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Constraints of instantaneous reactive power p–q theory

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Abstract: The instantaneous reactive power (IRP) p–q theory can be acknowledged as the first concept that provided fundamentals for switching compensator control and very often such a control is satisfactory. There are situations when this control can result in objectionable effects, however. Instantaneous active and reactive powers, p and q, defined in the IRP p–q theory, were introduced with the emphasis that the definitions of these powers are valid for any three-phase system, without any constraints as to the system properties with respect to the load structure and the supply voltages and load currents waveform. This may imply a conclusion that the instantaneous powers p and q specify the power properties of the three-phase systems regardless of such systems properties. This assurance regarding the lack of constraints has contributed to the dissemination of the IRP p–q theory, especially as a fundamental of the algorithms for switching compensator control. This study shows that such a conclusion has no ground, however. In fact, only at very specific properties of the three-phase system some conclusions on its power properties can be derived from the values of the instantaneous active and reactive powers. Also, it shows that the IRP p–q theory identifies the power properties of the three-phase loads correctly only on the condition that such loads are supplied with symmetrical and sinusoidal voltage.

1 Introduction

The instantaneous reactive power (IRP) p–q theory, developed by Akagi *et al.* [1, 2], can be acknowledged as the first concept that provided the fundamentals for switching compensator control and very often such a control is satisfactory. There are situations when this control can result in objectionable effects, however. It is one of the main power theories of the three-phase systems with nonsinusoidal voltages and currents.

It is usually expected from a power theory that it provides the interpretation of the energy flow phenomena in electrical systems and describes this flow in terms of powers. It is also expected that the power theory creates theoretical fundamentals for compensation in such systems. All these capabilities have at least the power theory of the single-phase systems with sinusoidal voltage and currents.

The importance of the three-phase systems for energy delivery along with the promises of the IRP p–q theory caused a tremendous interest in this concept and a huge number of journal and conference publications on this theory and its applications for the control of the pulse-width modulation inverter-based switching compensators, known commonly as active power filters. The number of the papers on the IRP p–q theory and its applications is now in the range of a few hundreds, this number continuously increases, and it is too high for citation in this paper, however. The author might only recommend the most recent presentations of the IRP p–q theory with conclusions by Herrera and Salmerón [3, 4], Morsi and El-Hawary [5] or Superti-Furga and Todeschini [6].

According to Akagi *et al.* [1, 2], there are no constraints on the conditions for which the IRP p-q Theory is valid and the instantaneous powers p and q can be calculated. This is evidently true, because the definitions of these powers are not confined by any condition as to the voltage and the current waveform and/or the symmetry. This conclusion may not apply, however, for using these powers in a compensator control algorithm. Without any restrictions these powers were used in Refs. [1, 2] as primary data for a compensator control.

The number of the IRP p–q theory related publications shows the level of its acceptance. There are papers that reported some problems with the IRP p–q theory and with its applications for compensator control, however.

The following issues are mainly a matter of concern in some publications on this theory.

(i) Some papers [7, 8] report a low performance of the compensators with the IRP p–q theory-based control in the presence of the supply asymmetry and/or harmonic distortion.
(ii) Possibility of an increase of the supply current distortion when a compensator is controlled with the IRP p–q theory-based algorithms, in [3, 9–12, 24, 25].
(iii) There are different opinions [3, 13–15] whether the IRP p–q theory makes instantaneous compensation possible or not.
(iv) There is still confusion, reported in [15, 16, 17, 23], as to the physical interpretation of the IRP q.

The first three issues, that is, (i)–(iii), have a practical importance, while the last, (iv), only a cognitive one.

Items (i) and (ii) imply a question: 'do p and q powers really convey information sufficient for a compensator control without any restrictions as to the three-phase system properties?' The conclusions of the papers [7–12, 24, 25] show that the answer to this question seems to be negative. It means that some restrictions on using the p and the q powers in the control algorithms are necessary. The question 'at what kind of restrictions respective three-phase systems properties the IRP $p-q$ theory describes power properties of such systems correctly?' is the subject of this paper.

2 Instantaneous *p* and *q* powers in terms of line voltages and currents

The instantaneous active and reactive powers p and q are calculated, according to the IRP $p-q$ theory, in terms of the load voltages and currents expressed, by using Clarke's transform, in the α and β coordinates. The voltages and the currents in the α and β coordinates are mathematical rather than physical entities, however. This creates some difficulties for physical interpretation of the instantaneous powers, in particular, the reactive one. Clarke's transform is not needed for calculation of the instantaneous powers p and q, however. It is easier to associate some features of the instantaneous powers with the system properties when these powers are expressed directly in terms of the three-phase voltages and currents, as shown in Fig. 1, rather than in terms of their values in the α and β coordinates.

Let us introduce a three-phase vector of the line voltages

$$\boldsymbol{u}(t) = [u_R(t),\, u_S(t),\, u_T(t)]^{\mathrm{T}} \stackrel{\mathrm{df}}{=} \boldsymbol{u} \tag{1}$$

where the voltages at the load terminals R, S and T, respectively, are measured with respect to an artificial zero, and similarly, a vector of the supply currents

$$\boldsymbol{i}(t) = [i_R(t),\, i_S(t),\, i_T(t)]^{\mathrm{T}} \stackrel{\mathrm{df}}{=} \boldsymbol{i} \tag{2}$$

Since

$$i_R(t) + i_S(t) + i_T(t) \equiv 0 \tag{3}$$

$$u_R(t) + u_S(t) + u_T(t) \equiv 0 \tag{4}$$

the formula for the calculation of the instantaneous power p can be simplified as follows

$$\begin{aligned} p &= \bar{p} + \tilde{p} = \frac{\mathrm{d}W}{\mathrm{d}t} = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = u_R i_R + u_S i_S + u_T i_T \\ &= (u_R - u_T) i_R + (u_S - u_T) i_S = u_{\mathrm{RT}} i_R + u_{\mathrm{ST}} i_S \end{aligned} \tag{5}$$

The instantaneous active and reactive powers p and q, are

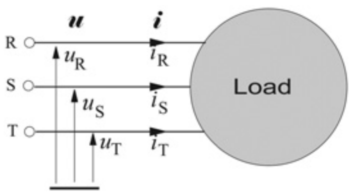

Fig. 1 *Three-phase load supplied with a three-wire line*

defined in the IRP $p-q$ theory in terms of the voltages and the currents in the α and β coordinates

$$\begin{aligned} p &= u_\alpha i_\alpha + u_\beta i_\beta \\ q &= u_\alpha i_\beta - u_\beta i_\alpha \end{aligned} \tag{6}$$

calculated by using Clarke's transform. This transform for the three-phase, three-wire systems with the voltages and the currents that satisfy the properties (3) and (4) can be rearranged to the form

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_R \\ u_S \end{bmatrix} \tag{7}$$

and similarly for the currents

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_R \\ i_S \end{bmatrix} \tag{8}$$

These formulae enable us to express the IRP q directly in terms of the supply voltages and the currents, meaning in the phase coordinates. Namely

$$\begin{aligned} q &= u_\alpha i_\beta - u_\beta i_\alpha \\ &= \sqrt{\frac{3}{2}} u_R \left(\frac{1}{\sqrt{2}} i_R + \sqrt{2} i_S \right) \\ &\quad - \left(\frac{1}{\sqrt{2}} u_R + \sqrt{2} u_S \right) \sqrt{\frac{3}{2}} i_R \\ &= \sqrt{3} (u_R i_S - u_S i_R) \end{aligned} \tag{9}$$

3 Instantaneous *p* and *q* powers of the purely resistive loads at the sinusoidal supply voltage

When an analysed system is complex, it is often difficult to interpret the results of this analysis. It is easier to interprete such results at a reduced system complexity. Purely resistive systems with harmonics generating loads (HGLs) are just such systems with a reduced complexity. The IRP $p-q$ theory was formulated, according to Akagi *et al.* [1, 2] without any constraints, thus also for purely resistive systems. Such systems are a subset of the set of the systems for which the conclusions of the IRP $p-q$ theory are valid. Therefore all these conclusions will apply to a system if it is purely resistive. At the same time, any conclusion that is not valid in a resistive subsystem will not be valid in the whole set of the systems for which the IRP $p-q$ theory was formulated. Therefore the restrictions of the IRP p−q theory are investigated in this paper for purely resistive systems. The restrictions of that theory which have to be satisfied for such resistive systems have to be obeyed, of course, by more complex systems.

The supply currents at the terminals of a harmonic generating load (HGL) are nonsinusoidal. They can have harmonics of the order n from a set of orders N. A three-phase vector of such currents can be expressed in the

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form of

$$\boldsymbol{i}(t) = \sum_{n \in N} \boldsymbol{i}_n(t) \quad (10)$$

The load generated current harmonics, because of the source internal impedance, can cause the load voltage distortion. When a source is strong, this distortion can be neglected. It is assumed in this section that indeed the supply source is strong enough, so that the load voltages are sinusoidal, meaning composed of only the fundamental harmonic, $\boldsymbol{u}_1(t)$. It is assumed, moreover, that these voltages are symmetrical and of the positive sequence.

At such assumptions the formula for the instantaneous active power p can be expressed as follows

$$p = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}_1^{\mathrm{T}} \sum_{n \in N} \boldsymbol{i}_n = \sum_{n \in N} \boldsymbol{u}_1^{\mathrm{T}} \boldsymbol{i}_n = \sum_{n \in N} p_n \quad (11)$$

where

$$p_n = u_{\mathrm{RT}1} i_{Rn} + u_{\mathrm{ST}1} i_{Sn} \quad (12)$$

Also, the IRP q can be expressed as the sum of this power for the individual harmonics, namely

$$\begin{aligned} q &= u_\alpha i_\beta - u_\beta i_\alpha \\ &= \sqrt{3}(u_R i_S - u_S i_R) \\ &= \sqrt{3}\left(u_{R1} \sum_{n \in N} i_{Sn} - u_{S1} \sum_{n \in N} i_{Rn}\right) \\ &= \sqrt{3}\left(\sum_{n \in N} u_{R1} i_{Sn} - \sum_{n \in N} u_{S1} i_{Rn}\right) \\ &= \sum_{n \in N} q_n \end{aligned} \quad (13)$$

where

$$q_n = \sqrt{3}\left(u_{R1} i_{Sn} - u_{S1} i_{Rn}\right) \quad (14)$$

Let us calculate the instantaneous powers of a resistive balanced load, shown in Fig. 2, supplied from a source of the sinusoidal symmetrical voltage such that the line-to-ground voltage at the terminal R is equal to

$$u_R = u_{R1} = \sqrt{2} U_1 \cos\ \omega_1 t \quad (15)$$

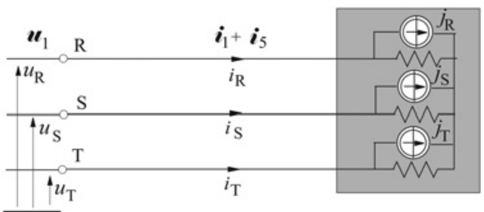

Fig. 2 *Resistive balanced HGL generating the fifth-order current harmonic*

that generates the fifth-order current harmonic, assuming that

$$i_R = \sqrt{2} I_1 \cos \omega_1 t + \sqrt{2} I_5 \cos(5\omega_1 t + \alpha_5) \quad (16)$$

At such assumptions, the line-to-line voltages have the waveforms

$$u_{\mathrm{RT}1} = \sqrt{6} U_1 \cos\left(\omega_1 t - 30^0\right)$$
$$u_{\mathrm{ST}1} = \sqrt{6} U_1 \cos\left(\omega_1 t - 90^0\right)$$

Since the fifth-order current harmonic is a negative sequence harmonic, then

$$i_S = \sqrt{2}[I_1 \cos(\omega_1 t - 120^0) + I_5 \cos(5\omega_1 t + \alpha_5 + 120^0)] \quad (17)$$

At such conditions, the instantaneous active power is equal to

$$\begin{aligned} p &= u_{\mathrm{RT}} i_R + u_{\mathrm{ST}} i_S \\ &= 2\sqrt{3} U_1 \cos(\omega_1 t - 30^0)[I_1 \cos \omega_1 t + I_5 \cos(5\omega_1 t + \alpha_5)] \\ &\quad + 2\sqrt{3} U_1 \cos(\omega_1 t - 90^0)[I_1 \cos(\omega_1 t - 120^0) \\ &\quad + I_5 \cos(5\omega_1 t + \alpha_5 + 120^0)] \\ &= 3U_1 I_1 + 3U_1 I_5 \cos(6\omega_1 t + \alpha_5) \\ &= p_1 + p_5 \\ &= \bar{p} + \tilde{p} \end{aligned} \quad (18)$$

The calculated instantaneous active power is decomposed in (18) directly into the constant and the oscillating components. These two components were obtained explicitly only because the waveforms of the voltages and the currents as well as the rms values U_1, I_1 and I_5 were assumed to be known. Without the Fourier analysis these values are not known explicitly, however. When the IRP p–q theory is used for a compensator control, these powers are calculated, according to the IRP p–q theory definition by a digital signal processing (DSP) system. It has a sequence of voltages and currents samples as the input data which are next used for calculating the voltages and the currents in the α and β coordinates, according to the matrix formulae (7) and (8). The constant and the oscillating components, $\bar{p}$ and $\tilde{p}$, are not seen explicitly in the instantaneous power p calculated by the DSP system, however. A low-pass or a high-pass filter is needed for the decomposition of this power into the constant and the oscillating components. The result of the filtering for the situation as discussed has the form

$$\bar{p} = A, \quad \tilde{p} = B \cos(6\omega_1 t + \alpha_5) \quad (19)$$

but it is not known what contributes to the A and B values.

The IRP of such a load, according to (13) is

$$q = q_1 + q_5 \quad (20)$$

where

$$q_1 = 0$$

because the load shown in Fig. 2 is balanced for the

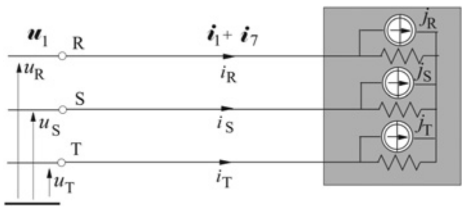

Fig. 3 *Balanced HGL generating the seventh-order current harmonic*

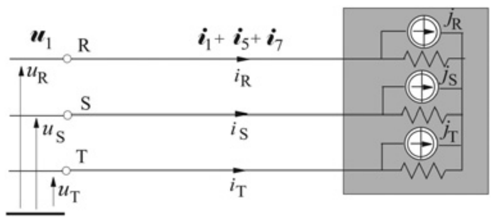

Fig. 4 *Resistive balanced HGL generating the fifth and the seventh-order current harmonic*

fundamental harmonic and is purely resistive, whereas according to formula (13)

$$\begin{aligned} q_5 &= \sqrt{3}(u_{R1}i_{S5} - u_{S1}i_{R5}) \\ &= 2\sqrt{3}U_1\{\cos\omega_1 t[I_5\cos(5\omega_1 t + \alpha_5 + 120^0)] \\ &\quad - \cos(\omega_1 t - 120^0)[I_5\cos(5\omega_1 t + \alpha_5)]\} \\ &= 3U_1 I_5 \sin(6\omega_1 t + \alpha_5 + \pi) \end{aligned} \tag{21}$$

To check how the order n of the load originated current harmonic affects the instantaneous powers, let us assume that all the other parameters of the circuit in Fig. 2 are kept unchanged, but instead of the fifth-order harmonic, the load shown in Fig. 3, generates the seventh-order current harmonic.

Since the seventh-order current harmonic is of the positive sequence the current in line S is

$$i_S = \sqrt{2}\left[I_1\cos(\omega_1 t - 120^0) + I_7\cos(7\omega_1 t + \alpha_7 - 120^0)\right] \tag{22}$$

The instantaneous active power of the fundamental harmonic, p_1, remains unchanged, whereas

$$\begin{aligned} p_7 &= u_{\rm RT1}i_{R7} + u_{\rm ST1}i_{S7} \\ &= 2\sqrt{3}U_1 I_7\{\cos(\omega_1 t - 30^0)\left[\cos(7\omega_1 t + \alpha_7)\right] \\ &\quad + \cos(\omega_1 t - 90^0)[\cos(7\omega_1 t + \alpha_7 - 120^0)]\} \\ &= 3U_1 I_7\cos(6\omega_1 t + \alpha_7) = \tilde{p} \end{aligned} \tag{23}$$

Thus, the change in the order of the load generated current harmonic from $n = 5$ to $n = 7$ does not affect the instantaneous active power. After filtering the DSP output signal we obtain as previously

$$\bar{p} = A, \quad \tilde{p} = B\cos(6\omega_1 t + \alpha_7) \tag{24}$$

thus, only the initial phase angle can be different, meaning, equal to α_5 or α_7.

The IRP associated with the presence of the seventh-order load originated current harmonic is

$$\begin{aligned} q_7 &= \sqrt{3}(u_{R1}i_{S7} - u_{S1}i_{R7}) \\ &= 2\sqrt{3}U_1\{\cos\omega_1 t[I_7\cos(7\omega_1 t + \alpha_7 - 120^0)] \\ &\quad - \cos(\omega_1 t - 120^0)[I_7\cos(7\omega_1 t + \alpha_7)]\} \\ &= 3U_1 I_7\sin(6\omega_1 t + \alpha_7) \end{aligned} \tag{25}$$

thus, only the phase of this power is affected, but not its frequency.

The loads in Figs. 2 and 3 are different loads. Unfortunately, they cannot be distinguished in terms of the IRP p–q theory, however. In terms of the p and the q powers these two loads are identical.

Now, let us assume that the load generates both the fifth and the seventh-order current harmonics. The circuit is shown in Fig. 4.

The oscillating component of the instantaneous power for such a load is

$$\begin{aligned} \tilde{p} &= p_5 + p_7 \\ &= 3U_1\left[I_5\cos(6\omega_1 t + \alpha_5) + I_7\cos(6\omega_1 t + \alpha_7)\right] \end{aligned} \tag{26}$$

while the IRP is

$$\begin{aligned} \tilde{q} &= q_5 + q_7 \\ &= 3U_1\left[I_5\sin\left(6\omega_1 t + \alpha_5 + \pi\right) + I_7\sin\left(6\omega_1 t + \alpha_7\right)\right] \end{aligned} \tag{27}$$

Formulae (26) and (27) show that the oscillating components of the instantaneous active and reactive powers are not associated with the particular power properties of the load, but only with the rms values of the harmonics, I_5, I_7, and their phases α_5, α_7. In particular, if

$$I_7 = I_5 \text{ and } \alpha_7 = \alpha_5 \tag{28}$$

then

$$\tilde{q} \equiv 0, \quad \tilde{p} = 2p_5 = 6U_1 I_5\cos(6\omega_1 t + \alpha_5) \tag{29}$$

while if

$$I_7 = I_5 \text{ and } a_7 = \alpha_5 + 180^0 \tag{30}$$

then

$$\tilde{p} \equiv 0, \quad \tilde{q} = 2q_5 = -6U_1 I_5\sin(6\omega_1 t + \alpha_5) \tag{31}$$

Observe that the conditions (28) and (30), for the initial phase angle of the load current harmonics, do not affect the power properties of the system. They affect the instantaneous powers p and q, however. If ‘plying’ with these angles changes the instantaneous p and q powers, then, it does not seem that these powers might be associated with any power property of the loads considered.

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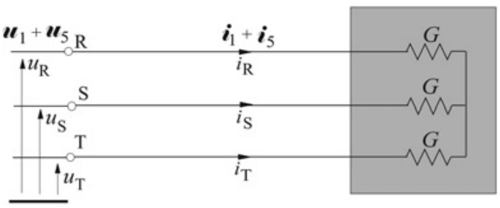

Fig. 5 *Resistive balanced load supplied with voltage composed of the fundamental and the fifth-order harmonic*

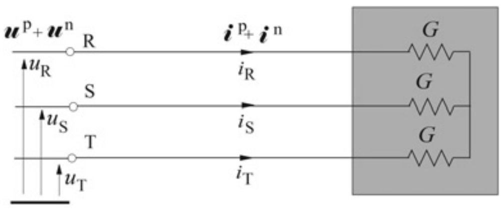

Fig. 6 *Balanced load with asymmetrical supply voltage*

4 Instantaneous *p* and *q* powers of the purely resistive loads at the nonsinusoidal supply voltage

Now, let a resistive balanced load, shown in Fig. 5, be supplied with the symmetrical voltage distorted by the fifth-order harmonic.

Assuming that the voltage at the terminal R is

$$u_R = \sqrt{2}U_1 \cos \omega_1 t + \sqrt{2}U_5 \cos 5\omega_1 t \tag{32}$$

the instantaneous active power of the load, according to formula (5) is equal to

$$\begin{aligned} p &= \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}}G\boldsymbol{u} = G[\boldsymbol{u}_1 + \boldsymbol{u}_5]^{\mathrm{T}}[\boldsymbol{u}_1 + \boldsymbol{u}_5] \\ &= G\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_1 + G\boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_5 + G\left(\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_1\right) \end{aligned} \tag{33}$$

The first two terms are the constant components of the instantaneous power

$$G\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_1 + G\boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_5 = G||\boldsymbol{u}_1||^2 + G||\boldsymbol{u}_5||^2 = P \tag{34}$$

The last term is equal to

$$\begin{aligned} G\left(\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_1\right) &= G\left[u_{1R}u_{5R} + u_{1S}u_{5S} + u_{1T}u_{5T}\right] \\ &\quad + G\left[u_{5R}u_{1R} + u_{5S}u_{1S} + u_{5T}u_{1T}\right] \\ &= 2G\left[u_{1R}u_{5R} + u_{1S}u_{5S} + u_{1T}u_{5T}\right] \\ &= 4GU_1U_5[\cos \omega_1 t \times \cos 5\omega_1 t \\ &\quad + \cos(\omega_1 t - 120^0) \times \cos(5\omega_1 t + 120^0) \\ &\quad + \cos(\omega_1 t + 120^0) \times \cos(5\omega_1 t - 120^0)] \\ &= 6GU_1U_5 \cos 6\omega_1 t \end{aligned} \tag{35}$$

Consequently, the instantaneous power of the load is

$$p = \bar{p} + \tilde{p} = P + 6GU_1U_5 \cos 6\omega_1 t \tag{36}$$

The IRP of such a resistive balanced load is, of course, equal to zero at any instant of time, meaning

$$\tilde{q} \equiv 0 \tag{37}$$

Thus, the instantaneous powers of the resistive load shown in Fig. 5, when supplied with the nonsinusoidal voltage given by formula (32), do not differ in terms of the IRP $p-q$ theory from an HGL as shown in Fig. 4, with the sinusoidal supply voltage, assuming that the generated current harmonics satisfy the condition (28) at $\alpha_5 = 0$.

After filtering the instantaneous p and q powers calculated according to the IRP $p-q$ theory, the results for the circuits shown in Figs. 4 and 5 have the same form, given by the formula (19) or (24), with the IRP equal to zero. Consequently, these two substantially different circuits cannot be distinguished in terms of the instantaneous p and q powers. This observation is important also for the IRP $p-q$ theory applications as a control algorithm. The loads in Figs. 4 and 5, having identical p and q powers, are substantially different as to the needs and the possibilities of their compensation.

5 Oscillating component of the instantaneous active *p* power

According to the IRP $p-q$ theory, the instantaneous active power p of the ideal loads, meaning the resistive balanced loads that do not generate the harmonics, is constant. The oscillating component, $\tilde{p}$, of this power is regarded, therefore as a component that occurs when the load is not ideal. It can be reduced according to Akagi *et al.* [1, 2], by a shunt compensator. This is indeed true for the loads shown in Figs. 2–4. The load in Fig. 5 is an ideal load, however, which supplied with a voltage composed of the fifth-order harmonic has a non-zero oscillating component of the instantaneous active power p, given by the formula (35). The same applies to the ideal loads supplied with asymmetrical voltage. To show this, let us calculate the instantaneous active power p of a balanced resistive load, shown in Fig. 6, supplied with the asymmetrical supply voltage composed of the positive sequence symmetrical component $\boldsymbol{u}^{\mathrm{p}}$ and the negative sequence symmetrical component $\boldsymbol{u}^{\mathrm{n}}$.

The instantaneous active power of the load is

$$\begin{aligned} p &= \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}}G\boldsymbol{u} = G[\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}]^{\mathrm{T}}[\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}] \\ &= G[\boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{n}}] + G[\boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{p}}] \end{aligned} \tag{38}$$

The first term of this expression is the sum of the active powers of the positive and the negative sequence symmetrical components, namely

$$G[\boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{n}}] = P^{\mathrm{p}} + P^{\mathrm{n}} = P \tag{39}$$

The second term of the formula (38) can be rearranged as follows

$$\begin{aligned} G\left(\boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{p}}\right) &= G\left(u_R^{\mathrm{p}}u_R^{\mathrm{n}} + u_S^{\mathrm{p}}u_S^{\mathrm{n}} + u_T^{\mathrm{p}}u_T^{\mathrm{n}}\right) \\ &\quad + G\left(u_R^{\mathrm{n}}u_R^{\mathrm{p}} + u_S^{\mathrm{n}}u_S^{\mathrm{p}} + u_T^{\mathrm{n}}u_T^{\mathrm{p}}\right) \\ &= 2G\left(u_R^{\mathrm{p}}u_R^{\mathrm{n}} + u_S^{\mathrm{p}}u_S^{\mathrm{n}} + u_T^{\mathrm{p}}u_T^{\mathrm{n}}\right) \\ &= 4GU^{\mathrm{p}}U^{\mathrm{n}}\left[\cos \omega_1 t \times \cos \omega_1 t\right. \\ &\quad + \cos\left(\omega_1 t - 120^0\right) \times \cos\left(\omega_1 t + 120^0\right) \\ &\quad \left.+ \cos(\omega_1 t + 120^0) \times \cos\left(\omega_1 t - 120^0\right)\right] \\ &= 6GU^{\mathrm{p}}U^{\mathrm{n}} \cos 2\omega_1 t \end{aligned} \tag{40}$$

Consequently

$$p = P + 6GU^{\text{p}}U^{\text{n}} \cos 2\omega_1 t \tag{41}$$

Inspite of the presence of the oscillating component in the instantaneous active power p, such a load does not need, of course, any compensation for improving its properties. It is the best load possible.

6 IRP *p–q* theory constraints

The IRP $p-q$ theory was developed in [1] for providing the fundamentals for the control algorithms of the shunt active power filters, capable of reducing the supply current harmonics originated HGL, the reactive current and the unbalanced current, it means, for reducing the harmful components of the supply current, caused by the specific unfavourable properties of the electrical loads.

Such algorithms are formulated by the IRP $p-q$ theory in terms of the instantaneous p and q powers, which are the linear forms, F, of various products of the supply voltages and the load currents or their transforms to the α and β coordinates, namely

$$p = \text{F}_p\{u_x \times i_y\}, \quad q = \text{F}_q\{u_x \times i_y\}$$

where the indices x and y can denote α, β, R, S, RT or ST.

Therefore these powers are affected in the same degree by the supply voltage and by the load current. Consequently, it is not possible to conclude without additional information, whether some components in these powers occur because of some features, such as the harmonics and/or the asymmetry of the load current or in the supply voltage. These components of the instantaneous p and q powers can be associated distinctively with the load properties, only on the condition that the supply voltage is sinusoidal and symmetrical. Only at such a condition the conclusions of the IRP $p-q$ theory when applied to shunt compensation are valid. The claim that this theory is valid for any condition is not true. Symmetry and the lack of the harmonics in the supply voltage is a necessary condition for drawing conclusions from the values of the instantaneous active and reactive p and q powers on the load power properties. Unfortunately, as shown in Sections 3 and 4, the symmetry and the lack of the harmonics in the supply voltage is not a sufficient condition for such conclusions. Even at the symmetrical and the sinusoidal supply voltage, different loads can have the same instantaneous active and reactive p and q powers. This is to some extent a debatable question, however. The question is: 'are loads that generate harmonics of different orders n, different or not for the IRP $p-q$ theory?'

7 IRP *q* interpretation

To have a cognitive value, a power theory should provide a physical interpretation of the quantities used by that theory for describing the power properties. This applies of course to the IRP q.

The physical meaning of the IRP q was not provided, in the early papers on the IRP $p-q$ theory [1, 2]. The only explanation of the physical meaning of the q-power, presented in [2] was:

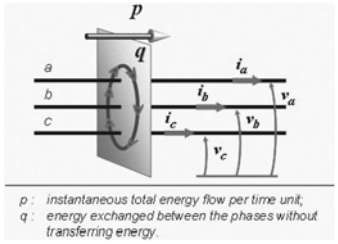

Fig. 7 *Physical meaning of the instantaneous active and reactive powers*

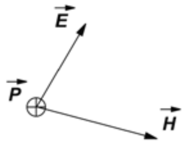

Fig. 8 *Orientation of the electric and the magnetic field intensities and the Poynting vector*

> 'The instantaneous imaginary power q was introduced on the same basis as the conventional instantaneous real power p in three-phase circuits, and then the IRP was defined with the focus on the physical meaning and the reason for naming.'

This sentence does not explain the physical meaning of the IRP q, however. Such an interpretation was provided in [18, 20], namely

> '…the imaginary power q is proportional to the quantity of energy that is being exchanged between the phases of the system…'
>
> 'figure'..' summarises the above explanations about the real and imaginary powers.'

This figure with the original caption, copied from Akagi *et al.* [20], is shown in Fig. 7.

The presented explanation does not fit Fig. 7, however. The arrow of the q-power suggests energy rotation rather than energy exchange between the phases. Since this is not clear, we should verify if any of these flows of energy is possible.

The flow of energy in the electromagnetic fields was described by J.H. Poynting in 1884. The energy flows along the Poynting vector $\vec{P}$, which is a vector product of the electric and the magnetic field intensities, $\vec{E}$ and $\vec{H}$, namely

$$\vec{P} = \vec{E} \times \vec{H} \tag{42}$$

It is perpendicular to each of them, as shown in Fig. 8.

The applications of the Poynting vector concept for describing the power properties and the energy flow in the power systems are analysed in [19].

The q-power cannot represent the energy rotation as suggested in Fig. 7, since the Poynting Vector cannot be parallel to the magnetic field intensity $\vec{H}$, which rotates around the supply lines, as shown in Fig. 9.

The question sign in Fig. 9 emphasises the question: 'is such a situation possible?' Of course, it is not.

Let us verify now whether 'energy is being exchanged between phases', as claimed in [18, 20]. When the conductors are ideal, meaning that their resistance can be

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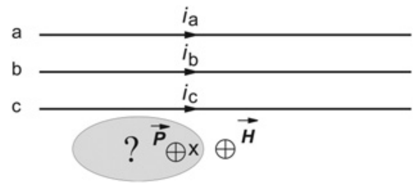

Fig. 9 *Orientation of the magnetic field intensity at the point x in the conductors plane*

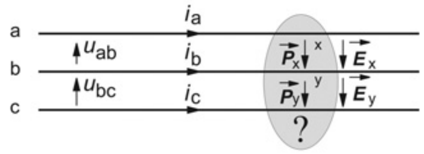

Fig. 10 *Orientation of the electric field intensity between the conductors of the three-phase line*

neglected, then the vector of the electric field intensity $\vec{E}$ has to be perpendicular to the conductor surface, as shown in Fig. 10.

The energy cannot flow from one line to another one, meaning, be 'exchanged between phases', because the Poynting Vector cannot be parallel to the vector of the electric field intensity $\vec{E}$, however. Consequently, the IRP q does not have a physical interpretation as suggested in [18, 20].

As was demonstrated in [17], this power in the systems with the sinusoidal voltages and currents can be expressed in terms of the power quantities used in the Currents' Physical Components-based power theory [17, 21] as

$$q = -Q - D\sin(2\omega_1 t + \psi) \qquad (43)$$

where Q is the common reactive power of the load, while D is the unbalanced power, defined in [22] and ψ denotes the phase angle of the load unbalanced admittance. This formula shows that the IRP q is not associated with a distinctively single phenomenon, but with two different phenomena. One of them is the phase shift between the supply voltage and the load current and consequently, the reactive power Q. The supply current asymmetry and consequently, the presence of the load unbalanced power D, is the second of these phenomena. Formula (43) does not apply to the situations where the voltages and the currents are nonsinusoidal, however. As to the author's of this paper best knowledge, it is highly unlikely that a physical interpretation of the q-power other than that, based on (43), but valid only at the sinusoidal voltages and current, could be found. Unfortunately, with the lack of a physical interpretation of the IRP q, the IRP $p-q$ theory has a major cognitive deficiency as a power theory. It does not explain the power related physical phenomena in electrical systems.

8 Conclusions

The IRP $p-q$ theory can be acknowledged as the first concept that provided the fundamentals for switching compensator control and very often such a control is satisfactory. There are situations when this control can result in objectionable effects, however. The IRP $p-q$ theory provides the fundamentals for the effective algorithms of compensator control, but it has some constraints that should be obeyed. The instantaneous active and reactive powers provide sufficient information for the compensator control only on the condition that the load is supplied with symmetrical and sinusoidal voltages. When these voltages are asymmetrical and/or nonsinusoidal then, the symmetrical and the sinusoidal component of the supply voltage can be separated by a DSP system for the compensator control. Instantaneous powers p and q calculated by using these filtered voltages are not equal to the instantaneous powers of the load, because the load is not supplied by the filtered voltages, however. Consequently, a compensator is not capable of fulfilling its objectives. In particular, the constant value of the instantaneous active power p could be an objective of compensation only if the supply voltages are sinusoidal and symmetrical. The oscillating component of the instantaneous active power should be compensated only if it is caused by the load asymmetry and/or the load generated harmonics, but not when it is caused by the supply voltage asymmetry and/or the harmonics.

The IRP $p-q$ theory has major deficiencies as a power theory from a cognitive perpective. The q-power is not associated with any physical phenomenon in the system. The oscillating components of the p and q powers can occur or disappear for a reason of secondary importance such as a change of a harmonic phase angle. Such a change does not affect the power properties of the electrical loads.

There is some level of awareness in the electrical engineering community involved in the development of switching compensators, that the supply voltage asymmetry and/or the harmonics can disturb the expected results of the compensation. These disturbances depend, of course, on the level of asymmetry and distortion. They could be neglectably small, accepted, or even an operator of the compensator might not be aware of them. This paper provides an explanation as to why these disturbances occur. By a contribution to a better comprehension of the IRP $p-q$ theory fundamentals, the paper should contribute to an improvement in the switching compensators' technology, includeing this, based on the IRP $p-q$ control.

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