

## CPC-based comparison of compensation goals in systems with nonsinusoidal voltages and currents

**Abstract.** *There is some level of confusion upon goals of compensation in systems with nonsinusoidal voltages and currents (NV&C), especially considering that compensation with switched compensators is a recursive process. This recursive nature of compensation is often ignored in discussions on compensation. This paper presents results of studies on recursive compensation with a switching compensator, aimed at reducing the supply current to its active component, defined according to Fryze, according to CPC power Theory, and to its working component. The paper also shows that the instantaneous reactive power p-q theory-based algorithm for switching compensator control, does not provide, in the presence of the supply voltage harmonics and/or asymmetry, right control of such compensators.*

**Streszczenie.** *Cele kompensacji w obwodach z niesinusoidalnymi przebiegami prądu i napięcia nie są jednoznacznie zdefiniowane i są ciągle przedmiotem dyskusji, szczególnie, że kompensacja jest procesem iteracyjnym. Ten iteracyjny charakter kompensacji jest często w dyskusjach nad kompensacją pomijany. Artykuł przedstawia wyniki badań nad iteracyjnym procesem kompensacji, którego celem jest minimalizacja prądu zasilania do prądu czynnego w sensie Fryzego i w sensie Teorii CPC, a także wtedy, gdy tym celem kompensacji jest redukcja prądu zasilania do prądu roboczego. Artykuł pokazuje, że teoria chwilowej mocy biernej, p-q, w warunkach zasilania odbiornika napięciem odkształconym lub asymetrycznym, nie tworzy poprawnych podstaw dla generowania sygnałów sterujących kompensatora.*

**Keywords:** switching compensators, active filters, CPC, active current, working current, detrimental current, instantaneous power, p-q.

**Słowa kluczowe:** kompensatory kluczujące, filtry aktywne, CPC, prąd czynny, prąd roboczy, prąd szkodliwy, moc chwilowa, p-q.

### Introduction

The ultimate goal of compensation in power systems is reduction of the cost of energy delivery and its utilization. Some elements of this cost, such as costs of investments, operation or cost of energy lost at delivery are relatively easy for estimation. However, cost of the life-span reduction by the supply voltage harmonics or asymmetry, related to overheating of the power equipment and/or effects of disturbances are rather difficult to evaluate in economic terms.

Therefore, the goals of shunt compensation are not formulated practically in economic terms, but rather in terms of reduction of electrical quantities that can be directly observed or measured. These could be some powers or harmful current components such as the reactive current or the current features such as its asymmetry and/or harmonic distortion.

Compensation can be envisioned in general as distributed over a power system with a hierarchy aimed at coordination of local compensators and their tasks. Having this in mind, we should observe that there is still a number of theoretical and practical questions related to local compensation and consequently, local compensation is the subject of this paper.

Observe that compensation goals are usually aimed at reduction of the cost of energy delivery, rather than on improvement of operation conditions of the compensated load. To improve the load operation conditions, impaired by the supply voltage harmonics and/or asymmetry, series compensators are needed. The subject of this paper is confined to shunt compensation, however.

The goals of shunt compensation are compared in this paper using the Currents' Physical Components (CPC) power theory [7] as a theoretical tool. It identifies power phenomena in systems with nonsinusoidal voltages and currents (NV&C) and therefore, it is well suited for studies on such systems.

### Controversy on compensation goals

There is some level of controversy and confusion with respect to the compensation goals. This is an outcome of a long existing confusion and controversy [5] on various approaches to power theory of systems with NV&C, espe-

cially that these goals, depending on the power theory, are formulated in different ways. Very often, these goals in papers on compensation are simply not specified, which contributes even more to the confusion.

These goals in the Instantaneous Reactive Power (IRP) p-q theory are formulated [1, 2, 6] in terms of instantaneous powers  $p$  and  $q$  or currents related to these powers. In the CPC power theory, these goals are formulated in terms of currents physical components.

In some cases, these are only apparently different goals, but in fact, these are the same goals, expressed only in a different way. Confusion can be generated also by the lack of specifics in some papers as to conditions of the compensation goals. Some papers are not sufficiently specific whether these goals are to be achieved at sinusoidal or nonsinusoidal supply voltage; at its symmetry or asymmetry, nor do these goals take into account the supply source impedance or, these goals of compensation are formulated for a load supplied from an infinite bus.

Three goals of compensation will be discussed in this paper, namely reduction of the supply current to its active component according to Fryze definition, according to CPC power theory and reduction of the supply current to its working component [7]. Discussion will be confined first to single-phase systems and next extended to three-phase, three-wire systems. Let us review the basic concepts.

### Active current according to Fryze definition

According to Fryze power theory, the minimum current needed at the terminal voltage  $u(t)$  for providing a single-phase load with the active power  $P$  is the active current

$$(1) \quad i_a(t) = G_e u(t), \quad \text{where} \quad G_e = \frac{P}{\|u\|^2}.$$

The remaining current component of the supply current

$$(2) \quad i_{rF}(t) \triangleq i(t) - i_a(t),$$

should be compensated to obtain a unity power factor.

A switching compensator, connected as shown in Fig. 1, controlled in such a way that the compensator current  $j(t)$  is equal to the negative value of the Fryze reactive current  $i_{rF}(t)$ , reduces the supply current to its active component.

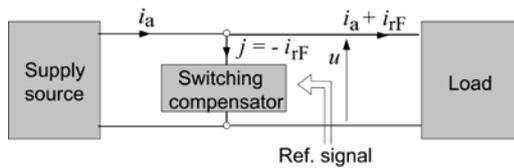


Fig. 1. System with compensator of Fryze reactive current

### Active current in the CPC power theory

According to Fryze power theory, the whole terminal voltage  $u(t)$  contributes to permanent flow of energy from a supply source to a load, meaning to its active power. This is true only on the condition that the load is not a source of harmonics. When this condition is not satisfied, then Fryze definition of the active current leads to results difficult to accept. Example of such a situation, taken from Ref. [7] is shown in Fig. 2. The supply voltage in this circuit was assumed to be

$$e = e_1 = 100\sqrt{2} \sin \omega_1 t \text{ V},$$

while the load contains a source of current harmonic

$$j = j_3 = 50\sqrt{2} \sin 3\omega_1 t \text{ A}.$$

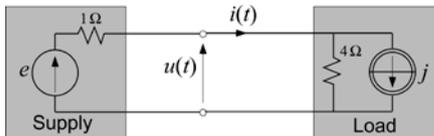


Fig. 2. Example of circuit with harmonic generating load

The voltage and current at the load terminals are equal to

$$u = u_1 + u_3 = 80\sqrt{2} \sin \omega_1 t - 40\sqrt{2} \sin 3\omega_1 t \text{ V},$$

$$i = i_1 + i_3 = 20\sqrt{2} \sin \omega_1 t + 40\sqrt{2} \sin 3\omega_1 t \text{ A}.$$

It is easy to observe that the load active power  $P$  is zero, so that, there is no active, but only the reactive current  $i_{rF}$  in this circuit. Thus, the first conclusion is that compensation of the whole current  $i(t)$  is the compensation goal. This is, of course, a wrong conclusion.

It was observed in the Currents' Physical Components power theory, that when current harmonics are originated in the load, due to its non-linearity or periodic time-variance, then a part of terminal voltage  $u(t)$  is only a response to injection of the current harmonics into the distribution system. The active power  $P_n$  of such harmonics can be negative, meaning the energy at the frequency of such harmonics flows from the load back to the supply source, reducing the load active power,  $P$ .

If  $P_D$  denotes the sum of positive harmonic active powers  $P_n$  and  $u_D(t)$  denotes the sum of the voltage harmonics for which these powers are positive, then the active current in the CPC power theory is defined as

$$(3) \quad i_{aD}(t) \triangleq G_{eD} u_D(t), \text{ where } G_{eD} \triangleq \frac{P_D}{\|u_D\|^2}.$$

For example, in the circuit shown in Fig. 2,  $P_D = 1600 \text{ W}$ ,

$$u_D = 80\sqrt{2} \sin \omega_1 t \text{ V}, \quad G_{eD} = 0.25 \text{ S},$$

and the active current according to CPC power theory is

$$i_{aD} = 20\sqrt{2} \sin \omega_1 t \text{ A}.$$

Thus, the compensator should eliminate the non-active component

$$i_b = i - i_{aD} = 40\sqrt{2} \sin 3\omega_1 t \text{ A},$$

from the supply current. It modifies the terminal voltage to  $u_D(t)$  component. Compensators should operate according to principle illustrated in Fig. 3.

Observe, that the non-active current  $i_b(t)$  is orthogonal to the active current  $i_{aD}(t)$ , thus, assuming that the compensator is lossless, it can compensate the system without any energy consumption. This is because its active power,  $P_c$ , as a scalar product, denoted as  $(\cdot, \cdot)$ , of the load voltage  $u_D(t)$  and the compensator current  $j(t)$ , is zero because

$$(4) \quad P_c = (u_D, j) = \square (u_D, i_b) = 0.$$

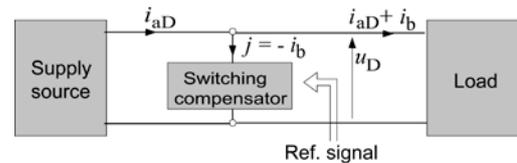


Fig. 3. System with compensator of CPC non-active current

The current  $i_b(t)$  is not referred to as a reactive current, because this adjective is reserved in the CPC power theory only for a current component that occurs due to a phase-shift between the voltage and current harmonics. In fact, current  $i_b(t)$  is composed [7] of the reactive, scattered and the load generated currents, namely

$$(5) \quad i_b(t) = i_r(t) + i_s(t) + i_c(t).$$

This decomposition has mainly a cognitive value, but also it is important for compensation more advanced than only with a switching compensator. For example, the reactive current  $i_r(t)$  can be compensated by shunt reactance compensators, while the scattered current  $i_s(t)$  is not affected by such compensators. The load generated current  $i_c(t)$  can be reduced by resonant harmonic filters. These properties can be taken into account at hybrid compensation, built as reactance and switching compensators. Also, the reactive current  $i_r(t)$  is usually a low frequency current, while the load generated harmonic current  $i_c(t)$  is a high frequency current. This property can be taken into account at construction of hybrid compensators built of a low frequency and a high frequency switching compensator.

### Working current

The active current is proportional to the supply voltage, meaning it is nonsinusoidal at distorted voltage and/or is asymmetrical when this voltage is asymmetrical.

Apart from energy absorbed by purely resistive loads, the energy conveyed by harmonics and/or negative sequence component of voltages and currents is not useful, but rather contributes to harmful effects in motors or electronic equipment. Only the positive sequence component of the current fundamental harmonic, which is in-phase with the voltage fundamental harmonic of the positive sequence, contributes to useful energy transmission to majority of present-day loads. This current component is referred to as a working current. In single-phase system

$$(6) \quad i_w(t) \triangleq i_{a1}(t) = G_w u_1(t),$$

where

$$(7) \quad G_w \triangleq G_1 \triangleq \frac{P_1}{\|u_1\|^2}.$$

The remaining part of the current can be referred to as a *detrimental* current

$$(8) \quad i_d(t) \triangleq i(t) - i_w(t).$$

According to this reasoning, elimination of the detrimental current by a compensator, meaning its current should be controlled in such a way that

$$(9) \quad j(t) = -i_d(t),$$

could be regarded as legitimate goal of compensation.

At such a compensation goal, assuming that the supply system is ideal, in the sense that its internal impedance is zero, so that the compensator does not affect terminal voltage, the compensator active power is equal to

$$(10) \quad P_c = (u, j) = (u, -i_d) = (u, -(i - i_1)) = \\ = -(u, i) + (u, i_1) = -P + P_1 = -P_h,$$

thus, the compensator has to deliver energy to the distribution system. Symbol  $P_h$  denotes the active power of all harmonics except the fundamental one. Switching compensators are not active, but passive devices, thus this energy has to be delivered to the compensator.

Assuming that the compensator is a lossless device, this energy can be delivered by an additional current, which is in-phase with the supply voltage fundamental harmonic,  $\Delta i_w(t)$ , such that

$$(u, \Delta i_w) = P_h.$$

It means that compensation of the detrimental current  $i_d(t)$  requires some increase of the working current. This is illustrated in Fig. 4.

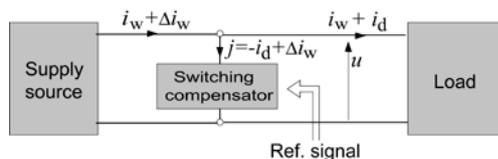


Fig. 4. System with a compensator of detrimental current  $i_d(t)$

This result has a clear interpretation: when the system is compensated with a goal aimed at reducing the supply current to a sinusoidal one, then the energy conveyed to the load by harmonics has to be converted to energy taken from the source by an additional sinusoidal current in-phase with the voltage fundamental harmonic. It means, an increase in the working current value as compared to its value before compensation.

### Compensation as a recursive process

Switching compensator can be regarded as a controlled current source. Its current is controlled, according to the assumed compensation goal, with a reference signal, generated by extracting information on the load powers properties from the terminal voltage and current. The compensator current affects the load voltage, current and powers and consequently, the reference signal. Thus, compensation in real systems is a recursive process.

A selected goal of compensation specifies the control algorithm in sequential steps of this recursive process, but not necessarily the final result. Convergence of this process and its final result may depend on the system properties. It is not clear for the authors of this paper, whether this convergence could be a real problem or it is only an academic one. It seems, however, that in situations common for distribution systems, problems with convergence of this process should not occur.

The recursive process of compensation with the goal of reducing the supply current to the active current according to Fryze definition, is illustrated on the purely resistive circuit shown in Fig. 5. The supply voltage in this circuit contains a relatively high, 5<sup>th</sup> order harmonic,

$$e = e_1 + e_5 = 100\sqrt{2} \sin \omega_1 t + 10\sqrt{2} \sin 5\omega_1 t \text{ V},$$

while the load contains a source of the 7<sup>th</sup> order current harmonic

$$j = j_7 = 15\sqrt{2} \sin 7\omega_1 t \text{ A}$$

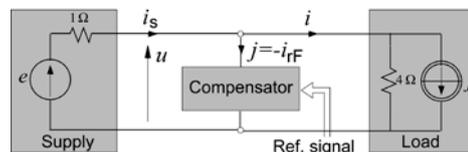


Fig.5. System with compensator of Fryze reactive current

Terminal voltage before compensation is

$$u = 80\sqrt{2} \sin \omega_1 t + 8\sqrt{2} \sin 5\omega_1 t - 12\sqrt{2} \sin 7\omega_1 t \text{ V},$$

and the current

$$i = 20\sqrt{2} \sin \omega_1 t + 2\sqrt{2} \sin 5\omega_1 t + 12\sqrt{2} \sin 7\omega_1 t \text{ A}.$$

The load active power, voltage rms value and the equivalent conductance are equal to, respectively

$$P = 1472 \text{ W}, \quad \|u\| = 82.29 \text{ V}, \quad G_e = 0.2228 \text{ S}.$$

hence, the active current is

$$i_a = 17.82\sqrt{2} \sin \omega_1 t + 1.78\sqrt{2} \sin 5\omega_1 t - 2.67\sqrt{2} \sin 7\omega_1 t \text{ A}.$$

The compensator current,  $j = i_a - i$ , is

$$j = -2.18\sqrt{2} \sin \omega_1 t - 0.22\sqrt{2} \sin 5\omega_1 t - 14.67\sqrt{2} \sin 7\omega_1 t \text{ A}.$$

It changes terminal voltage to

$$u = 82.18\sqrt{2} \sin \omega_1 t + 8.22\sqrt{2} \sin 5\omega_1 t + 2.67\sqrt{2} \sin 7\omega_1 t \text{ V},$$

and the load active power, voltage rms value and the equivalent conductance to, respectively

$$P = 1683 \text{ W}, \quad \|u\| = 82.15 \text{ V}, \quad G_e = 0.2494 \text{ S}.$$

This changes the active current to

$$i_a = 20.39\sqrt{2} \sin \omega_1 t + 2.04\sqrt{2} \sin 5\omega_1 t - 0.07\sqrt{2} \sin 7\omega_1 t \text{ A}.$$

In five steps of iteration the compensator current converges to

$$j = -15.0\sqrt{2} \sin 7\omega_1 t \text{ A},$$

and the terminal voltage and the supply current are finally modified to

$$u = 80\sqrt{2} \sin \omega_1 t + 8\sqrt{2} \sin 5\omega_1 t \text{ V},$$

$$i_s = 20\sqrt{2} \sin \omega_1 t + 2\sqrt{2} \sin 5\omega_1 t \text{ A}.$$

This current occurs to be not the active current according to Fryze definition, but defined according to the CPC power theory,  $i_{aD}(t)$ , and the terminal voltage has converged to voltage  $u_D(t)$ . Indeed, only this component of the voltage contributes to the positive active power. Thus

$$P_D = 1616 \text{ W}, \quad \|u_D\| = 82.40 \text{ V}, \quad G_{eD} = 0.250 \text{ S}.$$

so that, the active current according to CPC power theory is

$$i_{aD} = G_{eD} u_D = 20\sqrt{2} \sin \omega_1 t + 2\sqrt{2} \sin 5\omega_1 t \text{ A}.$$

Thus, the recursive process of the switching compensator control aimed at reducing the supply current to its active component, defined according to Fryze, has converged to the active current defined according to the CPC power theory. These two goals, different before compensation, have occurred to be identical after it.

Now, let us observe a compensation process when reduction of the supply current to its working component is the compensation goal. Since the load active power of the fundamental harmonic in the circuit considered is

$$P_1 = P_w = 1600 \text{ W},$$

the working current is equal to

$$i_w = i_1 = 20\sqrt{2} \sin \omega_1 t \text{ A}.$$

In the recursive process of the compensator control aimed at reducing the supply current  $i_s(t)$  to its working component, the compensator current converges to

$$j = 0.313\sqrt{2} \sin \omega_1 t - 2.5\sqrt{2} \sin 5\omega_1 t - 15.0\sqrt{2} \sin 7\omega_1 t \text{ A},$$

while the supply current and the voltage are modified to

$$i_s = 20.25\sqrt{2} \sin \omega_1 t \text{ A},$$

$$u = 79.75\sqrt{2} \sin \omega_1 t + 10\sqrt{2} \sin 5\omega_1 t \text{ V}.$$

The rms value of the working current is higher than that of the active one and consequently, there is higher energy loss at delivery. At first, compensation aimed at reducing the supply current to its working component, looks inferior to compensation to the active current. The increase in the cost of energy delivery could be regarded, however, as a sort of penalty for supplying the load with a distorted voltage. It might create an incentive for reducing voltage harmonics on the distribution side.

Effects of compensation aimed at reaching three discussed goals were illustrated with the circuit where the voltage harmonic on the distribution side,  $e_5$ , was orthogonal to the current harmonic generated in the load,  $j_7$ . Consequently, these harmonics, having different order, did not contribute to any permanent flow of energy between the supply source and the load. The conclusions that can be drawn from these illustrations can be generalized to systems where distribution and load generated harmonics do not have common orders.

Voltage harmonics in distribution systems and current harmonics generated in the load can have the same order, however. Because of this, such harmonics can contribute to permanent energy flow. This flow depends, of course, on magnitude of these harmonics and their mutual phase.

Distribution voltage harmonics and load generated harmonics occur as an effect of two different processes: one in the distribution system and the second in individual loads. With respect to phase-angles between the distribution voltage and load generated current harmonics they can be regarded as mutually random and consequently, orthogonal. Thus, in statistical terms, such harmonics, even of the same order, do not contribute to permanent energy flow between the load and distribution system. Nonetheless, compensation when these harmonics are not orthogonal can also be a matter of interest.

The worst case scenario in purely resistive circuits as considered, for example, in Fig. 5 occurs when the distribution voltage harmonics and the load generated harmonics are in-phase.

To study a recursive process of compensation for such a worst case situation, confined to distortion with only a single 3<sup>rd</sup> order harmonic, let us assume that in the system shown in Fig. 5, the distribution voltage harmonic and the load generated harmonic are in-phase and

$$e = e_1 + e_3 = 100\sqrt{2} \sin \omega_1 t + 10\sqrt{2} \sin 3\omega_1 t \text{ V}$$

$$j = j_3 = 20\sqrt{2} \sin 3\omega_1 t \text{ A}$$

Terminal voltage before compensation is

$$u = 80\sqrt{2} \sin \omega_1 t - 8\sqrt{2} \sin 3\omega_1 t \text{ V},$$

and the load current

$$i = 20\sqrt{2} \sin \omega_1 t + 18\sqrt{2} \sin 3\omega_1 t \text{ A}.$$

Let us assume that reducing the supply current to the active current according to Fryze definition, is the goal of compensation.

The load active power, voltage rms value and the equivalent conductance are equal to, respectively

$$P = 1456 \text{ W}, \quad \|u\| = 80.34 \text{ V}, \quad G_e = 0.2252 \text{ S}.$$

hence, reducing the supply current to its active component

$$i_a = 18.02\sqrt{2} \sin \omega_1 t - 1.8\sqrt{2} \sin 3\omega_1 t \text{ A},$$

is the goal of compensation.

The compensator current,  $j = i_a - i$ ,

$$j = -1.98\sqrt{2} \sin \omega_1 t - 19.8\sqrt{2} \sin 3\omega_1 t \text{ A},$$

changes terminal voltage to

$$u = 78.48\sqrt{2} \sin \omega_1 t + 7.85\sqrt{2} \sin 3\omega_1 t \text{ V},$$

so that, the active current changes. Eventually, the process converges to

$$i_s = 21.58\sqrt{2} \sin \omega_1 t + 2.16\sqrt{2} \sin 3\omega_1 t \text{ A}.$$

$$j = 1.98\sqrt{2} \sin \omega_1 t - 19.8\sqrt{2} \sin 3\omega_1 t \text{ A},$$

$$u = 78.42\sqrt{2} \sin \omega_1 t + 7.84\sqrt{2} \sin 3\omega_1 t \text{ V},$$

with

$$P = 1709 \text{ W}, \quad \|u\| = 78.81 \text{ V}, \quad G_e = 0.275 \text{ S}.$$

Thus, the compensation process does not converge to the active current of the load, but to a different current.

Let us assume that the supply current should be reduced to the active component, defined according to the CPC power theory. Since only the active power of the fundamental harmonic is positive, thus

$$u_D = 80\sqrt{2} \sin \omega_1 t \text{ V},$$

$$i_D = 20\sqrt{2} \sin \omega_1 t \text{ A},$$

so that

$$P_D = 1600 \text{ W}, \quad \|u_D\| = 80 \text{ V}, \quad G_{eD} = 0.25 \text{ S},$$

and the detrimental current is

$$i_d = 18\sqrt{2} \sin 3\omega_1 t \text{ A}.$$

A recursive process of compensation aimed at detrimental current compensation, but run in such a way that the compensator active power is zero, converges to

$$i_s = i_{aD} = 22.32\sqrt{2} \sin \omega_1 t \text{ A},$$

$$j = 2.90\sqrt{2} \sin \omega_1 t - 22.5\sqrt{2} \sin 3\omega_1 t \text{ A},$$

$$u = 77.78\sqrt{2} \sin \omega_1 t + 10\sqrt{2} \sin 3\omega_1 t \text{ V}.$$

The load active power after compensation is  $P = 1736 \text{ W}$ .

Observe, that the active current  $i_{aD}(t)$  in this particular situation is identical with the working current.

This numerical example illustrates a worst case scenario. Harmonics on the distribution side and generated in the load can be regarded as mutually orthogonal.



cuits, shown in Fig 6(a) and (b), cannot be distinguished in terms of instantaneous powers  $p$  and  $q$ .

When the instantaneous active power  $p(t)$  has a non-zero varying component, it can occur because the load generates current harmonics and this is a property of the load, or because the supply voltage is distorted, which is not the load property. The lack of information on the cause of this varying component is a major deficiency of the IRP p-q Theory as a fundamental for shunt switching compensator control. Such compensators are installed at the load terminals to compensate the load generated harmonics, but in the presence of the voltage harmonics the compensator attempts to compensate [9], not-existent the load originated current harmonics.

Instantaneous active and reactive powers, introduced in Ref. [1], were defined according to the authors of Ref. [2] "...for arbitrary voltages and current waveforms without any restrictions." Consequently, the question asked above on these powers under nonsinusoidal supply voltage is a legitimate question inside of the IRP p-q Theory. According to its authors, it was not restricted to systems with a sinusoidal supply voltage. It occurs, however, that the IRP p-q Theory is not capable of identifying power properties of loads with nonsinusoidal supply voltage. Conclusions of the IRP p-q Theory do not apply to systems supplied with such a voltage. This is the major limitation of that Theory, because one can expect, based on the authors assurance, that these conclusions are valid at arbitrary supply voltages.

This limitation can also be explained in the following way. Instantaneous powers  $p$  and  $q$  are algebraic forms of the supply voltages and the load currents, thus, their time features are affected both by the supply system excitation and by the load response. When only these two powers are known, it is not possible to conclude whether specific features of these powers come from the supply voltages or from the load currents. Consequently, no conclusions on the power properties of the load can be drawn.

This limitation has important consequences for formulating compensation goals in terms of instantaneous  $p$  and  $q$  powers. Although loads shown in Fig. 6(a) and 6(b) do not differ as to instantaneous powers, they have different power factor  $\lambda$ . The load in Fig. 6(a) can be compensated to a unity power factor with a resonant harmonic filter or with a switching compensator capable of reducing the 7<sup>th</sup> order harmonic. The load in Fig. 6(b) has, of course, a unity power factor. We can conclude this only from the load structure and parameters, but not from the instantaneous powers  $p$  &  $q$  values. An attempt of compensating the varying component of the instantaneous active power increases, as shown in paper [9] the supply current distortion and reduces the power factor, originally equal to  $\lambda = 1$ . Such a load might need an improvement in the supply quality with a series compensator or filter that would reduce the 7<sup>th</sup> order harmonic in the supply voltage, but for sure, not a shunt compensator. Also the supply voltage asymmetry can mess up conclusions drawn from instantaneous powers  $p$  and  $q$  features on the load power properties. Consequently, incorrect reference signals for compensator control can be generated. To show this, let us calculate the instantaneous reactive power  $q$  of a purely inductive balanced load, shown in Fig. 7(a), of the susceptance per phase equal to  $B = -1/\omega L$ , supplied with a sinusoidal symmetrical voltage of the positive sequence, equal at R terminal to

$$u_R^p = u_R = \sqrt{2} U^p \cos \omega_1 t,$$

thus, with current in the line R equal to

$$i_R^p = i_R = -\sqrt{2} B U^p \sin \omega_1 t.$$

The instantaneous reactive power  $q$  is equal to

$$(16) \quad q = \bar{q} + \tilde{q} = \sqrt{3} (u_R i_S - u_S i_R) = \\ = -2\sqrt{3} B U^{p2} [\cos \omega_1 t \sin(\omega_1 t - 120^\circ) - \\ - \cos(\omega_1 t - 120^\circ) \sin \omega_1 t] = 3 B U^{p2} = -Q,$$

meaning its average value is equal to the load reactive power negative value,  $-Q$ , in the common sense.

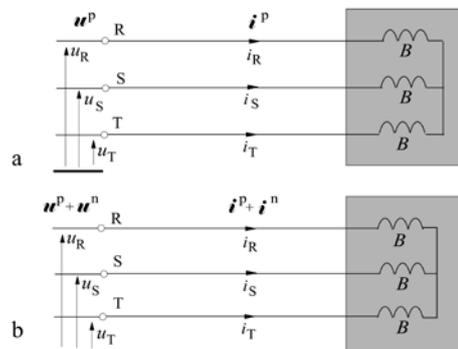


Fig.7(a). Inductive load with symmetrical supply voltage and (b) the same load with asymmetrical supply.

When the same load is supplied with asymmetrical voltage as shown in Fig. 7 (b), i.e., equal to the sum of the positive and negative sequence, so that

$$\mathbf{u} = \mathbf{u}^p + \mathbf{u}^n,$$

then assuming that

$$u_R = \sqrt{2} U^p \cos \omega_1 t + \sqrt{2} U^n \cos \omega_1 t,$$

the instantaneous reactive power  $q$  is equal to

$$(17) \quad q = \bar{q} + \tilde{q} = \sqrt{3} (u_R i_S - u_S i_R) = \\ = \sqrt{3} B \{ (U^p + U^n) \cos \omega_1 t \times \\ \times [U^p \sin(\omega_1 t - 120^\circ) + U^n \sin(\omega_1 t + 120^\circ)] - \\ - [U^p \cos(\omega_1 t - 120^\circ) + U^n \cos(\omega_1 t + 120^\circ)] \times \\ \times (U^p + U^n) \sin \omega_1 t \} = 3 B U^{p2} - 3 B U^{n2} = \\ = -Q^p + Q^n.$$

This, however, is not the (negative) value of the load reactive power of a balanced purely inductive load supplied with asymmetrical voltage, which is equal to

$$(18) \quad Q = -3 B U^{p2} - 3 B U^{n2} = Q^p + Q^n.$$

At voltage asymmetry such that  $U^p = U^n$ , the instantaneous reactive power  $q$  of an inductive load is zero. A switching compensator controlled with the IRP p-q Theory-based algorithm will not generate the right signal for such a purely inductive load compensation.

Similar problem occurs at compensation of a load imbalance, even of purely resistive loads, in the presence of supply voltage asymmetry.

Let us analyze such a situation with a purely resistive unbalanced load. When such a load is supplied, as shown in Fig. 8(a), with a sinusoidal symmetrical voltage of the positive sequence  $\mathbf{u}^p$ , such that

$$u_R^p = u_R = \sqrt{2} U^p \cos \omega_1 t,$$



Compensation of the detrimental component modifies the supply current to a sinusoidal and symmetrical one, needed for supplying the load with the working power. Compensation of the reactive current  $\mathbf{i}_{1r}^p$  modifies the load voltage positive sequence component to the working component  $\mathbf{u}_{w0}$ .

The active power of a lossless compensator at sinusoidal and symmetrical supply voltage is zero, because compensator current is orthogonal to such a voltage. However, when this voltage is nonsinusoidal and asymmetrical the compensator active power after the first step of the compensation process is

$$(31) \quad (\mathbf{u}, \mathbf{j}) = (\mathbf{u}_{w0} + \mathbf{e}_1^n + \mathbf{e}_h, -\mathbf{i}_{1r}^p - \mathbf{i}_1^n - \mathbf{i}_h) = -(\mathbf{e}_1^n, \mathbf{i}_1^n) - (\mathbf{e}_h, \mathbf{i}_h),$$

meaning it is not equal to zero. Thus, energy stored in the capacitor  $C$  and its voltage  $U_C$  decline. To keep this voltage constant, some amount of energy has to be delivered to the compensator at the fundamental frequency, which increases the working current. The change of the working current affects the working voltage, thus compensation is a recursive process which ends when

$$(32) \quad (\mathbf{u}_w, \Delta \mathbf{i}_w) - (\mathbf{e}_1^n, \mathbf{i}_1^n) - (\mathbf{e}_h, \mathbf{i}_h) = 0,$$

as it is shown in Fig. 9.

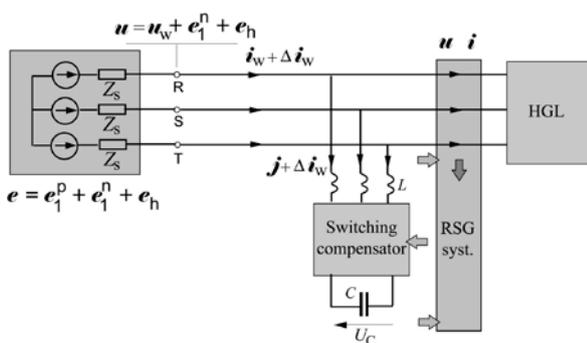


Fig. 9. Compensator of detrimental current

Eqn. (32) means that a compensator converts the active power of the negative sequence component of the voltage and current fundamental harmonic

$$(33) \quad P_1^n = (\mathbf{e}_1^n, \mathbf{i}_1^n),$$

and the active power of all higher order harmonics

$$(34) \quad P_h = (\mathbf{e}_h, \mathbf{i}_h),$$

into a working power

$$(35) \quad \Delta P_w = (\mathbf{u}_w, \Delta \mathbf{i}_w),$$

such that

$$(36) \quad \Delta P_w = P_1^n + P_h.$$

This conversion is illustrated in the active power diagram shown in Fig. 10.

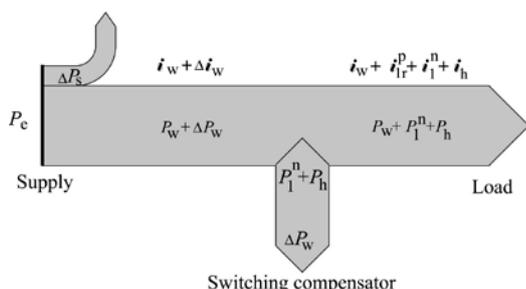


Fig. 10. Active power diagram

The rms value of the of the working current  $\|\mathbf{i}_w\|$  is, of course, higher than the rms value of the active current  $\|\mathbf{i}_a\|$  and consequently, the energy supplier with a compensator which reduces the supply current to its working component will experience higher energy loss at delivery as compared to that with a compensator that reduces the current to its active component. This could be regarded, however, as a sort of penalty for supplying customers with nonsinusoidal and/or asymmetrical voltage. Thus, an economic incentive can be created for improving the supply quality with respect to supply voltage symmetry and its harmonics.

## Conclusions

Electrical systems are compensated for economic purposes. In systems with nonsinusoidal voltages and currents it is not easy to express the goals of compensation in economic terms, however. Compensation is specified rather as reduction of some harmful quantities, but there is some level of ambiguity with respect to compensation goals, especially that compensation is a recursive process. In particular, the supply current compensation to the active current in Fryze's sense results in the active current as defined in the CPC-power theory. At the same time, the instantaneous reactive power p-q theory, in the presence of the supply voltage harmonics and asymmetry, does not specify goals of compensation, because it does not identify power properties of electrical loads. Expectations that the power theory identifies load properties in the presence of the supply voltage harmonics and asymmetry is particularly crucial for micro-grids, which could be weak systems with power electronics converters both on the supply and the customer side with a high share of single-phase harmonics generating loads.

## References

- [1] Akagi H., Kanazawa Y., Nabae A., (1984) Instantaneous reactive power compensators comprising switching devices without energy storage components, *IEEE Trans. IA*, IA-20, No. 3, 625-630.
- [2] Akagi H., Nabae A., (1993) The p-q theory in three-phase systems under non-sinusoidal conditions, *Europ. Trans. on Electr. Power, ETEP*, Vol. 3, No. 1, 27-31.
- [3] Montano J.-C., Salmeron P., (1999) Identification of instantaneous current components in three-phase systems, *IEE Proc. Science, Measur. and Technology*, Vol. 146, No. 5, 227-233.
- [4] Czarnecki L.S., (2004) On some misinterpretations of the Instantaneous Reactive Power p-q Theory, *IEEE Trans. on Power Electronics*, 19 No.3, 828-836.
- [5] Willems J.L., (2008) Critical comparison of various decomposition and compensation schemes in power systems, *Przeegląd Elektrotechniczny, (Electrical Review)*, R84, No. 11, 316-321.
- [6] Watanabe E. H., Akagi H., and Aredes M., (2008): Instantaneous p-q Power Theory for compensating nonsinusoidal systems, *Przeegląd Elektrotechniczny*, R84, No. 6, pp. 28-37.
- [7] Czarnecki L.S., (2008) Currents' Physical Components (CPC) concept: a fundamental for power theory, *Przeegląd Elektrotechniczny (Electrical Review)*, R84, No. 6, pp.28-37.
- [8] Czarnecki L.S., Pearce S. E., (2009) Compensation objectives and CPC-based generation of reference signals for shunt switching compensator control, *IET Power Electronics*, Vol. 2, No. 1, 33-41.
- [9] Czarnecki L.S., (2009) Effect of supply voltage harmonics on IRP p-q-based switching compensator control, *IEEE Trans. on Power Electronics*, Vol. 24, No. 2, 483-488.
- [10] Czarnecki L.S., (2010) Effect of supply voltage asymmetry on IRP p-q - based switching compensator control, *IET Power Electronics*, Vol. 3, No. 1, 11-17.

**Authors:** *prof. dr hab. Inż. Leszek S. Czarnecki, Fellow IEEE, Alfredo M. Lopez Distinguished Professor, ECE Dept., Louisiana State University, USA, 824 Louray Dr. Baton Rouge, LA 70808, [lsczar@cox.net](mailto:lsczar@cox.net), [www.lsczar.info](http://www.lsczar.info), Samuel E. Pearce, MS., ECE, LSU, 1305 Asphodel Dr., St. Gabriel, LA 70776, [spearc2@lsu.edu](mailto:spearc2@lsu.edu).*