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Powers in three-phase systems with neutral conductor at sinusoidal voltages and currents

Abstract. *Most of residential and industrial distribution systems as well as traction and distribution systems in commercial buildings are three-phase systems with a neutral conductor, denoted in this paper as 3pN systems. When loads are unbalanced then such systems cannot be now described as a whole in power terms because of a controversy regarding definition of the apparent, even if voltages and currents are sinusoidal. Consequently, it is not clear now how the apparent power and the power factor are affected by the load imbalance. The paper suggests definition of the apparent power for such systems and the power equation. It is based on the supply current decomposition into the Currents' Physical Components (CPC). The paper introduces two new powers that specify the effect of the load imbalance upon the apparent power of three-phase systems with a neutral conductor at sinusoidal voltages and currents.*

Streszczenie. *Sieci rozdzielcze w dzielnicach mieszkaniowych, zakładach przemysłowych, budynkach handlowych czy publicznych, a także sieci trakcyjne, są budowane zwykle jako sieci trójfazowe z przewodem zerowym. Oznaczane są one w tym artykule jako sieci 3pN. Wtedy, gdy odbiorniki zasilane z sieci 3pN nie są zrównoważone, nie można obecnie napisać równania mocy takich obwodów, trakto-wanych jako całość, a jedynie równania mocy poszczególnych faz. Przyczyną tego jest kontrowersja wokół definicji mocy pozornej w takich obwodach. Przedmiotem niniejszego artykułu jest propozycja definicji mocy pozornej, oraz równanie mocy w obwodach trójfazowych z przewodem zerowym i sinusoidalnymi przebiegami prądu i napięcia. Równanie to wynika z rozkładu prądu zasilania w takich obwodach na Składowe Fizyczne (ang.: CPC). (**Moce w obwodach trójfazowych z przzwodem zerowym i sinusoidalnymi przebiegami prądu i napięcia**)*

Keywords: Currents' Physical Components, CPC, power definitions, imbalanced systems, unbalanced power.
Słowa kluczowe: Składowe Fizyczne Prądu, CPC, definicje mocy, systemy niezrównoważone, moc niezrównoważenia.

Introduction

Loads in residential distribution systems, commercial buildings and industrial plants are a mixture of pure balanced three-phase loads such as three-phase motors or rectifiers, as well as single phase loads, as it is shown in Fig. 1. Because of this, power grids of such customers are built as three-phase systems with a neutral conductor. They will be denoted in this paper as 3pN grids. Maybe, the most distincive example of these sorts of grids is three-phase traction grid, since the driving carts can be supplied only from a single traction line, while rails form a neutral conductor.

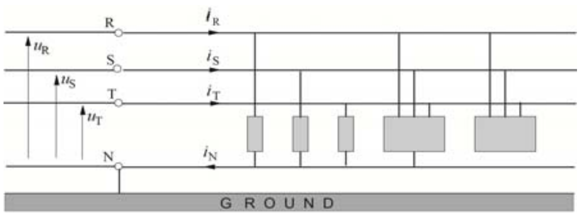

Fig. 1. Three-phase load with neutral conductor.

Although a substantial amount of electric energy is consumed in 3pN grids, we cannot describe them now in power terms as a whole, but only phase-by-phase, even if the voltages and currents are sinusoidal. This is because the power equation of three-phase systems

$$S^2 = P^2 + Q^2 \quad (1)$$

is valid only on the condition that the load is balanced.

It was known, as reported in Ref. [1] by Lyon in 1920, almost from the beginning of three-phase system development, that the load imbalance degrades the power factor, $\lambda = P/S$, thus it increases the apparent power of the source.

Fundamentals for studies of unbalanced systems were provided in 1918, with the development of the concept of symmetrical components by Fortescue [2], and a frame for these studies was established by AIEE [3] as well as Curtis and Silsbee [4] who defined basic power quantities.

The load imbalance and consequently, the supply current asymmetry is a property which is substantially different from a phase-shift between the voltage and current, which results in the reactive power Q.

Unfortunately, all attempts aimed at formulating a power equation of unbalanced three-phase systems, with a variety of different approaches, as presented in major publications on this subject in Refs. [5-10], provided the power equation only in the form of equation (1). Ref. [11] is a good source for studying various approaches to defining powers in unbalanced systems.

When a power equation is written in the form of eqn. (1) then effects of the phase-shift and the load imbalance upon apparent power S are not separated. Observe that the term denoted by "Q" in this equation is usually interpreted as the reactive power, but it has a non-zero value even for a purely resistive unbalanced load. Also it cannot be measured by a reactive power meter connected at terminals of such a load.

This is a major deficiency of the power equation in the form of eqn. (1), especially since it occurs even if voltages and currents are sinusoidal. Therefore, it is hard to imagine, that such equation could be a starting point for developing a power equation when these quantities are nonsinusoidal.

All that was said above applies to three-phase, three-wire systems, but such systems are a sub-set of three-phase systems with a neutral conductor.

The lack of a right definition of the apparent power for three-phase systems is one of the main reasons of eqn. (1) deficiency. According to Refs. [3, 4, 12], this definition can have one two following forms, namely

$$S = S_A = U_R I_R + U_S I_S + U_T I_T \quad (2)$$

known as an arithmetic apparent power, or

$$S = S_G = \sqrt{P^2 + Q^2} \quad (3)$$

known as a geometrical apparent power.

The issue of selecting a right definition of the apparent power S for three-phase, three-wire systems was solved in Ref. [13]. This selection was based on the answer to the following question asked in that reference: *"which apparent power definition provides the power factor $\lambda = P/S$ value that indeed characterizes the energy loss at its delivery?"*.

That study revealed that not definitions (2) and (3), but the definition suggested by Buchholz in Ref. [14]

$$S = S_B = \sqrt{U_R^2 + U_S^2 + U_T^2}\,\sqrt{I_R^2 + I_S^2 + I_T^2} \tag{4}$$

is the right definition of the apparent power in three-wire systems with sinusoidal voltages and currents. The arithmetic and geometric definitions of the apparent power provide the power factor values which do not specify the energy loss at its delivery in unbalanced systems correctly.

The selection of the definition of the apparent power, presented in Ref. [13], applies only to three-phase systems without a neutral conductor (3p systems), however. Since such systems are a sub-set of the set of 3pN systems, a definition that is not right in 3p systems cannot be right in 3pN systems. Consequently, the apparent power in 3pN unbalanced systems cannot be calculated according to the arithmetic or geometric definitions. Only definition (4) can be regarded as a "starting point" for its generalization for 3pN systems. Having a right definition of the apparent power, power properties of 3pN systems with unbalanced loads can be investigated and a right power equation of such systems can be developed. This is the subject of this paper.

Current three-phase rms value

Single-phase loads are supplied in 3pN systems with line-to-neutral voltages u_R, u_S and u_T. Three-phase equipment is supplied with line-to-line voltages u_{RS}, u_{ST} and u_{TR}. The star point of three-phase equipment may or may not be connected to the neutral conductor.

Unlike 3p systems, where the sum of line currents is equal to zero, in 3pN systems

$$i_R(t) + i_S(t) + i_T(t) = i_N(t).$$

Consequently, some definitions of power related quantities in 3pN systems could differ from comparable definitions for 3p systems.

Three-phase transmission equipment in three-wire systems is built to keep mutual symmetry of individual lines as much as possible. Therefore, it can be assumed that their impedances are mutually equal. In the case of equipment in systems with a neutral conductor, its impedance could be substantially different from the line impedance, however.

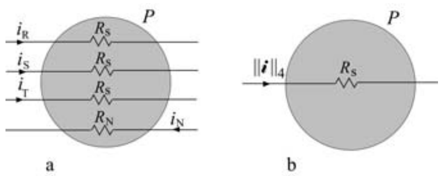

Fig. 2. (a) Three-phase four-wire device and (b) single-phase device, equivalent as to the active power.

Assuming that phase and neutral conductor resistances of a three-phase device, shown in Fig. 2, are, respectively, R_S and R_N, the active power of this device is equal to

$$P = R_S(||i_R||^2 + ||i_S||^2 + ||i_T||^2) + R_N ||i_N||^2$$

or

$$P = R_S ||\boldsymbol{i}||_3^2 + R_N ||i_N||^2.$$

Symbol $\boldsymbol{i}$ in this formula denotes a three-phase vector of line currents i_R, i_S and i_T. i.e., $\boldsymbol{i} = [i_R, i_S, i_T]^T$. The first term on the right side specifies the active power of the single-phase device with resistance R_s and line current I equal to the three-phase rms value, i.e., $I = ||\boldsymbol{i}||_3$

$$P = ||\boldsymbol{i}||_3^2 R_S .$$

Devices in Fig. 2a and b are mutually equivalent as to the active power P, if

$$||\boldsymbol{i}||_4^2 R_S = ||\boldsymbol{i}||_3^2 R_S + ||i_N||^2 R_N$$

hence

$$||\boldsymbol{i}||_4 = ||\boldsymbol{i}||_3 \sqrt{1 + \frac{R_N}{R_S}\left(\frac{||i_N||}{||\boldsymbol{i}||_3}\right)^2}\ . \tag{5}$$

This is the three-phase rms value of a three-phase current of a four-wire device. Formula (5) takes into account a fact that resistance R_N of the neutral conductor can differ from phase resistances R_s. This difference can be particularly visible for three-phase transformers. According to formula (5) the current three-phase rms value $||\boldsymbol{i}||_4$ cannot be calculated without information on the device resistance asymmetry, meaning the ratio R_N/R_s.

If this would not cause any confusion, this value will be denoted in this paper without index "4", meaning

$$||\boldsymbol{i}|| \overset{\text{df}}{=} ||\boldsymbol{i}||_4.$$

Formula (5) was developed without any restrictions as to the current waveform. Therefore, it applies not only to systems with sinusoidal currents, but also to systems with any nonsinusoidal, but periodic currents.

Observe, that when there is not dissipation of energy associated with the presence of a neutral conductor current, meaning $||i_N|| = 0$ or $R_N = 0$, then

$$||\boldsymbol{i}||_4 = ||\boldsymbol{i}||_3 = ||\boldsymbol{i}||. \tag{6}$$

Illustration 1. Let us calculate the supply current rms value $||\boldsymbol{i}||_4$ of a 3pN device shown in Fig. 3, if the line currents are

$$i_R = 50\sqrt{2}\sin\omega_1 t \text{ A}$$
$$i_S = 50\sqrt{2}\sin(\omega_1 t - 120^0) \text{ A}$$
$$i_T = 0$$

assuming that $R_s = \text{Re}\{\boldsymbol{Z}_s\} = 2\ \Omega$ and $R_N = 0.2\ \Omega$.

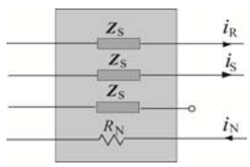

Fig. 3. Example of 3pN device with asymmetrical currents.

The neutral current is equal to

$$i_N = i_R + i_S + i_T = 50\sqrt{2}\sin(\omega_1 t - 60^0)\text{A}.$$

The rms value $||\boldsymbol{i}||_3$ is equal to

$$||\boldsymbol{i}||_3 = \sqrt{||i_R||^2 + ||i_S||^2 + ||i_T||^2} = \sqrt{50^2 + 50^2} = 70.7 \text{ A}$$

hence, according to formula (5), the three-phase rms value of the load current is

$$||\boldsymbol{i}||_4 = ||\boldsymbol{i}||_3 \sqrt{1 + \frac{R_N}{R_S}\left(\frac{||i_N||}{||\boldsymbol{i}||_3}\right)^2} = 70.7\sqrt{1 + \frac{0.2}{2}\left(\frac{50}{70.7}\right)^2} = 72.4 \text{ A}.$$

Thus, resistance of the neutral conductor contributes to the current three-phase rms value increase.

Voltages in 3pN systems

Single-phase loads in 3pN systems are supplied with line-to-neutral voltages. Line voltages in three-phase systems are referenced sometimes, or even measured with respect to different reference points, however. These could be a ground or artificial zero, as shown in Fig. 4. The choice of the reference point does not affect, of course, the energy flow in the system. Nonetheless, this choice of reference point can cause some confusion. To avoid it, this point should be distinctively specified and information on the reference even included in the voltage symbol as it was done in Fig. 4.

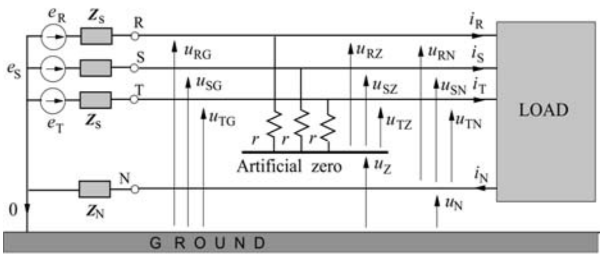

Fig. 4. Voltages in 3pN system.

It is assumed that internal voltages e_R, e_S and e_T, of the supply distribution system are mutually symmetrical of the positive sequence, meaning

$$e_S(t) = e_R(t - \frac{T}{3}), \quad e_T(t) = e_R(t - 2\frac{T}{3}) = e_R(t + \frac{T}{3})$$

and do not contain any DC component.

The line voltages measured with respect to the ground, u_{RG}, u_{SG} and u_{TG}, are reduced with respect to the internal voltages e_R, e_S and e_T of the distribution system by the voltage drop on the internal impedance, $\boldsymbol{Z}_s$, of the system. Because of load imbalance, load currents are, in general, asymmetrical, causing the voltages u_{RG}, u_{SG} and u_{TG} to also exhibit some level of asymmetry. These voltages are not applied to the load, however. The load is supplied by line-to-neutral voltages u_{RN}, u_{SN} and u_{TN}, reduced with respect to voltages u_{RG}, u_{SG} and u_{TG} by the voltage u_N, meaning by the voltage drop of the neutral current i_N on the neutral impedance $\boldsymbol{Z}_N$. For example, this voltage at R terminal is equal to

$$(7) \qquad \boldsymbol{U}_{RN} = \boldsymbol{E}_R - \boldsymbol{Z}_s \boldsymbol{I}_R - \boldsymbol{Z}_N \boldsymbol{I}_N .$$

Thus, individual line-to-neutral voltages u_{RN}, u_{SN} and u_{TN} are effected by the neutral current i_N, i.e., and consequently, by each of line currents i_R, i_S and i_T separately. It means that a single-phase load supplied, for example, from line S can affect the load supplied from line T and opposite. The higher the neutral conductor impedance Z_N the stronger interaction. Therefore, to reduce this interaction, 3pN systems are built to have the neutral conductor impedance Z_N as low as possible.

Energy conversion in the load and consequently, the load powers, are dependent on the line-to-neutral voltages u_{RN}, u_{SN} and u_{TN} and power properties of 3pN loads will be analyzed just at these supply voltages. Therefore, to simplify mathematical formulae, their symbols will be reduced in the remaining part of this paper to u_R, u_S and u_T.

Although the supply voltage in 3pN systems could be asymmetrical, the asymmetry of the supply current, caused by the load imbalance, is usually much higher, and it has a much greater effect on the condition of energy delivery than the voltage asymmetry. This paper focuses only on effects of the load imbalance on power properties of 3pN systems. Therefore, it is assumed that loads are supplied from an ideal source of a symmetrical and sinusoidal voltage. The three-phase rms value of such a voltage vector $\boldsymbol{u}$ can be defined [15] in the same way as for three-wire systems, meaning

$$(8) \qquad \|\boldsymbol{u}\|_4 = \|\boldsymbol{u}\|_3 = \|\boldsymbol{u}\| = \sqrt{\|u_R\|^2 + \|u_S\|^2 + \|u_T\|^2} = \sqrt{3}\,U_R .$$

Apparent power

As suggested in Ref. [15], the apparent power in three-phase systems should be defined as the product of three-phase rms values of the supply voltage and currents. This suggestion was later supported by results of analysis of various definitions of the apparent power presented in Ref. [13]. According to that suggestion, the apparent power in 3pN systems with an ideal source of symmetrical voltage, should be defined as

$$(9) \qquad S \overset{\text{df}}{=} \|\boldsymbol{u}\|_4 \|\boldsymbol{i}\|_4 = \|\boldsymbol{u}\|_3 \|\boldsymbol{i}\|_3 = \|\boldsymbol{u}\|\,\|\boldsymbol{i}\|.$$

Thus, at conditions as assumed above, there is no differrence between apparent power definition for 3p and 3pN systems.

Currents' Physical Components (CPC) in 3pN systems with LTI loads

The basic circuit for analyzing power phenomena in 3pN systems is a circuit with a linear, time-invariant (LTI) load supplied with sinusoidal and symmetrical voltage of the positive sequence from an ideal source.

Any three-phase LTI load configured in star (Y), shown in Fig. 5a, with an ideal (meaning with zero impedance) neutral conductor, is equivalent with respect to the active power P to a balanced resistive load, shown in Fig. 5b.

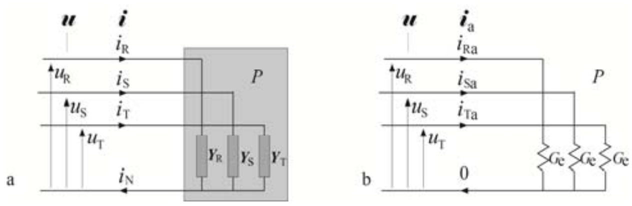

Fig. 5 (a) LTI load configured in Y and (b) its resistive balanced load equivalent as to the active power P.

Since the active power of the original load is

$$(10) \qquad P = \mathrm{Re}\{\boldsymbol{Y}_R^* + \boldsymbol{Y}_S^* + \boldsymbol{Y}_T^*\} U_R^2 = (G_R + G_S + G_T) U_R^2$$

thus, conductance G_e of a resistive balanced load, equivalent with respect to the active power is equal to

$$(11) \qquad G_e = \frac{P}{\|\boldsymbol{u}\|^2} = \frac{P}{3U_R^2} = \frac{1}{3}(G_R + G_S + G_T)$$

it will be referred to as an ***equivalent conductance*** of a load supplied with 3pN line. Such an equivalent resistive load draws a current, which is in-phase with the supply voltage $\boldsymbol{u}$ and can be regarded as the ***active current*** of the load, namely

(12) $$\boldsymbol{i}_{\rm a}(t) = G_{\rm e}\, \boldsymbol{u}(t).$$

It is the current of the lowest rms value such that the load at voltage $\boldsymbol{u}$ has active power P.

The original LTI load is equivalent with respect to the reactive power Q to a balanced reactive load, shown in Fig. 6b, of susceptance $B_{\rm e}$.

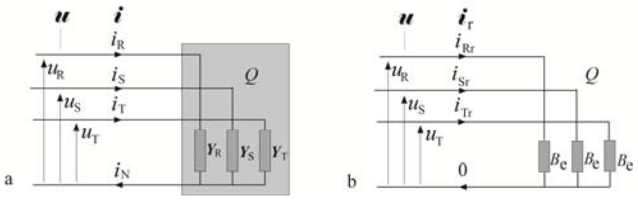

Fig. 6 (a) LTI load configured in Y and (b) its reactive balanced load equivalent as to the reactive power Q.

The reactive power Q of the original load is

(13) $$Q = {\rm Im}\{\boldsymbol{Y}_{\rm R}^* + \boldsymbol{Y}_{\rm S}^* + \boldsymbol{Y}_{\rm T}^*\}U_{\rm R}^2 = -(B_{\rm R} + B_{\rm S} + B_{\rm T})U_{\rm R}^2$$

thus, the susceptance of a reactive balanced load, equivalent to the original load with respect to the reactive power Q is equal to

(14) $$B_{\rm e} = -\frac{Q}{\|\boldsymbol{u}\|^2} = -\frac{1}{3}\frac{Q}{U_{\rm R}^2} = \frac{1}{3}(B_{\rm R} + B_{\rm S} + B_{\rm T}).$$

It will be referred to as the ***equivalent susceptance*** of loads supplied with 3pN lines. Such a reactive load draws a ***reactive current***

(15) $$\boldsymbol{i}_{\rm r} = B_{\rm e}\frac{d}{d(\omega_1 t)}\boldsymbol{u} = \sqrt{2}\,{\rm Re}\{\, jB_{\rm e}\begin{bmatrix}\boldsymbol{U}_{\rm R}\\ \boldsymbol{U}_{\rm S}\\ \boldsymbol{U}_{\rm R}\end{bmatrix} e^{j\omega_1 t}\}.$$

It is a symmetrical current of the same sequence as the supply voltage, meaning of the positive sequence.

The equivalent conductance and the equivalent susceptance can be combined together to form the ***equivalent admittance***

(16) $$\boldsymbol{Y}_{\rm e} \stackrel{\rm df}{=} G_{\rm e} + jB_{\rm e} = \frac{1}{3}(\boldsymbol{Y}_{\rm R} + \boldsymbol{Y}_{\rm S} + \boldsymbol{Y}_{\rm T}).$$

The residual component of the current occurs due to the load imbalance and is equal to

(17) $$\boldsymbol{i}_{\rm u} = \boldsymbol{i} - \boldsymbol{i}_{\rm a} - \boldsymbol{i}_{\rm r} = \sqrt{2}{\rm Re}\{\begin{bmatrix}(\boldsymbol{Y}_{\rm R} - G_{\rm e} - jB_{\rm e})\boldsymbol{U}_{\rm R}\\ (\boldsymbol{Y}_{\rm S} - G_{\rm e} - jB_{\rm e})\boldsymbol{U}_{\rm S}\\ (\boldsymbol{Y}_{\rm T} - G_{\rm e} - jB_{\rm e})\boldsymbol{U}_{\rm T}\end{bmatrix} e^{j\omega_1 t}\} =$$
$$= \sqrt{2}{\rm Re}\{\begin{bmatrix}(\boldsymbol{Y}_{\rm R} - G_{\rm e} - jB_{\rm e})\\ (\boldsymbol{Y}_{\rm S} - G_{\rm e} - jB_{\rm e})\alpha^*\\ (\boldsymbol{Y}_{\rm T} - G_{\rm e} - jB_{\rm e})\alpha\end{bmatrix}\boldsymbol{U}_{\rm R}\, e^{j\omega_1 t}\} \stackrel{\rm df}{=} \sqrt{2}{\rm Re}\{\begin{bmatrix}\boldsymbol{I}_{\rm Ru}\\ \boldsymbol{I}_{\rm Su}\\ \boldsymbol{I}_{\rm Tu}\end{bmatrix} e^{j\omega_1 t}\}.$$

The physical nature of the residual current $\boldsymbol{i}_{\rm u}$ is not clear at this moment. We can only say that this current is associated neither with the active nor with the reactive power. Let us calculate the crms value of the symmetrical component of the positive sequence of this current. It is equal to

(18) $$\boldsymbol{I}_{\rm u}^{\rm p} = \frac{1}{3}(\boldsymbol{I}_{\rm Ru} + \alpha\boldsymbol{I}_{\rm Su} + \alpha^*\boldsymbol{I}_{\rm Tu}) =$$
$$= \frac{1}{3}[(\boldsymbol{Y}_{\rm R} - G_{\rm e} - jB_{\rm e}) + \alpha(\boldsymbol{Y}_{\rm S} - G_{\rm e} - jB_{\rm e})\alpha^* + \alpha^*(\boldsymbol{Y}_{\rm T} - G_{\rm e} - jB_{\rm e})\alpha]\boldsymbol{U}_{\rm R} =$$
$$= \frac{1}{3}[(\boldsymbol{Y}_{\rm R} + \boldsymbol{Y}_{\rm S} + \boldsymbol{Y}_{\rm T}) - 3G_{\rm e} - j3B_{\rm e}]\boldsymbol{U}_{\rm R} = 0.$$

Thus, this current does not contain any component of the positive sequence, meaning it occurs due to the supply current asymmetry.

The crms value of the negative sequence component of this current is equal to

(19) $$\boldsymbol{I}_{\rm u}^{\rm n} = \frac{1}{3}(\boldsymbol{I}_{\rm Ru} + \alpha^*\boldsymbol{I}_{\rm Su} + \alpha\boldsymbol{I}_{\rm Tu}) =$$
$$= \frac{1}{3}[(\boldsymbol{Y}_{\rm R} - G_{\rm e} - jB_{\rm e}) + \alpha^*(\boldsymbol{Y}_{\rm S} - G_{\rm e} - jB_{\rm e})\alpha^* + \alpha(\boldsymbol{Y}_{\rm T} - G_{\rm e} - jB_{\rm e})\alpha]\boldsymbol{U}_{\rm R} =$$
$$= \frac{1}{3}(\boldsymbol{Y}_{\rm R} + \alpha\boldsymbol{Y}_{\rm S} + \alpha^*\boldsymbol{Y}_{\rm T})\,\boldsymbol{U}_{\rm R} \stackrel{\rm df}{=} \boldsymbol{A}^{\rm n}\boldsymbol{U}_{\rm R}$$

where

(20) $$\boldsymbol{A}^{\rm n} \stackrel{\rm df}{=} \frac{1}{3}(\boldsymbol{Y}_{\rm R} + \alpha\boldsymbol{Y}_{\rm S} + \alpha^*\boldsymbol{Y}_{\rm T})\,.$$

The crms value of the zero sequence component of the residual current is equal to

(21) $$\boldsymbol{I}_{\rm u}^{\rm z} = \frac{1}{3}(\boldsymbol{I}_{\rm Ru} + \boldsymbol{I}_{\rm Su} + \boldsymbol{I}_{\rm Tu}) =$$
$$= \frac{1}{3}[(\boldsymbol{Y}_{\rm R} - G_{\rm e} - jB_{\rm e}) + (\boldsymbol{Y}_{\rm S} - G_{\rm e} - jB_{\rm e})\alpha^* + (\boldsymbol{Y}_{\rm T} - G_{\rm e} - jB_{\rm e})\alpha]\boldsymbol{U}_{\rm R} =$$
$$= \frac{1}{3}(\boldsymbol{Y}_{\rm R} + \alpha^*\boldsymbol{Y}_{\rm S} + \alpha\boldsymbol{Y}_{\rm T})\,\boldsymbol{U}_{\rm R} \stackrel{\rm df}{=} \boldsymbol{A}^{\rm z}\boldsymbol{U}_{\rm R}$$

where

(22) $$\boldsymbol{A}^{\rm z} \stackrel{\rm df}{=} \frac{1}{3}(\boldsymbol{Y}_{\rm R} + \alpha^*\boldsymbol{Y}_{\rm S} + \alpha\boldsymbol{Y}_{\rm T}).$$

When phase-to-neutral admittances $\boldsymbol{Y}_{\rm R}$, $\boldsymbol{Y}_{\rm S}$ and $\boldsymbol{Y}_{\rm T}$ are mutually equal, meaning the load is balanced, then admitances $\boldsymbol{A}^{\rm n}$ and $\boldsymbol{A}^{\rm z}$ are equal to zero and consequently, the supply current does not contain $\boldsymbol{i}_{\rm u}$ component. It occurs only when the load is unbalanced. It means that the current $\boldsymbol{i}_{\rm u}$ stands for the unbalanced current. It is composed of the negative and positive sequence components,

(23) $$\boldsymbol{i}_{\rm u} = \boldsymbol{i}_{\rm u}^{\rm n} + \boldsymbol{i}_{\rm u}^{\rm z}\,,$$

where

(24) $$\boldsymbol{i}_{\rm u}^{\rm n} \stackrel{\rm df}{=} \sqrt{2}{\rm Re}\{\begin{bmatrix}\boldsymbol{I}_{\rm R}^{\rm n}\\ \boldsymbol{I}_{\rm S}^{\rm n}\\ \boldsymbol{I}_{\rm T}^{\rm n}\end{bmatrix} e^{j\omega_1 t}\} = \sqrt{2}{\rm Re}\{\begin{bmatrix}\boldsymbol{A}^{\rm n}\boldsymbol{U}_{\rm R}\\ \boldsymbol{A}^{\rm n}\boldsymbol{U}_{\rm T}\\ \boldsymbol{A}^{\rm n}\boldsymbol{U}_{\rm S}\end{bmatrix} e^{j\omega_1 t}\} =$$
$$= \sqrt{2}{\rm Re}\{\boldsymbol{A}^{\rm n}\boldsymbol{U}^{\#} e^{j\omega_1 t}\}$$

(25) $$\boldsymbol{i}_{\rm u}^{\rm z} \stackrel{\rm df}{=} \sqrt{2}{\rm Re}\{\begin{bmatrix}\boldsymbol{I}_{\rm R}^{\rm z}\\ \boldsymbol{I}_{\rm S}^{\rm z}\\ \boldsymbol{I}_{\rm T}^{\rm z}\end{bmatrix} e^{j\omega_1 t}\} = \sqrt{2}{\rm Re}\{\begin{bmatrix}\boldsymbol{A}^{\rm z}\boldsymbol{U}_{\rm R}\\ \boldsymbol{A}^{\rm z}\boldsymbol{U}_{\rm R}\\ \boldsymbol{A}^{\rm z}\boldsymbol{U}_{\rm R}\end{bmatrix} e^{j\omega_1 t}\} =$$
$$= \sqrt{2}{\rm Re}\{\boldsymbol{A}^{\rm z}\boldsymbol{U}_{\rm R} e^{j\omega_1 t}\}.$$

The symbol $\boldsymbol{U}^{\#}$ in formula (26) denotes a vector of crms values of line-to-neutral voltages with switched $\boldsymbol{U}_{\rm S}$ and $\boldsymbol{U}_{\rm T}$ entries, while $\boldsymbol{U}_{\rm R}$ in formula (27) denotes vector of the same crms values $\boldsymbol{U}_{\rm R}$.

These two currents will be called ***negative*** and ***zero sequence unbalanced currents*** of a LTI load, respectively, and consequently, the complex number $\boldsymbol{A}^{\rm n}$ will be called ***a negative sequence unbalanced admittance,*** while $\boldsymbol{A}^{\rm z}$ will be called ***a zero sequence unbalanced admittance*** of the load.

Unbalanced admittances $\boldsymbol{A}^{\rm n}$ and $\boldsymbol{A}^{\rm z}$ are equal to zero when admittances $\boldsymbol{Y}_{\rm R}$, $\boldsymbol{Y}_{\rm S}$ and $\boldsymbol{Y}_{\rm T}$ are mutually equal, but this is only a sufficient, but not a necessary condition to have zero unbalanced admittances. They can be zero even if

$$\boldsymbol{Y}_{\rm R} \neq \boldsymbol{Y}_{\rm S} \neq \boldsymbol{Y}_{\rm T}$$

but in such a situation only one of them $\boldsymbol{A}^{\rm n}$ or $\boldsymbol{A}^{\rm z}$ can be equal to zero. When the load is unbalanced then the supply current has to contain at least one of two unbalanced currents $\boldsymbol{i}_{\rm u}^{\rm n}$, or $\boldsymbol{i}_{\rm u}^{\rm z}$.

With formula (23) the current decomposition expressed with formula (17) can be rewritten as

$$\boldsymbol{i} = \boldsymbol{i}_{\rm a} + \boldsymbol{i}_{\rm r} + \boldsymbol{i}_{\rm u}^{\rm n} + \boldsymbol{i}_{\rm u}^{\rm z}. \tag{26}$$

Currents $\boldsymbol{i}_{\rm a}$, $\boldsymbol{i}_{\rm r}$, $\boldsymbol{i}_{\rm u}^{\rm n}$ and $\boldsymbol{i}_{\rm u}^{\rm z}$ in this decomposition can be regarded as the Currents' Physical Components, CPC, of three-phase LTI loads with neutral conductor, supplied from a source of symmetrical sinusoidal voltage. Physical interpretation of the active and reactive currents, $\boldsymbol{i}_{\rm a}$, and $\boldsymbol{i}_{\rm r}$, is exactly the same as in 3p systems. The ***active current*** $\boldsymbol{i}_{\rm a}$ is associated distinctively with the phenomenon of permanent energy transmission and consequently, with the load active power P. The ***reactive current*** $\boldsymbol{i}_{\rm r}$ is associated distinctively with the phenomenon of the phase-shift between the supply voltage and current and consequently, with the load reactive power Q. The ***negative sequence unbalanced current*** $\boldsymbol{i}_{\rm u}^{\rm n}$ is an effect of the supply current asymmetry due to the load imbalance, but it does not require any neutral conductor for its presence. The ***zero sequence unbalanced current*** $\boldsymbol{i}_{\rm u}^{\rm z}$ is also an effect of the supply current asymmetry caused by the load imbalance, but it cannot occur in the supply current if the load is not equipped with the neutral conductor.

One should observe that in spite of the adjective '*physical*', Currents' Physical Components do not exist as physical quantities. They are nothing other than mathematical entities, obtained by a specific decomposition of the supply current. These entities are **only associated** with distinctive physical phenomena in the load. For example, when a load causes a phase-shift between the supply voltage and the load current, then the supply current can be decomposed into components, such that one of them is the reactive current. There are an infinite number of different decompositions of the same current, without any reactive component, however.

Equivalent circuit of 3pN loads

Decomposition (26) means that a LTI load supplied with 3pN line from a source of sinusoidal and symmetrical voltage has an equivalent circuit composed of four parallel circuits that draw individual $\boldsymbol{i}_{\rm a}, \boldsymbol{i}_{\rm r}, \boldsymbol{i}_{\rm u}^{\rm n}$ and $\boldsymbol{i}_{\rm u}^{\rm z}$ currents, as shown in Fig 7.

The crms value of the negative sequence component $\boldsymbol{i}_{\rm u}^{\rm n}$ of the supply current in line S in the equivalent circuit shown in Fig. 7 is

$$\boldsymbol{I}_{\rm Su}^{\rm n} = \boldsymbol{A}^{\rm n}\boldsymbol{U}_{\rm T} = \boldsymbol{A}^{\rm n}\alpha^{*}\boldsymbol{U}_{\rm S} = (\alpha^{*}\boldsymbol{A}^{\rm n})\,\boldsymbol{U}_{\rm S} \tag{27}$$

and similarly, in line T

$$\boldsymbol{I}_{\rm Tu}^{\rm n} = \boldsymbol{A}^{\rm n}\boldsymbol{U}_{\rm S} = \boldsymbol{A}^{\rm n}\alpha\boldsymbol{U}_{\rm T} = (\alpha\boldsymbol{A}^{\rm n})\,\boldsymbol{U}_{\rm T}\,. \tag{28}$$

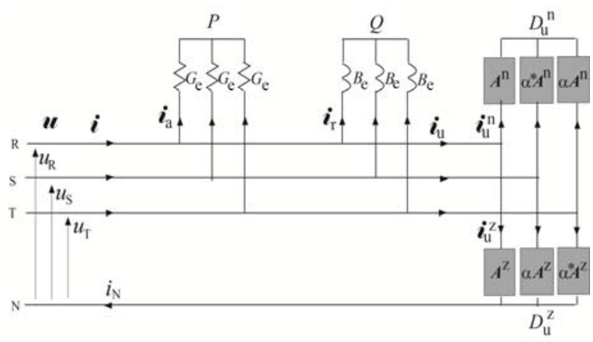

Fig. 7. Equivalent circuit of LTI load supplied from 3pN line with symmetrical sinusoidal voltage.

The crms value of the zero sequence component $\boldsymbol{i}_{\rm u}^{\rm z}$ of the current in line S in the circuit, shown in Fig. 7 is

$$\boldsymbol{I}_{\rm Su}^{\rm z} = \boldsymbol{A}^{\rm z}\boldsymbol{U}_{\rm R} = \boldsymbol{A}^{\rm z}\alpha\,\boldsymbol{U}_{\rm S} = (\alpha\boldsymbol{A}^{\rm z})\,\boldsymbol{U}_{\rm S} \tag{29}$$

and similarly, in line T

$$\boldsymbol{I}_{\rm Tu}^{\rm z} = \boldsymbol{A}^{\rm z}\boldsymbol{U}_{\rm R} = \boldsymbol{A}^{\rm z}\alpha^{*}\,\boldsymbol{U}_{\rm T} = (\alpha^{*}\boldsymbol{A}^{\rm z})\,\boldsymbol{U}_{\rm T}\,. \tag{30}$$

The four physical components of the supply current are mutually orthogonal hence their rms values fulfill the relationship

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\rm a}||^2 + ||\boldsymbol{i}_{\rm r}||^2 + ||\boldsymbol{i}_{\rm u}^{\rm n}||^2 + ||\boldsymbol{i}_{\rm u}^{\rm z}||^2. \tag{31}$$

Orthogonality of unbalanced currents $\boldsymbol{i}_{\rm u}^{\rm n}$ and $\boldsymbol{i}_{\rm u}^{\rm z}$ between themselves and orthogonality to other components result from differences in their sequence. Current $\boldsymbol{i}_{\rm u}^{\rm n}$ is of negative sequence; current $\boldsymbol{i}_{\rm u}^{\rm z}$ is of zero sequence, while currents $\boldsymbol{i}_{\rm a}$ and $\boldsymbol{i}_{\rm r}$ are of positive sequence. The relationship (31) is illustrated in Fig. 8.

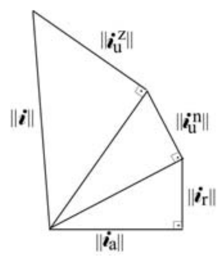

Fig. 8. Diagram of three-phase rms values of CPC.

Three-phase rms values of particular components are equal to

$$||\boldsymbol{i}_{\rm a}|| = G_{\rm e}\,||\boldsymbol{u}|| \tag{32}$$

$$||\boldsymbol{i}_{\rm r}|| = |B_{\rm e}|\,||\boldsymbol{u}|| \tag{33}$$

$$||\boldsymbol{i}_{\rm u}^{\rm n}|| = A^{\rm n}\,||\boldsymbol{u}|| \tag{34}$$

$$||\boldsymbol{i}_{\rm u}^{\rm z}|| = A^{\rm z}\,||\boldsymbol{u}||. \tag{35}$$

The active and reactive currents $\boldsymbol{i}_{\rm a}$ and $\boldsymbol{i}_{\rm r}$ are symmetrical of the positive sequence, while the unbalanced current $\boldsymbol{i}_{\rm u}^{\rm n}$ is symmetrical of the negative sequence. Thus, these three currents in supply lines R, S and T add up to zero, not contributing to the neutral current. The neutral current $i_{\rm N}$ is an effect of the presence of the zero sequence component of the unbalanced current, namely

$$i_{\rm N} = 3 i^{\rm z}_{\rm Ru} = 3\sqrt{2}\,{\rm Re}\{\boldsymbol{I}^{\rm z}_{\rm u}\, e^{j\omega_1 t}\} = 3\sqrt{2}\,{\rm Re}\{\boldsymbol{A}^{\rm z}\boldsymbol{U}_{\rm R}\, e^{j\omega_1 t}\} \tag{36}$$

and its rms value is equal to

$$||i_{\rm N}|| = 3A^{\rm z}||\boldsymbol{u}||. \tag{37}$$

Illustration 2. Let us calculate the rms values of the supply Current Physical Components for the load shown in Fig. 9, assuming that the supply voltage is symmetrical and the rms value of the voltage at terminal R is $U_{\rm R}$ = 120 V.

The line-to-neutral admittances of the load are equal to

$$\boldsymbol{Y}_{\rm R} = \frac{1}{0.5 + j0.87} = 0.5 - j0.87\ {\rm S},\quad \boldsymbol{Y}_{\rm S} = 1\ {\rm S},\quad \boldsymbol{Y}_{\rm T} = 0$$

thus, the equivalent admittance of the load equals to

$$\boldsymbol{Y}_{\rm e} = G_{\rm e} + jB_{\rm e} = \frac{1}{3}(0.5 - j0.87 + 1) = 0.50 - j0.29\ {\rm S}.$$

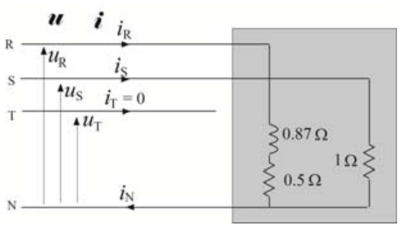

Fig. 9. Example of an unbalanced load supplied with 3pN line.

The negative sequence unbalanced admittance is equal to zero, since

$$\boldsymbol{A}^{\rm n} = \frac{1}{3}(\boldsymbol{Y}_{\rm R} + \alpha\boldsymbol{Y}_{\rm S} + \alpha^*\boldsymbol{Y}_{\rm T}) = \frac{1}{3}(0.5 - j0.87 + 1e^{j120^0}\times 1) = 0$$

while the zero sequence unbalanced admittance is equal to

$$\boldsymbol{A}^{\rm z} = \frac{1}{3}(\boldsymbol{Y}_{\rm R} + \alpha^*\boldsymbol{Y}_{\rm S} + \alpha\boldsymbol{Y}_{\rm T}) = \frac{1}{3}(0.5 - j0.87 + 1e^{-j120^0}\times 1) =$$
$$= 0.58e^{-j90^0}{\rm S}.$$

Thus, the supply current contains $\boldsymbol{i}^{\rm z}_{\rm u}$, but not $\boldsymbol{i}^{\rm n}_{\rm u}$ component. Since the three-phase rms value of the supply voltage is

$$||\boldsymbol{u}|| = \sqrt{3}\,U_{\rm R} = \sqrt{3}\times 120 = 207.8\ {\rm V}$$

three-phase rms values of the supply current physical components are equal to

$$||\boldsymbol{i}_{\rm a}|| = G_{\rm e}||\boldsymbol{u}|| = 0.50\times 207.8 = 103.9\ {\rm A}$$

$$||\boldsymbol{i}_{\rm r}|| = |B_{\rm e}|\,||\boldsymbol{u}|| = 0.29\times 207.8 = 60.3\ {\rm A}$$

$$||\boldsymbol{i}^{\rm n}_{\rm u}|| = A^{\rm n}||\boldsymbol{u}|| = 0$$

$$||\boldsymbol{i}^{\rm z}_{\rm u}|| = A^{\rm z}||\boldsymbol{u}|| = 0.58\times 207.8 = 120.5\ {\rm A}.$$

The crms value of the zero sequence unbalanced current in line R, $i^{\rm z}_{\rm Ru}$, is equal to

$$\boldsymbol{I}^{\rm z}_{\rm Rn} = \boldsymbol{A}^{\rm z}\boldsymbol{U}_{\rm R} = 0.58e^{-j90^0}\times 120 = 69.6e^{-j90^0}\ {\rm A}.$$

The zero sequence currents in lines S and T, of course, have the same crms value. The crms value of the neutral current is

$$\boldsymbol{I}_{\rm N} = 3\boldsymbol{I}^{\rm z}_{\rm Rn} = 3\times 69.6e^{-j90^0} = 208.8e^{-j90^0}\ {\rm A}.$$

Powers of 3pN loads

Decomposition of the supply current of an LTI load supplied with symmetrical sinusoidal voltage of the positive sequence into the CPCs, leads directly to the power equation of such a load. Multiplying eqn. (31) by the supply voltage $\boldsymbol{u}$ three-phase rms value $||\boldsymbol{u}||$, we obtain

$$S^2 = P^2 + Q^2 + D^{\rm n\,2}_{\rm u} + D^{\rm z\,2}_{\rm u} \tag{38}$$

with

$$P = ||\boldsymbol{i}_{\rm a}||\,||\boldsymbol{u}|| = G_{\rm e}||\boldsymbol{u}||^2 \tag{39}$$

$$Q \stackrel{\rm df}{=} \pm||\boldsymbol{i}_{\rm r}||\,||\boldsymbol{u}|| = -B_{\rm e}||\boldsymbol{u}||^2 \tag{40}$$

$$D^{\rm n}_{\rm u} \stackrel{\rm df}{=} ||\boldsymbol{i}^{\rm n}_{\rm u}||\,||\boldsymbol{u}|| = A^{\rm n}||\boldsymbol{u}||^2 \tag{41}$$

$$D^{\rm z}_{\rm u} \stackrel{\rm df}{=} ||\boldsymbol{i}^{\rm z}_{\rm u}||\,||\boldsymbol{u}|| = A^{\rm z}||\boldsymbol{u}||^2. \tag{42}$$

This power equation contains two new power quantities, $D^{\rm n}_{\rm u}$ and $D^{\rm z}_{\rm u}$. These two powers are associated with the presence of the negative and zero sequence unbalanced components in the supply current. Therefore, they will be called ***negative sequence unbalanced power*** and ***zero sequence unbalanced power***, respectively. The power equation is illustrated geometrically with a diagram shown in Fig. 10.

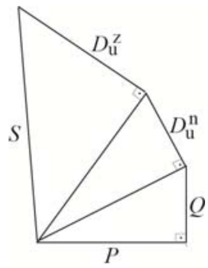

Fig. 10. Diagram of powers of LTI load supplied with a symmetrical sinusoidal voltage with 3pN line.

The power equation (38) describes the relationship between powers of an LTI load supplied by 3pN lines from a symmetrical source of sinusoidal voltages.

Illustration 3. Let us calculate the active, reactive and both unbalanced powers for the unbalanced load shown in Fig. 11, assuming that $U_{\rm R}$ = 120 V.

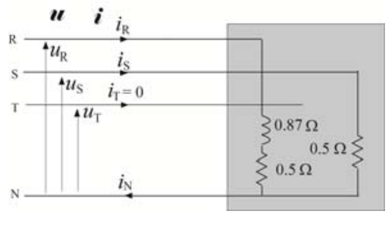

Fig. 11. Example of an unbalanced load.

For such a load, the equivalent admittance is equal to

$$\boldsymbol{Y}_{\rm e} = G_{\rm e} + jB_{\rm e} = \frac{1}{3}(0.50 - j0.87 + 2.0) = 0.83 - j0.29\ {\rm S}.$$

The negative sequence unbalanced admittance is

$$\boldsymbol{A}^{\rm n} = \frac{1}{3}(\boldsymbol{Y}_{\rm R} + \alpha\boldsymbol{Y}_{\rm S} + \alpha^*\boldsymbol{Y}_{\rm T}) = \frac{1}{3}(0.5 - j0.87 + 1e^{-j120^0}\times 0.5) =$$
$$= 0.33e^{-120.1^0}\ {\rm S}$$

while the zero sequence unbalanced admittance has the value

$$A^z = \frac{1}{3}(Y_R + \alpha^* Y_S + \alpha Y_T) = \frac{1}{3}(0.5 - j\,0.87 + 1e^{j120^0} \times 0.5) =$$
$$= 0.88 e^{-j101^0}\ \mathrm{S}.$$

Since

$$||\boldsymbol{u}|| = \sqrt{3}\, U_R = \sqrt{3} \times 120 = 207.8\ \mathrm{V}$$

the particular powers are equal to

$$P = G_e ||\boldsymbol{u}||^2 = 0.83 \times (207.8)^2 = 36.0\ \mathrm{kW}$$

$$Q = -B_e ||\boldsymbol{u}||^2 = 0.29 \times (207.8)^2 = 12.5\ \mathrm{kvar}$$

$$D_u^n = A^n ||\boldsymbol{u}||^2 = 0.33 \times (207.8)^2 = 7.2\ \mathrm{kVA}$$

$$D_u^z = A^z ||\boldsymbol{u}||^2 = 0.88 \times (207.8)^2 = 38.0\ \mathrm{kVA}.$$

Power factor

The power factor of LTI loads supplied with symmetrical sinusoidal voltage in 3pN systems is equal to

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2 + D_u^{n2} + D_u^{z2}}} \tag{43}$$

thus, not only the reactive power Q, but also both unbalanced powers, D_u^n and D_u^z contribute to the load power factor degradation. The power factor can be expressed not only in terms of powers, but also in terms of three-phase rms values of CPCs of the supply current, namely

$$\lambda = \frac{P}{S} = \frac{||\boldsymbol{i}_a||}{||\boldsymbol{i}||} = \frac{||\boldsymbol{i}_a||}{\sqrt{||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u^n||^2 + ||\boldsymbol{i}_u^z||^2}}. \tag{44}$$

Particularly important is the possibility of expressing the power factor in terms of the load parameters, in particular, in terms of the equivalent conductance, G_e, susceptance, B_e and the magnitude of unbalanced admittances A^n and A^z

$$\lambda = \frac{||\boldsymbol{i}_a||}{||\boldsymbol{i}||} = \frac{G_e}{\sqrt{G_e^2 + B_e^2 + A^{n2} + A^{z2}}}. \tag{45}$$

Thus, the power factor of 3pN loads declines from unity value because of non-zero equivalent susceptance B_e of the load, the negative sequence unbalanced admittance A^n and the zero sequence unbalanced admittance A^z. This last formula emphasizes the fact that the power factor depends only on the load properties, but not on voltages, currents or powers. It is defined in terms of the active and apparent powers, but eventually, only the load properties specify the power factor value. Also, in a case of reactive compensation, only a change by means of such a compensator of the parameters as seen by the supply makes the power factor improvement possible.

Conclusions

The paper demonstrates that three-phase systems with a neutral conductor and linear, time invariant loads supplied from a source of sinusoidal and symmetrical voltage can be described, not only phase-by-phase, but as a whole, in terms of active, reactive and two unbalanced powers. The obtained power equation has more power terms than commonly used equation (1), and these powers are associated with distinctive properties of the load.

Equally important is a conclusion that the vector of the supply current $\boldsymbol{i}$ can be decomposed into four mutually orthogonal Physical Components, uniquely associated with distinctive phenomena and properties of the load. These components are specified in terms of their three-phase rms values ||.|| and can be expressed in terms of the load parameters. Thus, the effect of each of these properties, separately, on the energy loss at its delivery can be investigated. Therefore, this paper provides a starting point for studies on reactive compensation in 3pN systems.

More advanced issues such as the effect of waveform distortion, both on the supply side and caused by harmonics generating loads, on the power properties of the system, are not the subject of this paper.

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