Currents' Physical Components of Unbalanced Three-Phase LTI Loads at Asymmetrical Nonsinusoidal Supply Voltage

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Abstract— **Energy flow - related phenomena in three-phase unbalanced, linear, time-invariant (LTI) loads, supplied with asymmetrical and nonsinusoidal voltages are investigated in the paper. It is demonstrated that the load current can be decomposed into Currents' Physical Components (CPC), associated with distinctive physical phenomena in the load. It is also shown how the CPC can be expressed in terms of the supply voltage and equivalent parameters of the load. This decomposition provides solid fundamentals for defining powers of such loads.**

Keywords-power definitions; orthogonal decomposition; scattered current; scattered power; power theory; CPC..

I. INTRODUCTION

Residential, industrial and commercial loads have a form of aggregates of single-phase loads supplied from Δ/y transformer, as shown in Fig.1. The same applies to traction systems. Such loads as seen from the primary side of the transformer stand for three-wire loads. The lines on the secondary side of the supply transformer are mainly loaded phase-by-phase and consequently, such a three-phase load can be unbalanced. This applies particularly to three-phase AC arc furnaces, where each arc stands for an individual single-phase load, sometimes of very high power.
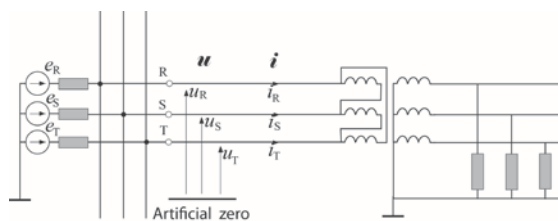

Fig. 1. Three-phase, three-wire system with aggregates of single-phase loads

It is known how to describe such unbalanced load in terms of powers when the supply voltage is nonsinusoidal, but symmetrical [xx] or when the supply voltage is asymmetrical, but sinusoidal [yy]. Description of such loads in terms of powers when, at the same time, the supply voltage is both asymmetrical and nonsinusoidal is currently not known.

Power properties of three-phase loads at nonsinusoidal, but symmetrical voltage and at asymmetrical, but sinusoidal voltage were explained in [xx] and [yy] by means of the load current decomposition into the Currents' Physical Components (CPC). The objective of this paper is explanation of the power properties of three-phase unbalanced loads at asymmetrical and nonsinusoidal voltage using the concept of the load current decomposition into CPC, at the assumption that the load is linear and time-invariant (LTI), however. This assumption means that the load cannot generate current harmonics. They can occur only as a response to the supply voltage harmonics.

II. LOAD LINE-TO-LINE ADMITTANCES

Power properties of a three-phase load have to be determined having available only voltages and currents at the load terminals. Let these are line-to-line voltages, $v_{RS}(t)$, $v_{ST}(t)$ and $v_{TR}(t)$ and line currents $i_R(t)$, $i_S(t)$ and $i_T(t)$. They can be arranged into vectors

$$\boldsymbol{v}(t) \overset{df}{=} \left[v_{RS}(t),\ v_{ST}(t),\ v_{TR}(t)\right]^T,\quad \boldsymbol{i} \overset{df}{=} \left[i_R(t),\ i_S(t),\ i_T(t)\right]^T. \quad (1)$$

We assume that all these quantities, denoted generally by $x(t)$, are periodic and can be expressed in terms of their harmonics $x_n(t)$

$$x(t) = \sum_{n \in N} x_n(t) = X_0 + \sqrt{2}\ \mathrm{Re} \sum_{n \in N} X_n e^{jn\omega_1 t} \quad (2)$$

where N denotes the set of harmonic orders n and

$$\boldsymbol{X}_n = X_n e^{j\alpha_n} = \frac{\sqrt{2}}{T} \int_0^T x(t) e^{-jn\omega_1 t} dt \quad (3)$$

is the complex rms (crms) value of the nth order harmonic. Modern measurement instruments calculate these values digitally. Instead of using (3), they process samples of voltages or currents x_k, provided by voltage or current sensors and analog to digital (A/D) converters. Such digital instruments can calculate the crms values X_n using Discrete Fourier Transform (DFT), namely

$$\boldsymbol{X}_n = \frac{\sqrt{2}}{K} \sum_{k=0}^{k=K-1} x_k e^{-j\frac{2\pi n}{K}k} \quad (4)$$

where K denotes the number of samples in one period T of the supply voltage variability.

Assuming that voltages and currents are nonsinusoidal, but periodic, the voltage and current vectors can be expres-

sed as a sum of harmonics of the order n from a set N, namely

$$\mathbf{v}(t) \overset{df}{=} \begin{bmatrix} v_{RS}(t) \\ v_{ST}(t) \\ v_{TR}(t) \end{bmatrix} = \sum_{n \in N} \mathbf{v}_n(t) = \sqrt{2} \operatorname{Re} \sum_{n \in N} \begin{bmatrix} V_{RSn} \\ V_{STn} \\ V_{TRn} \end{bmatrix} e^{jn\omega_1 t} =$$
$$= \sqrt{2} \operatorname{Re} \sum_{n \in N} \mathbf{V}_n e^{jn\omega_1 t} \qquad (5)$$

$$\mathbf{i}(t) \overset{df}{=} \begin{bmatrix} i_R(t) \\ i_S(t) \\ i_T(t) \end{bmatrix} = \sum_{n \in N} \mathbf{i}_n(t) = \sqrt{2} \operatorname{Re} \sum_{n \in N} \begin{bmatrix} I_{Rn} \\ I_{Sn} \\ I_{Tn} \end{bmatrix} e^{jn\omega_1 t} =$$
$$= \sqrt{2} \operatorname{Re} \sum_{n \in N} \mathbf{I}_n e^{jn\omega_1 t} \qquad (6)$$

Having the complex rms (crms) values of the voltage and current harmonics at the load terminals, the following question can be asked: *is it possible to calculate equivalent line-to-line admittances of a three-phase load for harmonic frequencies?*

This question applies to any harmonic, therefore, in an attempt to answer it, we can neglect the harmonic order n, assuming that $n\omega_1 = \omega$. Thus the above question simplifies to: *having the crms values of voltages and currents at terminals of the load shown in Fig. 2(a), can we find equivalent line-to-line admittances in the circuit in Δ structure shown in Fig. 2(b)?*
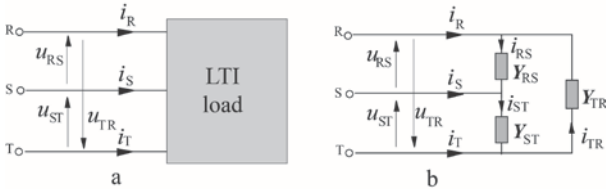

Fig. 2. Three-phase LTI load and its equivalent of Δ structure

If P and Q are the active and reactive powers measured at the load terminals, then the complex power of the load in Fig. 2 (b) is

$$C = P + jQ = U_{RS}I_{RS}^* + U_{ST}I_{ST}^* + U_{TR}I_{TR}^* =$$
$$= Y_{RS}^* U_{RS}^2 + Y_{ST}^* U_{ST}^2 + Y_{TR}^* U_{TR}^2 \qquad (7)$$

while the crms values of the line-to-line currents in the equivalent load have to satisfy node equations:

$$I_R = Y_{RS}U_{RS} - Y_{TR}U_{TR} \qquad (8)$$

$$I_S = Y_{ST}U_{ST} - Y_{RS}U_{RS} \qquad (9)$$

Equations (7) − (9), after some minor re-arrangements, can be written in the matrix form

$$\begin{bmatrix} U_{RS}^2 & U_{ST}^2 & U_{TR}^2 \\ U_{RS} & 0 & -U_{TR} \\ -U_{RS} & U_{ST} & 0 \end{bmatrix} \begin{bmatrix} Y_{RS} \\ Y_{ST} \\ Y_{TR} \end{bmatrix} = \begin{bmatrix} C^* \\ I_R \\ I_S \end{bmatrix} \qquad (10)$$

with three unknown line-to-line admittances Y_{RS}, Y_{ST} and Y_{TR}. Observe that the conjugate complex power of the load $C^* = P + jQ$ is a known quantity. It can be measured at the load terminals. Let us denote the square matrix on the left side by $\mathbf{U}$. Equation (10) has a unique solution on the condition that determinant of matrix $\mathbf{U}$ is not equal to zero. Let us calculate it

$$\operatorname{Det}\{\mathbf{U}\} = U_{ST}^2 U_{TR} U_{RS} + U_{TR}^2 U_{RS} U_{ST} + U_{RS}^2 U_{TR} =$$
$$= U_{RS} U_{ST} U_{TR} (U_{ST}^* + U_{TR}^* + U_{RS}^*) = 0 \qquad (11)$$

which means that (10) does not a have unique solution. It has an infinite number of solutions. One of three line-to-line admittances, Y_{RS}, Y_{ST} or Y_{TR} can have any value. Thus, there is an infinite number of loads of Δ structure equivalent to the original load with respect to currents at the load terminals and consequently, with respect to the load active and reactive powers P and Q.

Illustration 1. The load shown in Fig. 3 at the supply voltage of the crms values $U_{RS} = 100$V, $U_{ST} = j100$V and $U_{TR} = (-100 - j100)$ V has the active power $P = 40$ kW.
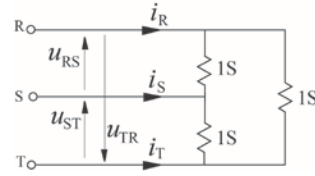

Fig. 3. Example of a load

The crms values of the line currents are:

$$\mathbf{I}_R = (200 + j100) \text{ A}$$

$$\mathbf{I}_S = (-100 + j100) \text{ A}$$

$$\mathbf{I}_T = (-100 - j200) \text{ A.}$$

We can find a load equivalent to that in Fig. 3, assuming any value for one of the three line-to-line admittances. Let assume that $Y_{RS} = 0$, then from (8) and (9) we obtain:

$$Y_{ST} = (1.0 + j1.0) \text{ S}$$

$$Y_{TR} = (1.5 - j0.5) \text{ S}$$
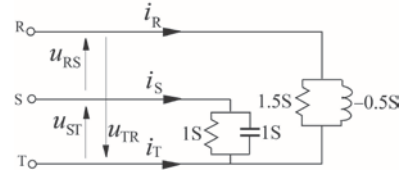
i.e., equivalent load, as shown in Fig. 4.

Fig. 4. A load equivalent to that in Fig. 3.

This load at the same supply voltages has identical line currents as that in Fig. 3 and the same active and reactive powers, $P = 40$kW, $Q = 0$.

III.   THE LOAD CURRENT DECOMPOSITION

The internal voltage of the system

$$\mathbf{e}(t) \overset{df}{=} \begin{bmatrix} e_R(t) \\ e_S(t) \\ e_T(t) \end{bmatrix} = \sum_{n \in N} \begin{bmatrix} e_{Rn}(t) \\ e_{Sn}(t) \\ e_{Tn}(t) \end{bmatrix} = \sum_{n \in N} \mathbf{e}_n(t) \qquad (12)$$

is assumed to be asymmetrical. Thus, each harmonic of this voltage can contain symmetrical components of the positive, negative and the zero order:

$$\mathbf{e}(t) = \sum_{n \in N} \mathbf{e}_n = \sum_{n \in N} (\mathbf{e}_n^p + \mathbf{e}_n^n + \mathbf{e}_n^z) = \mathbf{e}^p + \mathbf{e}^n + \mathbf{e}^z \qquad (13)$$

The zero sequence component $\mathbf{e}^z(t)$ of the internal voltage cannot cause any current in three-wire systems, but it contributes to an increase of the three-phase rms value of the supply voltage, thus it increases the apparent power S and reduces the load power factor. Even an ideal resistive

balanced load supplied with a voltage that contains the symmetrical component of the zero sequence would have the power factor lower than one. To avoid this, the zero sequence component of the supply voltage has to be eliminated by referencing the voltage to an artificial zero, as this is shown in Fig. 1. Such a voltage can be composed of symmetrical components of only the positive and the negative sequence:

$$\boldsymbol{u}(t) = \sum_{n\in N} \boldsymbol{u}_n = \sum_{n\in N} (\boldsymbol{u}_n^{\mathrm{p}} + \boldsymbol{u}_n^{\mathrm{n}}) = \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}} \qquad (14)$$

The crms values of the symmetrical components of the supply voltage harmonics U_n^{p}, U_n^{n} and U_n^{z} are calculated in the same way as for sinusoidal quantities, namely

$$\begin{bmatrix} U_n^{\mathrm{z}} \\ U_n^{\mathrm{p}} \\ U_n^{\mathrm{n}} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix} \begin{bmatrix} U_{Rn} \\ U_{Sn} \\ U_{Tn} \end{bmatrix} \qquad (15)$$

When the supply voltage is symmetrical, then harmonics are also symmetrical and of a specified sequence. Harmonics of the order $n = 3k + 1$ are of positive sequence; harmonics of the order $n = 3k - 1$ are of negative sequence, while harmonics of the order $n = 3k$ are of zero sequence. When the supply voltage is asymmetrical, then this property is no longer valid, however. In particular, the third order harmonic can exist both in the supply voltage and in the load current, because at the supply voltage asymmetry, the third order harmonic is not zero sequenc e. The third order harmonic can contain symmetrical components of the positive and negative sequence.

The supply voltage, referenced to the artificial zero, arranged in a three-phase vector, can be presented in the form

$$\boldsymbol{u}(t) \overset{\mathrm{df}}{=} \begin{bmatrix} u_R(t) \\ u_S(t) \\ u_T(t) \end{bmatrix} = \sum_{n\in N} \boldsymbol{u}_n(t) = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} \begin{bmatrix} U_{Rn} \\ U_{Sn} \\ U_{Tn} \end{bmatrix} e^{jn\omega_1 t} =$$
$$= \sqrt{2}\,\mathrm{Re} \sum_{n\in N} \boldsymbol{U}_n e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} (\boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{U}_n^{\mathrm{n}}) e^{jn\omega_1 t} \qquad (16)$$

The vectors of the positive and the negative sequence voltages in this formula can be expressed in terms the crms values U_n^{p} and U_n^{n} using unit three-phase vectors $\mathbf{1}^{\mathrm{p}}$ and $\mathbf{1}^{\mathrm{n}}$ defined as follows

$$\begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix} \overset{\mathrm{df}}{=} \mathbf{1}^{\mathrm{p}}, \qquad \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} \overset{\mathrm{df}}{=} \mathbf{1}^{\mathrm{n}} \qquad (17)$$

and illustrated in Fig. 5
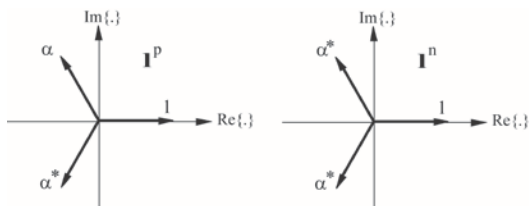
Fig. 5. Unit three-phase vectors $\mathbf{1}^{\mathrm{p}}$ and $\mathbf{1}^{\mathrm{n}}$.

Using these unit vectors, the supply voltage can be presented in the form

$$\boldsymbol{u}(t) = \sum_{n\in N} \boldsymbol{u}_n(t) = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} (\boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{U}_n^{\mathrm{n}}) e^{jn\omega_1 t} =$$
$$= \sqrt{2}\,\mathrm{Re} \sum_{n\in N} (\mathbf{1}^{\mathrm{p}} U_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}} U_n^{\mathrm{n}}) e^{jn\omega_1 t} \qquad (18)$$

The load in Fig. 6(a) is equivalent with respect to active power P to a balanced resistive load, shown in Fig. 6(b),
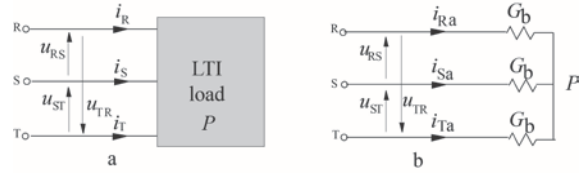

Fig. 6. A three-phase load and a balanced resistive load equivalent with respect to active power P.

of conductance

$$G_{\mathrm{b}} = \frac{P}{\|\boldsymbol{u}\|^2} \qquad (19)$$

where $\|\boldsymbol{u}\|$ denotes three-phase rms value of the supply voltage, equal to the root of sum of squares of line voltages rms values, namely

$$\|\boldsymbol{u}\| = \sqrt{\|u_R\|^2 + \|u_S\|^2 + \|u_T\|^2} \qquad (20)$$

The current of such an equivalent load

$$\boldsymbol{i}_{\mathrm{a}} = G_{\mathrm{b}} \boldsymbol{u} = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} G_{\mathrm{b}} (\mathbf{1}^{\mathrm{p}} U_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}} U_n^{\mathrm{n}}) e^{jn\omega_1 t} \qquad (21)$$

will be referred to as **the active current** of the load.

According to the assumption in this paper, the load is linear, so that the current response of the load to this voltage can be calculated harmonic-by-harmonic.

The load at each harmonic frequency has the active and reactive powers. For the harmonic of the nth order

$$P_n = \mathrm{Re}\{\boldsymbol{U}_n^{\mathrm{T}} \boldsymbol{I}_n^*\}, \qquad Q_n = \mathrm{Im}\{\boldsymbol{U}_n^{\mathrm{T}} \boldsymbol{I}_n^*\} \qquad (22)$$

The load can be unbalanced for the nth order harmonic, but with respect to the active and reactive powers P_n and Q_n at voltage $\boldsymbol{u}_n$ such a load is equivalent to a balanced load of the phase admittance

$$Y_{n\mathrm{b}} = G_{n\mathrm{b}} + jB_{n\mathrm{b}} = \frac{P_n - jQ_n}{\|\boldsymbol{u}_n\|^2} = \frac{C_n^*}{\|\boldsymbol{u}_n\|^2} \qquad (23)$$

where $\|\boldsymbol{u}_n\|$ denotes the three-phase rms value of the nth order voltage harmonic, equal to

$$\|\boldsymbol{u}_n\| = \sqrt{U_{Rn}^2 + U_{Sn}^2 + U_{Tn}^2} \qquad (24)$$

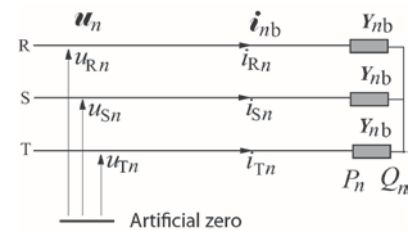Such equivalent load is shown in Fig. 7.

Fig. 7. A balanced load equivalent to the original with respect to P_n and Q_n powers of the nth order harmonic.

The line current of such an equivalent load is composed of the active current

$$\boldsymbol{i}_{na} = G_{nb}\,\boldsymbol{u}_n = \sqrt{2}\,\mathrm{Re}\{G_{nb}\,(\boldsymbol{U}_n^p + \boldsymbol{U}_n^n)e^{jn\omega_1 t}\} =$$
$$= \sqrt{2}\,\mathrm{Re}\{G_{nb}(\mathbf{1}^p U_n^p + \mathbf{1}^n U_n^n)e^{jn\omega_1 t}\} \tag{25}$$

and the reactive current

$$\boldsymbol{i}_{nr} = B_{nb}\,\boldsymbol{u}_n(t+\frac{T}{4n}) = \sqrt{2}\,\mathrm{Re}\{jB_{nb}(\boldsymbol{U}_n^p + \boldsymbol{U}_n^n)e^{jn\omega_1 t}\} =$$
$$= \sqrt{2}\,\mathrm{Re}\{jB_{nb}(\mathbf{1}^p U_n^p + \mathbf{1}^n U_n^n)e^{jn\omega_1 t}\} \tag{26}$$

The admittance Y_{nb} is admittance of equivalent balanced load, while the load can be unbalanced for the nth order harmonic. Consequently, the nth order harmonic of the line current $\boldsymbol{i}_n$ can contain the unbalanced current

$$\boldsymbol{i}_{nu} = \boldsymbol{i}_n - \boldsymbol{i}_{nb} = \boldsymbol{i}_n - (\boldsymbol{i}_{na} + \boldsymbol{i}_{nr}) =$$
$$= \sqrt{2}\,\mathrm{Re}\{\boldsymbol{I}_n - Y_{nb}(\mathbf{1}^p U_n^p + \mathbf{1}^n U_n^n)e^{jn\omega_1 t}\} \tag{27}$$

It means that each current harmonic $\boldsymbol{i}_n$ can be regarded as a sum of three components

$$\boldsymbol{i}_n = \boldsymbol{i}_{na} + \boldsymbol{i}_{nr} + \boldsymbol{i}_{nu} \tag{28}$$

and consequently, the load current (6) is equal to

$$\boldsymbol{i}(t) = \begin{bmatrix} i_R(t) \\ i_S(t) \\ i_T(t) \end{bmatrix} = \sum_{n\in N}\boldsymbol{i}_n(t) = \sum_{n\in N}(\boldsymbol{i}_{na} + \boldsymbol{i}_{nr} + \boldsymbol{i}_{nu}) \tag{29}$$

The current

$$\sum_{n\in N}\boldsymbol{i}_{nr} \overset{df}{=} \boldsymbol{i}_r = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_{nb}(\mathbf{1}^p U_n^p + \mathbf{1}^n U_n^n)e^{jn\omega_1 t} \tag{30}$$

occurs in the load current because of the phase shift of the current harmonics with respect to the supply voltage harmonic. Therefore it can be regarded as a reactive current of the load.

The current

$$\sum_{n\in N}\boldsymbol{i}_{nu} \overset{df}{=} \boldsymbol{i}_u \tag{31}$$

occurs in the load current because of the load imbalance for harmonic frequencies.

The current

$$\sum_{n\in N}\boldsymbol{i}_{na} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} G_{nb}(\mathbf{1}^p U_n^p + \mathbf{1}^n U_n^n)e^{jn\omega_1 t} \tag{32}$$

is not the active current $\boldsymbol{i}_a$ of the load, however. These two currents differ by

$$\sum_{n\in N}\boldsymbol{i}_{na} - \boldsymbol{i}_a =$$
$$= \sqrt{2}\,\mathrm{Re}\sum_{n\in N}(G_{nb} - G_b)(\mathbf{1}^p U_n^p + \mathbf{1}^n U_n^n)e^{jn\omega_1 t} \overset{df}{=} \boldsymbol{i}_s \tag{33}$$

This difference occurs when the conductance G_b of equivalent balanced load differs from conductances G_{nb} for harmonic frequencies. The current $\boldsymbol{i}_s$ will be referenced to as a scattered current.

Combining (29) – (33), the load current can be expressed as follows

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_s + \boldsymbol{i}_r + \boldsymbol{i}_u \tag{34}$$

Each of these currents is associated with a different phenomenon in the load. The active current $\boldsymbol{i}_a$ is associated with permanent energy delivery with active power P to the load. Any other current contribute to this transfer. The scattered current $\boldsymbol{i}_s$ is associated with the phenomenon of a change of the load conductance G_{nb} with harmonic order n. The reactive current $\boldsymbol{i}_s$ is associated with the phase-shift of the load current harmonics with respect to the supply voltage harmonics. The unbalanced current $\boldsymbol{i}_u$ is associated with the load imbalance for harmonic frequencies.

IV. ORTHOGONALITY

The currents in decomposition (34) affect the three-phase rms value $\|\boldsymbol{i}\|$ of the load current on the condition that they are mutually orthogonal, meaning their scalar product

$$(\boldsymbol{i}_x, \boldsymbol{i}_v) = \frac{1}{T}\int_0^T \boldsymbol{i}_x^T(t)\,\boldsymbol{i}_v(t)\,dt \tag{35}$$

is equal to zero. Harmonics of different order n, including are mutually orthogonal. Therefore, calculation of the three-phase rms value of quantities defined as a sum of harmonics is straightforward. In particular,

$$\|\boldsymbol{i}_r\| = \sqrt{\sum_{n\in N}\|\boldsymbol{i}_{nr}\|^2} = \sqrt{\sum_{n\in N} B_{nb}^2\|\boldsymbol{u}_n\|^2} \tag{36}$$

$$\|\boldsymbol{i}_s\| = \sqrt{\sum_{n\in N}\|\boldsymbol{i}_{ns}\|^2} = \sqrt{\sum_{n\in N}(G_{nb}-G_b)^2\|\boldsymbol{u}_n\|^2} \tag{37}$$

$$\|\boldsymbol{i}_u\| = \sqrt{\sum_{n\in N}\|\boldsymbol{i}_{nu}\|^2} \tag{38}$$

Orthogonality of the active, reactive and unbalanced currents in systems with sinusoidal quantities was proven in [..]. This applies, of course, to individual harmonic of any order n. Therefore, the active, reactive and unbalanced components of the nth order harmonic of the load current $\boldsymbol{i}_n$ in decomposition (28) are mutually orthogonal, i.e.,

$$(\boldsymbol{i}_{na}, \boldsymbol{i}_{nr}) = (\boldsymbol{i}_{na}, \boldsymbol{i}_{nu}) = (\boldsymbol{i}_{nr}, \boldsymbol{i}_{nu}) = 0 \tag{39}$$

Current components in decomposition (29) are sums of harmonics. Because harmonics of different order r and s, are mutually orthogonal, then the scalar products of two currents, which are sums of harmonics, can be expressed generally as

$$(\boldsymbol{i}_x, \boldsymbol{i}_v) = (\sum_{r\in N}\boldsymbol{i}_{rx}, \sum_{s\in N}\boldsymbol{i}_{sv}) = \sum_{n\in N}(\boldsymbol{i}_{nx}, \boldsymbol{i}_{nv}). \tag{40}$$

Thus, if harmonics of the same order n of two currents are mutually orthogonal, i.e., $(\boldsymbol{i}_{nx}, \boldsymbol{i}_{nv}) = 0$, then such currents are orthogonal as well. Consequently, all terms on the right side of (29) are mutually orthogonal, hence

$$\|\boldsymbol{i}\|^2 = \|\sum_{n\in N}\boldsymbol{i}_{na}\|^2 + \|\boldsymbol{i}_r\|^2 + \|\boldsymbol{i}_u\|^2 \tag{41}$$

Since

$$\sum_{n\in N}\boldsymbol{i}_{na} = \boldsymbol{i}_a + \boldsymbol{i}_s \tag{42}$$

the question arises, are the active and scattered currents mutually orthogonal?

need to create these components, incorporating the applicable criteria that follow.

V. EASE OF USE

For the nth order harmonic

$$Y_{\mathrm{b}n} = G_{\mathrm{b}n} + jB_{\mathrm{b}n} = \frac{P_n - jQ_n}{\|\boldsymbol{u}_n\|^2} = \frac{C_n^*}{\|\boldsymbol{u}_n\|^2}$$

$$\boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n}.$$

$$\boldsymbol{i}_{\mathrm{a}n} = G_{\mathrm{b}n}\,\boldsymbol{u}_n = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{b}n}(\boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{U}_n^{\mathrm{n}})e^{jn\omega_1 t}\} = $$
$$= \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{b}n}(\mathbf{1}^{\mathrm{p}}U_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}U_n^{\mathrm{n}})e^{jn\omega_1 t}\}$$

$$\boldsymbol{i}_{\mathrm{r}n} = B_{\mathrm{b}n}\,\boldsymbol{u}_n(t+T/n/4) = \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{b}n}(\boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{U}_n^{\mathrm{n}})e^{jn\omega_1 t}\} = $$
$$= \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{b}n}(\mathbf{1}^{\mathrm{p}}U_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}U_n^{\mathrm{n}})e^{jn\omega_1 t}\}$$

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Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, sc,

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dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".

- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.

- Do not mix complete spellings and abbreviations of units: "Wb/m2" or "webers per square meter", not "webers/m2". Spell out units when they appear in text: ". . . a few henries", not ". . . a few H".

- Use a zero before decimal points: "0.25", not ".25". Use "cm3", not "cc". (bullet list)

C. Equations

The equations are an exception to the prescribed specifications of this template. You will need to determine whether or not your equation should be typed using either the Times New Roman or the Symbol font (please no other font). To create multileveled equations, it may be necessary to treat the equation as a graphic and insert it into the text after your paper is styled.

Number equations consecutively. Equation numbers, within parentheses, are to position flush right, as in (1), using a right tab stop. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in

$$\alpha + \beta = \chi. \tag{1}$$

Note that the equation is centered using a center tab stop. Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(1)", not "Eq. (1)" or "equation (1)", except at the beginning of a sentence: "Equation (1) is . . ."

D. Some Common Mistakes

- The word "data" is plural, not singular.

- The subscript for the permeability of vacuum $\square0$, and other common scientific constants, is zero with subscript formatting, not a lowercase letter "o".

- In American English, commas, semi-/colons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used,

instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)

- A graph within a graph is an "inset", not an "insert". The word alternatively is preferred to the word "alternately" (unless you really mean something that alternates).

- Do not use the word "essentially" to mean "approximately" or "effectively".

- In your paper title, if the words "that uses" can accurately replace the word "using", capitalize the "u"; if not, keep using lower-cased.

- Be aware of the different meanings of the homophones "affect" and "effect", "complement" and "compliment", "discreet" and "discrete", "principal" and "principle".

- Do not confuse "imply" and "infer".

- The prefix "non" is not a word; it should be joined to the word it modifies, usually without a hyphen.

- There is no period after the "et" in the Latin abbreviation "et al.".

- The abbreviation "i.e." means "that is", and the abbreviation "e.g." means "for example".

An excellent style manual for science writers is [7].

VII. USING THE TEMPLATE

After the text edit has been completed, the paper is ready for the template. Duplicate the template file by using the Save As command, and use the naming convention prescribed by your conference for the name of your paper. In this newly created file, highlight all of the contents and import your prepared text file. You are now ready to style your paper; use the scroll down window on the left of the MS Word Formatting toolbar.

A. Authors and Affiliations

The template is designed so that author affiliations are not repeated each time for multiple authors of the same affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization). This template was designed for two affiliations.

1) For author/s of only one affiliation (Heading 3): To change the default, adjust the template as follows.

a) Selection (Heading 4): Highlight all author and affiliation lines.

b) Change number of columns: Select the Columns icon from the MS Word Standard toolbar and then select "1 Column" from the selection palette.

c) Deletion: Delete the author and affiliation lines for the second affiliation.

d) For author/s of more than two affiliations: To change the default, adjust the template as follows.

e) Selection: Highlight all author and affiliation lines.

f) Change number of columns: Select the "Columns" icon from the MS Word Standard toolbar and then select "1 Column" from the selection palette.

g) Highlight author and affiliation lines of affiliation 1 and copy this selection.

h) Formatting: Insert one hard return immediately after the last character of the last affiliation line. Then paste down the copy of affiliation 1. Repeat as necessary for each additional affiliation.

i) Reassign number of columns: Place your cursor to the right of the last character of the last affiliation line of an even numbered affiliation (e.g., if there are five affiliations, place your cursor at end of fourth affiliation). Drag the cursor up to highlight all of the above author and affiliation lines. Go to Column icon and select "2 Columns". If you have an odd number of affiliations, the final affiliation will be centered on the page; all previous will be in two columns.

B. Identify the Headings

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is "Heading 5". Use "figure caption" for your Figure captions, and "table head" for your table title. Run-in heads, such as "Abstract", will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced. Styles named "Heading 1", "Heading 2", "Heading 3", and "Heading 4" are prescribed.

C. Figures and Tables

1) Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation "Fig. 1", even at the beginning of a sentence.

TABLE I.   TABLE TYPE STYLES

Table Head	Table Column Head		
	Table column subhead	*Subhead*	*Subhead*
copy	More table copy[a]		

a. Sample of a Table footnote. (Table footnote)

We suggest that you use a text box to insert a graphic (which is ideally a 300 dpi TIFF or EPS file, with all fonts embedded) because, in an MSW document, this method is somewhat more stable than directly inserting a picture.

To have non-visible rules on your frame, use the MSWord "Format" pull-down menu, select Text Box >

Figure 1.   Example of a figure caption. (figure caption)

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity "Magnetization", or "Magnetization, M", not just "M". If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write "Magnetization (A/m)" or "Magnetization {A[m(1)]}", not just "A/m". Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

ACKNOWLEDGMENT (HEADING 5)

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted expression, "One of us (R. B. G.) thanks . . ." Instead, try "R. B. G. thanks". Put sponsor acknowledgments in the unnum-bered footnote on the first page.

REFERENCES

The template will number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use "Ref. [3]" or "reference [3]" except at the beginning of a sentence: "Reference [3] was the first . . ."

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the reference list. Use letters for table footnotes.

Unless there are six authors or more give all authors' names; do not use "et al.". Papers that have not been published, even if they have been submitted for publication, should be cited as "unpublished" [4]. Papers that have been accepted for publication should be cited as "in press" [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

[1]   G. Eason, B. Noble, and I. N. Sneddon, "On certain integrals of Lipschitz-Hankel type involving products of Bessel functions," Phil. Trans. Roy. Soc. London, vol. A247, pp. 529–551, April 1955. (references)

[2]   J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.

[3]   I. S. Jacobs and C. P. Bean, "Fine particles, thin films and exchange anisotropy," in Magnetism, vol. III, G. T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271–350.

[4]   K. Elissa, "Title of paper if known," unpublished.

[5]   R. Nicole, "Title of paper with only first word capitalized," J. Name Stand. Abbrev., in press.

[6]   Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, "Electron spectroscopy studies on magneto-optical media and plastic substrate interface," IEEE Transl. J. Magn. Japan, vol. 2, pp. 740–741, August 1987 [Digests 9th Annual Conf. Magnetics Japan, p. 301, 1982].

[7]   M. Young, The Technical Writer's Handbook. Mill Valley, CA: University Science, 1989.