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Currents' Physical Components (CPC) concept: a fundamental of Power Theory

Abstract. *The paper introduces Readers to currently the most advanced power theory of electric systems with nonsinusoidal voltages and currents, along with fundamentals of their compensation, based on the concept of Currents' Physical Components (CPC). It includes single-phase systems and unbalanced three-phase systems with linear, time-invariant (LTI) and harmonic generating loads (HGLs).*

Streszczenie. *Artykuł jest wprowadzeniem do najbardziej obecnie zaawansowanej teorii mocy systemów elektrycznych z niesinusoidalnymi przebiegami prądu i napięcia, wraz z podstawami ich kompensacji, opartej na koncepcji Składowych Fizycznych Prądu (Currents' Physical Components – CPC). Obejmuje on układy jednofazowe oraz niezrównoważone układy trójfazowe, zasilane trójprzewodowo, z liniowymi odbiornikami stacjonarnymi (LTI) oraz z odbiornikami generującymi harmoniczne prądu (HGL).*

Keywords: power definitions, reactive power, scattered power, unbalanced power, harmonic generating power, filtry mocy.
Słowa kluczowe: definicje mocy, moc bierna, moc rozrzutu, moc generowana harmonicznych, moc niezrównoważenia, active filters.

Introduction

The Currents' Physical Components (CPC) concept is presently the most advanced form of the power theory of systems with periodic and with semi-periodic voltages and currents. It applies to the most broad as compared to other concepts, class of systems: to single-phase and to three-wire unbalanced systems with linear, time-invariant (LTI) loads and with harmonic generating loads (HGLs).

The CPC approach is a product of 25 years of development. Its presentation within the limits of one paper is not possible. This paper explains only a "backbone" of the CPC concept, while its details can be found in references. Some of them, in *.pdf format, and other not referenced papers related to the subject, can be found at www.lsczar.info.

The CPC concept is the first approach to the power theory that has provided complete physical interpretation of power phenomena in electric systems. It is also the first one that provided fundamentals for power factor improvement with reactive compensators under nonsinusoidal conditions, both in single-phase and in three-wire systems. It also introduced a number of new concepts to electrical engineering, such as scattered current and power [12, 18] load generated current [17] unbalanced current and power, equivalent and unbalanced admittances [16], working current and power [37], concept of semi-periodic quantities [28] or the concept of quasi-instantaneous compensation [36].

Also it has proven to be a very effective tool for the investigations of various concepts of the power theory and it has identified a number of misconceptions that have occurred during this theory development. It includes studies on Budeanu [14], Fryze [25], Kusters and Moore [11] power theories; on the Instantaneous Reactive Power p-q Theory [31], as well as on apparent power definitions for three-phase systems [27], or on the Poynting Theorem as a hypothetical fundamental of the power theory [32].

Power theory

Power properties of electric systems are only apparently simple. They have been debated for more than a century; the number of publications be can only roughly estimated. It can be a thousand of them. Several papers on powers are published each year. Several "schools" that explain these properties in a specific way have emerged.

The reason for this interest is obvious. The rate of energy delivery, its effectiveness and equipment ratings are specified in terms of powers and are affected by power related phenomena. Taking into account the huge amount of electric energy we use, the interest of electrical engineering in power properties of electric systems is evident.

As electrical engineers, we should understand them. We should know how to modify electrical systems to improve the effectiveness of energy delivery and utilization. Therefore, motivations for studies of power properties of electric systems are both *cognitive* and very *practical* in nature. Cognitive, or *why* issues and practical, or *how* issues are, of course, strongly interrelated.

All these issues are the subject of the power theory of electric systems, although the term "power theory" is vague and consequently, it can be comprehended in various ways. Therefore, before fundamentals of power theory are discussed, the meaning of the term "power theory" and its subject, as used in this paper, have to be explained.

What it is "power theory"?

Power theory, for sure, is concerned with electric energy flow. Does it concern for example, however, with energy radiation by antennas?

As long as the subject of "power theory" is not specified, such a question cannot be answered. To avoid any subjecttive definition, we should look into its meaning when this term was introduced.

The term "power theory" was coined in 1931, by Fryze [5], when he undertook an attempt at an explanation of the difference between the active and apparent powers, P and S. This was a response to Steinmetz observation [1], that the apparent power in circuits with an electric arc is higher than the active power. It was also a response to conclusions [4] by Budeanu, that this difference can be explained in terms of the reactive and distortion powers. Thus, the explanation of the difference between the active and apparent powers was the original subject of the power theory.

The power theory being developed by Budeanu [4] or Fryze, [5], Quade [6], Rozenzwig [7], Depenbrock [9], and numerous other scientists, was confined to pure cognitive, *why* issues. They were extended by practical, *how* issues, when compensation in systems under nonsinusoidal conditions became more urgent.

Power theory being developed by Shepherd and Zakikhani [8], Kusters and Moore [10], Czarnecki, Nabae and Akagi [13], Depenbrock [20], Tenti [30] and numerous other scientists, includes both *why* and *how* questions.

Answers to these questions, given by various scientists, have occurred to be mutually different. Sorts of "schools" around various concepts have emerged, identified as schools of Budeanu, Fryze, Shepherd and Zakikhani, Czarnecki, or Nabae and Akagi power theories.

Referencing to different concepts of power theory by names provides a convenient short-cuts in debates. At the some time, however, it diminishes numerous overlaps of various concepts and preserves some misconceptions.

Therefore, it seems that instead of several different power theories, it should be regarded as a unique intellectual entity. The entity that represents our knowledge on power properties of electric systems. It could be seen as a sort of database of all true statements, both verbal or in a form of mathematical relations, on power properties of such systems. Each valid conclusion on these properties contributes to this theory development.

Although the term "power theory" in the title of this paper is used in the latest meaning, this term associated with names will be used, for brevity, as well.

Subject of power theory

The subject of the power theory can be described as focused on the difference between the active and apparent powers, P and S, only if a very concise statement on power theory is needed. In fact, this subject includes several issues. One could expect that the power theory will provide:

(i) – an explanation and physical interpretation of power phenomena that accompany energy delivery;

(ii) – a definition of power quantities which describe energy flow and its utilization, as well as can specify power ratings of the power equipment;

(iii) - fundamentals for energy accounts between energy producers and customers;

(iv) - fundamentals for studies on effectiveness of energy delivery;

(v) - fundamentals for design and control of equipment for power factor improvement;

(vi) - fundamentals for design and control of equipment for loading and supply quality improvement.

Observe moreover, that in case of items (v) and (vi), the equipment for power factor and/or quality improvement can be built in a form of reactance harmonic filters or compensators or as PWM Inverter-based switching compensators. Power theory should provide fundamentals for design and control of these substantially different devices.

Various "schools" of the power theory can be judged by a degree of fulfillment of expectations compiled above, being aware of such a judgment, however, that these expectations have emerged gradually. These are present day, but not original expectations.

Confines of power theory

It would be trivial to say, that power theory, as a part of electrical engineering, has to obey rules of mathematics and physics. It is located at their junction, as illustrated in Fig. 1.

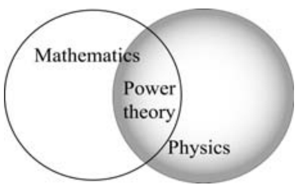

Fig. 1 Power theory as confined by mathematics and physics

While mathematical entities are usually distinctively defined, physical entities are founded on experimental observations, however. Their meaning and interpretation can be vague.

The history of power theory development demonstrates long efforts aimed at fulfillment of this scheme. While all commonly known concepts of power theory are mathematically correct, most of them have deficiencies from physical point of view. Some quantities, claimed to be physical, often do not specify any physical phenomenon.

Expectation that quantities used for describing power properties of electric systems should have a physical meaning in the sense illustrated in Fig. 1, should be treated carefully, however. Observe that the power equation of single-phase LTI load supplied with sinusoidal voltage,

$$P^2 + Q^2 = S^2 ,$$

does not fulfill expectations illustrated in Fig. 1. Only the active power P can be regarded as a physical quantity. Physical meaning of the reactive power Q is debatable [35], while the apparent power S is evidently not a physical quantity. A physical phenomenon that is characterized by apparent power S does not exist.

Power theory serves technology; and technology often needs conventional quantities. The apparent power S is just such a conventional, but of a great importance quantity. The same is with the power factor, $\lambda = P/S.$

CPC methodology

There were attempts aimed at developing power theory as a whole that would include all situations possible in power systems. At the same time, it was not capable of explaining power properties of LTI single-phase RL loads at nonsinusoidal voltage and improve their power factor.

Century long attempts aimed at understanding power properties of electric systems have shown that this is a very tangled issue. The CPC concept, used to untangle it, follows therefore, very old but reliable methodology, introduced to physics by Galileo Galilei. The theory is built up, step-by-step, starting from the most simple situations, using simplified models of real systems. We can proceed to a level with higher complexity only after the theory at the lower level is thoroughly verified. Such a most simple situation is in single-phase circuits with LTI loads. Observe however, that all attempts aimed at the power theory development, before CPC-concept, have failed to explain power properties of even such simple circuits and provide any fundamentals for power factor correction.

CPC in single-phase circuits with LTI loads

Development of the CPC-based power theory started in 1984 in Ref. [12], with explanation of power properties of single-phase circuits with LTI loads supplied with nonsinusoidal voltage, followed by the first method of compensation of reactive current.

Let us assume that a linear time-invariant load, shown in Fig. 2, is supplied with voltage

$$u(t) = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n e^{jn\omega_1 t} .$$

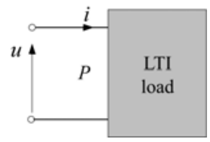

Fig. 2 Linear time-invariant (LTI) load

The load can be characterized by admittance for harmonic frequencies

$$\boldsymbol{Y}_n \triangleq G_n + jB_n ,$$

thus, the load current can be expressed as

$$i(t) = Y_0 U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{Y}_n \boldsymbol{U}_n\, e^{jn\omega_1 t}\ .$$

We can assume, after Fryze [5], that the load current contains active component, known as the ***active current***, i_a. It is the current component proportional to the supply voltage $u(t)$, of minimum value needed for energy permanent delivery to the load with the average rate equal to the active power P. It can be expressed as

(1) $$i_\mathrm{a}(t) \triangleq G_\mathrm{e}\, u(t) = G_\mathrm{e} U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} G_\mathrm{e} \boldsymbol{U}_n\, e^{jn\omega_1 t}\ .$$

Since the active power of a resistive load is $P = G\|u\|^2$, such a load is equivalent with respect to the active power P at voltage $u(t)$ to the original load, if its conductance is

$$G \triangleq G_\mathrm{e} = \frac{P}{\|u\|^2}\ .$$

It is referred to as ***equivalent conductance***. The remaining component of the load current, after subtracting the active current, is equal to

$$i(t) - i_\mathrm{a}(t) = (Y_0 - G_\mathrm{e})U_0 + \sqrt{2}\ \mathrm{Re}\sum_{n\in N} (\boldsymbol{Y}_n - G_\mathrm{e})\boldsymbol{U}_n\, e^{jn\omega_1 t} =$$
$$= (Y_0 - G_\mathrm{e})U_0 + \sqrt{2}\ \mathrm{Re}\sum_{n\in N} (G_n + jB_n - G_\mathrm{e})\boldsymbol{U}_n\, e^{jn\omega_1 t}\ .$$

It can be decomposed into the following components

(2) $$\sqrt{2}\ \mathrm{Re}\sum_{n\in N} jB_n \boldsymbol{U}_n\, e^{jn\omega_1 t} \triangleq i_\mathrm{r}(t)\ ,$$

(3) $$(G_0 - G_\mathrm{e})U_0 + \sqrt{2}\ \mathrm{Re}\sum_{n\in N} (G_n - G_\mathrm{e})\boldsymbol{U}_n\, e^{jn\omega_1 t} \triangleq i_\mathrm{s}(t)\ .$$

This decomposition reveals that two entirely different phenomena contribute to ths useless component of the supply current of LTI loads at nonsinusoidal supply voltage:

1. The presence of the current harmonic component shifted by 90^0 with respect to the voltage harmonics. Their sum is the ***reactive current***, $i_\mathrm{r}(t)$. It is identical with the Shepherd and Zakikhani's [8] quadrature current.
2. The difference of the load conductance G_n for harmonic frequency from the load equivalent conductance, G_e. The sum of these components creates current $i_\mathrm{s}(t)$. Since it occurs when conductances G_n are scattered around the equivalent conductance G_e, this current was called in Ref. [12] a ***scattered current***. The presence of the scattered current in the supply current of LTI loads is a new phenomenon reveled by this decomposition.

Thus, the supply current of LTI loads at nonsinusoidal supply voltage can be expressed as

(4) $$i(t) = i_\mathrm{a}(t) + i_\mathrm{s}(t) + i_\mathrm{r}(t)\ ,$$

The change of the conductance is not an "exotic" property, but a common property of LTI loads. Consider, for example, the RL load shown in Fig. 3. Its conductance

$$G_n = \mathrm{Im}\{\boldsymbol{Y}_n\} = \mathrm{Im}\frac{1}{R + jn\omega_1 L} = \frac{R}{R^2 + (n\omega_1 L)^2}\ ,$$

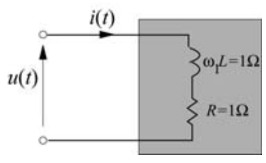

Fig. 3 Example of RL load

This conductance for parameters shown in Fig. 3 is

$$G_0 = 1\ \mathrm{S},\ G_1 = 0.5\ \mathrm{S},\ G_2 = 0.2\ \mathrm{S},\ G_3 = 0.1\ \mathrm{S},\ G_4 = 0.06\ \mathrm{S},$$

thus, it changes with harmonic order. The load current of such a common load at nonsinusoidal supply voltage has to contain a scattered current.

Thus, the current of LTI loads at nonsinusoidal supply voltage contains three components associated with three distinctively different physical phenomena in the load:

1. Permanent energy conversion: - active current, $i_\mathrm{a}(t)$.
2. Change of the load conductance G_n with harmonic order - scattered current, $i_\mathrm{s}(t)$.
3. Phase-shift between the voltage and current harmonics: - reactive current, $i_\mathrm{r}(t)$.

Therefore, these currents were called ***Current's Physical Components (CPC)***. The adjective "physical" does not mean however, that these currents do exists physically. They do not exist as physical entities, but like harmonics, only as mathematical entities, associated with some physical phenomena in the load.

Their rms value is equal to

(5) $$\|i_\mathrm{a}\| = G_\mathrm{e}\|u\| = \frac{P}{\|u\|}\ ,$$

(6) $$\|i_\mathrm{s}\| = \sqrt{\sum_{n\in N_0} (G_n - G_\mathrm{e})^2 U_n^2}\ ,$$

(7) $$\|i_\mathrm{r}\| = \sqrt{\sum_{n\in N} B_n^2 U_n^2} = \sqrt{\sum_{n\in N} \left(\frac{Q_n}{U_n}\right)^2}\ .$$

Symbol N_0 denotes set N of orders n, along with $n = 0$.

The possibility of calculating the supply current rms value having rms values of currents i_a, i_s and i_r depends on orthogonality of these currents, i.e., whether their scalar products are equal to zero.

The scalar product of periodic quantities $x(t)$ and $y(t)$ is equal to

$$(x, y) \triangleq \frac{1}{T}\int_0^T x(t)\, y(t)\, dt = \mathrm{Re}\sum_{n\in N_0} \boldsymbol{X}_n \boldsymbol{Y}_n^*\ ,$$

hence

$$(i_\mathrm{r}, i_\mathrm{a}) = \mathrm{Re}\sum_{n\in N} jB_\mathrm{e} \boldsymbol{U}_n G_\mathrm{e} \boldsymbol{U}_n^* = \mathrm{Re}\sum_{n\in N} jB_\mathrm{e} G_\mathrm{e} U_n^2 = 0\ ,$$

$$(i_\mathrm{r}, i_\mathrm{s}) = \mathrm{Re}\sum_{n\in N} jB_\mathrm{e} \boldsymbol{U}_n (G_n - G_\mathrm{e})\boldsymbol{U}_n^* = 0\ ,$$

$$(i_\mathrm{s}, i_\mathrm{a}) = \mathrm{Re}\sum_{n\in N_0} (G_n - G_\mathrm{e})\boldsymbol{U}_n G_\mathrm{e}\boldsymbol{U}_n^* = G_\mathrm{e}\sum_{n\in N_0} (G_n - G_\mathrm{e})U_n^2 =$$
$$= G_\mathrm{e}\Big(\sum_{n\in N_0} G_n U_n^2 - G_\mathrm{e}\sum_{n\in N_0} U_n^2\Big) = G_\mathrm{e}(P - G_\mathrm{e}\|u\|^2) = 0\ .$$

Thus CPC are mutually orthogonal and consequently

(8) $$\|i\|^2 = \|i_\mathrm{a}\|^2 + \|i_\mathrm{s}\|^2 + \|i_\mathrm{r}\|^2\ .$$

This relationship can be visualized with a rectangular box shown in Fig. 4.

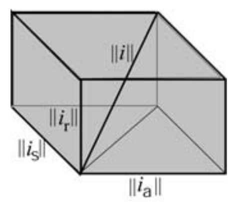

Fig 4. Rectangular box of rms values of CPC

If the length of the box edges is proportional to rms values of current's physical components, then the box diagonal is proportional to the load current rms value, $\|i\|$.

Multiplying eqn. (8) by the square of the voltage rms value, the power equation of LTI loads is obtained

(9) $$S^2 = P^2 + D_s^2 + Q^2\ ,$$

where

(10) $$D_s \triangleq ||u||\,||i_s|| \quad \text{and} \quad Q \triangleq ||u||\,||i_r||\ ,$$

are *scattered power* and *reactive power*, respectively.

Equation (9) resembles Budeanu's equation only apparently. In fact, the reactive power Q is defined in a different way than the Budeanu's reactive power. The scattered power D_s has nothing in common with the distortion power.

Illustration 1. Let us calculate the rms value of the supply current physical components of the load shown in Fig. 5, if the supply voltage is

$$u = 50 + \sqrt{2}\,\mathrm{Re}\{100 e^{j\omega_1 t} + 20 e^{j5\omega_1 t}\}\,\mathrm{V},\ \ \omega_1 = 1\,\mathrm{rd/s}\ .$$

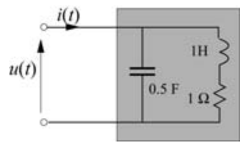

Fig. 5 Example of LTI load

The supply voltage rms value is

$$||u|| = 113.58\ \mathrm{V}.$$

The set of voltage harmonics is N_0 = {0,1,5} and the load admittance for harmonic orders from this set is equal to

$$Y_0 = 1\ \mathrm{S},\quad \boldsymbol{Y}_1 = 0.5\ \mathrm{S},\quad \boldsymbol{Y}_5 = 0.04 + j2.31\ \mathrm{S},$$

and consequently, the supply current is

$$i = 50 + \sqrt{2}\,\mathrm{Re}\{50 e^{j\omega_1 t} + 46.2\, e^{j89^\circ} e^{j5\omega_1 t}\}\,\mathrm{A}\ ,$$

and its rms value is equal to

$$||i|| = \sqrt{I_0^2 + I_1^2 + I_5^2} = \sqrt{50^2 + 50^2 + 46.2^2} = 84.47\ \mathrm{A}\ .$$

To decompose the supply current into physical components and calculate their rms value, the active power and equivalent conductance of the load have to be calculated. The active power is

$$P = \sum_{n \in 0,1,5} G_n U_n^2 = 7.516\ \mathrm{kW}\ ,$$

so that, the equivalent conductance of the load has the value

$$G_e = \frac{P}{||u||^2} = \frac{7516}{113.58^2} = 0.5826\ \mathrm{S}\ .$$

The rms values of the supply current physical components are equal to

$$||i_a|| = G_a\,||u|| = 66.17\ \mathrm{A}\ ,$$

$$||i_s|| = \sqrt{\sum_{n \in 0,1,5} (G_n - G_e)^2 U_n^2} = 24.93\ \mathrm{A}\ ,$$

$$||i_r|| = \sqrt{\sum_{n \in 1,5} B_n^2 U_n^2} = 46.2\ \mathrm{A}\ .$$

It is easy to verify that the calculated rms values satisfy relationship (8) and indeed

$$||i|| = \sqrt{||i_a||^2 + ||i_s||^2 + ||i_r||^2} = \sqrt{66.17^2 + 24.93^2 + 46.2^2} = 84.47\mathrm{A}\ .$$

Thus, decomposition (4) is strictly satisfied even for very high distortion. This conclusion is trivial to some degree, because decomposition (4) was not founded on any approximations. The scattered and reactive powers of the load shown in Fig. 5 are equal to

$$D_s = 2.83\ \mathrm{kVA} \quad \text{and} \quad Q = 5.25\ \mathrm{kVAr}.$$

Compensation of LTI loads

Traditionally, a compensator was a reactance device connected at the load terminals, for example, a capacitor, or even an overexcited synchronous machine, that made a reduction of the reactive current rms value, thus an improvement of the power factor, possible. Now, this term has a much wider meaning, both as to objectives and technical tools. Not only the reactive current, but also load current harmonics, load imbalance, or power variation can be objectives of compensation. The supply voltage asymmetry or this voltage harmonic distortion can also be compensated. Compensators can be built not only as reactance devices, but also as controlled current sources that inject compensating current into supply lines or as controlled voltage sources that inject compensating voltage. They can be built with fixed parameters or as adaptive devices, in simple or in hybrid structure.

In spite of complexity of the present-day objectives [37] and technical tools available, the question: *whether the power factor of a load under nonsinusoidal conditions can be improved by a reactance compensator or not?* was for a long time, and it is even now, a major practical issue of the power theory. The CPC-based power theory was the first to provide the confirmative answer, with very simple, as follows, reasoning.

A lossless reactance compensator, connected as shown in Fig. 6, of admittance for harmonic frequencies $\boldsymbol{Y}_{Cn} = jB_{Cn}$, assuming that it does not affect the supply voltage, does not affect the active and scattered currents.

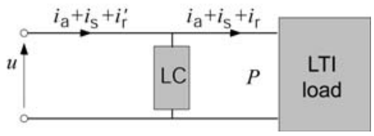

Fig. 6 LTI load with LC compensator

It modifies only the reactive current to

$$i_r' = \sqrt{2}\,\mathrm{Re}\sum_{n \in N} j\,(B_n + B_{Cn})\,\boldsymbol{U}_n e^{jn\omega_1 t}\ ,$$

and its rms value to

$$||i_r'|| = \sqrt{\sum_{n \in N} (B_n + B_{Cn})^2 U_n^2}\ .$$

In particular, this current is compensated entirely, on the condition that for each $n \in N$,

(11) $$B_{Cn} = -B_n\ .$$

Thus, the maximum value of the power factor of LTI loads at reactance shunt compensation cannot be higher than

(12) $$\lambda \triangleq \frac{P}{S} = \lambda_{max} = \frac{||i_a||}{\sqrt{||i_a||^2 + ||i_s||^2}}\ .$$

The question: *whether LTI loads can be compensated to unity power factor with reactance compensator or not*? was solved in Ref. [18]. Such compensation is possible. It requires, however, that a reactance compensator has not only a shunt, but also a series branch. Due to such compensator complexity, its practical value is rather questionable. The confirmative answer to the question formulated above has only some theoretical merit.

CPC in single-phase circuits with HGLs

Linear time-invariant (LTI) loads cannot be sources of harmonics. To check how load-generated current harmonics affect power properties of a circuit, let us consider a purely resistive circuit shown in Fig. 7. It is assumed that at the supply voltage

$$e = e_1 = 100\sqrt{2}\sin\omega_1 t \quad \text{V},$$

the third order current harmonic

$$j = j_3 = 50\sqrt{2}\sin 3\omega_1 t \quad \text{A},$$

is generated in the load

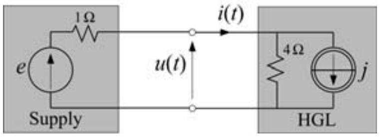

Fig. 7 Resistive circuit with HGL

The voltage and current at the load terminals are equal to

$$u = u_1 + u_3 = 80\sqrt{2}\sin\omega_1 t - 40\sqrt{2}\sin 3\omega_1 t \quad \text{V},$$

$$i = i_1 + i_3 = 20\sqrt{2}\sin\omega_1 t + 40\sqrt{2}\sin 3\omega_1 t \quad \text{A},$$

thus, the active power is zero, since

$$P = \frac{1}{T}\int_0^T u\, i\, dt = \sum_{n=1,3} U_n\, I_n \cos\varphi_n = 1600 - 1600 = 0\,.$$

Observe that there is neither a phase-shift between voltage and current harmonics, nor any change of the load conductance with harmonic order, thus no reactive and scattered currents and powers. The apparent power S at the supply terminals is equal to

$$S = ||u||\, ||i|| = 89.44 \times 44.72 = 4000 \text{ VA},$$

but we are not able to write the power equation in form (9).

It is easy to find the reason for it. Namely, it is caused by the presence of the active power associated with the third order voltage and current harmonics, of the negative value, $P_3 = -1600$ W. When a current harmonic is generated in the load, it is a source of energy flow from the load to the supply source, where this energy is dissipated on the supply source resistance. Thus, there is a current component in the supply current, which cannot be interpreted as a reactive or a scattered current, but it does not contribute to the load active power P. Quite opposite, it reduces that power. This component can be associated with energy flow in the opposite direction to the normal flow, meaning, back from the load to the supply source. Thus, generation of current harmonics in the load, due to its nonlinearity or periodic time-variance, has to be considered [17] as a phenomenon that affects power properties of electric circuits.

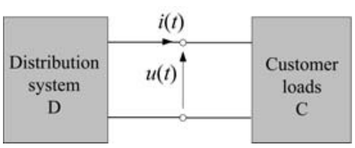

Fig. 8 Cross-section between distribution system (D) and customer load (C)

The presence of current harmonics generated in the load can be identified by measuring the phase angle, φ_n, between the voltage and current harmonics, u_n and i_n, at the cross-section between distribution system (D) and the customer (C) load, as shown in Fig. 8. Since the active power of the n^{th} order harmonic is equal to

$$P_n = U_n\, I_n \cos\varphi_n\,,$$

then, if

$$|\varphi_n| < \pi/2,$$

there is an average component of energy flow at the n^{th} order harmonic from the supply towards the load, and if

$$|\varphi_n| > \pi/2,$$

there is an average component of energy flow at n^{th} order harmonic from the load back the supply source.

With this observation, the set N of all harmonic orders n can be decomposed into sub-sets N_D and N_C, as follows

$$\text{if } |\varphi_n| \le \pi/2, \text{ then } n \in N_D, \qquad \text{if } |\varphi_n| > \pi/2, \text{ then } n \in N_C. \tag{13}$$

It enables the voltage and current decomposition into components with harmonics from sub-sets N_D and N_C,

$$i = \sum_{n\in N} i_n = \sum_{n\in N_D} i_n + \sum_{n\in N_C} i_n \triangleq i_D + i_C\,, \tag{14}$$

$$u = \sum_{n\in N} u_n = \sum_{n\in N_D} u_n + \sum_{n\in N_C} u_n \triangleq u_D - u_C\,. \tag{15}$$

The voltage u_C is defined as the negative sum of voltage harmonics because, as a supply source response to load generated current, i_C, it has the opposite sign as compared to the sign of the distribution system originated voltage harmonics. The same applies to harmonic active power, thus

$$P = \sum_{n\in N} P_n = \sum_{n\in N_D} P_n + \sum_{n\in N_C} P_n \triangleq P_D - P_C\,. \tag{16}$$

Sub-sets N_D and N_C do not contain common harmonic orders n, thus currents i_D and i_C are mutually orthogonal. Hence their rms values satisfy the relationship

$$||i||^2 = ||i_D||^2 + ||i_C||^2\,.$$

The same applies to the voltage rms values, namely

$$||u||^2 = ||u_D||^2 + ||u_C||^2\,.$$

Decomposition (13) of harmonic orders and the voltage and current according to (14) and (15), mean that the system, as presented in Fig. 7, can be described as superposition of two systems. The first, shown in Fig. 9a, has a LTI load and the second, shown in Fig. 9b, has only a current source on the customer side while the distribution system is a passive energy receiver.

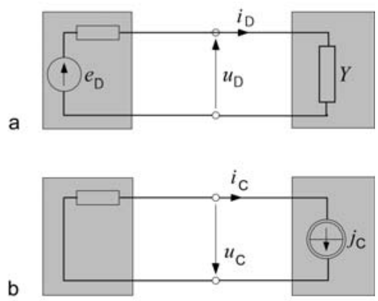

Fig. 9 (a) Equivalent circuit for harmonics $n \in N_D$ and (b) equivalent circuit for harmonics $n \in N_C$

The circuit in Fig. 9a, as a system with a LTI load, can be described according to the CPC approach. Namely, if the equivalent admittance is

$$\boldsymbol{Y}_n = G_n + jB_n = \frac{\boldsymbol{I}_n}{\boldsymbol{U}_n}\,,$$

and the equivalent conductance:

$$G_{eD} \triangleq \frac{P_D}{||u_D||^2}\,,$$

then the current i_D can be decomposed into the active, scattered and reactive components, namely

$$i_D = i_a + i_s + i_r,$$

and eventually, the load current can be decomposed into four physical components,

$$i = i_a + i_s + i_r + i_C. \tag{17}$$

These currents are mutually orthogonal, hence

$$||\,i\,||^2 = ||\,i_a||^2 + ||\,i_s||^2 + ||\,i_r||^2 + ||\,i_C||^2\,. \tag{18}$$

This relation can be visualized on the diagram shown in Fig. 10.

One can observe, however, that the active current in decomposition (17) is not defined according to Fryze, but as

$$i_a \triangleq G_{eD}\, u_D\,. \tag{19}$$

It is equal to the conventional Fryze's active current only if harmonics are not generated in the load, i.e., only if sub-set N_C is empty, thus $N = N_D$.

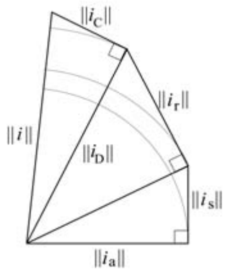

Fig. 10 Diagram of CPC rms values in a circuit with HGL

Confusion with powers in three-phase systems

A Reader of this paper should be aware that in spite of long lasting investigations on powers in three-phase systems, which stared in 1920 with Lyon [2], Buchholz [3], Quade [6], Rosenzweig [7], power properties of such systems, even under sinusoidal conditions, were not correctly identified by the last decade. There is confusion [26] even on how the apparent power, S, in three-wire systems should be defined, since three different definitions coexist in electrical engineering now, namely,

$$S \triangleq U_R I_R + U_S I_S + U_T I_T = S_A\,,$$

$$S \triangleq \sqrt{P^2 + Q^2} = S_G\,,$$

$$S \triangleq \sqrt{U_R^2 + U_S^2 + U_T^2}\ \sqrt{I_R^2 + I_S^2 + I_T^2} = S_B\,, \tag{20}$$

These definitions result in the same value of the apparent power S, only if the line currents are sinusoidal and symmetrical. Otherwise these values are different and consequently, the power factor, $\lambda = P/S$, has different values.

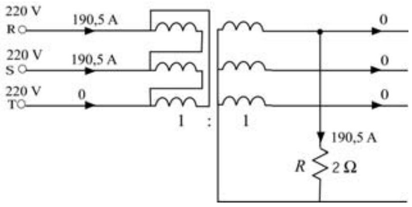

Fig. 11 Example of a three-wire system

The apparent power S in the system shown in Fig. 11 is equal to

$$S_A = 83.8\ \text{kVA}, \qquad S_G = 72.6\ \text{kVA}, \qquad S_B = 102.7\ \text{kVA}.$$

Since the load active power is P = 72.6 kW, the power factor, depending on the apparent power definition, is

$$\lambda_A = 0.86, \qquad \lambda_G = 1, \qquad \lambda_B = 0.71.$$

A reasoning that has enabled selection of the right definition of the apparent power S was presented in Ref. [27] in 1999. It was demonstrated in that paper that energy loss at its delivery and consequently, the power factor value are calculated correctly if the apparent power is defined according to definition (20), suggested by Buchholz [3] in 1922.

The main issue is that the apparent power calculated according to Buchholz's definition does not fulfill the common power equation of three-phase systems:

$$S^2 = P^2 + Q^2\,.$$

These powers in the system shown in Fig. 10 are equal to: $S = S_B$ = 102.7 kVA, P = 72.6 kW, and Q = 0. Unfortunately, none of the approaches to power theory before the CPC development have managed to solve this problem.

CPC in three-wire systems with sinusoidal voltages and currents

Let us consider an equivalent circuit configured in Δ of a three-phase three-wire load shown in Fig. 12.

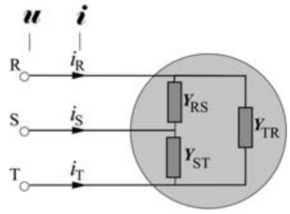

Fig. 12 Equivalent circuit of a three-phase load

Supply currents of such a load can be arranged in a three-phase vector and expressed as

$$\boldsymbol{i} \triangleq \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} = \sqrt{2}\text{Re}\begin{bmatrix} \boldsymbol{I}_R \\ \boldsymbol{I}_S \\ \boldsymbol{I}_T \end{bmatrix} e^{j\omega t} \triangleq \sqrt{2}\text{Re}\,\{\boldsymbol{I}\, e^{j\omega t}\}\,.$$

At a symmetrical supply voltage of positive sequence, the supply current can be expressed in the form

$$\boldsymbol{i} = \sqrt{2}\text{Re}\,\{[(G_e + jB_e)\,\boldsymbol{U} + \boldsymbol{A}\boldsymbol{U}^{\#}]\,e^{j\omega t}\}\,, \tag{21}$$

where

$$\boldsymbol{U} \triangleq \begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_S \\ \boldsymbol{U}_T \end{bmatrix}, \qquad \boldsymbol{U}^{\#} \triangleq \begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_T \\ \boldsymbol{U}_S \end{bmatrix},$$

$$G_e + jB_e = \boldsymbol{Y}_{RS} + \boldsymbol{Y}_{ST} + \boldsymbol{Y}_{TR} \triangleq \boldsymbol{Y}_e\,, \tag{22}$$

is ***equivalent admittance***, while

$$-(\boldsymbol{Y}_{ST} + \alpha\,\boldsymbol{Y}_{TR} + \alpha^{*}\boldsymbol{Y}_{RS}) \triangleq \boldsymbol{A} \triangleq Ae^{j\psi}\,, \tag{23}$$

is ***unbalanced admittance*** of the load. Having the supply current $\boldsymbol{i}$ expressed in the form (21), we can define three current components, namely, the ***active current***

$$\boldsymbol{i}_a \triangleq \sqrt{2}\,\text{Re}\,\{G_e \boldsymbol{U}\ e^{j\omega t}\}\,, \tag{24}$$

the ***reactive current***

$$\boldsymbol{i}_r \triangleq \sqrt{2}\,\text{Re}\,\{\,jB_e \boldsymbol{U}\, e^{j\omega t}\}\,, \tag{25}$$

and the ***unbalanced current***

$$\boldsymbol{i}_u \triangleq \sqrt{2}\,\text{Re}\,\{\boldsymbol{A}\,\boldsymbol{U}^{\#} e^{j\omega t}\}\,, \tag{26}$$

thus, the supply current of LTI three-phase loads is equal to

(27) $$\boldsymbol{i} = \boldsymbol{i}_{\rm a} + \boldsymbol{i}_{\rm r} + \boldsymbol{i}_{\rm u}.$$

These three currents are associated with distinctively different phenomena:

1. permanent energy conversion - active current,
2. phase-shift - reactive current,
3. load imbalance - unbalanced current,

therefore, they can be regarded as physical components of the supply currents.

To verify whether these currents affect energy loss at delivery independently of each other or not, concepts of scalar product, introduced in [15], and orthogonality of three-phase quantities are needed.

The scalar product of three-phase quantities was defined for three-phase vectors $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ as

$$(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^{\rm T}\boldsymbol{y}\,dt\,,$$

and by analogy to single phase systems, the ***three-phase rms value*** was defined in Ref. [15] as

$$||\boldsymbol{x}|| \triangleq \sqrt{(\boldsymbol{x},\boldsymbol{x})} = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{x}^{\rm T}\boldsymbol{x}\,dt} = \sqrt{\boldsymbol{X}^{\rm T}\boldsymbol{X}^{*}}\,,$$

This three-phase rms values for the active, reactive and unbalance currents are equal to

$$||\boldsymbol{i}_{\rm a}|| = G_{\rm e}\,||\boldsymbol{u}||,$$

$$||\boldsymbol{i}_{\rm r}|| = |B_{\rm e}|\,||\boldsymbol{u}||,$$

$$||\boldsymbol{i}_{\rm u}|| = A\,||\boldsymbol{u}||.$$

These three currents are mutually orthogonal on the condition that their scalar products are equal to zero. The scalar product for sinusoidal quantities can be expressed as

$$(\boldsymbol{x},\boldsymbol{y}) = {\rm Re}\{\boldsymbol{X}^{\rm T}\boldsymbol{Y}^{*}\}\,.$$

Scalar products of CPC defined with (24-26) are

$$(\boldsymbol{i}_{\rm a},\boldsymbol{i}_{\rm r}) = {\rm Re}\{\boldsymbol{I}_{\rm a}^{\rm T}\,\boldsymbol{I}_{\rm r}^{*}\} = {\rm Re}\{G_{\rm e}\,\boldsymbol{U}^{\rm T}(jB_{\rm e}\,\boldsymbol{U})^{*}\} = {\rm Re}\{-jB_{\rm e}G_{\rm e}\,||\boldsymbol{u}||^2\} = 0\,,$$

$$(\boldsymbol{i}_{\rm a},\boldsymbol{i}_{\rm r}) = {\rm Re}\{\boldsymbol{I}_{\rm a}^{\rm T}\,\boldsymbol{I}_{\rm r}^{*}\} = {\rm Re}\{G_{\rm e}\,\boldsymbol{U}^{\rm T}(jB_{\rm e}\,\boldsymbol{U})^{*}\} = {\rm Re}\{-jB_{\rm e}G_{\rm e}\,||\boldsymbol{u}||^2\} = 0\,,$$

$$(\boldsymbol{i}_{\rm a},\boldsymbol{i}_{\rm u}) = {\rm Re}\{\boldsymbol{I}_{\rm a}^{\rm T}\,\boldsymbol{I}_{\rm u}^{*}\} = {\rm Re}\{G_{\rm e}\,\boldsymbol{U}^{\rm T}(A\,\boldsymbol{U}^{\#})^{*}\} = 0\,,$$

thus, they are orthogonal. Consequently, their three-phase rms values satisfy relationship

(28) $$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\rm a}||^2 + ||\boldsymbol{i}_{\rm r}||^2 + ||\boldsymbol{i}_{\rm u}||^2\,.$$

Multiplying this equation by $||\boldsymbol{u}||^2$, the power equation of three-wire systems with LTI loads is obtained

(29) $$S^2 = P^2 + Q^2 + D_{\rm u}^2\,.$$

This equation, as compared to the conventional power equation, contains a new power,

(30) $$D_{\rm u} \triangleq ||\boldsymbol{i}_{\rm u}||\,||\boldsymbol{u}|| = A\,||\boldsymbol{u}||^2,$$

named in Ref. [15], as the ***unbalanced power***. Only such an equation satisfies the power relation when the apparent power S is defined according to Buchholz's definition. The active and reactive powers:

(31) $$P \triangleq ||\boldsymbol{i}_{\rm a}||\,||\boldsymbol{u}|| = G_{\rm e}\,||\boldsymbol{u}||^2,$$

(32) $$Q \triangleq \pm||\boldsymbol{i}_{\rm r}||\,||\boldsymbol{u}|| = -B_{\rm e}\,||\boldsymbol{u}||^2,$$

can be directly related to the load equivalent parameters. For example, for the load shown in Fig. 11,

$\boldsymbol{Y}_{\rm e} = \boldsymbol{Y}_{\rm RS} = 0.5$ S, $G_{\rm e} = 0.5$ S, $B_{\rm e} = 0$, $\boldsymbol{A} = -\alpha^{*}\boldsymbol{Y}_{\rm RS} = 0.5\,e^{j60}$ S, $||\boldsymbol{u}|| = 220\sqrt{3}$ V, and hence: $P = 0.5\,(220\sqrt{3})^2 = 72.6$ kW, $Q = 0$, $D_{\rm u} = 0.5\,(220\sqrt{3})^2 = 72.6$ kVA. With these powers, the power equation (29) is satisfied, since

$$102.7^2 \approx 72.6^2 + 0 + 72.6^2.$$

Balancing compensators

The CPC concept provides an exceptionally simple solution to the problem of design of reactance compensators for three-wire unbalanced systems.

The reactive and unbalanced components of the supply current can be entirely compensated, on the condition that the equivalent susceptance $B_{\rm Ce}$ and unbalanced admittance $\boldsymbol{A}_{\rm C}$ of the compensator satisfy relations

(33) $$B_{\rm e} + B_{\rm Ce} = 0, \quad \boldsymbol{A} + \boldsymbol{A}_{\rm C} = 0\,.$$

These conditions can be fulfilled by a reactance ***balancing compensator*** of the structure shown in Fig. 13.

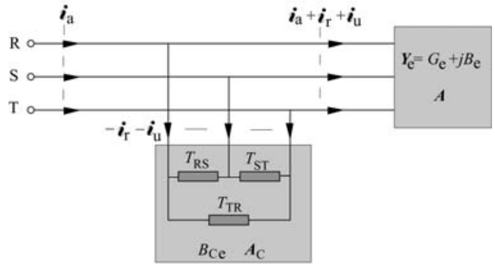

Fig. 13 Circuit with balancing compensator

It is enough [16] for this purpose that the line-to-line susceptances of the balancing compensator, $T_{\rm XY}$, are equal to

$$T_{\rm RS} = (\sqrt{3}{\rm Re}\,\boldsymbol{A} - {\rm Im}\,\boldsymbol{A} - B_{\rm e})/3\,,$$

(34) $$T_{\rm ST} = (2\,{\rm Im}\,\boldsymbol{A} - B_{\rm e})/3\,,$$

$$T_{\rm TR} = (-\sqrt{3}{\rm Re}\,\boldsymbol{A} - {\rm Im}\,\boldsymbol{A} - B_{\rm e})/3\,.$$

Such a compensator can be built with fixed parameters or as an adaptive device [23] with thyristor controlled inductors (TCI) used for susceptances $T_{\rm XY}$ control. Such control with TCI in various structures are discussed in Ref. [21].

CPC in three-wire systems with LTI loads and nonsinusoidal voltages and currents

When the supply voltage in three-wire systems is symmetrical, positive sequence and nonsinusoidal, meaning

$$\boldsymbol{u} = \sum_{n\in N}\boldsymbol{u}_n = \sqrt{2}\,{\rm Re}\sum_{n\in N}\boldsymbol{U}_n\,e^{jn\omega_1 t}\,,$$

but without zero sequence harmonics, then the active, reactive and unbalanced components of the supply current preserve their physical meaning, only all of them are a form of the sum of harmonics , namely

(35) $$\boldsymbol{i}_{\rm a} \triangleq \sqrt{2}\,{\rm Re}\sum_{n\in N} G_{\rm e}\,\boldsymbol{U}_n\,e^{jn\omega_1 t},$$

(36) $$\boldsymbol{i}_{\rm r} \triangleq \sqrt{2}\,{\rm Re}\sum_{n\in N} jB_{{\rm e}n}\boldsymbol{U}_n\,e^{jn\omega_1 t},$$

(37) $$\boldsymbol{i}_{\rm u} \triangleq \sqrt{2}\,{\rm Re}\sum_{n\in N} \boldsymbol{A}_n\,\boldsymbol{U}_n^{\#}\,e^{jn\omega_1 t},$$

and new components occur [15] in the supply current, namely, the scattered current, defined as

(38) $$\boldsymbol{i}_{\rm s} \triangleq \sqrt{2}\,{\rm Re}\sum_{n\in N} (G_{{\rm e}n} - G_{\rm e})\,\boldsymbol{U}_n\,e^{jn\omega_1 t},$$

where the load conductances $G_{{\rm e}n}$ and $G_{\rm e}$ are defined as

(39) $$G_{\rm e} \triangleq \frac{P}{||\boldsymbol{u}||^2}, \qquad G_{{\rm e}n} \triangleq \frac{P_n}{||\boldsymbol{u}_n||^2}\,.$$

Each of these four currents is associated with distinctively different phenomenon in the three-phase load, thus they are CPC of the supply currents, such that

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_s + \boldsymbol{i}_r + \boldsymbol{i}_u \,. \tag{40}$$

Their three-phase rms values are equal to, respectively,

$$||\boldsymbol{i}_a|| = G_e ||\boldsymbol{u}||\,, \tag{41}$$

$$||\boldsymbol{i}_s|| = \sqrt{\sum_{n\in N} (G_{en} - G_e)^2 ||\boldsymbol{u}_n||^2}\,, \tag{42}$$

$$||\boldsymbol{i}_r|| = \sqrt{\sum_{n\in N} B_{en}^2 ||\boldsymbol{u}_n||^2}\,, \tag{43}$$

$$||\boldsymbol{i}_u|| = \sqrt{\sum_{n\in N} A_n^2 ||\boldsymbol{u}_n||^2}\,. \tag{44}$$

These currents are mutually orthogonal, so that

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2\,, \tag{45}$$

while the power equation has the form

$$S^2 = P^2 + D_s^2 + Q^2 + D_u^2\,. \tag{46}$$

Similarly, as in single-phase systems, the scattered current cannot be compensated by any shunt reactance compensator. The reactive and unbalanced currents can be compensated entirely by shunt compensator which may have the structure shown in Fig. 12. Unfortunately, as it was demonstrated in Ref. [16], entire compensation may require very complex compensators and consequently, are not practical. These currents can be minimized very effectively, however, [19] by compensators composed of no more than two LC elements in each compensator branch.

CPC in three-wire systems with HGLs

Similarly, as in single-phase systems, current harmonics generated in a three-phase load, due to its nonlinearity or periodic time-variance, can cause the load, normally an energy consumer, to become the source of energy at some harmonic frequencies, meaning the harmonic active power P_n becomes negative. The load generated current $\boldsymbol{i}_C$ occurs in such a case in the supply current $\boldsymbol{i}$. This modifies the supply current decomposition to the form

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_s + \boldsymbol{i}_r + \boldsymbol{i}_u + \boldsymbol{i}_C\,. \tag{47}$$

Calculation of particular currents requires, as it was done for single-phase circuits with HGLs, that the set of harmonic orders N is decomposed into sub-sets N_D and N_C. It was enough in single-phase circuit to observe for that purpose the value of phase angle φ_n and follow condition (13). This condition, for three-phase systems with HGLs, has to be modified. The sign of the harmonic active current

$$I_{an} \triangleq I_{Rn}\cos\varphi_{Rn} + I_{Sn}\cos\varphi_{Sn} + I_{Tn}\cos\varphi_{Tn}\,,$$

has to be calculated for that purpose. Next,

$$\begin{aligned} &\text{if}\quad I_{an} \geq 0,\ \text{then}\ n\in N_D,\\ &\text{if}\quad I_{an} < 0,\ \text{then}\ n\in N_C. \end{aligned} \tag{48}$$

Remaining calculations do not differ substantially from those for single-phase systems with HGLs, only performed on three-phase vectors. Moreover, the unbalanced current has to be calculated. All these currents are orthogonal, thus

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2 + ||\boldsymbol{i}_C||^2\,, \tag{49}$$

with

$$||\boldsymbol{i}_C|| = \sqrt{\sum_{n\in N_C} ||\boldsymbol{i}_n||^2}\,.$$

Comments on CPC - based compensation

Attention in this paper is focused on cognitive, rather then on practical issues, mainly related to compensation. It is worthy to emphasize, however, that the CPC concept, by providing explanation of power phenomena, provides solid fundamentals for compensation [24], [33, 34], [36, 37].

The issue of compensation is much too broad to be covered here. The paper, in this respect, is confined to only a few remarks, important for prospective applications of the CPC-based power theory for PWM-based switching compensator control.

There is a myth that the CPC-based power theory, as being based on the frequency-domain approach, is computationally too demanding to have any practical value for a compensator control. It is not true. In fact, the number of multiplications needed for the reference signal generation for the compensator control is in the order of ten, so that, CPC approach enables a ***quasi-instantaneous*** control [36] of switching compensators.

Owing to a deep insight into power phenomena, capabilities of the CPC approach to compensation are in a striking contrast to that of the Instantaneous Reactive Power (IRP) p-q Theory which misinterprets [31] power phenomena. The CPC-based approach enables compensation with different objectives [37] in simple or various hybrid structures, [24], or even to build compensator as a programmable devices with objectives tailored to specific needs. The IRP p-q based approach does not provide such capabilities. Moreover, as observed in Ref. [29] and analyzed in details in Ref. [38], in the presence of the supply voltage harmonics and/or asymmetry, the IRP p-q based algorithms generate an erroneous reference signal for the compensator control.

There are a few concepts rooted in the CPC concept, but which go further, that enable quasi-instantaneous control of PWM inverter-based switching compensators. These are the concepts of working current and working power, semi-periodic quantities and a recursive formula for Discrete Fourier Transform (DFT) calculation. These concepts are drafted below.

Working current and power

Decomposition (47) provides information on how distinctive, power related phenomena contribute to the current three-phase rms value and consequently, energy loss on delivery. Power loss on the supply system resistance, assumeing that its change with harmonic frequency is neglected, is equal to

$$\Delta P_s = R_s ||\boldsymbol{i}||^2 = R_s\,(||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2 + ||\boldsymbol{i}_C||^2)\,,$$

thus, each current, and consequently each phenomenon associated with such a current, contributes to this loss independently on each other.

This conclusion has evident cognitive value. One could ask, however, whether it has a practical value for formulating objectives of compensation or not.

The reactive and unbalanced currents, $\boldsymbol{i}_r$ and $\boldsymbol{i}_u$, can be entirely compensated or reduced by shunt reactance compensators. The scattered and generated currents, $\boldsymbol{i}_s$ and $\boldsymbol{i}_C$, are not affected by such compensators, however. The generated current can be filtered out by resonant harmonic filters or compensated by switching compensators, which can also compensate other useless currents, meaning the scattered, reactive and unbalanced currents. In effect, only the active current $\boldsymbol{i}_a$ can remain after compensation and indeed, this is often regarded, even in this paper, as an ultimate objective of compensation. The supply source is loaded in such a case only with the active power P, or P_D, when HGLs are entirely compensated. Observe however, that in systems with nonsinusoidal and asymmetrical supply voltage, the active current

$$\boldsymbol{i}_{\mathrm{a}} \triangleq G_{\mathrm{e}}\,\boldsymbol{u}\ ,$$

is nonsinusoidal and asymmetrical. *Should however such a current be an ultimate objective of compensation?*

A similar question can be asked about the active power, P. The term "active" power is regarded usually as a synonym for a "useful" power, but the active power can contain both useful and harmful components. Only the active power of the positive sequence voltage and current component of the fundamental harmonic is a useful power of a synchronous or induction motor. Active power of the negative sequence and active power of harmonics cannot be converted to mechanical power on the motor shaft, but only it contributes to the motor temperature and/or mechanical vibration. Shunt compensation objectives should take these conclusions into account.

For that purpose, the vector of the supply voltages u_{R}, u_{S} and u_{T}, of a three-wire system, in general asymmetrical and nonsinusoidal, can be decomposed into vectors of the positive and negative sequence components of the fundamental harmonic $\boldsymbol{u}_1^{\mathrm{p}}$, $\boldsymbol{u}_1^{\mathrm{n}}$, and a harmonic component $\boldsymbol{u}_{\mathrm{h}}$,

$$\boldsymbol{u} = \boldsymbol{u}_1 + \sum_{n\in N'} \boldsymbol{u}_n = \boldsymbol{u}_1^{\mathrm{p}} + \boldsymbol{u}_1^{\mathrm{n}} + \boldsymbol{u}_{\mathrm{h}}\ . \tag{50}$$

where N' denotes set N without n = 1. The active power P of a load supplied with such a voltage is composed of active powers of the fundamental P_1 and all higher order harmonics, P_n, i.e.,

$$P = P_1 + \sum_{n\in N'} P_n \triangleq P_1 + P_{\mathrm{h}} = P_1^{\mathrm{p}} + P_1^{\mathrm{n}} + P_{\mathrm{h}}\ , \tag{51}$$

while the active power of the fundamental harmonic is composed of active power of the positive and negative voltage and current sequences.

The active power of harmonics, P_{h}, and that of the negative sequence, P_1^{n}, is useful only for resistive loads, which do not represent usually the main loads in power systems. Most of the energy is converted to mechanical energy by electric motors. It amounts up to 2/3 in US power system. Only the active power of the positive sequence, P_1^{p}, stands for such motors as useful power. Also for various electronic-types of loads, harmonics and voltage asymmetry contribute to extra heat, disturbances or malfunctions, rather than to useful work.

A characteristic name that would clearly distinguish power P_1^{p}, from other ones, is needed. It was suggested in Ref. [37] to name it ***working power*** and denote by P_{w}, thus

$$P_{\mathrm{w}} \triangleq P_1^{\mathrm{p}}\ .$$

A current proportional to the positive sequence component of the supply voltage fundamental harmonic $\boldsymbol{u}_1^{\mathrm{p}} \triangleq \boldsymbol{u}_{\mathrm{w}}$, and of minimum value needed at working power P_{w},

$$\boldsymbol{i}_{\mathrm{w}} \triangleq G_{\mathrm{w}}\,\boldsymbol{u}_{\mathrm{w}}\ , \tag{52}$$

with

$$G_{\mathrm{w}} \triangleq \frac{P_{\mathrm{w}}}{\|\boldsymbol{u}_{\mathrm{w}}\|^2}\ , \tag{53}$$

was called in [37] a ***working current***. The remaining part of the supply current

$$\boldsymbol{i} - \boldsymbol{i}_{\mathrm{w}} \triangleq \boldsymbol{i}_{\mathrm{d}}\ , \tag{54}$$

does not contribute to working power and can be regarded as a ***detrimental current***. When this current is compensated, the supply source is loaded only with symmetrical and sinusoidal working current.

Observe, that if $\boldsymbol{U}_1^{\mathrm{p}}$ and $\boldsymbol{I}_1^{\mathrm{p}}$ denote complex rms value of the positive sequence components of the load voltage and current fundamental harmonic, $\boldsymbol{u}_1$ and $\boldsymbol{i}_1$, or simply, crms values, $\boldsymbol{U}_{\mathrm{w}}$ and $\boldsymbol{I}_{\mathrm{w}}$, of the load working voltage and currents, then

$$G_{\mathrm{w}} \triangleq \frac{P_{\mathrm{w}}}{\|\boldsymbol{u}_{\mathrm{w}}\|^2} = \frac{3U_1^{\mathrm{p}} I_1^{\mathrm{p}} \cos\varphi_1^{\mathrm{p}}}{3U_1^{\mathrm{p}\,2}} = \frac{I_{\mathrm{w}}}{U_{\mathrm{w}}}\cos\varphi_{\mathrm{w}}\ . \tag{55}$$

Semi-periodic quantities

Quantities in electric distribution systems are only apparently periodic. Each event disturbs periodicity, while the harmonic approach to power theory is based on the assumption that voltages and currents are periodic. Indeed, they are non-periodic. Powers, rms values and crms values of harmonics are not defined for such quantities.

Voltages and currents in distribution systems, although non-periodic, have specific properties. Power system generators provide almost sinusoidal voltage, so that energy to such system is delivered from generators only at the fundamental frequency. Non-periodic voltages occur mainly as a system response to non-periodic currents, while such currents occur due to transient or permanent disturbances of their periodicity. It was suggested in Ref. [28] to refer to such quantities as ***semi-periodic***.

Energy is delivered to systems with semi-periodic quantities at the fundamental frequency with period T of the generated voltage. This period can be detected and serve as a main time-frame for power properties analysis.

All functionals, such as rms value, $\|x\|$, active power, P, or crms values of harmonics, $\boldsymbol{X}_n$, which for periodic quantities $x(t)$ are constant numbers, for semi-periodic quantities become functions of time. Calculation of all these functionals includes averaging over period T, thus, they are slowly varying functions of time. Harmonics with varying amplitude and phase have to be regarded as ***quasi-harmonics***.

For calculating the working current in three-wire systems with supply voltage without any zero sequence components, the crms values of the fundamental quasi-harmonics of two voltages and two currents are needed. Its value at the end t of the T-long observation window is equal to

$$\boldsymbol{X}_1(t) = \frac{\sqrt{2}}{T}\int_{t-T}^{t} x(t)\, e^{-j\omega_1 t}\, dt\ .$$

This value at instant $t = kT_s$ in the observation window, shown in Fig. 14, with N equidistant samples x_n,

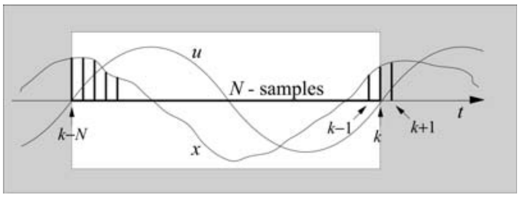

Fig. 14. Observation window

can be calculated using DFT, namely

$$\boldsymbol{X}_{1k} = \frac{\sqrt{2}}{N}\sum_{n=k-N+1}^{n=k} x_n\, e^{-j\frac{2\pi}{N}n}\ .$$

The amount of calculation, for the last formula requires $2N$ multiplications, using recursive formula [22],

$$\boldsymbol{X}_{1k} = \boldsymbol{X}_{1k-1} + (x_k - x_{k-N})\boldsymbol{W}_k, \tag{56}$$

with terms

$$\boldsymbol{W}_k \triangleq \frac{\sqrt{2}}{N}e^{-j\frac{2\pi}{N}k}\ ,$$

stored in a look-up table, can be reduced to only two multiplications.

Observe that the CPC approach as applied to compensation is not a pure frequency-domain approach. Although the working current is calculated in the frequency–domain, the detrimental current is specified in a time-domain. Thus it is a ***frequency/time-***, or a ***hybrid-domain*** approach.

Conclusions

Readers of this paper, written as a guide over the CPC-concept, can easily conclude that all what was presented is a sort of scaffolding for a full fledged power theory of systems with nonsinusoidal voltages and currents, which is still under development. Some issues, such as four-wire and multi-phase systems were not even touched. The CPC concept was not popular, for apparent reasons, explained above, as a fundamental for switching compensator control, thus a number of application oriented issues are still waiting for investigations. In spite of all these still open questions, it seems that there is currently no more powerful tool than the CPC concept that could handle both cognitive and practical issues of systems with nonsinusoidal voltages and currents.

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