\mathrm{S}} \end{bmatrix} = \boldsymbol{U}^{\#}\ , \tag{13}$$

then, the line current's components in formula (12) are defined as follows

$$\boldsymbol{i}_{\mathrm{a}} = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}}\ \boldsymbol{U}\ e^{j\omega_1 t}\}\ , \tag{14}$$

is the active current,

$$\boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re}\{\,jB_{\mathrm{e}}\ \boldsymbol{U}e^{j\omega_1 t}\}\ , \tag{15}$$

is the reactive current and

$$\boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\ \boldsymbol{U}^{\#}e^{j\omega_1 t}\}\ , \tag{16}$$

is the unbalanced current. Since equivalent conductance G_{e} in formulae (6) and (14) is equal to

$$G_{\mathrm{e}} = \frac{P}{||\boldsymbol{u}||^2}\ , \tag{17}$$

the active current $\boldsymbol{i}_{\mathrm{a}}$ occurs in supply lines only if there is a permanent flow of energy to the load, i.e., if its active power P is not equal to zero.

Equivalent susceptance B_{e} in formulae (6) and (15) is equal to

$$B_{\mathrm{e}} = -\frac{Q}{||\boldsymbol{u}||^2}\ , \tag{18}$$

thus, the reactive current $\boldsymbol{i}_{\mathrm{r}}$ occurs in supply lines only if the load has a non-zero reactive power Q, i.e., only if there is a phase-shift between the supply voltage and the line current.

The unbalanced current $\boldsymbol{i}_{\mathrm{u}}$ occurs in supply lines only if unbalanced admittance $\boldsymbol{A}$ in formula (16) is not equal to zero, i.e., when line-to-line admittances $\boldsymbol{Y}_{\mathrm{RS}}$, $\boldsymbol{Y}_{\mathrm{ST}}$ and $\boldsymbol{Y}_{\mathrm{TR}}$ of the load are not equal and consequently, there is asymmetry of the line currents.

Components $\boldsymbol{i}_{\mathrm{a}}$, $\boldsymbol{i}_{\mathrm{r}}$ and $\boldsymbol{i}_{\mathrm{u}}$ of the line currents occur due to distinctive physical phenomena, namely, permanent flow of energy, current phase-shift and the line current asymmetry. Therefore, these components are referred to as *current's physical components*. These three currents are mutually orthogonal and consequently, their RMS values fulfill the relationship

$$||\boldsymbol{i}\,||^2 = ||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2\ , \tag{19}$$

where

$$||\boldsymbol{i}_{\mathrm{a}}|| = G_{\mathrm{e}}\,||\boldsymbol{u}||\ ,\quad ||\boldsymbol{i}_{\mathrm{r}}|| = |B_{\mathrm{e}}|\,||\boldsymbol{u}||\ ,\quad ||\boldsymbol{i}_{\mathrm{u}}|| = A\,||\boldsymbol{u}||\ . \tag{20}$$

The power equation of such a system is obtained by multiplication of eqn. (19) by the square of the supply voltage RMS value, $||\boldsymbol{u}||$. It has the form

$$S^2 = P^2 + Q^2 + D^2\ . \tag{21}$$

In this equation

$$P = ||\boldsymbol{u}||\cdot||\boldsymbol{i}_{\mathrm{a}}|| = G_{\mathrm{e}}\,||\boldsymbol{u}||^2,\quad Q = \pm\,||\boldsymbol{u}||\cdot||\boldsymbol{i}_{\mathrm{r}}|| = -B_{\mathrm{e}}\,||\boldsymbol{u}||^2, \tag{22}$$

are the active and reactive powers, while

$$D = ||\boldsymbol{u}||\cdot||\boldsymbol{i}_{\mathrm{u}}|| = A||\boldsymbol{u}||^2\ , \tag{23}$$

is the unbalanced power of the load.

Observe, that the apparent power S in power equation (21) is not defined according to conventional definitions as

$$S = U_R I_R + U_S I_S + U_T I_T\,, \tag{24}$$

or

$$S = \sqrt{P^2 + Q^2}\,. \tag{25}$$

It is defined as a product of three-phase RMS values of the supply voltage and the line current,

$$S = ||\boldsymbol{u}||\cdot||\boldsymbol{i}||. \tag{26}$$

It was demonstrated in paper [5] that definitions (24) and (25) when applied to unbalanced systems, do not result in a correct value of the apparent power of the supply and the load power factor, $\lambda = P/S$.

Only the active, reactive and unbalanced currents have clear physical interpretations in the CPC power theory. Reactive and unbalanced powers, like the apparent power S in single-phase systems, are defined only as products of voltage and current RMS values. Nonetheless, even these powers are associated with distinctive power phenomena in the system.

VI. POWERS AND THE POYNTING VECTOR

If indeed the PV and PT approach can be considered as fundamental for power theory, the PV should provide information on powers and on their relation specified by the power equation. Thus, let us try to answer the question: *could powers in three-phase systems be expressed in terms of the Poynting Vector?* An affirmative answer to this question is crucial for the thesis that power theory could be based on the Poynting Vector.

To answer this question, let us consider a balanced load shown in Fig. 6, supplied with a symmetrical sinusoidal voltage of the positive sequence, with line R-to-ground voltage $u_R = \sqrt{2}\,U\cos\omega t$.

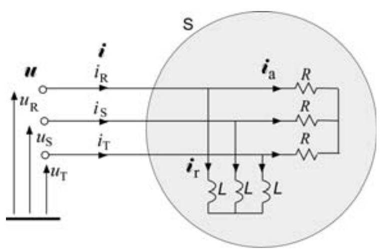

Fig. 6. Balanced RL load

The unbalanced current of such a load, $\boldsymbol{i}_u = 0$, thus, the flux of the Poynting Vector through surface S is equal to the active power. It means, it is constant, because

$$\oiint_S \vec{\boldsymbol{P}} \bullet d\vec{\boldsymbol{S}} = p(t) = \boldsymbol{u}^T \boldsymbol{i} = \boldsymbol{u}^T(\boldsymbol{i}_a + \boldsymbol{i}_r) = \boldsymbol{u}^T \boldsymbol{i}_a = P\,, \tag{27}$$

since

$$\boldsymbol{u}^T\boldsymbol{i}_r = \sqrt{2}\,U \begin{bmatrix} \cos\omega t \\ \cos(\omega t - 120^0) \\ \cos(\omega t + 120^0) \end{bmatrix}^T \sqrt{2}\,I_r \begin{bmatrix} \sin\omega t \\ \sin(\omega t - 120^0) \\ \sin(\omega t + 120^0) \end{bmatrix} = \tag{28}$$

$$= U I_r\,[\sin 2\omega t + \sin(2\omega t + 120^0) + \sin(2\omega t - 120^0)] \equiv 0.$$

The same value of the Poynting Vector flux is obtained, of course, when the load is purely resistive, as shown in Fig. 7.

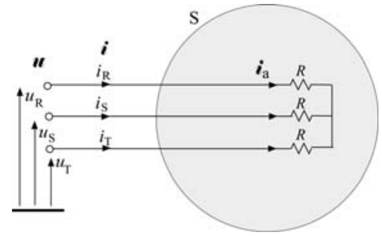

Fig. 7. Balanced resistive load

It means that the loads in Fig. 6 and in Fig. 7, different with respect to reactive and apparent powers, Q and S, and power factor λ, cannot be distinguished in terms of the Poynting Vector flux through the surface S. Thus, there is no relationship between the flux of the Poynting Vector through a surface that encloses a three-phase load and the reactive power Q of such a load.

In single-phase systems under sinusoidal conditions, when the supply voltage and current are specified in terms of complex RMS values $\boldsymbol{U}$ and $\boldsymbol{I}$, the notion of the complex apparent power can be introduced. It is defined as

$$\boldsymbol{S} = S e^{j\varphi} = \boldsymbol{U}\boldsymbol{I}^* = P + jQ\,. \tag{29}$$

Using such an approach, the electric and magnetic field intensities and consequently, the Poynting Vector, could be specified as complex vectors. The flux of the imaginary part of the Poynting Vector is equal to the reactive power thus, there is a relation between this vector and the reactive power, Q. Unfortunately, all attempts aimed at application of such an approach to nonsinusoidal systems or three-phase unbalanced systems have failed. Such attempts have resulted in erroneous definitions of reactive and apparent powers and in erroneous power equations. Moreover, such an approach did not reveal the presence of the unbalanced power, D, in the power equation. Therefore, there is no ground for extrapolation of the relationship between the Poynting Vector and the reactive power, valid in single-phase sinusoidal systems, to a similar relationship in nonsinusoidal and three-phase unbalanced systems.

Also, observe that the presence of a non-zero Poynting Vector around a conductor does not indicate that energy delivery is associated with such a conductor. The magnetic field intensity $\vec{\boldsymbol{H}}$ around the R conductor in the circuit shown in Figure 8 depends on the conductors' geometrical configuration and could be of the same order as to the magnitude as that

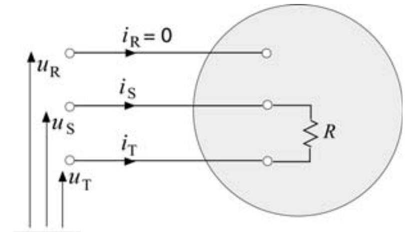

Fig. 8. Unbalanced resistive load

around the remaining S and T conductors and it is perpendicular to the electric field intensity $\vec{\boldsymbol{E}}$ between supply lines. Thus, the Poynting Vector depends on the supply voltage u_R and could have any value, without affecting the energy flow, dependent only on the voltage difference $u_S - u_T$ and the load resistance R. Nothing changes with respect to this flow when the voltage u_R changes. Energy flow depends only on the total flux of the Poynting Vector through the surface that encloses the load, but this is only the conventional instantaneous power $p(t)$ of the load and nothing else. In particular, there is no relation of the Poynting Vector with the unbalanced power, D, the only power that, apart from the active power, P, characterizes the load shown in Figure 8.

It should be noticed at this point that there is little hope that the apparent, reactive and unbalanced powers could be related to the Poynting Vector. To justify such an opinion, let us consider, for example, the apparent power. According to conclusions presented in papers [5] and [7], the apparent power in three-phase, three-wire systems should be defined by formula (26), i.e., as the product of RMS values of three-phase voltage and current vectors. For systems with sinusoidal voltages and currents, this formula can be simplified to the form

$$S = \sqrt{U_R^2 + U_S^2 + U_T^2} \bullet \sqrt{I_R^2 + I_S^2 + I_T^2} \ . \tag{30}$$

Let us suppose that the supply source is ideal, i.e., the supply voltages are independent of the supply current. In such a case, one of two factors that specify the apparent power

$$||\boldsymbol{u}|| = \sqrt{U_R^2 + U_S^2 + U_T^2} \ , \tag{31}$$

is independent of the magnetic field intensity, $\vec{\boldsymbol{H}}$, thus it is independent of the Poynting Vector, $\vec{\boldsymbol{P}}$. It means that the apparent power cannot be expressed in terms of the Poynting Vector. Similarly, the reactive and unbalanced powers also cannot be expressed in terms of the PV. Therefore, the flux of the Poynting Vector over a load boundary, which provides only information on the load instantaneous power, does not provide information on the apparent, reactive and unbalanced powers of three-phase loads.

VII. CONCLUSIONS

The suggestion that power theory should be founded on the Poynting Vector was analyzed in this paper. It was concluded from this analysis that the PV does not provide any information on power properties and power related phenomena in power systems other than on the instantaneous and the active powers. The apparent, reactive and unbalanced powers cannot be expressed in terms of the PV. Consequently, power theory cannot be founded on the concept of the Poynting Vector and its properties. Also, the Poynting Vector and its flux do not provide any information useful for interpretation of power properties of electric circuits and for practical applications of power theory in electrical systems.

The Poynting Theorem and the Poynting Vector are fundamental mathematical tools for calculating energy flow, its storage and dissipation in electromagnetic fields. Their applications in various areas of electrical engineering are countless. However, the PV and PT fail to contribute to the power theory, mainly because this theory is based not only on physical phenomena, such as energy flow, but also on conventional quantities, such as the apparent power or the power factor. The difference between the active and apparent powers cannot be related and explained in terms of the Poynting Vector. The same is true for reduction of this difference by means of a compensator.

REFERENCES

[1]. Z. Cekareski and A.E. Emanuel, (2001) "Poynting Vector and the power quality of electric energy," *European Trans. on Electrical Power, ETEP*, vol. 11, no. 6, pp. 375-382.

[2]. Z. Cekareski and A.E. Emanuel, (1999) "On the physical meaning of nonactive powers in three-phase systems," *Power Engineering Review, IEEE*, vol.19, no.7, pp. 46-47.

[3]. A. Ferrero, S. Leva, A.P. Morando (2001) "An approach to the non-active power concept in terms of the Poynting-Park Vector." *European Trans. on Electrical Power, ETEP*, vol. 11, no. 5: 301-308.

[4]. L.S. Czarnecki, (1994) "Misinterpretations of some power properties of electric circuits," *IEEE Trans. on Power Delivery*, vol. 9, no. 4, pp. 1760-1770.

[5]. L.S. Czarnecki, (1999) "Energy flow and power phenomena in electric circuits: illusions and reality. *Archiv für Elektrotechnik*," (82), no. 4, pp. 10-15.

[6]. S. Fryze, (1932) "Active, reactive and apparent powers in electrical circuits with distorted voltage and current waveforms." *ETZ*, no. 7: 193-203, no. 8: 225-234, no. 22, pp. 673-676.

[7]. L.S. Czarnecki, (1988) "Orthogonal decomposition of the current in a three-phase non-linear asymmetrical circuit with nonsinusoidal voltage," *IEEE Trans. IM.*, Vol. IM-37, No. 1, pp. 30-34.

[8]. L.S. Czarnecki, (1995) "Power related phenomena in three-phase unbalanced systems," *IEEE Trans. on Power Delivery*, Vol. 10, No. 3, pp. 1168-1176.

[9]. L.S. Czarnecki, (2000) "Harmonics and power phenomena," *Wiley Encyclopedia of Electrical and Electronics Engineering*, John Wiley & Sons, Inc., Supplement 1, pp. 195-218.

[10]. L.S. Czarnecki, (2004) "On some misinterpretations of the Instantaneous Reactive Power p-q Theory," *IEEE Trans. on Power Electronics*, Vol. 19, No.3, pp. 828-836.

BIOGRAPHY

Leszek S. Czarnecki (F'96), Alfredo M. Lopez Distinguished Professor, received the M.Sc. and Ph.D. degrees in electrical engineering and Habil. Ph.D. degree from Silesian University of Technology, Gliwice, Poland, in 1963, 1969 and 1984, respectively, where he was employed as an Assistant Professor. Beginning in 1984 he worked for two years at the Power Engineering Section, Division of Electrical Engineering, National Research Council (NRC) of Canada as a Research Officer. In 1987 he joined the Electrical Engineering Dept. at Zielona Gora University of Technology, Poland. In 1989 Dr. Czarnecki joined the Electrical and Computer Engineering Dept. at Louisiana State University, Baton Rouge, where he is a Professor of Electrical Engineering now. His research interests include network analysis and synthesis, power phenomena in nonsinusoidal systems, compensation and supply quality improvement in such systems.

For developing a power theory of three-phase nonsinusoidal unbalanced systems and methods of compensation of such systems, Dr. Czarnecki was elected to the grade of Fellow IEEE in 1996.