Working and Reflected Active Powers of Three-Phase Loads

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***Abstract*—Unbalanced and/or harmonic generating loads have to be supplied with a working active power, which is higher than the active power measured at the load terminals. Consequently, energy providers of such loads are under paid for the energy delivered. The difference between the working active power and the common active power is referred to as the reflected active power. The reflected active power increases with the decline of the short circuit to load power ratio. Therefore, the reflected active power could be easily visible in weaker power systems, such as micro-grids in islanded mode or on terminals of very high power industrial loads, such as pulsing loads or arc furnaces. The load generated harmonics and/or load imbalance are the causes for the reflected active power. The paper presents results of studies on the dependence of the reflected active power on the relative short circuit power of the supply source.**

Keywords–Unbalanced loads, harmonics, power theory, power definitions, Currents Physical Components, CPC

I. INTRODUCTION

A study on the energy flow in systems with harmonic generating loads (HGLs) and/or unbalanced loads (ULs) are summarized in [2] resulting in a conclusion that this flow can be decomposed into two streams of the energy flowing in the opposite directions. Assuming that the energy source provides a symmetrical and sinusoidal voltage (S&SV), only the current fundamental harmonic of the positive sequence in phase with the generated voltage delivers, along with this voltage, permanent energy to the load. This is the first and the major stream of the energy flow. The second stream of energy flow occurs due to loads that generate current harmonics due to their non-linearity or periodic switching and current asymmetry due to imbalanced loading. Additionally, this second stream of energy flow can occur due to load voltage distortion and/or asymmetry caused by supply source impedance with the load generated current harmonics and/or the asymmetrical current. This second stream cannot convey the energy back to the internal voltage $e(t)$ source, because this voltage is sinusoidal and symmetrical. Therefore, energy has to be dissipated on the supply system resistance.

The average rate of these two streams of energy flow, calculated over the period T, specifies the working active power $P_{\rm w}$ and the reflected active power $P_{\rm r}$. The conventional active power is the difference of these two powers,

$$P = P_{\rm w} - P_{\rm r}\,. \tag{1}$$

The reflected active power occurs in the system because of the non-zero resistance $R_{\rm s}$ of the supply source and consequently, non-zero short circuit power. To reduce the load voltage rms variation with load power changes, the relative value of the short circuit power $S_{\rm sc}$ in residential or commercial distribution systems should be sufficiently high, at least 20-times higher than the maximum load power. In such a situation, the relative value of the supply source resistance $R_{\rm s}$ is low and the reflected active power $P_{\rm r}$ can be in the range, as demonstrated in [3], of a single percent of the active power P or even lower. This sufficiently high value of the short circuit power $S_{\rm sc}$ is needed to protect all users connected at the point of the common supply (PCS) against the supply voltage rms variations. This short circuit power $S_{\rm sc}$ is mainly specified by the power ratings and consequently, the cost of the transformer which supplies the PCS.

There are situations when a varying load that could be sensitive to the supply voltage rms variations is isolated from the rest of the loads. These could be traction systems or some high power industrial loads. Thus, cost optimization and technological constraints may justify a lower relative value of the short circuit power at such load terminals. In the case of arc furnace supplies, the short-circuit power at the supply could be only twice as high as the furnace power. The difference between the active power P and the reflected active power $P_{\rm r}$ can substantially increase in such situations. Also micro-grids operated in islanded mode could have a relatively low short-circuit power [5-8] and can be mainly composed of single-phase harmonic generating loads (HGL).

The phenomenon of the energy flow from harmonic generating or unbalanced loads back to the supply source was first reported in [1]. The concept of the ***working active power*** $P_{\rm w}$ and the ***reflected active power*** $P_{\rm r}$, along with these names was suggested in [2]. It was introduced, for clarity reasons, as a conclusion from the analysis of energy flow in a purely resistive system. The presence of a reactance and the phase-shift between the supply voltage and the load current obscures the reflected energy flow phenomenon. Thus, phenomena in resistive-reactive three-phase three-wire systems will be analyzed in this paper.

II. WORKING AND REFLECTED ACTIVE POWERS OF HARMONIC GENERATING LOADS

To preserve clarity of the presented analysis, balanced and unbalanced three-phase loads supplied with a three-wire line, will be analyzed separately. The internal voltage of the distribution system, arranged in vector format is,

$$\boldsymbol{e}(t) = [e_{\mathrm{R}}(t),\, e_{\mathrm{S}}(t),\, e_{\mathrm{T}}(t)]^{\mathrm{T}} \qquad (2)$$

and assumed to be sinusoidal and symmetrical of the positive sequence only.

Let a balanced harmonic generating load (HGL) be supplied as shown in Fig. 1.

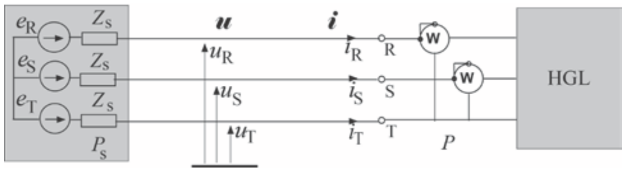

Figure 1. Circuit with Harmonic Generating Load (HGL).

Decomposition of the current vector $\boldsymbol{i}$ into Currents' Physical Components (CPC) in a circuit with a HGL requires detecting the sign of harmonic active powers P_n at the load terminals. When the internal voltage $\boldsymbol{e}$ is sinusoidal, then only the active power of the fundamental harmonic P_1 is positive [2]. The remaining ones are negative.

If $\boldsymbol{u}_{\mathrm{D}}$ and $\boldsymbol{i}_{\mathrm{D}}$ denote the load voltage and current components composed of harmonics for which the active power P_n is positive, then in the situation considered,

$$\boldsymbol{u}_{\mathrm{D}} = \boldsymbol{u}_1 \text{ and } \boldsymbol{i}_{\mathrm{D}} = \boldsymbol{i}_1. \qquad (3)$$

The remaining components are denoted by $\boldsymbol{u}_{\mathrm{C}}$ and $\boldsymbol{i}_{\mathrm{C}}$, such that

$$\boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_{\mathrm{C}}, \qquad \boldsymbol{i} = \boldsymbol{i}_1 + \boldsymbol{i}_{\mathrm{C}}\,. \qquad (4)$$

The load equivalent admittance $\boldsymbol{Y}_{\mathrm{e}}$ for a balanced load can be calculated having complex rms (crms) values of the load voltage and current $\boldsymbol{U}_{\mathrm{R1}}$ and $\boldsymbol{I}_{\mathrm{R1}}$ of the fundamental harmonic, namely

$$\boldsymbol{Y}_{\mathrm{e1}} = G_{\mathrm{e1}} + jB_{\mathrm{e1}} = \frac{\boldsymbol{I}_{\mathrm{R1}}}{\boldsymbol{U}_{\mathrm{R1}}}\,. \qquad (5)$$

In such a case, the current decomposition has the form

$$\boldsymbol{i} = \boldsymbol{i}_1 + \boldsymbol{i}_{\mathrm{C}} = \boldsymbol{i}_{\mathrm{a1}} + \boldsymbol{i}_{\mathrm{r1}} + \boldsymbol{i}_{\mathrm{C}} \qquad (6)$$

where

$$\boldsymbol{i}_{\mathrm{a1}} = G_{\mathrm{e}}\,\boldsymbol{u}_1 = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e1}}\boldsymbol{U}_1\,e^{j\omega_1 t}\} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{I}_{\mathrm{a1}}\,e^{j\omega_1 t}\} \qquad (7)$$

is the active current defined according to the CPC power theory, with $\boldsymbol{U}_1 = [\boldsymbol{U}_{\mathrm{R1}}, \boldsymbol{U}_{\mathrm{S1}}, \boldsymbol{U}_{\mathrm{T1}}]^{\mathrm{T}}$;

$$\boldsymbol{i}_{\mathrm{r1}} = \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{e1}}\boldsymbol{U}_1\,e^{j\omega_1 t}\} \qquad (8)$$

is the reactive current , while

$$\boldsymbol{i}_{\mathrm{C}} = \sum_{n\neq 1}\boldsymbol{i}_n = \sqrt{2}\,\mathrm{Re}\sum_{n=2}^{\infty}\boldsymbol{I}_n\,e^{jn\omega_1 t} \qquad (9)$$

is the load generated harmonic current.

The active power P measured at the load terminals is

$$P = \frac{1}{T}\int_0^T \boldsymbol{u}^{\mathrm{T}}(t)\,\boldsymbol{i}(t)\,dt = (\boldsymbol{u},\boldsymbol{i}) = (\boldsymbol{u}_1+\boldsymbol{u}_{\mathrm{C}},\boldsymbol{i}_1+\boldsymbol{i}_{\mathrm{C}}) = (\boldsymbol{u}_1,\boldsymbol{i}_1) + (\boldsymbol{u}_{\mathrm{C}},\boldsymbol{i}_{\mathrm{C}}) \qquad (10)$$

since quantities with indices "1" and "C" do not have common harmonic orders, they are mutually orthogonal.

The first scalar product in (10) is

$$(\boldsymbol{u}_1,\boldsymbol{i}_1) = (\boldsymbol{u}_1,\boldsymbol{i}_{\mathrm{a1}} + \boldsymbol{i}_{\mathrm{r1}}) = (\boldsymbol{u}_1,\boldsymbol{i}_{\mathrm{a1}}) = P_1 \qquad (11)$$

because

$$(\boldsymbol{u}_1,\boldsymbol{i}_{\mathrm{a1}}) = \mathrm{Re}\{\boldsymbol{U}_1^{\mathrm{T}}\boldsymbol{I}_{\mathrm{a1}}^{*}\} = \mathrm{Re}\{\boldsymbol{U}_1^{\mathrm{T}}G_{\mathrm{e1}}\boldsymbol{U}_1^{*}\} = G_{\mathrm{e1}}(U_{\mathrm{R1}}^2 + U_{\mathrm{S1}}^2 + U_{\mathrm{T1}}^2) = G_{\mathrm{e1}}\,||\boldsymbol{u}_1||^2 = P_1\,. \qquad (12)$$

The load voltage component $\boldsymbol{u}_{\mathrm{C}}$ is the supply source response to the load generated harmonic current

$$\boldsymbol{u}_{\mathrm{C}} = \sum_{n\neq 1}\boldsymbol{u}_n = \sqrt{2}\,\mathrm{Re}\sum_{n=2}^{\infty}\boldsymbol{U}_n\,e^{jn\omega_1 t} = -\sqrt{2}\,\mathrm{Re}\sum_{n=2}^{\infty}\boldsymbol{Z}_{\mathrm{s}n}\boldsymbol{I}_n\,e^{jn\omega_1 t}. \qquad (13)$$

The second scalar product in (10) is equal to

$$(\boldsymbol{i}_{\mathrm{C}},\boldsymbol{u}_{\mathrm{C}}) = \mathrm{Re}\sum_{n=2}^{\infty}\boldsymbol{I}_n^{\mathrm{T}}\boldsymbol{U}_n^{*} = \mathrm{Re}\sum_{n=2}^{\infty}\boldsymbol{I}_n^{\mathrm{T}}\boldsymbol{Z}_{\mathrm{s}n}\boldsymbol{I}_n^{*} = -\mathrm{Re}\sum_{n=2}^{\infty}\boldsymbol{Z}_{\mathrm{s}n}(I_{\mathrm{R}n}^2 + I_{\mathrm{S}n}^2 + I_{\mathrm{T}n}^2) = -R_{\mathrm{s}}\,||\boldsymbol{i}_{\mathrm{C}}||^2 = -P_{\mathrm{r}}\,. \qquad (14)$$

The power P_{r} occurs as the effect of the load generated current $\boldsymbol{i}_{\mathrm{C}}$ flowing in the supply system resistance R_{s} which is the reflected active power.

The active power P of the load, specified by (10), with (12) and (14), is decomposed into two components

$$P = (\boldsymbol{u}_1,\boldsymbol{i}_1) + (\boldsymbol{u}_{\mathrm{C}},\boldsymbol{i}_{\mathrm{C}}) = P_1 - P_{\mathrm{r}}\,. \qquad (15)$$

The active power of the fundamental harmonic P_1 is higher than the active power of the load P. Thus, the fundamental harmonic of the load voltages and currents provides the HGL with higher average rate of the energy flow than the active power P. This extra amount of energy is reflected back to the supply source, where it is dissipated in the resistance R_{s}. The active power P_1 needed to operate HGL with power P is referred to as the working active power P_{W}, i.e., $P_1 = P_{\mathrm{W}}$. It means that the active power decomposition (1) into the working active power P_{w} and the reflected active power P_{r}, obtained in [2] for purely resistive single-phase systems, is valid also for three-phase resistive-reactive systems.

The reflected active power P_{r} is the difference between the working active power P_{w} and the active power of the load P, namely

$$P_{\mathrm{r}} = P_{\mathrm{w}} - P = R_{\mathrm{s}}\,||\boldsymbol{i}_{\mathrm{C}}||^2\,. \qquad (16)$$

Thus, this property can be used for calculating the internal resistance of the supply source:

$$R_{\mathrm{s}} = \frac{P_{\mathrm{w}} - P}{||\boldsymbol{i}_{\mathrm{C}}||^2}\,. \qquad (17)$$

Observe that the three-phase rms value $||\boldsymbol{i}_{\mathrm{C}}||$ does not have to be calculated harmonic by harmonic, as formula (14) suggests. Since the currents $\boldsymbol{i}_{\mathrm{D}} = \boldsymbol{i}_1$ and $\boldsymbol{i}_{\mathrm{C}}$ are orthogonal, then

$$||\boldsymbol{i}_{\mathrm{C}}||^2 = ||\boldsymbol{i}||^2 - ||\boldsymbol{i}_1||^2\,. \qquad (18)$$

In systems with symmetrical voltages and currents, the three phase rms values of the line current $||\boldsymbol{i}||$ and its fundamental harmonic $||\boldsymbol{i}_1||$ are higher by a factor of $\sqrt{3}$ more than the rms values of line currents and their fundamental harmonics. In such a situation, it is enough to measure the rms value of the line current, say in phase R, $||i_{\rm R}||$ and the crms value of its fundamental harmonic $\boldsymbol{I}_{\rm R1}$.

If an A/D converter provides K current samples $i_{\rm k}$ in a period T, then the square of the current rms value is

$$||i||^2 = \frac{1}{K}\sum_{k=0}^{K-1} i_k^2 \,. \qquad (19)$$

and the crms value of the current fundamental harmonic is

$$\boldsymbol{I}_1 = \frac{\sqrt{2}}{K}\sum_{k=0}^{K-1} i_k\, e^{j\frac{2\pi}{K}k} \,. \qquad (20)$$

The reflected active power $P_{\rm r}$ contributes to energy loss at the supply system resistance. It can be easily calculated as the difference between the working active power $P_{\rm w}$ and the conventional active power P. This is the more visible component of extra energy loss when HGLs are supplied. The second component is less visible in which the HGL has to be supplied with a current of higher rms value of the fundamental harmonic than a LTI load of the same active power P. Consequently, more energy is lost at the supply source resistance. For clarity, this is illustrated below by an analysis of energy flow in purely resistive, single-phase circuits with HGL and LTI loads of the same active power, shown in Fig. 2.

Numerical illustration 1. Let us analyze circuits with the internal voltage of the supply source equal to

$$e(t) = 100\sqrt{2}\sin\omega_1 t \ \ \text{V} \,.$$

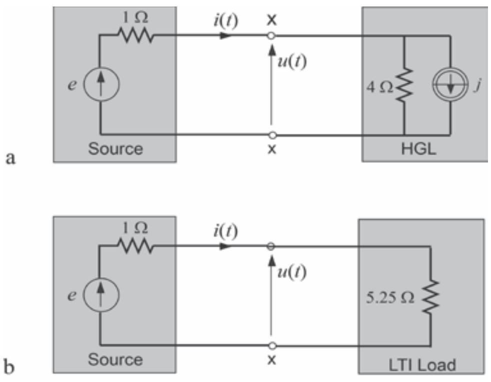

Figure 2. Circuits with resistive HGL and LTI loads.

The HGL is composed of a resistor and current source of the third order harmonic equal to

$$j(t) = 20\sqrt{2}\sin 3\omega_1 t \ \ \text{A} \,.$$

The current and the voltage at the load terminals are

$$i(t) = 20\sqrt{2}\sin\omega_1 t \ + 16\sqrt{2}\sin 3\omega_1 t \ \ \text{A}$$

$$u(t) = 80\sqrt{2}\sin\omega_1 t - 16\sqrt{2}\sin 3\omega_1 t \ \ \text{V} \,.$$

The active power of the load is

$$P = (u, i) = 80\times 20 - 16\times 16 = 1{,}344 \ \text{W} \,.$$

The working active power is

$$P_{\rm w} = (u_1 \,,\, i_1) = 80\times 20 = 1{,}600 \ \text{W}$$

and the reflected active power

$$P_{\rm r} = P_{\rm w} - P = 256\text{W} \,.$$

Active power of the supply source resistance, $R_{\rm s} = 1\Omega$, is

$$P_{\rm s} = R_{\rm s}\,||i||^2 = 1\times(20^2 + 16^2) = 656\text{W} \,.$$

The load shown in Fig. 2(b) is equivalent with respect to the active power P = 1,344 W to the load in Fig. 2(b). The current and the voltage at its load terminals are

$$i(t) = 16\sqrt{2}\sin\omega_1 t \ \ \text{A}, \qquad u(t) = 84\sqrt{2}\sin\omega_1 t \ \text{V} \,.$$

The active power of the supply source resistance in the circuit shown in Fig. 2(b) is $P_{\rm s}$ = 256 W. The difference between the active power of the internal resistance of the source in both circuit is equal to 400W. It contains the reflected active power $P_{\rm r}$ = 256W and the effect of the current fundamental harmonic rms value increase from 16A to 20A, because the circuit with HGL has to be supplied with working active power $P_{\rm w}$ = 1,600 W, which is higher than the load active power P = 1344 W. This increases

$$\Delta P_{\rm s} = 1\times 20^2 - 1\times 16^2 = 144\text{W} \,.$$

The relative difference between the reflected active power and the working active power depends mainly on the internal resistance of the supply source. This dependence was investigated experimentally in laboratory conditions using a three phase circuit shown in Fig. 3.

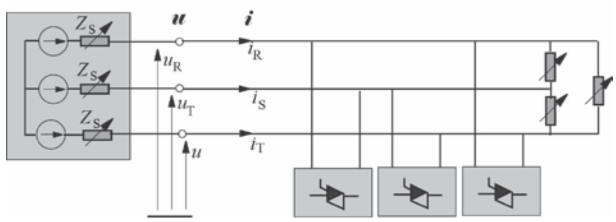

Figure 3. Experimental three-phase circuit with HGL.

Three single–phase rectifiers were used for controlling the load current harmonic distortion. The modification of the load impedance $Z_{\rm S}$ made the relative short circuit power possible. A LabView-based digital instrument was used in the experiment power measurements.

Measurement results for a balanced load of the power factor for the fundamental harmonic λ_1 = 0.7 and the load current total harmonic distortion (THD) equal to $\delta_{\rm i}$ = 100% for different relative short circuit power $S_{\rm sc}$ are in Table 1.

TABLE I Powers in circuit with HGL

$S_{\rm sc}$	P [W]	$P_{\rm w}$ [W]	$P_{\rm r}$ [W]	$P_{\rm w}/P$
20	228.5	232.8	4.30	1.02
15	230.0	239.8	9.80	1.04
10	210.3	221.7	11.4	1.05
5	204.4	223.9	19.5	1.10
4	224.4	256.6	32.2	1.14
3	214.3	250.1	35.8	1.17
2	150.0	189.3	39.3	1.26

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These results demonstrate that the difference between the working active power and the traditional active power increases, as expected, with the relative short circuit power reduction.

III. WORKING AND REFLECTED ACTIVE POWERS OF UNBALANCED LTI LOADS

When the load is unbalanced and the line currents are asymmetrical, the voltage at the load terminals $\boldsymbol{u}$ is also asymmetrical because the voltage drops across the internal supply source impedance can be different for each line.

The unbalanced LTI load current decomposition into the Currents' Physical Components (CPC) at sinusoidal, but asymmetrical supply voltage has the form [4]

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{b}} + \boldsymbol{i}_{\mathrm{u}} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}} \tag{21}$$

where the current $\boldsymbol{i}_{\mathrm{b}}$ is a current of a balanced load equivalent to the original one with respect to the active and reactive powers P and Q of the phase admittance

$$Y_{\mathrm{b}} = G_{\mathrm{b}} + jB_{\mathrm{b}} = \frac{P - jQ}{\|\boldsymbol{u}\|^2}. \tag{22}$$

Of the three currents on the right side of (21) only the first one $\boldsymbol{i}_{\mathrm{a}}$ contributes to permanent energy flow, meaning to the active power P of the load. Since the load current $\boldsymbol{i}$ is composed of the positive $\boldsymbol{i}^{\mathrm{p}}$ and negative $\boldsymbol{i}^{\mathrm{n}}$ sequence components and consequently, also the voltage is composed of such components, $\boldsymbol{u}^{\mathrm{p}}$ and $\boldsymbol{u}^{\mathrm{n}}$, the load power is

$$\begin{aligned} P = (\boldsymbol{u},\boldsymbol{i}) = (\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}, \boldsymbol{i}^{\mathrm{p}} + \boldsymbol{i}^{\mathrm{n}}) = (\boldsymbol{u}^{\mathrm{p}},\boldsymbol{i}^{\mathrm{p}}) + (\boldsymbol{u}^{\mathrm{n}},\boldsymbol{i}^{\mathrm{n}}) = \\ = P^{\mathrm{p}} - R_{\mathrm{s}}(\boldsymbol{i}^{\mathrm{n}},\boldsymbol{i}^{\mathrm{n}}) = P^{\mathrm{p}} - R_{\mathrm{s}}\|\boldsymbol{i}^{\mathrm{n}}\|^2 = P^{\mathrm{p}} - P^{\mathrm{n}}. \end{aligned} \tag{23}$$

The last term represents the active of the supply source resistance R_{s}. This is the reflected active power, $P_{\mathrm{r}} = P^{\mathrm{n}}$, which occurs as the effect of the load current asymmetry because of the load imbalance. Thus the unbalanced load has to be supplied with the power P^{p}, which is higher than the load active power P. Thus, this power is just the working active power, $P_{\mathrm{w}} = P^{\mathrm{p}}$, of the unbalanced load. It means that the active power decomposition (1), namely

$$P = P_{\mathrm{w}} - P_{\mathrm{r}} \tag{24}$$

is also valid for LTI unbalanced resistive-reactive loads.

Similar to the case of HGL, this property enables the calculation of the internal resistance of the supply source

$$R_{\mathrm{s}} = \frac{P_{\mathrm{w}} - P}{\|\boldsymbol{i}^{\mathrm{n}}\|^2}. \tag{25}$$

As it was similarity discussed previously for HGLs, the reflected active power P_{r} amounts to only a part of increase of the active power of the supply source resistance due to the load imbalance. This imbalance causes the load to be supplied with a higher power than the load active power P, meaning the working active power P_{w}. Consequently, the energy loss at its delivery is higher than the loss of a balanced load of the same active power. This is illustrated by analysis of two loads of the same active power. One of them is unbalanced, as shown in Fig. 4(a), the second, shown in Fig. 4(b), is balanced.

Numerical illustration 2. Let us analyze two circuits supplied with symmetrical voltage, which for phase R is equal to

$$e_{\mathrm{R}}(t) = 220\sqrt{2}\sin\omega_1 t \ \ \mathrm{V}$$

and unbalanced and balanced loads, shown in Fig. 4(a) and 4(b), of the same active power P.

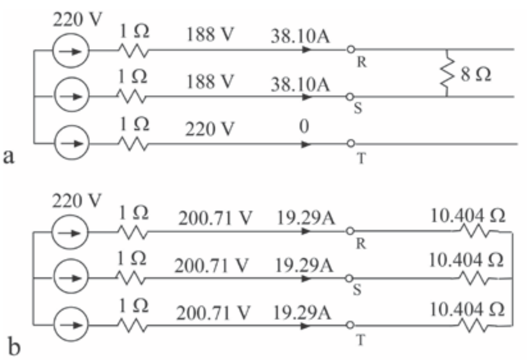

Figure 4. Circuits with balanced and unbalanced load.

The active power P of both loads is P = 11,616 W. The crms values of the symmetrical components of the load voltage in circuit with the unbalanced load are

$$\boldsymbol{U}^{\mathrm{p}} = 198.0 \ \mathrm{V}, \quad \boldsymbol{U}^{\mathrm{n}} = 21.87 e^{-j119.8^{\mathrm{o}}} \ \mathrm{V}$$

and the crms values of the load currents symmetrical components

$$\boldsymbol{I}^{\mathrm{p}} = 22.0 \ \mathrm{A}, \quad \boldsymbol{I}^{\mathrm{n}} = 22.0 e^{-j60^{\mathrm{o}}} \ \mathrm{A}.$$

The working active power of the load is

$$P_{\mathrm{w}} = P^{\mathrm{p}} = 3U^{\mathrm{p}}I^{\mathrm{p}}\cos\varphi^{\mathrm{p}} = 13{,}068 \ \mathrm{W}$$

and the reflected active power

$$P_{\mathrm{r}} = -P^{\mathrm{n}} = -3U^{\mathrm{n}}I^{\mathrm{n}}\cos\varphi^{\mathrm{n}} = 1{,}452 \ \mathrm{W}.$$

The active power loss of the internal resistance

$$P_{\mathrm{s}} = P_{\mathrm{su}} = 2R_{\mathrm{s}}I_{\mathrm{R}}^2 = 2{,}903 \ \mathrm{W}.$$

In the circuit shown in Fig. 4(b) with balanced load which has the same active power, the active power of the supply source resistance is

$$P_{\mathrm{s}} = P_{\mathrm{sb}} = 3R_{\mathrm{s}}I_{\mathrm{R}}^2 = 1{,}116 \ \mathrm{W}.$$

In the system with the unbalanced load, even if the reflected active power is subtracted, the active power of this resistance

$$P_{\mathrm{su}} - P_{\mathrm{r}} = 2{,}903 - 1{,}452 = 1{,}451 \ \mathrm{W}$$

is higher than this power in a system with equivalent balanced load. Thus the hidden increase of this power loss is

$$\Delta P_{\mathrm{s}} = P_{\mathrm{su}} - P_{\mathrm{su}} = 1{,}451 - 1{,}116 = 335\mathrm{W}.$$

Measurement results for an unbalanced load of the power factor λ = 0.7 in the circuit as shown in Fig. 3, but with THD equal to δ_{i} = 0%, and only one line-to-line single phase LTI load and various relative short-circuit powers S_{sc} are compiled in Table II.

TABLE II Powers in circuit with one line-to-line LTI load

S_{sc}	P [W]	P_w [W]	P_r [W]	P_w/P
20	77.2	77.9	0.7	1.01
15	79.9	81.7	1.8	1.02
10	74.5	78.4	3.9	1.05
5	78.2	84.6	6.4	1.08
4	85.2	95	9.8	1.11
3	82.4	94.2	11.8	1.14
2	88.7	104.8	16.1	1.18

Measurement results in a similar experiment, but with two line-to-line single phase LTI loads, are compiled in Table III.

TABLE III Powers in circuit with two line-to line LTI loads

S_{sc}	P [W]	P_w [W]	P_r [W]	P_w/P
20	152.7	153.3	0.6	1.004
15	156.1	157.6	1.5	1.010
10	143.6	146.8	3.2	1.022
5	146.3	151.5	5.2	1.036
4	153.4	159.8	6.4	1.042
3	153.3	160.6	7.3	1.048
2	162.3	171.1	8.8	1.054

IV. WORKING AND REFLECTED ACTIVE POWERS OF UNBALANCED HGLS

When the internal voltage $\boldsymbol{e}(t)$ of the supply system is symmetrical and sinusoidal then the effects of the load imbalance and generation of the current harmonics could be superimposed. This is because both the load imbalance and generated harmonics contribute to the reflected active power.

In such systems with unbalanced load, only the active power of the fundamental harmonic is positive. Thus, the load current can be decomposed into three-phase vectors of the current fundamental harmonic $\boldsymbol{i}_1$ and of all load generated harmonics $\boldsymbol{i}_C$, namely

$$\boldsymbol{i} = \boldsymbol{i}_1 + \boldsymbol{i}_C \,. \tag{26}$$

If the current fundamental harmonic $\boldsymbol{i}_1$ is composed of the active, reactive and unbalanced currents, then

$$\boldsymbol{i} = \boldsymbol{i}_{a1} + \boldsymbol{i}_{r1} + \boldsymbol{i}_{u1} + \boldsymbol{i}_C \,. \tag{27}$$

The active and reactive currents are defined by (7) and (8), while

$$\boldsymbol{i}_{u1} = \boldsymbol{i}_1 - (\boldsymbol{i}_{a1} + \boldsymbol{i}_{r1}) \,. \tag{28}$$

Currents $\boldsymbol{i}_{a1}$ and $\boldsymbol{i}_{r1}$ are of the positive sequence, while the current $\boldsymbol{i}_{u1}$ is of the negative sequence, so that in a response to this current the load voltage is equal to

$$\boldsymbol{u} = \boldsymbol{u}_1^p + \boldsymbol{u}_1^n + \boldsymbol{u}_C \,. \tag{29}$$

The active power of the load can be expressed as

$$\begin{aligned} P &= (\boldsymbol{u},\boldsymbol{i}) = (\boldsymbol{u}_1^p + \boldsymbol{u}_1^n + \boldsymbol{u}_C, \boldsymbol{i}_{a1} + \boldsymbol{i}_{r1} + \boldsymbol{i}_{u1} + \boldsymbol{i}_C) = \\ &= (\boldsymbol{u}_1^p, \boldsymbol{i}_{a1}) + (\boldsymbol{u}_1^n, \boldsymbol{i}_{u1}) + (\boldsymbol{u}_C, \boldsymbol{i}_C) = \\ &= P^p - R_s(\boldsymbol{i}_{u1}, \boldsymbol{i}_{u1}) - R_s(\boldsymbol{i}_C, \boldsymbol{i}_C) = \\ &= P^p - R_s\|\boldsymbol{i}_{u1}\|^2 - R_s\|\boldsymbol{i}_C\|^2 = P_w - P_r \end{aligned} \tag{30}$$

with

$$P_r = R_s\|\boldsymbol{i}_{u1}\|^2 + R_s\|\boldsymbol{i}_C\|^2 \,. \tag{31}$$

Formula (31) shows that the unbalanced and the load generated harmonic currents contribute to the reflected active power independently of each other.

Measurement results for an unbalanced load of the power factor $\lambda = 0.7$ in the circuit as shown in Fig. 3, with THD equal to $\delta_i = 100\%$, and only one line-to-line single phase LTI load are compiled in Table IV.

TABLE IV Powers in circuit with one line-to line HGL

S_{sc}	P [W]	P_w [W]	P_r [W]	P_w/P
20	76.0	79.8	3.8	1.05
15	76.4	83.0	6.6	1.09
10	70.3	78.1	7.8	1.11
5	69.7	82.5	12.8	1.18
4	75.3	95.7	20.4	1.27
3	73.1	99.7	26.6	1.36
2	50.8	76.6	25.8	1.51

V. CONCLUSIONS

The paper demonstrates that the active power P decomposition into the working active power P_w and the reflected active power P_r, introduced originally for purely resistive systems, is valid for resistive-reactive systems with unbalanced LTI loads and HGLs.

REFERENCES

[1] L.S. Czarnecki, "Comments on active power flow and energy accounts in electrical systems with nonsinusoidal voltage and asymmetry," IEEE Trans. on Power Delivery, Vol. 11, No. 3, pp. 1244-1250, 1996.

[2] L.S. Czarnecki, "Working, reflected and detrimental active powers," IET on Generation, Transmission and Distribution, Vol. 6, No. 3, pp. 223-239, 2012.

[3] L.S. Czarnecki, T.N. Toups, "Working and reflected active powers of harmonics generating single-phase loads," Przegląd Elektrotechniczny), R. 89, No. 10, 2014.

[4] L.S. Czarnecki, B. Prashanna, "Currents' Physical Components (CPC) in three-phase systems with asymmetrical voltage," To be published in Przegląd Elektrotechniczny, 2015.

[5] A. Ipalchi, F. Albyeh, "Grid of the future," IEEE Power and Energy Magazine, pp. 52-62, March/April 2009.

[6] J. McDonald, "Leader or follower: developing the smart grid business case," IEEE Power & Energy Magazine, pp. 18-24, Nov./Dec. 2008.

[7] S. Chen at al., "Service-oriented Advanced Metering Infrastructure for Smart Grids," 2010 Asia-Pacific Power and Energy Conference, APPEC, pp. 1-4.

[8] G. Mauri, D. Moneta, C. Bettoni, "Energy conservation and smart grids: new challenge for multimetering infrastructures," IEEE Power Tech. Conference, Bucharest, pp. 1-5, 2009.

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