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What is Wrong with the Conservative Power Theory (CPT)

Leszek S. Czarnecki, IEEE Life Fellow, IEEE

Abstract – **The Conservative Power Theory (CPT) is one of the latest approaches to definitions of powers and compensation in systems with nonsinusoidal voltages and currents. It is shown in this paper that the CPT misinterprets power phenomena in electrical systems, however, and it does not create right fundamentals for the power factor improvement by reactive compensation. In particular, capacitive compensation of the "reactive energy" *W*, as defined in the CPT, can even degrade the power factor. It is also demonstrated in the paper that the unbalanced current in the CPT is wrongly defined.**

Keywords – power definitions; reactive current; reactive power; distortion power; Currents' Physical Components; CPC.

I. Introduction

The Conservative Power Theory (CPT) seems to be the latest in the long chain of attempts aimed at developing power theory of systems with nonsinusoidal voltages and currents, initiated by Steinmetz's observation [1] that the apparent power S could be higher than the active power P, in spite of the lack of the voltage and current phase-shift.

Its development started in 2003 in paper [8], where mathematical fundamentals of the CPT were presented for single-phase systems with an extension to poly-phase networks. Later the CPT was focused mainly on three-phase systems [10-12]. It has now disseminated in electrical engineering and provides CPT-based interpretations of the power phenomena in electrical systems and fundamentals for their compensation.

Taking into account the number of publications based on the CPT, it is important that the CPT describes and interprets the power related phenomena in a right way and provides right fundamentals for compensator design. Unfortunately, as it will be shown in this paper, the power quantities and the supply current components introduced by the CPT are not associated with specific physical phenomena in the load. It applies to the quantity introduced in the CPT and called the "reactive energy", the reactive and void currents as well the unbalanced current. These new quantities defined in the CPT contribute to misinterpretations of power phenomena and to erroneous conclusions as to methods of compensator design.

The CPT has the same deficiencies as the Budeanu power theory, exposed in [6] and [7]. Moreover, the adjective "*conservative*", which is pivotal for the CPT to such a degree, that it is used in its name, can be applied, as it was demonstrated in [13], in the same sense to the Budeanu reactive power Q_{B}, which does not have any physical interpretation and any practical application. In both cases *conservativeness* has nothing in common with the Law of Conservation of Energy (LCE). The conservation property of the "reactive energy" W in the CPT and the reactive power Q_{B} in the Budeanu power theory has only mathematical, but not physical fundamentals.

Conclusions on interpretations of very confusing power properties, drawn from studies of real and complex systems, where various phenomena are superimposed, might not be credible. These studies should be done on systems, where the number of different power related phenomena is reduced as much as possible. It means that to be valid and credible in poly-phase systems with a full complexity, these interpretations, definitions and conclusions have to be credible when applied to single-phase and even to purely reactive systems. A statement to be valid in the whole set of power systems has to be valid in every sub-set of such systems. Single-phase and purely reactive loads are just sub-sets of the set of three-phase loads. Therefore, to obtain credible conclusions, this paper investigates how the CPT interprets the power related phenomena in such, strongly simplified systems.

II. "Reactive Energy" *W*

The reactive current in the CPT is defined as

$$i_{\mathrm{rT}}(t) \stackrel{\mathrm{df}}{=} \frac{W}{\|\hat{u}\|^2}\hat{u}(t) \tag{1}$$

where

$$W \stackrel{\mathrm{df}}{=} (\hat{u}, i) \stackrel{\mathrm{df}}{=} \frac{1}{T}\int_0^T \hat{u}(t)\, i(t)\, dt \tag{2}$$

denotes "***a reactive energy***" as defined in the CPT**.** This term is written in quotation marks because the quantity W for a capacitor is negative, while energy cannot be negative. Any quantity that can be negative cannot be regarded as "*energy*". The quantity $\hat{u}(t)$ denotes the unbiased integral of the supply voltage. Index "T" in the definition (1) was used in this paper to differentiate the reactive current as defined in the CPT from the reactive currents defined in other power theories.

Introduced by the CPT, a new concept of the reactive current $i_{\mathrm{rT}}(t)$, as defined by (1), has the physical interpretation entirely founded on the physical interpretation of the "reactive energy" W. Thus, what is the "reactive energy"?

This term does not exist in the first papers on the CPT, meaning in [8] and [10]. Its mathematical definition was provided without any physical interpretation. Its interpretation can be found in [12], namely

> "*…the reactive energy accounts for inductive and capacitive energy stored in the load circuit*."

To verify this interpretation of the "reactive energy", let us calculate the energy E stored in an ideal LC load, shown in Fig. 1, supplied with a sinusoidal voltage

$$u(t) = \sqrt{2}\, U \cos\omega_1 t\,.$$

The energy stored in such a reactive load is

$$E = \frac{1}{2}L i_{\mathrm{L}}^2(t) + \frac{1}{2}Cu^2(t) = \frac{U^2}{\omega^2 L}\sin^2\omega t + CU^2\cos^2\omega t. \qquad (3)$$

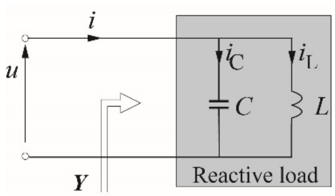

Fig. 1. Ideal reactive load.

Now, let us calculate the “reactive energy” W of the same reactive load. The unbiased voltage integral is equal to

$$\hat{u}(t) = \sqrt{2}\,\frac{U}{\omega}\sin\omega t \qquad (4)$$

thus the “reactive energy” W of such a reactive load is

$$W = (\hat{u}, i) = \frac{1}{T}\int_0^T \hat{u}(t)[i_{\mathrm{L}}(t) + i_{\mathrm{C}}(t)]dt = (\frac{1}{\omega^2 L} - C)U^2. \qquad (5)$$

This is not the energy E stored, as specified by (3), in the LC load, shown in Fig. 1. Thus the interpretation of the “reactive energy” W, as presented in [12], is not right. It is even more visible at a resonance in that load, when $1/\omega L = \omega C$. At such a condition the “reactive energy” W is zero, while the energy stored in the load is

$$E = \frac{U^2}{\omega}(\frac{1}{\omega L}\sin^2\omega t + \omega C\cos^2\omega t) = \frac{1}{\omega^2 L}U^2. \qquad (6)$$

Doubts about whether the opinion expressed in [12] is right can be strengthened by results of analysis of a purely resistive circuit with a TRIAC, shown in Fig. 2.

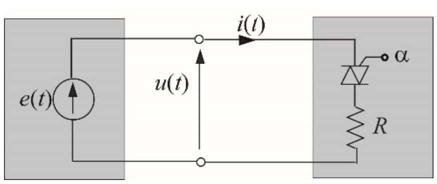

Fig. 2. Resistive load with periodic switch

At sinusoidal supply voltage

$$u(t) = \sqrt{2}\,U\sin\omega_1 t$$

the load current at the TRIAC firing angle α has the waveform as shown in Fig. 3. The load current in such a circuit can be decomposed into harmonics

$$i(t) = \sum_{k=1}^{\infty} i_n(t) = i_1(t) + \sum_{k=2}^{\infty} i_n(t) \qquad (7)$$

with the current fundamental harmonic

$$i_1(t) = \sqrt{2}\,I_1\sin(\omega_1 t - \varphi_1) \qquad (8)$$

i.e., shifted with respect to the voltage as shown in Fig. 3. The unbiased integral of the supply voltage is

$$\hat{u}(t) = -\sqrt{2}\,\frac{U}{\omega}\cos\omega_1 t \qquad (9)$$

and consequently, the “reactive energy” W is equal to

$$W = (\hat{u}, i) = \sum_{n=1}^{\infty}(\hat{u}_n, i_n) = (\hat{u}, i_1) = \frac{1}{T}\int_0^T \hat{u}(t)\, i_1(t)\, dt = \frac{U I_1}{\omega_1}\sin\varphi_1. \qquad (10)$$

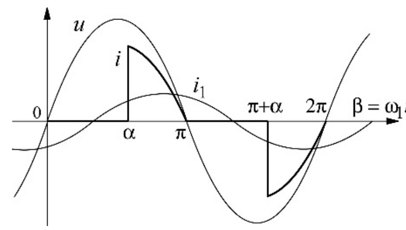

Fig. 3. Voltage, current and the current fundamental harmonic i_1 waveforms in resistive circuit with TRIAC

Thus, loads without any capability of energy storage could have a “reactive energy” W. This confirms the previous conclusion that the “reactive energy” W is not associated with the phenomenon of energy storage.

The “reactive energy” W is defined originally by (2) in the time-domain. In such a way the CPT follows Fryze’s concept [4] of defining power quantities without any use of harmonics. This confines insight into the meaning of this quantity, however.

Thus, let us express the “reactive energy” W of a purely reactive load in the frequency-domain, assuming that the supply voltage is nonsinusoidal and composed of harmonics of the order n from a set N, namely

$$u(t) = \sum_{n\in N} u_n(t) = \sqrt{2}\sum_{n\in N} U_n\cos n\omega_1 t. \qquad (11)$$

The unbiased integral of such a voltage is

$$\hat{u}(t) = \sum_{n\in N}\hat{u}_n(t) = \sqrt{2}\sum_{n\in N}\frac{U_n}{n\,\omega_1}\sin n\omega_1 t. \qquad (12)$$

A purely reactive load has the admittance for harmonic frequency of the n^{th} order harmonic equal to

$$\mathbf{Y}_n = G_n + jB_n = jB_n$$

i.e., with $G_n = 0$. If for the n^{th} order harmonic the load is inductive, then $B_n < 0$ and

$$i_n(t) = \sqrt{2}\,|B_n|U_n\sin n\omega_1 t\,.$$

If for such a harmonic the load is capacitive, i.e., $B_n > 0$, then

$$i_n(t) = -\sqrt{2}\,|B_n|U_n\sin n\omega_1 t\,.$$

Therefore, the current of a purely reactive load can be expressed in the form

$$i(t) = \sum_{n\in N} i_n(t) = -\sqrt{2}\sum_{n\in N}\mathrm{sgn}\{B_n\}|B_n|U_n\sin n\omega_1 t. \qquad (13)$$

The “reactive energy” W of such a reactive LC load is

$$W = (\hat{u}, i) = \sum_{n\in N}(\hat{u}_n, i_n) = \sum_{n\in N} W_n = -\sum_{n\in N}\mathrm{sgn}\{B_n\}|B_n|\frac{U_n^2}{n\,\omega_1}. \qquad (14)$$

Individual terms W_n of this sum can be, depending on the sign of the load susceptance B_n, positive or negative, thus they can cancel mutually. This mutual cancelation of the harmonic “reactive energies” W_n resembles mutual cancelation of harmonic reactive powers Q_n in the Budeanu definition [3] of the reactive power Q_B.

$$Q_B = \sum_{n\in N} U_n I_n \sin\varphi_n = \sum_{n\in N} Q_n . \tag{15}$$

This mutual cancelation was of one of the major deficiencies of the Budeanu reactive power [6, 7], for which it was eventually abandoned in the power theory.

Formula (14) for the "reactive energy" W has a strong analogy with definition of the reactive power Q_B. This is particularly visible if (15) is rearranged for reactive loads to the form.

$$Q_B = \sum_{n\in N} U_n I_n \sin\varphi_n = -\sum_{n\in N} \mathrm{sgn}\{B_n\}|B_n| U_n^2 . \tag{16}$$

Individual terms in the Budeanu definition of the reactive power Q_B stand for the amplitude of the energy oscilation at the frequency of individual harmonics, since the bidirectional component of the instantaneous power $p(t)$ of the n^{th} order harmonic is equal to

$$\tilde{p}_n = U_n I_n \sin\varphi_n \sin 2n\omega_1 t = Q_n \sin 2n\omega_1 t. \tag{17}$$

The sum (15) of these amplitudes Q_n, i.e., the Budeanu reactive power Q_B, does not specify, as shown in [6], any physical phenomenon in the circuit, however.

Thus the "reactive energy" W, when expressed in the frequency-domain", look a lot like power quantities suggested at the beginning of the power theory development. In particular, it occurs to be almost identical with the reactive power Q_I defined in 1925 [2] by Illovici.

Namely, according to Illovici, the reactive power should be defined as the quantity measured by a wattmeter with the resistor in the voltage branch replaced by an inductor L.

Such a device, assuming that it is ideal and lossless, measures the quantity

$$Q_I = \sum_{n\in N} \frac{1}{n} U_n I_n \sin\varphi_n . \tag{18}$$

According to Illovici just this is one of the quantities that should be regarded as the reactive power at nonsinusoidal supply voltage.

Assuming that the voltage branch is lossless, then at terminals of a purely reactive LC load such an instrument measures the quantity

$$Q_I = \sum_{n\in N} \frac{1}{n} U_n I_n \sin\varphi_n = -\sum_{n\in N} \mathrm{sgn}\{B_n\}|B_n| \frac{U_n^2}{n} = \omega_1 W . \tag{19}$$

Thus, the Illovici reactive power Q_I and the "reactive energy" W differ mutually only by the dimensional coefficient ω_1. Consequently, there is no physical phenomenon in the load that could be characterized by the quantity W, called in the CPT "a reactive energy".

III. The Reactive Current $i_{rT}(t)$

The previous section has demonstrated that the physical interpretation of reactive current $i_{rT}(t)$ in the CPT cannot be founded on the "reactive energy", since it does not have such interpretation. Thus, what it is the reactive current $i_{rT}(t)$?

Definition (1) of this current shows that it can be regarded as a current of an ideal inductor, since

$$i_{rT}(t) = \frac{W}{\|\hat{u}\|^2}\hat{u}(t) = \frac{1}{L_e}\hat{u}(t) \tag{20}$$

where

$$L_e = \frac{\|\hat{u}\|^2}{W} . \tag{21}$$

It means that with respect to the "reactive energy" W at the supply voltage $u(t)$, the purely reactive load is equivalent to an inductor of inductance L_e. Such an inductor draws the current $i_{rT}(t)$ from the supply source.

Since the physical meaning of the "reactive energy" W in the CPT is not clear, not clear is also the physical meaning of the reactive current $i_{rT}(t)$. Its meaning can be clarified using the Currents' Physical Components (CPC) power theory. Namely, at the supply voltage

$$u(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n e^{jn\omega_1 t} \tag{22}$$

the reactive current defined in the CPT is

$$i_{rT}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \frac{1}{jn\omega_1 L_e}\boldsymbol{U}_n e^{jn\omega_1 t}. \tag{23}$$

This is not the reactive current as defined by Shepherd and Zakikhani [5], namely the current

$$i_r(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_n \boldsymbol{U}_n e^{jn\omega_1 t}. \tag{24}$$

meaning, the current which occurs in the supply lines due to a phase-shift between the voltage and current harmonics. The current $i_{rT}(t)$ is only a part of that reactive current $i_r(t)$.

According to the CPT the reactive current $i_{rT}(t)$ can be compensated entirely by a capacitor connected as shown in Fig. 4.

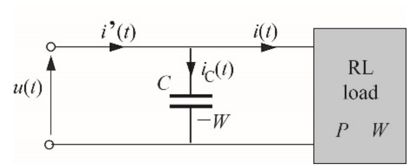

Fig. 4. RL load with a capacitor which compensates the "reactive energy" W.

The "reactive energy" of the capacitor is

$$W_C = (\hat{u}, i_C) = -\sum_{n\in N} \frac{n\omega_1 C}{n\omega_1} U_n^2 = -C\sum_{n\in N} U_n^2 = -C\|u\|^2. \tag{25}$$

Thus a shunt capacitor of capacitance

$$C = \frac{W}{\|u\|^2} \tag{26}$$

compensates the "reactive energy" W entirely. It changes the CPT reactive current $i_{rT}(t)$ to

$$i'_{rT}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j(n\omega_1 C - \frac{1}{n\omega_1 L_e})\boldsymbol{U}_n e^{jn\omega_1 t}. \tag{27}$$

The susceptance of the capacitor C changes with the harmonic order in a different way than the susceptance of the equivalent inductance L_e, however. Thus, reduction of the reactive current $i_{rT}(t)$ does not result from (27), but from reduction of the "reactive energy" W to zero and an increase of the equivalent inductance L_e to infinity. The true reactive current $i_r(t)$, as defined by (24), is not compensated, however. The CPT ignores the fact that the compensating capacitor can affect also the void current.

IV. The Void Current $i_v(t)$

The load current according to the CPT is composed of the active, reactive and the void currents

$$i(t) = i_a(t) + i_{rT}(t) + i_v(t) \tag{28}$$

where the void current is defined as

$$i_v(t) = i(t) - [i_a(t) + i_{rT}(t)] \,. \tag{29}$$

The void current $i_v(t)$, as defined by (29), is not expressed in terms of voltage and the load parameters, which are specified in the frequency-domain, however, but in the time-domain. The physical meaning of this current is not clear. This meaning can be clarified in the frequency-domain, with the CPC–based power theory.

Since the active current $i_a(t)$ is equal to

$$i_a(t) \stackrel{\text{df}}{=} G_e u(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} G_e \boldsymbol{U}_n e^{jn\omega_1 t}, \quad G_e = \frac{P}{\|u\|^2}\,. \tag{30}$$

while the reactive current i_{rT}(t) is given by (23), thus the void current can be expressed as

$$i_v = i - i_a - i_{rT} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}[(G_n + jB_n) - G_e - \frac{1}{jn\omega_1 L_e})\boldsymbol{U}_n e^{jn\omega_1 t}\,. \tag{31}$$

This formula shows that the void current is in fact a compound quantity. It contains in-phase component

$$i_s(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}[(G_n - G_e)\boldsymbol{U}_n e^{jn\omega_1 t} \tag{32}$$

revealed in CPC and called *scattered current*. It contains also a quadrature component, i.e., composed of current harmonics shifted by $\pi/2$ with respect to the voltage harmonics

$$i_{vr}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j\,(B_n + \frac{1}{n\omega_1 L_e})\boldsymbol{U}_n e^{jn\omega_1 t}\,. \tag{33}$$

Thus,

$$i_v(t) = i_s(t) + i_{vr}(t)\,. \tag{34}$$

The quadrature component of the void current has the rms value

$$\|i_{vr}\| = \sqrt{\sum_{n\in N}(B_n + \frac{1}{n\omega_1 L_e})^2 U_n^2}\,. \tag{35}$$

When a capacitor is connected as shown in Fig. 4 to compensate the "reactive energy" W, then the supply current does not contain the reactive current $i_{rT}(t)$. The quadrature component of the void current changes to

$$i'_{vr}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j\,(B_n + n\omega_1 C)\boldsymbol{U}_n e^{jn\omega_1 t}. \tag{36}$$

Its rms value changes to

$$\|i'_{vr}\| = \sqrt{\sum_{n\in N}(B_n + n\omega_1 C)^2 U_n^2}\,. \tag{37}$$

Thus capacitive compensation of the reactive current $i_{rT}(t)$ changes the void current rms value. Moreover, this change increases with the harmonic order n. Thus, compensation of the reactive current $i_{rT}(t)$ cannot be separated from its effect on the void current $i_{vr}(t)$ rms value increase. This is illustrated numerically on an example of effects of compensation of the "reactive energy" W of RL load shown in Fig. 5. To have these effects clearly visible, it was assumed that the fifth order harmonic of the supply voltage has the rms value U_5 equal to the fundamental harmonic rms value U_1. It is, of course, unrealistically strong distortion, but we could expect that conclusions of the CPT are valid irrespective of the level of the supply voltage distortion.

At the supply voltage harmonics complex rms (crms) values

$$\boldsymbol{U}_1 = \boldsymbol{U}_5 = 100\,e^{j0^\circ}\ \mathrm{V}, \quad \|u\| = 100\sqrt{2}\ \mathrm{V}$$

the crms values of the load current harmonics are

$$\boldsymbol{I}_1 = 70.7\,e^{-j45^\circ}\ \mathrm{A}, \quad \boldsymbol{I}_5 = 19.9\,e^{-j79^\circ}\ \mathrm{A}, \quad \|i\| = 73.4\ \mathrm{A}$$

so that, assuming that the supply voltage frequency is normalized to ω_1 = 1 rad/s, the "reactive energy, is

$$W = \mathrm{Re}\sum_{n=1,5}\frac{\boldsymbol{U}_n}{jn\omega_1}(\boldsymbol{Y}_n\boldsymbol{U}_n)^* = 0.538\times 10^4\ \mathrm{J}.$$

Capacitance of a shunt capacitor for the "reactive energy" W of the load compensation is equal to

$$C = \frac{W}{\|u\|^2} = 0.269\ \mathrm{F}\,.$$

The capacitor compensates the "reactive energy" W of the load, but it changes the crms values of the supply current harmonics to

$$\boldsymbol{I'}_1 = 55.1\,e^{-j24.8^\circ}\ \mathrm{A}, \quad \boldsymbol{I'}_5 = 115.3\,e^{j88.0^\circ}\ \mathrm{A}, \quad \|i'\| = 127.8\ \mathrm{A}.$$

The results of compensation of this "energy" are shown in Fig. 5.

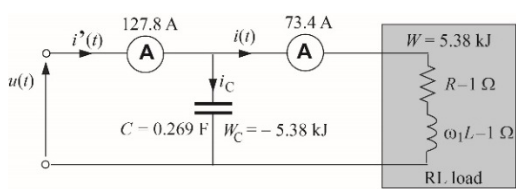

Fig. 5. Results of compensation of the "reactive energy" W of RL load

The "reactive energy" W of the compensated load is zero, but the compensator increases the void current rms value. Consequently, instead of improving the power factor, it was worsened.

V. Distortion Power

According to CPT, the load current of a purely reactive single-phase LC load is composed only of the reactive $i_{rT}(t)$ current and the void $i_v(t)$ current.

$$i(t) = i_{rT}(t) + i_v(t)\,. \tag{38}$$

The supply current of a purely reactive load contains neither the active current, as defined in the Fryze Power Theory [4], nor the scattered current, as defined in the CPC–based power theory [9].

The reactive and void currents are mutually orthogonal, so that their rms values satisfy the relationship

$$||i||^2 = ||i_{\rm rT}||^2 + ||i_{\rm v}||^2 . \quad (39)$$

Multiplying this formula by the square of the supply voltage rms value $||u||$, the power equation of reactive loads is obtained. It has the form

$$S^2 = Q_{\rm T}^2 + D_{\rm T}^2 . \quad (40)$$

According to [12], the quantity

$$D_{\rm T} = ||i_{\rm v}||\times||u|| \quad (41)$$

is a distortion power of the load. In some papers on the CPT, such as [10], this quantity is called a void power.

The concept of a distortion power occurred for the first time in Budeanu's power theory. It was defined as

$$D_{\rm B} \stackrel{\rm df}{=} \sqrt{S^2 - P^2 - Q_{\rm B}^2} . \quad (42)$$

Indices T and B were used in (42 – 44) to distinguish distortion powers in Budeanu and CPT power theories. Despite having the same name, these are two different quantities.

Distortion power $D_{\rm B}$ is interpreted as a measure of the effect of the voltage and current mutual distortion on the apparent power S of the load. This interpretation was challenged in [6, 7], where it was demonstrated that such interpretation was not right. There is no relation between distortion power $D_{\rm B}$ and the voltage and current mutual distortion.

Let us check whether distortion power $D_{\rm T}$ defined in the CPT is related to the load voltage and current mutual distortion. This is done below with a numerical analysis of a purely reactive load shown in Fig. 6

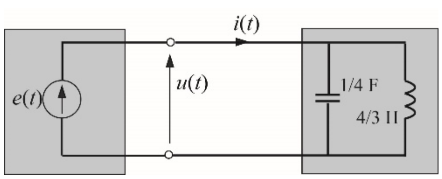

Fig. 6. Circuit with reactive load.

supplied with the voltage:

$$u(t) = \sqrt{2}\,(100\sin\omega_1 t + 30\sin 3\omega_1 t)\,{\rm V}, \qquad \omega_1 = 1 \text{ rad/s}.$$

The admittances of such a load for the voltage harmonics are $\boldsymbol{Y}_1 = -j1/2$ S and $\boldsymbol{Y}_3 = j1/2$ S. The "reactive energy" W of such load is equal to

$$W = -\sum_{n\in\{1,\,3\}} {\rm sgn}\{B_n\}|B_n|\frac{U_n^2}{n\omega_1} = 4.85 \text{ kJ}.$$

Since

$$||\hat{u}|| = \sqrt{\sum_{n\in\{1,\,3\}} \left(\frac{U_n}{n\omega_1}\right)^2} = \sqrt{\left(\frac{U_1}{\omega_1}\right)^2 + \left(\frac{U_3}{3\omega_1}\right)^2} = 100.50 \text{ Vt}$$

the rms value of the reactive current $i_{\rm rT}(t)$ is

$$||i_{\rm rT}|| = ||\frac{W}{||\hat{u}||^2}\hat{u}(t)|| = \frac{|W|}{||\hat{u}||} = 48.26 \text{ A}.$$

The load current rms value is

$$||i|| = \sqrt{\sum_{n\in\{1,\,3\}} (Y_n U_n)^2} = \sqrt{(0.5\times100)^2 + (0.5\times30)^2} = 52.20\text{A}.$$

Since the active current does not exist, the rms value of the void current is equal to

$$||i_{\rm v}|| = \sqrt{||i||^2 - ||i_{\rm rT}||^2} = \sqrt{52.2^2 - 48.26^2} = 19.90\text{A}$$

so that the distortion power

$$D_{\rm T} = ||i_{\rm v}||\,||u|| = 19.90\times104.40 = 2.08 \text{ kVA}.$$

The load current is equal to

$$i(t) = \sqrt{2}\,[50\sin(\omega_1 t - \frac{\pi}{2}) + 15\sin(3\omega_1 t + \frac{\pi}{2})] =$$

$$= \sqrt{2}[50\sin\omega_1(t - \frac{T}{4}) + 15\sin 3\omega_1(t - \frac{T}{4})]\text{A} = \frac{1}{2}u(t - \frac{T}{4}).$$

The load current is only shifted versus the voltage by $T/4$, as shown in Fig. 7. In spite of non-zero distortion power $D_{\rm T}$, the voltage and current are not mutually distorted. It demonstrates that there is no relation between distortion power $D_{\rm T}$ and distortion of the load current with respect to the supply voltage.

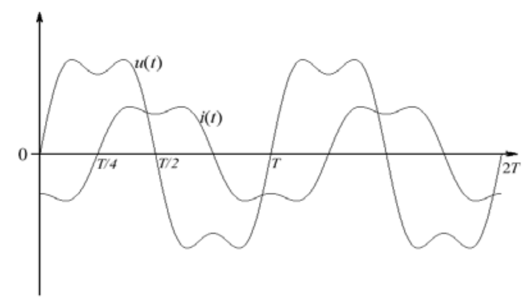

Fig. 7. Waveforms of the voltage and current.

This conclusion has a strong analogy to the conclusion on the distortion power $D_{\rm B}$ in the Budeanu Power Theory. Both in the CPT and in the Budeanu power theory, the name "distortion power" of $D_{\rm B}$ and $D_{\rm T}$ quantities suggests a relationship between these powers and the voltage and current mutual distortion. There is not such a relationship between these powers and the voltage and current distortion, however. The concept of these powers in both cases contributes to misinterpretation of power related phenomena in systems with nonsinusoidal voltage.

VI. Unbalanced Current

The unbalanced current in the CPT has two components, the active and reactive ones. This concept is investigated assuming that the load is purely resistive, so that only the active component of the unbalanced current can occur. Its three-phase rms value can be calculated according to [11] formula (12a) as

$$||\boldsymbol{i}_{\rm uT}|| = \boldsymbol{I}_{\rm a}^{\rm u} = \sqrt{\sum_{n=1}^{3}\left(\frac{P_n}{U_n}\right)^2 - \left(\frac{P}{\boldsymbol{U}}\right)^2} \quad (43)$$

where P_n denotes the active power of the $n^{\rm th}$ phase of the load, U_n is the rms value of that phase voltage and $\boldsymbol{U}$ is in [11] a symbol of a collective rms value, which is equal to, defined in [9], three-phase rms value $||\boldsymbol{u}||$

$$||\boldsymbol{u}|| = \boldsymbol{U} = \sqrt{U_1^2 + U_2^2 + U_3^2}\ . \tag{44}$$

The formula (45) does not provide the right value of the unbalanced current, however. To show this, let us consider a purely resistive load shown in Fig. 8 with a sinusoidal and symmetrical supply voltage of the crms values

$$\boldsymbol{E}_1 = 100e^{j0^{\circ}}\text{V},\ \boldsymbol{E}_2 = 100e^{-j120^{\circ}}\text{V},\ \boldsymbol{E}_3 = 100e^{j120^{\circ}}\text{V}.$$

For parameters as shown in Fig. 8 we have

$$P_1 = 0,\quad P_2 = 15\ \text{kW},\quad P_3 = 15\ \text{kW},\quad P = 30\ \text{kW}$$

$$U_1 = U_2 = U_3 = 100\ \text{V},\quad \boldsymbol{U} = ||\boldsymbol{u}|| = \sqrt{3}U = 173.20\ \text{V}.$$

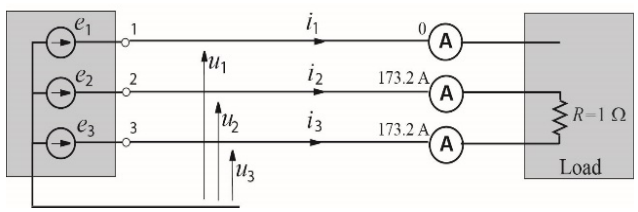

Fig. 8. Resistive circuit with unbalanced load.

Consequently, formula (43), meaning (12a) in [11], results in

$$||\boldsymbol{i}_{\rm uT}|| = \boldsymbol{I}_{\rm a}^{\rm u} = \sqrt{\sum_{n=1}^{3}(\frac{P_n}{U_n})^2 - (\frac{P}{\boldsymbol{U}})^2} = 122.4\ \text{A}\ . \tag{45}$$

This is not the right three-phase rms value of the unbalanced current in such a circuit, however. The vector of the load current $\boldsymbol{i} = [i_1,\ i_2,\ i_3]^{\rm T}$ in such purely resistive circuit is composed of only the active $\boldsymbol{i}_{\rm a}$ and the unbalanced $\boldsymbol{i}_{\rm u}$ currents, namely.

$$\boldsymbol{i}(t) = \boldsymbol{i}_{\rm a}(t) + \boldsymbol{i}_{\rm u}(t). \tag{46}$$

These two currents are mutually orthogonal, so that their three-phase (collective) rms values have to satisfy the relation

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\rm a}||^2 + ||\boldsymbol{i}_{\rm u}||^2 \tag{47}$$

where in circuit shown in Fig. 8,

$$||\boldsymbol{i}|| = \sqrt{I_1^2 + I_2^2 + I_3^2} = 173.2\sqrt{2} = 244.95\ \text{A}$$

$$||\boldsymbol{i}_{\rm a}|| = \frac{P}{||\boldsymbol{u}||} = \frac{30\times 10^3}{173.2} = 173.20\ \text{A}\ .$$

Hence the three-phase rms value of the unbalanced current is

$$||\boldsymbol{i}_{\rm u}|| = \sqrt{||\boldsymbol{i}||^2 - ||\boldsymbol{i}_{\rm a}||^2} = 173.20\ \text{A}\ . \tag{48}$$

This is not the value 122.4 A obtained by (45) from (12b) in [11]. The numerical result (48) can be confirmed by decomposition of the load current into symmetrical components. Active current $\boldsymbol{i}_{\rm a}(t)$ in the situation shown in Fig. 8 is identical with the positive sequence component of the load current, while the unbalanced current $\boldsymbol{i}_{\rm u}(t)$ is identical with the negative sequence component of that current.

VII. CONCLUSIONS

The Conservative Power Theory, the latest step in the power theory development, occurs to be a sort of return to its initial phase, to Illovici and Budeanu concepts. Although, unlike the Budeanu power theory, it is formulated in the time-domain and generalized to unbalanced three-phase loads, it has all deficiencies of the Budeanu power theory. CPT also follows the Fryze approach, meaning it is based on the current orthogonal decomposition, but repeats some of its deficiencies. Namely, just as Fryze's concept did not explain the physical meaning of the reactive current, $i_{\rm rF}$(t), the CPT also does not provide physical interpretation of the reactive current $i_{\rm rF}$(t), because the "reactive energy" W is not a physical quantity. Consequently, the void current $i_{\rm v}$(t) also does not have any physical meaning. It is associated in the CPT with distortion power $D_{\rm T}$, but similarly as it was with Budeanu distortion power $D_{\rm B}$, there is no relationship between distortion power and the voltage and current mutual distortion. It means that the Conservative Power Theory misinterprets power related phenomena in electrical circuits. Moreover, the Budeanu Power Theory is not less "conservative" than the Conservative Power Theory.

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