# Why the Electric Arc Nonlinearity Improves the Power Factor of Ac Arc Furnaces?

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*Abstract:* – The arcs in ac arc furnaces are stabilized by line inductors, at the cost of the power factor decline. It seems that in common opinions also the arc nonlinearity contributes to this decline. It occurs, however, that at the same parameters, the furnace, regarded as a linear device, has a lower power factor as compared to its value, when the arc nonlinearity is taken into account.

The value of ac arc furnaces power factor (PF) is estimated in this paper at the assumption that the dc voltage on the arc has a constant value and a furnace is in a steady state. The PF of ac arc furnaces was evaluated for three states of the furnace operation, namely, for a balanced operation; for a two-arcs operation and for a furnace with a unidirectional arc. The analysis presented in the paper and modeling demonstrate that the arc nonlinearity improves the furnace power factor.

Index terms: active and reactive powers; compensation, Currents Physical Components, CPC.

#### I. INTRODUCTION

Due to a presence of inductors in supply lines of ac arc furnaces (AF), installed for arcs stabilization, such furnaces operate at a relatively low power factor,  $\lambda$ . To improve it, compensators of the reactive power, which often operate also as harmonic filters, are installed. Due to technical reasons, such compensators are installed usually on the primary side of the AF transformer [1, 2, 5, 8, 11]. However, just this transformer, connected as shown in Fig. 1, is the main piece of the arc furnace assembly that is affected by the low power factor  $\lambda$  and current harmonics generated by the furnace arcs. A low power factor and the current harmonics cause extra energy loss in this transformer and contribute to needed transformer power ratings.



Fig. 1. The electrical structure of an ac arc furnace supply with a furnace transformer.

The power factor and the harmonic distortion of the AF current are usually evaluated and/or measured on the primary

side of the furnace transformer [5, 11, 14]. Consequently, the source of the power factor degradation and the current harmonic distortion, meaning the arc furnace, is partially hidden and affected by the transformer. Therefore, to clarify the effect of arcs on the power factor  $\lambda$ , it is evaluated in this paper on the secondary side of the AF transformer.

Due to a movement of the AF charge and turbulence in the arc plasma, arcs ignition and extinction are random processes, especially in the boring and the melting modes of the AF operation. Consequently, also the power factor  $\lambda$  of the AF changes randomly. Therefore, only some statistical measures of this random process specify eventually [1-3] the arc furnace performance as an electrical load.

Nonetheless, the knowledge on the effect of the state of arc furnace on the power factor can have a cognitive merit. An AF can operate with all three arcs ignited, with only two arcs ignited and even with two bidirectional and one unidirectional arc. Such states of the AF operation differ substantially as to the power factor  $\lambda$  value and the furnace current distortion.

The electric arc nonlinearity is caused mainly by two agents. These are the voltage needed for the arc ignition,  $U_0$ , and the dependence of the arc plasma resistance  $R_p$  on the arc current.

The voltage needed for the arc ignition  $U_0$  depends on the distance of the electrode from the melted charge. When the charge moves, the voltage  $U_0$  can change each period *T*.

The plasma resistance  $R_p$  depends on the arc current rms value and the plasma space properties. Therefore, it can be assumed that from the perspective of a single period *T* of the supply voltage variability, the resistance  $R_p$  is a slowly varying parameter of the arc. Therefore, when arcs are ignited, then there is a ground for the assumption that parameters of the arc furnace do not change over a single period *T*.

The power factor of the AF is evaluated in this paper, by modeling a steady state of the furnace in a single period T. Three states of the AF are investigated: (i) its balanced operation, i.e., with three bi-directional arcs, (ii) its operation with only two bidirectional arcs, and (iii) its operation with two bidirectional arcs and one unidirectional arc.

Parameters of AF are not easily available by measurements [2, 3, 10, 14] and they are known with very low accuracy especially that they change during the AF operation. These parameters were fixed for modeling in this paper, however. Therefore, the values of the power factor obtained can be regarded only as a very rough approximation of the true values. Nonetheless, this modeling provides some information on the effect of the furnace state, on the power factor  $\lambda$  change.

# II. A LINEAR MODEL

Physical phenomena inside of the cage of an arc furnace are very complex; possibilities of their observation and measurement are very limited. Nonetheless, for the arc furnace analysis, an equivalent circuit of the AF is needed.

The furnace line resistance *R* is the main parameter of such a model. This resistance is composed of the supply line and the inductor resistance  $R_0$ , the arc plasma resistance  $R_p$  and the resistance of the current path in the steel charge below electrodes, denoted by  $R_s$ . Thus, the line resistance is

$$R = R_0 + R_p + R_s \,. \tag{1}$$

The plasma resistance  $R_p$  is the most important component of the line resistance R. The resistance of the arc plasma can be evaluated using Cassie [6] and Mayr [7] methods, but the accuracy of such evaluations is not high. Only some metrological experiments on operating arc furnaces can provide more credible values of these resistances. In spite of that, when the furnace currents and powers are calculated, sometimes the arc is approximated even by a short-circuit [5, 14], as shown in Fig. 2, although the plasma resistance is included in the line resistance R.



Fig. 2. An equivalent circuit of an arc furnace with the arc approximated by a short-circuit.

Inductors connected in the supply lines of the arc furnace, needed for maintaining the arc current continuity. Their reactance X is usually selected to be comparable with the line resistance R. This reactance has usually a fixed value, while the line resistance changes with the furnace current rms value, so that this condition cannot be satisfied without some level of the reactance control, however.

The equivalent circuit of an arc furnace, as shown in Fig. 2, stands for a linear model of the furnace. Sometimes such a model is used for the most basic evaluation of the AF performance as an electrical load. In particular, when the inductance L value is selected such that  $\omega_1 L = R$ , then at such an approximation, the power factor of the arc furnace in a balanced mode is  $\lambda = P/S = 0.71$ . This value will be used later in this paper as assort of a reference. The following question arises, however: *Is this value of the power factor of the AF, with real nonlinear arcs? In other words, does the arc nonlinearity reduces or increases the power factor value?* 

## III. A BASIC NONLINEAR MODEL OF THE ARC

To ignite the arc, the voltage between the electrode and the charge has to reach a sufficient value. After ignition, the voltage between the electrode and the furnace charge has an ac component, equal to the product of the arc current and the plasma resistance  $R_p$ , and a dc component. This dc voltage declines a bit from a constant value, denoted here by  $U_0$ , with the arc current increase [4]. It depends mainly on the arc length. The arc is approximated [4] as shown in Fig. 3, with the constant voltage on the arc assumed to be  $U_0 = 300$  V.



Fig. 3. A circuit (b) that approximates the arc (a).

The equivalent circuit, shown in Fig. 3, simplifies the relation between the arc voltage and current. It preserves the main feature of this relationship, however. Namely, the arc cannot be ignited before the arc voltage has a sufficiently high value.

# IV. POWER FACTOR AT A BALANCED MODE

Even if the plasma resistance  $R_p$  is shifted from the arc to the line resistance R, the dc voltage on the arc cannot be neglected in the equivalent circuit of the arc furnace.

When the arc is approximated as shown in Fig. 2, then the equivalent circuit has a structure as shown in Fig. 4.



Fig. 4. An equivalent circuit of an arc furnace with the arc approximated as shown in Fig. 2.

At sufficiently high line inductances *L* all arcs are ignited and the line currents are continuous. There is a dc voltage  $a_x U_0$ , in each line, with coefficient  $a_x = \pm 1$  (x = R, S, T), dependent on the current sign. Namely, if  $i_x > 0$ , then  $a_x = 1$ , if  $i_x < 0$ , then  $a_x = -1$ . If G = 1/R denotes the line conductance, then the voltage  $v_0$  of the joining point of arcs, i.e., the melted steel, has to satisfy for a dc voltage a nodal equation

$$v_{A0} = -\frac{Ga_{\rm R}U_0 + Ga_{\rm S}U_0 + Ga_{\rm T}U_0}{3G} = -\frac{1}{3}(a_{\rm R} + a_{\rm S} + a_{\rm T})U_0.$$
 (2)

In a three-wire system

$$\dot{i}_{\rm R}(t) + \dot{i}_{\rm S}(t) + \dot{i}_{\rm T}(t) \equiv 0$$
 (3)

thus at least one line current has to be negative, meaning at least one coefficient  $a_x$  is negative, i.e.,  $a_x = -1$ . Since in one period *T* each of the line currents changes its sign twice, thus the voltage of the joining point of arcs

$$v_{\rm A0} = \pm \frac{1}{3} U_0 \tag{4}$$

and changes the sign six times in period T, as shown in Fig. 5.

Taking into account the variation of the voltage on the joining point of arcs, the voltage on the arc in line R is

$$v_{\rm R} = a_{\rm R} U_0 + v_{\rm A0} = \frac{1}{3} (2 a_{\rm R} - a_{\rm S} - a_{\rm T}) U_0 \,. \tag{5}$$



Fig. 5. Variation of the voltage at the joining point of arcs.

The variation of the voltage  $v_R$  is shown in Fig. 6.



Fig. 6. Variation of the voltage at the arc in line R.

One should observe, that the voltages  $v_{R}$ ,  $v_{S}$  and  $v_{T}$ , shown in Fig. 4, are not voltages of the arc furnace electrodes, because the arc plasma resistance  $R_{p}$  is included in the line resistance R. Thus, these are only fictitious voltages. There are no physical points in the furnace that have such voltages. Nonetheless, such fictitious voltages enable us to separate two main features of arcs nonlinearity. These are: - (a) a change of the arc plasma resistance with the current value, and - (b) a dc voltage on the arc. Consequently, there are two different mechanisms of the heat release in the arc. The energy released in the furnace on this resistance is proportional to the active power

$$P_{\rm p} = 3R_{\rm p} ||i_{\rm R}||^2 \,. \tag{6}$$

The dc voltage on arcs causes the energy release proportional to the active power

$$P_{\rm d} = 3 \frac{1}{T} \int_{0}^{T} v_{\rm R}(t) \, i_{\rm R}(t) \, dt \, . \tag{7}$$

**Illustration 1.** Arc furnace assemblies differ from the electrical supply perspective mainly as to the furnace power, the supply voltage rms value and the power of the furnace transformer. The power presently built arc furnaces reaches 750MVA, the furnace supply voltage rms value can be between 400 V and 1300 V. At such high power of the furnace, transformers could have a power comparable with the furnace power or only a few times higher.

This paper is to provide some general conclusions on the effect of the arc nonlinearity on the power factor. The power of the furnace is irrelevant for studies on such an issue. Because it is much more convenient to analyze a furnace of relatively low power, it was assumed in this paper that the arc furnace has the line resistance, as defined by (1), equal to  $R = 1 \Omega$  and the inductor for the arc stabilization has a reactance for the fundamental frequency equal to  $X = R = 1 \Omega$ . It was assumed also, that the furnace is supplied from a transformer with the secondary voltage of the rms value U = 700 V and with the

power ratings only twice much higher than the arc furnace apparent power. The reactance-to-resistance ratio of the transformer was assumed to be  $X_t/R_s = 5$ . The electrical parameters of the arc furnace and the transformer are shown in Fig. 7.



Fig. 7. Arc furnace and transformer parameters in illustration 1.

The waveforms of the internal voltage of the supply system, recalculated to the secondary side of the transformer,  $e_{\rm R}$ , the voltage on the supply terminals of the furnace  $u_{\rm R}$ , the fictitious voltage  $v_{\rm R}$ , and the line current  $i_{\rm R}$ , obtained from a computer modeling, are shown in Fig. 8.



Fig. 8. Waveforms of voltages and currents in line R.

Because voltages and currents are nonsinusoidal, powers in the studied circuit were calculated by a discrete approach, calculating values of *K* equidistance samples of the current  $i_R$ and the voltage  $v_R$ . Having these samples  $i_{Rk}$  and  $v_{Rk}$ , and assuming that K = 32, the active power  $P_d$ , defined by (7) is equal to

$$P_{\rm d} = 3\frac{1}{T}\int_{0}^{T} v_{\rm R}(t) \ i_{\rm R}(t) \ dt \approx 3\frac{1}{K}\sum_{k=0}^{k=K-1} v_{\rm Rk} \ i_{\rm Rk} = 198.0 \ \rm kW \ . \tag{8}$$

The current rms value in line R is

$$||i_{\rm R}|| = \sqrt{\frac{1}{T}} \int_{0}^{T} i_{\rm R}^{2}(t) \ dt \approx \sqrt{\frac{1}{K}} \sum_{k=0}^{k=K-1} i_{\rm Rk}^{2} = 245.7 \ {\rm A}$$
(9)

hence, the active power of the arc resistance is equal to

$$P_{\rm r} = 3R ||i_{\rm R}||^2 = 181.1 \,\rm kW$$
 (10)

As it can be observed from Fig. 8, the line current are distorted. The total harmonic distortion (THD), defined as

$$\delta_{i} = \frac{\|i_{h}\|}{\|i_{l}\|} = \sqrt{\sum_{n,n\neq 1}^{\infty} (\frac{I_{n}}{I_{1}})^{2}}$$
(11)

is equal to  $\delta_1 = 2.6\%$ . At such low level of distortion, the apparent power *S* of the arc furnace is almost equal to the apparent power of the fundamental harmonic *S*<sub>1</sub>

$$S \approx S_1 = 3 U_1 I_1 \tag{12}$$

The complex rms (crms) values  $U_1$  and  $I_1$  of the voltage and current fundamental harmonic in this illustration are

$$U_1 = U_1 e^{j\alpha_1} = 578.1 e^{-j11.1^\circ} \text{V}, \ I_1 = I_1 e^{j\beta_1} = 245.6 e^{-j38.2^\circ} \text{A}$$
  
hence

$$S_1 = 3 U_1 I_1 = = 425.9 \text{ kVA}.$$

The power factor of the furnace can be approximated by

$$\lambda = \frac{P}{S} \approx \frac{P}{S_1} = \frac{P_r + P_d}{S_1} = 0.89.$$
 (13)

This result shows that the power factor of the arc furnace in a balanced mode of operation is higher than that calculated under the assumption that the arc is regarded as a short-circuit. It is because not only the current flow through the arc plasma resistance contributes to the electric energy conversion to the heat in the furnace, but also the presence of the dc voltage on the arc contributes to this conversion.

# IV. POWER FACTOR AT TWO ARCS

In the boring or melting mode of operation, due to the furnace charge movement, one arc could extinct. An operator action might be required to restore the arc and return to a balanced mode of operation. Lowering the electrode to touch the melted steel can ignite the arc again. Meantime the furnace operates with only two arcs and this affects the furnace power and the power factor value. Let us evaluate the power factor value at such a mode of operation, assuming that the arc in the line S is not ignited.

Let us suppose that the ignited arcs are regarded as shortcircuits, as it is shown in Fig. 9.



Fig. 9. An equivalent circuit of an arc furnace with two arcs approximated by a short-circuit.

As it was shown in [13], when a load is unbalanced then the arithmetic and geometric definitions of the apparent power result in an erroneous value of the power factor. The apparent power S definition suggested by Buchholz [12] should be used instead, namely

$$S = \sqrt{U_{\rm R}^2 + U_{\rm S}^2 + U_{\rm T}^2} \sqrt{I_{\rm R}^2 + I_{\rm S}^2 + I_{\rm T}^2}$$
(14)

Assuming that the supply voltage is symmetrical, the power factor of the furnace at two arcs mode operation is

$$\lambda = \frac{P}{S} = \frac{2RI_{\rm R}^2}{\sqrt{3}U_{\rm R}\sqrt{2}I_{\rm R}} = \sqrt{\frac{2}{3}}\frac{R}{U_{\rm R}}I_{\rm R} = \sqrt{\frac{2}{3}}\frac{R}{U_{\rm R}}\sqrt{\frac{2}{3}U_{\rm R}} = \sqrt{\frac{2}{3}}\frac{R}{U_{\rm R}}\frac{\sqrt{3}U_{\rm R}}{\sqrt{(2R)^2 + (2X)^2}} = \frac{1}{\sqrt{2}}\frac{R}{\sqrt{R^2 + X^2}}$$
(15)

When the reactance is equal to the line resistance, i.e., X = R, then the power factor is  $\lambda = 0.5$ .

Instead of regarding the arc as a short-circuit, let us assume that it is approximated as discussed previously and the arcs are approximated as shown in Fig. 2. At such an approximation, the equivalent circuit of the furnace at two arcs mode of operation is shown in Fig. 10.



Fig. 10. An equivalent circuit at two arcs mode operation

Assuming that the line inductance is sufficiently high to preserve the current continuity, the voltage  $v_{RT}$  is equal to

$$v_{\rm RT} = 2a_{\rm R}U_0 \tag{16}$$

As it was in a balanced mode of operation with the voltage  $v_R$ , the voltage  $v_{RT}$  is only a fictitious voltage, because the arc plasma resistance  $R_p$  was moved to the line resistance R. The variation of the voltage  $v_{RT}$  is shown in Fig. 11.



Fig. 11. Variation of the voltage at arcs in lines R and T.

The energy released on the arcs due to the dc voltage on them is proportional to the active power

$$P_{\rm d} = \frac{1}{T} \int_{0}^{T} v_{\rm RT}(t) \ i_{\rm R}(t) \ dt \tag{17}$$

while the electric energy released in arcs plasma resistance is proportional to

$$P_{\rm p} = 2R_{\rm p} \|i_{\rm R}\|^2 \,. \tag{18}$$

*Illustration 2.* For the same parameters of the arc furnace and its supply as shown in Illustration 1, the waveforms of voltages and the line R current at two arcs mode of operation are shown in Fig. 12.



Fig. 12. Waveforms of voltages and current in line R at two arcs mode of operation.

Having samples  $i_{Rk}$  and  $v_{RTk}$ , and assuming that K = 32, the active power  $P_d$  is equal to

$$P_{\rm d} = \frac{1}{T} \int_{0}^{T} v_{\rm RT}(t) \ i_{\rm R}(t) \ dt = \frac{1}{K} \sum_{k=0}^{k=K-1} v_{\rm RTk} \ i_{\rm Rk} = 99.1 \ \rm kW$$

The line R current rms value is equal to

$$\|i_{\mathbf{R}}\| = \sqrt{\frac{1}{T}} \int_{0}^{T} i_{\mathbf{R}}^{2}(t) \ dt \approx \sqrt{\frac{1}{K}} \sum_{k=0}^{k=K-1} i_{\mathbf{R}k}^{2} = 188.1 \text{ A}$$

so that, the active power at the furnace resistance

$$P_{\rm r} = 2 R ||i_{\rm R}||^2 = 70.8 \text{ kW}.$$

Since the voltages and currents of arc furnaces are nonsinusoidal, the Buchholz definition of the apparent power S, introduced in [12] for systems with sinusoidal quantities, has to be first generalized to systems with nonsinusoidal voltages and currents, as it was suggested in [9], namely

$$S = \sqrt{||u_{\rm R}||^2 + ||u_{\rm S}||^2 + ||u_{\rm T}||^2} \cdot \sqrt{||i_{\rm R}||^2 + ||i_{\rm S}||^2 + ||i_{\rm T}||^2}$$
(19)

The line R voltage rms value is

$$||u_{\rm R}|| = \sqrt{\frac{1}{T}} \int_{0}^{T} u_{\rm R}^{2}(t) dt \approx \sqrt{\frac{1}{K}} \sum_{k=0}^{k=K-1} u_{\rm Rk}^{2} = 565.3 \,\rm V \,.$$

The line S is not loaded, so that  $||u_S|| = 700 \text{ V}$ , while for line T

$$||u_{\rm T}|| = \sqrt{\frac{1}{T} \int_{0}^{T} u_{\rm T}^2(t) \ dt} \approx \sqrt{\frac{1}{K} \sum_{k=0}^{k=K-1} u_{\rm Tk}^2} = 673.3 \, \rm V \, .$$

At such rms values, the formula (21) results in the apparent power of the furnace at two-arcs operation S = 298.9 kVA, and consequently, the power factor is equal to

$$\lambda = \frac{P}{S} = \frac{P_{\rm r} + P_{\rm d}}{S} = 0.57$$

This power factor is higher than that, calculated previously, considering arcs as short-circuits. Similarly, as it was in the case of a balanced operation of the furnace, the dc voltage on the arc contributes to a higher value of arc furnaces power factor.

# V. POWER FACTOR AT UNIDIRECTIONAL ARC

Arcs in arc furnaces during a quiet mode of the furnace operation are bidirectional. During such a mode of operation, the line currents can be approximated with a high accuracy by quantities with a negative symmetry with respect to the values shifted by a half of the period T, i.e.,

$$x(t - T/2) = -x(t)$$
(20)

Quantities with such a property do not contain harmonics of the even order n. Moreover, due to current symmetry and three-wire without neutral supply lines, the third order harmonic cannot occur in the supply current. Consequently, the 5<sup>th</sup> order is the lowest order harmonic.

Published reports, based on measurements performed on

arc furnaces [8, 10, 11], show that the arc current can contain harmonics of the even order, however. The second order current harmonic could reach even a level above of 50% of the fundamental harmonic. This can be explained only if there are intervals of time when an arc in the furnace is unidirectional, meaning (20) is no longer valid.

At assumptions as previously, the arc furnace with one, unidirectional arc, say in line S with a positive current, can be approximated as shown in Fig. 13.



Fig. 13. An equivalent circuit of an arc furnace with a unidirectional arc in line S.

When the arc in line S is ignited, then the furnace behaves as a balanced load, with the voltage  $v_A$  of the joining point of arcs as shown in Fig. 5.

When the arc in line S is not ignited, i.e., when the current in line S reaches zero, then the dc voltage at the joining point of arcs is zero, because coefficients  $a_R$  and  $a_T$  have to be of the opposite sign, and hence

$$v_{A0} = -\frac{Ga_{R}U_{0} + Ga_{T}U_{0}}{2G} = -\frac{1}{2}(a_{R} + a_{T})U_{0} = 0.$$
 (21)

Only ac voltage occurs at the point A in this state of the furnace operation. The crms value of this voltage fundamental harmonic is

$$V_{\rm A1} = \frac{Y_1 E_{\rm R} + Y_1 E_{\rm T}}{2Y_1} = \frac{1}{2} (E_{\rm R} + E_{\rm T}) = -\frac{1}{2} E_{\rm S}$$
(22)

meaning it changes as  $-e_{\rm S}(t)/2$ . The time interval with this voltage ends when the distribution system internal voltage  $e_{\rm S}(t)$ , reaches the value  $v_{\rm A} + U_0$ , and consequently, the arc in line S is ignited again. The time variance of particular voltages and the current in the line S are shown in Fig. 14. These plots are drawn for the furnace parameters as shown in Fig. 7.



Fig. 14. Waveforms of voltages and current in line S at a unidirectional arc.

The state of the arc in one line affects the remaining ones because it changes the voltage  $v_A$  of the joining points of arcs. In particular, when the arc in line S is unidirectional with a positive current, then the current of the arc in line R changes as shown in Fig. 15.



Fig. 15. Waveforms of voltages and current in line R at unidirectional arc in line S.

The energy released on arcs due to the dc voltage is proportional to the active power

$$P_{\rm d} = \frac{1}{T} \int_{0}^{T} (v_{\rm R} i_{\rm R} + v_{\rm S} i_{\rm S} + v_{\rm T} i_{\rm T}) dt$$
(23)

while that on the arcs plasma resistance

$$P_{\rm p} = R_{\rm p} \left\| \boldsymbol{i} \right\|^2 \tag{24}$$

where

$$\|\boldsymbol{i}\| = \sqrt{\|\boldsymbol{i}_{\mathrm{R}}\|^{2} + \|\boldsymbol{i}_{\mathrm{S}}\|^{2} + \|\boldsymbol{i}_{\mathrm{T}}\|^{2}}$$
(25)

is the furnace current three-phase rms value. The active power of the total resistance R of the furnace is

$$P_{\rm r} = R ||\mathbf{i}||^2 \tag{26}$$

so that, the power factor of the furnace is

$$\lambda = \frac{P}{S} = \frac{P_{\rm r} + P_{\rm d}}{||\boldsymbol{u}|||\boldsymbol{i}||}$$
(27)

**Illustration 3.** For the same parameters of the arc furnace and its supply as in Illustration 1, having K = 32 discrete values of voltages and currents, the quantities that specify the power factor  $\lambda$ , were calculated as follows.

$$P_{\rm d} \approx \frac{1}{K} \sum_{k=0}^{k=K-1} [v_{\rm Rk} \, i_{\rm Rk} + v_{\rm Sk} \, i_{\rm Sk} + v_{\rm Tk} \, i_{\rm Tk}] = 152.7 \, \rm kW$$
$$||\mathbf{i}|| \approx \sqrt{\frac{1}{K} \sum_{k=0}^{k=K-1} [i_{\rm Rk}^2 + i_{\rm Sk}^2 + i_{\rm Tk}^2]} = 361.7 \,\rm A$$

hence

$$P_{\rm r} = R ||\mathbf{i}||^2 = 130.8 \, \rm kW$$
.

Since

$$|\mathbf{u}|| \approx \sqrt{\frac{1}{K} \sum_{k=0}^{k=K-1} [u_{Rk}^2 + u_{Sk}^2 + u_{Tk}^2]} = 1067.5 \text{V}$$

the power factor at unidirectional arc is approximately equal to

$$\lambda = \frac{P_{\rm r} + P_{\rm d}}{\|\boldsymbol{u}\|\|\boldsymbol{i}\|} \approx 0.73$$

#### VII. CONCLUSIONS

The paper was focused on a decomposition of the electric energy conversion into the heat energy in arc furnaces, into two phenomena. Namely, a heat is released in arc plasma due to its resistance and due to a dc voltage on the arc. This dc voltage contributes to a furnace active power increase thus it elevates the power factor, which is reduced by a reactive power of the line inductors.

The modeling of the arc furnace was based on the assumption that the furnace parameters do not change in a single period T, thus it can be fixed in each of the modeled states. The states can change randomly or remain unchanged by time intervals of random duration. Moreover, transients occur when states are changed. Consequently, the power factor and the powers calculated in this paper change randomly. Nonetheless, the presented results can provide rough information on currents rms values and on harmonics in specific states of the arc furnace operation.

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