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Power Properties of Four-Wire Systems at Nonsinusoidal Supply Voltage

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Abstract **– Powers of three-phase aggregates of single-phase static linear (ASPSL) loads supplied with a nonsinusoidal voltage are the subject of the paper. It is demonstrated in the paper that the supply current of such systems with a nonsinusoidal voltage can be decomposed into six mutually orthogonal components associated with distinctive physical phenomena in the load, two of which do not exist at a sinusoidal voltage. Such decomposition enables evaluation of contribution of these phenomena to the supply current rms value increase and the power factor decline. A power equation and definitions of powers in such systems in the presence of the voltage harmonics are proposed. The paper shows that all powers can be expressed in terms of the load parameters, which can be identified by a measurement at the load terminals.**

Index terms **– Power definitions, unbalanced systems, power theory, Currents' Physical Components, CPC.**

I. INTRODUCTION

Power properties of three-phase four-wire systems composed of aggregates of single-phase static linear (ASPSL) loads, as shown in Fig. 1, with nonsinusoidal supply voltage is the subject of this paper.

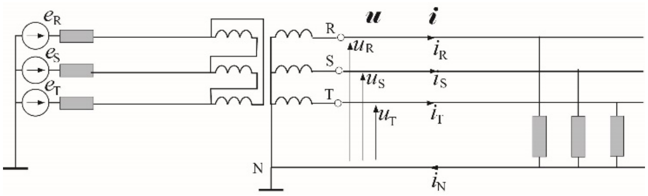

Fig. 1. Diagram of three-phase system with neutral conductor and aggregates of single-phase static linear (ASPSL) loads.

Such systems with sinusoidal voltage were described in terms of powers in [20]. Their generalization to systems with nonsinusoidal voltage might look trivial: nonsinusoidal voltage is only a sum of sinusoidal components. It is not trivial, however. Powers in single-phase linear loads with sinusoidal voltage were known at the end of XIX century. Their generalization for nonsinusoidal voltage has required almost a century long studies. Apart from the active power P, other powers have occurred to be not a sum of powers known in sinusoidal systems. New powers had to be defined at nonsinusoidal voltage. All of these apply to powers of ASPSL loads with nonsinusoidal voltage, which will be discussed in this paper. The load current decomposition and power equation developed in [20] are no longer valid. At nonsinusoidal supply voltage all power quantities defined in [20] have to be re-defined and new ones have to be introduced.

The structure in Fig.1 approximates structures of distribution systems in residential subdivisions or in commercial buildings. Three-phase loads, such as motors, are usually dominated in such systems by aggregates of single-phase loads. This structure would not be appropriate for industrial systems, dominated by three-phase loads: motors or drives, however.

Systems with ASPSL loads are usually much more prone to be unbalanced than industrial power systems. Traction systems, with cart motors supplied from only one line of a three-phase grid, can be strongly unbalanced, thus they could be closer to systems with ASPSL loads, than to industrial systems, however.

Powers in systems with ASPSL loads are often calculated phase-by-phase, meaning by calculating the active, reactive and apparent powers for individual phases, let for R phase:

$$P_R = U_R I_R \cos\varphi_R, \quad Q_R = U_R I_R \sin\varphi_R, \quad S_R = U_R I_R \tag{1}$$

and combining them for the whole three-phase system as follows:

$$P = P_R + P_S + P_T, \quad Q = Q_R + Q_S + Q_T, \quad S = S_R + S_S + S_T \tag{2}$$

or

$$S = \sqrt{P^2 + Q^2}\ . \tag{3}$$

Unfortunately, formulas (2) and (3) in the presence of the line currents asymmetry result, as shown in [13], in different values of the apparent power S and both of them result in a wrong value of the power factor $\lambda = P/S$. Moreover, even formulas (1) for the active, reactive and apparent powers of a single-phase load in the presence of harmonic distortion are not valid.

Attempts aimed at defining powers in three-phase systems with nonsinusoidal voltages and currents have not only long history, which started in thirties of the previous century [3], but are a focus of attention even now [14-20].

There are three "families" of approaches to power definitions in three-phase systems with nonsinusoidal voltages and currents:

(i) ***Time-domain approaches*** based on averaging of electrical quantities over the period of the voltage and current variability [2, 5, 9].

(ii) ***Frequency-domain approaches*** based on the concept of harmonics [3, 4, 7, 10, 19].

(iii) ***Instantaneous approaches*** based on definitions of powers as instantaneous quantities [6, 8, 11, 12, 15].

Power properties in this paper are studied and powers are defined not only for cognitive reasons, but also to provide fundamentals for design of reactive compensators. Such compensators should have specific frequency properties, meaning specific admittances for harmonic frequencies. To provide such fundamentals, the power properties of the load should be specified in the frequency-domain. Therefore, studies in this paper belong to (ii) family, i.e., they are based on the frequency-domain approach.

Apart from the adopted system structure, in Fig. 1, there is also other major assumption in this paper. Namely, it is assumed that loads are stationary, linear, time-invariant, meaning that loads do not generate current harmonics. In fact, there are many harmonics generating loads (HGL) both in residential grids and in commercial buildings. Consequently, the system studied in the paper differs substantially from real systems. The reason for such an assumption is explained below.

The authors' previous paper [20] provides an explanation of power properties and definitions of powers of three-phase four-wire systems at conditions that: (i) the supply voltage is sinusoidal and (*ii*) the loads are linear. An increase in the complexity of the studied system by removal, at the same time, of two above conditions would be against the basic rules of the methodology of empirical sciences. The complexity of investigated systems should increase gradually, step-by-step. Leaps over a few steps could lead to not credible results. Therefore, only condition (*i*) is removed in this paper. Results obtained in this paper should create a ground for the next step of studies, in particular, studies on power properties of four-wire systems with removed (*ii*) condition, i.e., with harmonics generating loads.

Voltage harmonics in systems shown in Fig. 1 can be conveyed from the primary side of the transformer. It does not apply to harmonics of the zero sequence, of the order $n = 3k$, mainly to the 3rd order harmonic, however. Symmetrical harmonics of the zero sequence are not conveyed to the secondary side by a transformer with windings configured in Δ.

Such harmonics can occur in the supply voltage $\boldsymbol{u}$ only if there are loads on the secondary side of the transformer that generate the zero sequence harmonic currents. The existence of such loads would contradict the assumption that loads in the studied system are LTI, however. Nonetheless, to include also the effect of the zero sequence voltage harmonics on the power properties of the system in this study, we will assume that there are sources of such harmonics on the load side, but only properties of three-phase ASPSL loads will be studied.

Since this paper is a continuation of [20], most symbols, originally introduced in [7], have the same meanings. Only some of them are modified, when applied to harmonics, by the index of the harmonic order n. Therefore, it is recommended that the reader of this paper is acquainted with [20].

The subject of this paper fits the long lasting attempts aimed at description and interpretation of energy flow phenomena in electrical systems and in particular, in three-phase four-wire systems with nonsinusoidal voltages and currents.

II. LOAD CURRENT DECOMPOSITION

Let us express the vector of the line-to-neutral supply voltage $\boldsymbol{u}(t)$, or shortly the voltage $\boldsymbol{u}(t)$, as a sum of harmonics $\boldsymbol{u}_n(t)$ of the order n from a set N

$$\boldsymbol{u}(t) \stackrel{\text{df}}{=} \begin{bmatrix} u_{\mathrm{R}}(t) \\ u_{\mathrm{S}}(t) \\ u_{\mathrm{T}}(t) \end{bmatrix} = \sum_{n\in N} \boldsymbol{u}_n(t) \tag{4}$$

where

$$\boldsymbol{u}_n(t) \stackrel{\text{df}}{=} \begin{bmatrix} u_{\mathrm{R}n}(t) \\ u_{\mathrm{S}n}(t) \\ u_{\mathrm{T}n}(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t} \stackrel{\text{df}}{=} \sqrt{2}\,\mathrm{Re}\{U_n e^{jn\omega_1 t}\}. \tag{5}$$

Symbol U_n denotes a vector of complex rms (crms) values of line voltage harmonics $\boldsymbol{U}_{\mathrm{R}n}$, $\boldsymbol{U}_{\mathrm{S}n}$, and $\boldsymbol{U}_{\mathrm{T}n}$. The vector of the load line currents $\boldsymbol{i}(t)$, or shortly the load current $\boldsymbol{i}(t)$, can be expressed as a sum of current harmonics

$$\boldsymbol{i}(t) \stackrel{\text{df}}{=} \begin{bmatrix} i_{\mathrm{R}}(t) \\ i_{\mathrm{S}}(t) \\ i_{\mathrm{T}}(t) \end{bmatrix} = \sum_{n\in N} \boldsymbol{i}_n(t) \tag{6}$$

where

$$\boldsymbol{i}_n(t) \stackrel{\text{df}}{=} \begin{bmatrix} i_{\mathrm{R}n}(t) \\ i_{\mathrm{S}n}(t) \\ i_{\mathrm{T}n}(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{I}_{\mathrm{R}n} \\ \boldsymbol{I}_{\mathrm{S}n} \\ \boldsymbol{I}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t} \stackrel{\text{df}}{=} \sqrt{2}\,\mathrm{Re}\{I_n e^{jn\omega_1 t}\}. \tag{7}$$

To simplify notation, the dependence of voltages and currents on time (t) can be neglected, thus these vectors will be also written as

$$\boldsymbol{u} = \boldsymbol{u}(t),\ \boldsymbol{u}_n = \boldsymbol{u}_n(t),\ \boldsymbol{i} = \boldsymbol{i}(t),\ \boldsymbol{i}_n = \boldsymbol{i}_n(t).$$

With respect to the active power P at the voltage $\boldsymbol{u}$, the load is equivalent to a purely resistive balanced load, shown in Fig. 2,

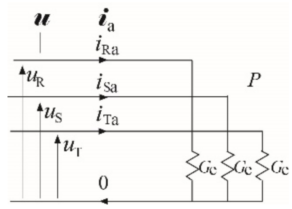

Fig. 2. Resistive balanced load equivalent to the original load with respect to its active power *P*.

of conductance

$$G_{\mathrm{e}} = \frac{P}{||\boldsymbol{u}||^2} \tag{8}$$

referred to as an ***equivalent conductance*** of the load. Such an equivalent load draws the ***active current*** from the source, meaning the current proportional to the supply voltage

$$\boldsymbol{i}_{\mathrm{a}} \stackrel{\text{df}}{=} \begin{bmatrix} i_{\mathrm{Ra}} \\ i_{\mathrm{Sa}} \\ i_{\mathrm{Ta}} \end{bmatrix} = G_{\mathrm{e}}\,\boldsymbol{u} = G_{\mathrm{e}} \sum_{n\in N} \boldsymbol{u}_n = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} G_{\mathrm{e}}\, U_n\, e^{jn\omega_1 t}. \tag{9}$$

Its three-phase rms value is equal to

$$||\boldsymbol{i}_{\mathrm{a}}|| = G_{\mathrm{e}}\,||\boldsymbol{u}|| = \frac{P}{||\boldsymbol{u}||}. \tag{10}$$

This is the smallest current three-phase rms value needed for the load with the active power P. The concept of this current was introduced to the power theory by Fryze for single-phase loads in [2] and for three-phase loads in [5].

The remaining part of the load current, after the active current is separated, namely

$$\boldsymbol{i} - \boldsymbol{i}_{\mathrm{a}} = \sum_{n\in N} \boldsymbol{i}_n - G_{\mathrm{e}}\,\boldsymbol{u} = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} (I_n - G_{\mathrm{e}}\, U_n) e^{jn\omega_1 t} \tag{11}$$

is a useless current.

Linear loads satisfy superposition principle and consequently, such loads at nonsinusoidal supply voltage can be analyzed harmonic-by-harmonic. Therefore, the load current decomposition obtained in [20] is valid for each individual harmonic.

The load shown in Fig. 1 has for each individual harmonic an equivalent circuit shown in Fig. 3, with line-to-neutral admittances equal to

$$\boldsymbol{Y}_{\mathrm{R}n} = \frac{\boldsymbol{I}_{\mathrm{R}n}}{\boldsymbol{U}_{\mathrm{R}n}}, \qquad \boldsymbol{Y}_{\mathrm{S}n} = \frac{\boldsymbol{I}_{\mathrm{S}n}}{\boldsymbol{U}_{\mathrm{S}n}}, \qquad \boldsymbol{Y}_{\mathrm{T}n} = \frac{\boldsymbol{I}_{\mathrm{T}n}}{\boldsymbol{U}_{\mathrm{T}n}}. \tag{12}$$

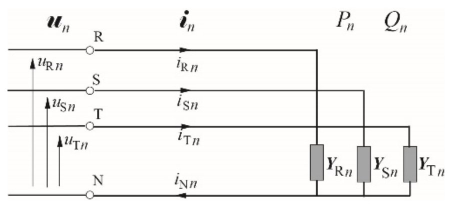

Fig. 3. Equivalent circuit of the load for the n^{th} order harmonic

One could observe that this equivalent circuit with admittances specified by (12) is valid at any load configuration and it takes into account impedances of the supply conductors.

The load current harmonic of the n order is composed, according to [20], of an active current

$$\boldsymbol{i}_{\mathrm{a}n}(t) = G_{\mathrm{e}n}\,\boldsymbol{u}_n(t) = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}n}\, U_n\, e^{jn\omega_1 t}\} \tag{13}$$

where

$$G_{\mathrm{e}n} = \frac{P_n}{||\boldsymbol{u}_n||^2} = \frac{1}{3}\mathrm{Re}\{\boldsymbol{Y}_{\mathrm{R}n} + \boldsymbol{Y}_{\mathrm{S}n} + \boldsymbol{Y}_{\mathrm{T}n}\} \tag{14}$$

is the equivalent conductance of the load for the n^{th} order harmonic.

The load current harmonic contains also a reactive current

$$\boldsymbol{i}_{\mathrm{r}n}(t) = B_{\mathrm{e}n}\frac{d}{d(n\omega_1 t)}\boldsymbol{u}_n(t) = \sqrt{2}\,\mathrm{Re}\{\,jB_{\mathrm{e}n}\,U_n\,e^{jn\omega_1 t}\} \tag{15}$$

where

$$B_{\mathrm{e}n} = -\frac{Q_n}{\|\boldsymbol{u}_n\|^2} = \frac{1}{3}\mathrm{Im}\{\boldsymbol{Y}_{\mathrm{R}n}+\boldsymbol{Y}_{\mathrm{S}n}+\boldsymbol{Y}_{\mathrm{T}n}\}\,. \tag{16}$$

When the load is unbalanced, then the n^{th} order current harmonic contains moreover an unbalanced current

$$\boldsymbol{i}_{\mathrm{u}n} = \boldsymbol{i}_n - \boldsymbol{i}_{\mathrm{a}n} - \boldsymbol{i}_{\mathrm{r}n}\,. \tag{17}$$

It can be expressed in terms of the load equivalent parameters as follows

$$\boldsymbol{i}_{\mathrm{u}n} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix}\boldsymbol{I}_{\mathrm{Ru}n}\\ \boldsymbol{I}_{\mathrm{Su}n}\\ \boldsymbol{I}_{\mathrm{Tu}n}\end{bmatrix}e^{jn\omega_1 t}\} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix}(\boldsymbol{Y}_{\mathrm{R}n}-\boldsymbol{Y}_{\mathrm{e}n})\boldsymbol{U}_{\mathrm{R}n}\\ (\boldsymbol{Y}_{\mathrm{S}n}-\boldsymbol{Y}_{\mathrm{e}n})\boldsymbol{U}_{\mathrm{S}n}\\ (\boldsymbol{Y}_{\mathrm{T}n}-\boldsymbol{Y}_{\mathrm{e}n})\boldsymbol{U}_{\mathrm{T}n}\end{bmatrix}e^{jn\omega_1 t}\} \tag{18}$$

where

$$\boldsymbol{Y}_{\mathrm{e}n} \overset{\mathrm{df}}{=} G_{\mathrm{e}n} + jB_{\mathrm{e}n} = \frac{1}{3}(\boldsymbol{Y}_{\mathrm{R}n}+\boldsymbol{Y}_{\mathrm{S}n}+\boldsymbol{Y}_{\mathrm{T}n}) \tag{19}$$

is equivalent admittance of the load for the n^{th} order harmonic.

The supply voltage harmonics are three-phase symmetrical quantities of different sequence, which depends on their order n, and this order affects the unbalanced current.

Harmonics of the order $n = 3k + 1$, i.e., $n = 1, 4, 7,...$are of the positive sequence, thus, assuming that

$$\alpha = 1e^{j2\pi/3}, \qquad \alpha^* = 1e^{-j2\pi/3}$$

$\boldsymbol{U}_{\mathrm{S}n} = \alpha^*\,\boldsymbol{U}_{\mathrm{R}n}$, $\boldsymbol{U}_{\mathrm{T}n} = \alpha\,\boldsymbol{U}_{\mathrm{R}n}$, hence, the unbalanced current is

$$\boldsymbol{i}_{\mathrm{u}n} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix}\boldsymbol{I}_{\mathrm{Ru}n}\\ \boldsymbol{I}_{\mathrm{Su}n}\\ \boldsymbol{I}_{\mathrm{Tu}n}\end{bmatrix}e^{jn\omega_1 t}\} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix}(\boldsymbol{Y}_{\mathrm{R}n}-\boldsymbol{Y}_{\mathrm{e}n})\\ (\boldsymbol{Y}_{\mathrm{S}n}-\boldsymbol{Y}_{\mathrm{e}n})\alpha^*\\ (\boldsymbol{Y}_{\mathrm{T}n}-\boldsymbol{Y}_{\mathrm{e}n})\alpha\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}\}. \tag{20}$$

Harmonics of the order $n = 3k - 1$, i.e., $n = 2, 5, 8,\ldots$are of the negative sequence, so that $\boldsymbol{U}_{\mathrm{S}n} = \alpha\,\boldsymbol{U}_{\mathrm{R}n}$, $\boldsymbol{U}_{\mathrm{T}n} = \alpha^*\,\boldsymbol{U}_{\mathrm{R}n}$, hence

$$\boldsymbol{i}_{\mathrm{u}n} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix}\boldsymbol{I}_{\mathrm{Ru}n}\\ \boldsymbol{I}_{\mathrm{Su}n}\\ \boldsymbol{I}_{\mathrm{Tu}n}\end{bmatrix}e^{jn\omega_1 t}\} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix}(\boldsymbol{Y}_{\mathrm{R}n}-\boldsymbol{Y}_{\mathrm{e}n})\\ (\boldsymbol{Y}_{\mathrm{S}n}-\boldsymbol{Y}_{\mathrm{e}n})\alpha\\ (\boldsymbol{Y}_{\mathrm{T}n}-\boldsymbol{Y}_{\mathrm{e}n})\alpha^*\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}\}. \tag{21}$$

Harmonics of the order $n = 3k$, i.e., $n = 3, 6, 9,\ldots$are of the zero sequence, so that $\boldsymbol{U}_{\mathrm{R}n} = \boldsymbol{U}_{\mathrm{S}n} = \boldsymbol{U}_{\mathrm{T}n}$, hence

$$\boldsymbol{i}_{\mathrm{u}n} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix}\boldsymbol{I}_{\mathrm{Ru}n}\\ \boldsymbol{I}_{\mathrm{Su}n}\\ \boldsymbol{I}_{\mathrm{Tu}n}\end{bmatrix}e^{jn\omega_1 t}\} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix}\boldsymbol{Y}_{\mathrm{R}n}-\boldsymbol{Y}_{\mathrm{e}n}\\ \boldsymbol{Y}_{\mathrm{S}n}-\boldsymbol{Y}_{\mathrm{e}n}\\ \boldsymbol{Y}_{\mathrm{T}n}-\boldsymbol{Y}_{\mathrm{e}n}\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}\}. \tag{22}$$

The unbalanced component of the load current harmonic of the n^{th} order is usually asymmetrical, so that it can be decomposed into symmetrical components of the positive, negative and the zero sequences, namely

$$\boldsymbol{i}_{\mathrm{u}n} = \boldsymbol{i}^{\mathrm{p}}_{\mathrm{u}n} + \boldsymbol{i}^{\mathrm{n}}_{\mathrm{u}n} + \boldsymbol{i}^{\mathrm{z}}_{\mathrm{u}n} \tag{23}$$

with the crms values of these components equal to

$$\begin{bmatrix}\boldsymbol{I}^{\mathrm{z}}_{\mathrm{u}n}\\ \boldsymbol{I}^{\mathrm{p}}_{\mathrm{u}n}\\ \boldsymbol{I}^{\mathrm{n}}_{\mathrm{u}n}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^*\\ 1 & \alpha^* & \alpha\end{bmatrix}\begin{bmatrix}\boldsymbol{I}_{\mathrm{Ru}n}\\ \boldsymbol{I}_{\mathrm{Su}n}\\ \boldsymbol{I}_{\mathrm{Tu}n}\end{bmatrix}. \tag{24}$$

For harmonics of the positive sequence

$$\begin{bmatrix}\boldsymbol{I}^{\mathrm{z}}_{\mathrm{u}n}\\ \boldsymbol{I}^{\mathrm{p}}_{\mathrm{u}n}\\ \boldsymbol{I}^{\mathrm{n}}_{\mathrm{u}n}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^*\\ 1 & \alpha^* & \alpha\end{bmatrix}\begin{bmatrix}(\boldsymbol{Y}_{\mathrm{R}n}-\boldsymbol{Y}_{\mathrm{e}n})\\ (\boldsymbol{Y}_{\mathrm{S}n}-\boldsymbol{Y}_{\mathrm{e}n})\alpha^*\\ (\boldsymbol{Y}_{\mathrm{T}n}-\boldsymbol{Y}_{\mathrm{e}n})\alpha\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n} =$$

$$= \frac{1}{3}\begin{bmatrix}\boldsymbol{Y}_{\mathrm{R}n}+\alpha^*\boldsymbol{Y}_{\mathrm{S}n}+\alpha\boldsymbol{Y}_{\mathrm{T}n}\\ 0\\ \boldsymbol{Y}_{\mathrm{R}n}+\alpha\boldsymbol{Y}_{\mathrm{S}n}+\alpha^*\boldsymbol{Y}_{\mathrm{T}n}\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n} \overset{\mathrm{df}}{=} \begin{bmatrix}\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}\\ 0\\ \boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n}\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n}. \tag{25}$$

For harmonics of the negative sequence

$$\begin{bmatrix}\boldsymbol{I}^{\mathrm{z}}_{\mathrm{u}n}\\ \boldsymbol{I}^{\mathrm{p}}_{\mathrm{u}n}\\ \boldsymbol{I}^{\mathrm{n}}_{\mathrm{u}n}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^*\\ 1 & \alpha^* & \alpha\end{bmatrix}\begin{bmatrix}(\boldsymbol{Y}_{\mathrm{R}n}-\boldsymbol{Y}_{\mathrm{e}n})\\ (\boldsymbol{Y}_{\mathrm{S}n}-\boldsymbol{Y}_{\mathrm{e}n})\alpha\\ (\boldsymbol{Y}_{\mathrm{T}n}-\boldsymbol{Y}_{\mathrm{e}n})\alpha^*\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n} =$$

$$= \frac{1}{3}\begin{bmatrix}\boldsymbol{Y}_{\mathrm{R}n}+\alpha\boldsymbol{Y}_{\mathrm{S}n}+\alpha^*\boldsymbol{Y}_{\mathrm{T}n}\\ \boldsymbol{Y}_{\mathrm{R}n}+\alpha^*\boldsymbol{Y}_{\mathrm{S}n}+\alpha\boldsymbol{Y}_{\mathrm{T}n}\\ 0\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n} \overset{\mathrm{df}}{=} \begin{bmatrix}\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}\\ \boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}\\ 0\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n}. \tag{26}$$

For harmonics of the zero sequence

$$\begin{bmatrix}\boldsymbol{I}^{\mathrm{z}}_{\mathrm{u}n}\\ \boldsymbol{I}^{\mathrm{p}}_{\mathrm{u}n}\\ \boldsymbol{I}^{\mathrm{n}}_{\mathrm{u}n}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^*\\ 1 & \alpha^* & \alpha\end{bmatrix}\begin{bmatrix}\boldsymbol{Y}_{\mathrm{R}n}-\boldsymbol{Y}_{\mathrm{e}n}\\ \boldsymbol{Y}_{\mathrm{S}n}-\boldsymbol{Y}_{\mathrm{e}n}\\ \boldsymbol{Y}_{\mathrm{T}n}-\boldsymbol{Y}_{\mathrm{e}n}\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n} =$$

$$= \frac{1}{3}\begin{bmatrix}0\\ \boldsymbol{Y}_{\mathrm{R}n}+\alpha\boldsymbol{Y}_{\mathrm{S}n}+\alpha^*\boldsymbol{Y}_{\mathrm{T}n}\\ \boldsymbol{Y}_{\mathrm{R}n}+\alpha^*\boldsymbol{Y}_{\mathrm{S}n}+\alpha\boldsymbol{Y}_{\mathrm{T}n}\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n} \overset{\mathrm{df}}{=} \begin{bmatrix}0\\ \boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}\\ \boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n}\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n}. \tag{27}$$

It means that the unbalanced current of individual harmonics of a specified order n and consequently, specified sequence, can be composed of only components of the remaining sequence. For example, the unbalanced current of the 5^{th} order harmonic, i.e., the harmonic of the negative sequence can have only components of the positive and the zero sequences.

Three equations (25-27) could be reduced to just one equation by introducing a ***generalized complex rotation coefficient***, defined as

$$\beta \overset{\mathrm{df}}{=} (\alpha^*)^n = \begin{cases}1, & \text{for} \quad n = 3k\\ \alpha^*, & \text{for} \quad n = 3k+1\,.\\ \alpha, & \text{for} \quad n = 3k-1\end{cases} \tag{28}$$

With this coefficient, eqs. (25-27) can be rewritten as

$$\begin{bmatrix}\boldsymbol{I}^{\mathrm{z}}_{\mathrm{u}n}\\ \boldsymbol{I}^{\mathrm{p}}_{\mathrm{u}n}\\ \boldsymbol{I}^{\mathrm{n}}_{\mathrm{u}n}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^*\\ 1 & \alpha^* & \alpha\end{bmatrix}\begin{bmatrix}(\boldsymbol{Y}_{\mathrm{R}n}-\boldsymbol{Y}_{\mathrm{e}n})\\ (\boldsymbol{Y}_{\mathrm{S}n}-\boldsymbol{Y}_{\mathrm{e}n})\beta\\ (\boldsymbol{Y}_{\mathrm{T}n}-\boldsymbol{Y}_{\mathrm{e}n})\beta^*\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n} = \begin{bmatrix}\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}\\ \boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}\\ \boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n}\end{bmatrix}\boldsymbol{U}_{\mathrm{R}n} \tag{29}$$

where

$$\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n} = \frac{1}{3}[(\boldsymbol{Y}_{\mathrm{R}n}+\beta\boldsymbol{Y}_{\mathrm{S}n}+\beta^*\boldsymbol{Y}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{e}n}(1+\beta+\beta^*)] \tag{30}$$

$$\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n} = \frac{1}{3}[(\boldsymbol{Y}_{\mathrm{R}n}+\alpha\beta\boldsymbol{Y}_{\mathrm{S}n}+\alpha^*\beta^*\boldsymbol{Y}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{e}n}(1+\alpha\beta+\alpha^*\beta^*)] \tag{31}$$

$$\boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n} = \frac{1}{3}[(\boldsymbol{Y}_{\mathrm{R}n}+\alpha^*\beta\boldsymbol{Y}_{\mathrm{S}n}+\alpha\beta^*\boldsymbol{Y}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{e}n}(1+\alpha^*\beta+\alpha\beta^*)] \tag{32}$$

are unbalanced admittances of the load for the zero, positive and the negative sequence harmonics. Their values depend not only on the load admittances for harmonics, but also on the harmonic sequence.

Decomposition (23) of the unbalanced component of a current harmonic of the $n^{\rm th}$ order can rearranged using ***unit symmetrical vectors***, defined as

$$\begin{bmatrix}1\\ \alpha^*\\ \alpha\end{bmatrix} \overset{\rm df}{=} 1^{\rm p},\quad \begin{bmatrix}1\\ \alpha\\ \alpha^*\end{bmatrix} \overset{\rm df}{=} 1^{\rm n},\quad \begin{bmatrix}1\\ 1\\ 1\end{bmatrix} \overset{\rm df}{=} 1^{\rm z} \tag{33}$$

and shown in Fig. 4.

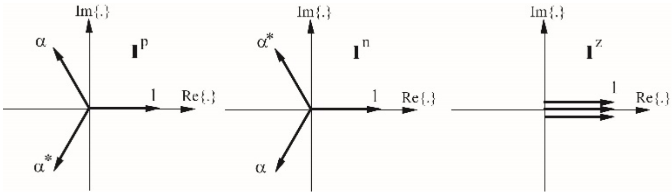

Fig. 4. Unit symmetrical vectors

With these vectors, decomposition (23) can be rewritten to the form

$$\boldsymbol{i}_{{\rm u}n} = \sqrt{2}{\rm Re}\{(\boldsymbol{Y}^{\rm p}_{{\rm u}n}1^{\rm p} + \boldsymbol{Y}^{\rm n}_{{\rm u}n}1^{\rm n} + \boldsymbol{Y}^{\rm z}_{{\rm u}n}1^{\rm z})\boldsymbol{U}_{{\rm R}n}e^{jn\omega_1 t}\}. \tag{34}$$

Observe that due to eqs. (25-27) for each harmonic order n only two of three admittances

$$\boldsymbol{Y}^{\rm p}_{{\rm u}n},\quad \boldsymbol{Y}^{\rm n}_{{\rm u}n},\quad \boldsymbol{Y}^{\rm z}_{{\rm u}n}$$

could have a non-zero value.

Components of the unbalanced current harmonic decomposition (23) are symmetrical, which is emphasized by (34). Their three-phase rms values are equal to

$$||\boldsymbol{i}^{\rm p}_{{\rm u}n}|| = \sqrt{3}Y^{\rm p}_{{\rm u}n}U_{{\rm R}n},\, ||\boldsymbol{i}^{\rm n}_{{\rm u}n}|| = \sqrt{3}Y^{\rm n}_{{\rm u}n}U_{{\rm R}n},\, ||\boldsymbol{i}^{\rm z}_{{\rm u}n}|| = \sqrt{3}Y^{\rm z}_{{\rm u}n}U_{{\rm R}n}\,. \tag{35}$$

Because they have different sequences, they are mutually orthogonal, and consequently, the square of the three-phase rms value of the unbalanced current of the $n^{\rm th}$ order harmonic is

$$||\boldsymbol{i}_{{\rm u}n}||^2 = ||\boldsymbol{i}^{\rm p}_{{\rm u}n}||^2 + ||\boldsymbol{i}^{\rm n}_{{\rm u}n}||^2 + ||\boldsymbol{i}^{\rm z}_{{\rm u}n}||^2. \tag{36}$$

Since formula (17) for the $n^{\rm th}$ order load current harmonic can be rewritten to the form

$$\boldsymbol{i}_n = \boldsymbol{i}_{{\rm a}n} + \boldsymbol{i}_{{\rm r}n} + \boldsymbol{i}_{{\rm u}n} \tag{37}$$

thus the load current can be expressed as

$$\boldsymbol{i} = \sum_{n\in N}\boldsymbol{i}_n = \sum_{n\in N}(\boldsymbol{i}_{{\rm a}n} + \boldsymbol{i}_{{\rm r}n} + \boldsymbol{i}_{{\rm u}n})\,. \tag{38}$$

The first term on the right side is not the active current $\boldsymbol{i}_{\rm a}$ defined by (9), however. It can be rearranged to the form

$$\sum_{n\in N}\boldsymbol{i}_{{\rm a}n} = \sqrt{2}{\rm Re}\sum_{n\in N}G_{{\rm e}n}U_n e^{jn\omega_1 t} =$$
$$= \sqrt{2}{\rm Re}\sum_{n\in N}G_{\rm e}U_n e^{jn\omega_1 t} + \sqrt{2}{\rm Re}\sum_{n\in N}(G_{{\rm e}n}-G_{\rm e})U_n e^{jn\omega_1 t} =$$
$$= \boldsymbol{i}_{\rm a} + \boldsymbol{i}_{\rm s} \tag{39}$$

where the current component

$$\boldsymbol{i}_{\rm s} = \sqrt{2}{\rm Re}\sum_{n\in N}(G_{{\rm e}n}-G_{\rm e})U_n e^{jn\omega_1 t} \tag{40}$$

occurs because the equivalent conductance for harmonic frequencies $G_{{\rm e}n}$ of the load is usually different than its equivalent conductance $G_{\rm e}$. If different, then conductances $G_{{\rm e}n}$ of the load are scattered around $G_{\rm e}$ value and therefore, the current $\boldsymbol{i}_{\rm s}(t)$ will be referred to as a ***scattered current***. This is a symmetrical current of the three-phase rms value equal to

$$||\boldsymbol{i}_{\rm s}|| = \sqrt{3}\sqrt{\sum_{n\in N}(G_{{\rm e}n}-G_{\rm e})^2U^2_{{\rm R}n}}\,. \tag{41}$$

With (39), the load current decomposition can be rewritten to the form

$$\boldsymbol{i} = \boldsymbol{i}_{\rm a} + \boldsymbol{i}_{\rm s} + \boldsymbol{i}_{\rm r} + \boldsymbol{i}_{\rm u} \tag{42}$$

where

$$\boldsymbol{i}_{\rm r} = \sum_{n\in N}\boldsymbol{i}_{{\rm r}n} = \sqrt{2}\,{\rm Re}\sum_{n\in N}jB_{{\rm e}n}U_n e^{jn\omega_1 t} \tag{43}$$

is the ***reactive current***. It is a symmetrical current of three-phase rms value equal to

$$||\boldsymbol{i}_{\rm r}|| = \sqrt{3}\sqrt{\sum_{n\in N}B^2_{{\rm e}n}U^2_{{\rm R}n}}\,. \tag{44}$$

The last component in decomposition (42)

$$\boldsymbol{i}_{\rm u} = \sum_{n\in N}\boldsymbol{i}_{{\rm u}n} = \sqrt{2}\,{\rm Re}\sum_{n\in N}(\boldsymbol{Y}^{\rm p}_{{\rm u}n}1^{\rm p} + \boldsymbol{Y}^{\rm n}_{{\rm u}n}1^{\rm n} + \boldsymbol{Y}^{\rm z}_{{\rm u}n}1^{\rm z})\boldsymbol{U}_{{\rm R}n}e^{jn\omega_1 t} =$$
$$= \boldsymbol{i}^{\rm p}_{\rm u} + \boldsymbol{i}^{\rm n}_{\rm u} + \boldsymbol{i}^{\rm z}_{\rm u} \tag{45}$$

is the ***unbalanced current*** of the load. In this formula

$$\boldsymbol{i}^{\rm s}_{\rm u} = \sqrt{2}\,{\rm Re}\sum_{n\in N}\boldsymbol{Y}^{\rm s}_{{\rm u}n}1^{\rm s}\boldsymbol{U}_{{\rm R}n}e^{jn\omega_1 t} \tag{46}$$

where the upper index s denotes the current sequence, i.e., elements from the set {p, n, z}. The three-phase rms value of the unbalanced current is

$$||\boldsymbol{i}_{\rm u}|| = \sqrt{\sum_{n\in N}||\boldsymbol{i}_{{\rm u}n}||^2} = \sqrt{\sum_{\rm s\,=\,p,n,z}||\boldsymbol{i}^{\rm s}_{\rm u}||^2} \tag{47}$$

where

$$||\boldsymbol{i}^{\rm s}_{\rm u}|| = \sqrt{\sum_{n\in N}||\boldsymbol{i}^{\rm s}_{{\rm u}n}||^2} = \sqrt{3}\sqrt{\sum_{n\in N}(Y^{\rm s}_{{\rm u}n})^2U^2_{{\rm R}n}}\,. \tag{48}$$

Components of the load current in decomposition (42) are associated with distinctive physical phenomena, which can be identified by measurements at the load terminals. The active current $\boldsymbol{i}_{\rm a}$ is associated with permanent transfer of energy. The reactive current $\boldsymbol{i}_{\rm r}$ is associated with the phase-shift of the current harmonics with respect to the voltage harmonics. The scattered current $\boldsymbol{i}_{\rm s}$ is associated with a phenomenon of the load conductance change with harmonic order. The unbalanced current $\boldsymbol{i}_{\rm u}$ is associated with asymmetry of the load current harmonics. Therefore, these four currents can be regarded as the Current Physical Components (CPC). It has to be stressed, however, that in spite of their names, these currents do not exist physically – they are only an outcome of a mathematical decomposition. Thus they are mathematical, but not physical entities. Nonetheless, each of them is ***associated*** with a distinctive physical phenomenon in the load. Therefore, this decomposition provides an important insight into physical phenomena in the load.

To calculate the three-phase rms value $||\boldsymbol{i}||$ of the load current as a root of the sum of squares of three-phase rms values of CPC, they have to be mutually orthogonal, meaning their scalar product, defined for vectors $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ as

$$(\boldsymbol{x},\boldsymbol{y}) \overset{\rm df}{=} \frac{1}{T}\int_0^T \boldsymbol{x}^{\rm T}(t)\,\boldsymbol{y}(t)\,dt \tag{49}$$

has to be equal to zero.

The three-phase rms values of the current components in (42) satisfy the relationship

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\rm a}||^2 + ||\boldsymbol{i}_{\rm s}||^2 + ||\boldsymbol{i}_{\rm r}||^2 + ||\boldsymbol{i}_{\rm u}||^2 \tag{50}$$

on the condition that the scalar products

$$(\boldsymbol{i}_{\rm a},\boldsymbol{i}_{\rm s}),\ (\boldsymbol{i}_{\rm a},\boldsymbol{i}_{\rm r}),\ (\boldsymbol{i}_{\rm a},\boldsymbol{i}_{\rm u}),\ (\boldsymbol{i}_{\rm s},\boldsymbol{i}_{\rm r}),\ (\boldsymbol{i}_{\rm s},\boldsymbol{i}_{\rm u}),\ (\boldsymbol{i}_{\rm r},\boldsymbol{i}_{\rm u})$$

are equal to zero.

The scalar product, originally defined in the time-domain, can be calculated in the frequency-domain with vectors X_n and Y_n of crms values of harmonics of quantities $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$, since

$$\frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\, \boldsymbol{y}(t)\, dt = \mathrm{Re}\sum_{n\in N} X_n^{\mathrm{T}} Y_n^{*} \qquad (51)$$

where the upper index T denotes a transposed vector, while the asterisk * denotes a vector of conjugate crms values.

It was proven in Appendices A - F that the scalar products of all current components in (42) are indeed equal to zero, thus these four components are mutually orthogonal. It means that each of them contributes to the three-phase rms value of the load current $||\boldsymbol{i}||$ according to relationship (50). Thus each of CPC contributes to the three-phase rms value increase independently of each other.

This relationship can be illustrated, along with (47), with a polygon shown in Fig. 5, with side length proportional to the three-phase rms values of individual components of the load current.

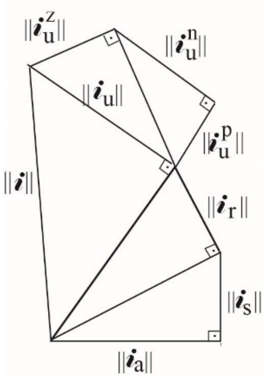

Fig. 5. Diagram of three-phase rms values of the current physical components.

Orthogonality of six components cannot be illustrated, of course, on a plane. Only two sides can be drawn on a plane as orthogonal and these are only two first terms on the right side of (50). Its sequence can be changed, however, without affecting the length of diagonal, meaning $||\boldsymbol{i}||$.

Numerical illustration. Let us assume that the load shown in Fig. 6 is supplied with a symmetrical voltage of the fundamental harmonic rms value U_1 = 240 V distorted with the 3$^{\mathrm{rd}}$, 5$^{\mathrm{th}}$ and 7$^{\mathrm{th}}$ order harmonics of relative rms value U_3 = 2% U_1, U_5 =3% U_1 and U_7 = 1.5% U_1.

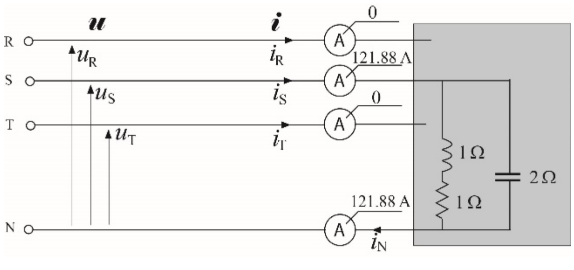

Fig. 6. Example of unbalanced load

The rms values of line-to-neutral voltage harmonics and the load admittance for harmonic frequencies are compiled in Table1.

Table 1. Rms values of voltage harmonics and load admittance

n	U_n [V]	$\boldsymbol{Y}_{\mathrm{S}n} = G_{\mathrm{S}n}+jB_{\mathrm{S}n}$ [S]
1	240	0.5
3	4.8	0.1+j1.2
5	7.2	0.0385+j2.308
7	3.6	0.02+j3.36

The active power of the load is

$$P = \sum_{n\in N} G_{\mathrm{S}n} U_{\mathrm{S}n}^2 = 28.804\ \mathrm{kW}\,.$$

The supply voltage three rms value

$$||\boldsymbol{u}|| = \sqrt{\sum_{n\in N} ||\boldsymbol{u}_n||^2} = \sqrt{3\sum_{n\in N} U_n^2} = 416.01\ \mathrm{V}\,.$$

Thus the equivalent conductance of the load is

$$G_{\mathrm{e}} = \frac{P}{||\boldsymbol{u}||^2} = 0.165\ \mathrm{S}$$

and the three-phase rms value of the active current is

$$||\boldsymbol{i}_{\mathrm{a}}|| = G_{\mathrm{e}}\, ||\boldsymbol{u}|| = \frac{P}{||\boldsymbol{u}||} = 69.224\ \mathrm{A}\,.$$

The values of equivalent conductance $G_{\mathrm{e}n}$, susceptance $B_{\mathrm{e}n}$, and magnitude of unbalanced admittances $\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}, \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}$ and $\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}}$ for harmonic frequencies, calculated according to (14), (16) and (30-32), are compiled in Table 2.

Table 2. Equivalent parameters of the load for harmonics

n	$G_{\mathrm{e}n}$[S]	$B_{\mathrm{e}n}$[S]	$Y_{\mathrm{u}n}^{\mathrm{p}}$ [S]	$Y_{\mathrm{u}n}^{\mathrm{n}}$ [S]	$Y_{\mathrm{u}n}^{\mathrm{z}}$ [S]
1	0.167	0	0	0.167	0.167
3	0.033	0.400	0.401	0.401	0
5	0.013	0.769	0.769	0	0.769
7	0.007	1.120	0	1.12	1.12

The three-phase rms value of the scattered current, calculated with (41) is

$$||\boldsymbol{i}_{\mathrm{s}}|| = \sqrt{3\sum_{n\in N} [(G_{\mathrm{e}n} - G_{\mathrm{e}})U_n]^2} = 2.428\ \mathrm{A}$$

and the reactive current, calculated with (44)

$$||\boldsymbol{i}_{\mathrm{r}}|| = \sqrt{3\sum_{n\in N} (B_{\mathrm{e}n} U_n)^2} = 12.323\ \mathrm{A}.$$

The three-phase rms value of the unbalanced current components calculated with (48) are

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}|| = \sqrt{3\sum_{n\in N} (Y_{\mathrm{u}n}^{\mathrm{p}} U_n)^2} = 10.158\ \mathrm{A}$$

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}|| = \sqrt{3\sum_{n\in N} (Y_{\mathrm{u}n}^{\mathrm{n}} U_n)^2} = 69.713\ \mathrm{A}$$

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}|| = \sqrt{3\sum_{n\in N} (Y_{\mathrm{u}n}^{\mathrm{z}} U_n)^2} = 70.291\ \mathrm{A}$$

and consequently, the three-phase rms value of the unbalanced current is

$$||\boldsymbol{i}_{\mathrm{u}}|| = \sqrt{||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}||^2} = 99.518\ \mathrm{A}\,.$$

The three-phase rms value of the load current, calculated as the root of sum of squares of rms values of the line currents, is

$$||\boldsymbol{i}|| = \sqrt{||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2} = ||i_{\mathrm{S}}||^2 = \sqrt{\sum_{n\in N} (Y_{Sn} U_n)^2} = 121.88\ \mathrm{A}\,.$$

This value can be used for verification of the decomposition of the load current into the active, scattered, reactive and the unbalanced currents, since the root of sum of squares their three-phase rms values should result in the same value of $||\boldsymbol{i}||$. Indeed

$$||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{s}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2} = 121.88\ \mathrm{A}$$

which confirms numerical correctness of calculations.

III. Powers and the Power Factor

"Power properties" of a load can be explained, which sounds strange, without any power equation, but in terms of only the current decompositions (42) and (45) and the relationships for their three-phase rms values (47) and (50). This is because only one factor in each power definition: the current, depends on the load properties. The second factor: the voltage is not dependent on the load, assuming of course, that source impedance is zero.

Current components in (42) and (45) are associated with distinctive physical phenomena in the load and its structure. These components can be identified by measurement of the load equivalent parameters on the load terminals. Each of them contributes to the load current three-phase rms value increase, according to (47) and (50), independently of each other and this is the essence of the CPC-based power theory.

Nonetheless, the EE community is accustomed to using powers rather than the load current components. To meet this expectation, let us multiply (50) by the square of the three-phase rms value of the load voltage $||\boldsymbol{u}||$ and the power equation

$$S^2 = P^2 + D_s^2 + Q^2 + D_u^2 \qquad (52)$$

with

$$P = ||\boldsymbol{u}||\,||\boldsymbol{i}_a||, \quad D_s \stackrel{\text{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_s||, \quad Q \stackrel{\text{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_r||, \quad D_u \stackrel{\text{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_u||. \qquad (53)$$

is obtained with the apparent power S defined in the sense of Buchholtz's [1] definition. Observe that apart from powers known in systems with sinusoidal voltages and currents, meaning the active, reactive and unbalanced powers, a new power D_s has occurred in the power equation. It will be referred to ***as a scattered power***.

Because the unbalanced current $\boldsymbol{i}_u$ components in (45), as currents of different sequence, are mutually orthogonal, they satisfy (47). Multiplying (47) by the voltage three-phase rms value $||\boldsymbol{u}||$, the unbalanced power can be expressed as

$$D_u^2 = D_u^{p2} + D_u^{n2} + D_u^{z2} \qquad (54)$$

with

$$D_u \stackrel{\text{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_u||, \quad D_u^p \stackrel{\text{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_u^p||, \quad D_u^n \stackrel{\text{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_u^n||, \quad D_u^z \stackrel{\text{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_u^z||. \qquad (55)$$

Relationships (52) and (54) can be illustrated with a polygon shown in Fig. 7.

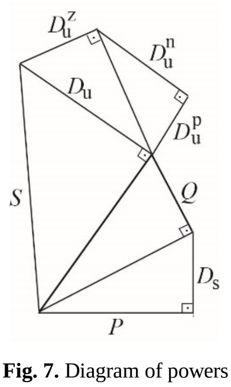

Fig. 7. Diagram of powers

It visualizes the contribution of particular powers to the apparent power S of the load.

The power factor (PF) of the load supplied in a four-wire system with a symmetrical, but nonsinusoidal voltage is equal to

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + D_s^2 + Q^2 + D_u^{p2} + D_u^{n2} + D_u^{z2}}} \qquad (56)$$

thus, not only the reactive power Q, but the scattered power D_s and unbalanced powers, D_u^p, D_u^n and D_u^z contribute to the load PF degradation. Observe, however, that the PF can be expressed not only in terms of powers, but also in terms of three-phase rms values of CPCs of the supply current, namely

$$\lambda = \frac{P}{S} = \frac{||\boldsymbol{i}_a||}{||\boldsymbol{i}||} = \frac{||\boldsymbol{i}_a||}{\sqrt{||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u^p||^2 + ||\boldsymbol{i}_u^n||^2 + ||\boldsymbol{i}_u^z||^2}}. \qquad (57)$$

IV. Conclusions

The paper demonstrates that the complex rms values of harmonics of voltages and currents, measured at the supply three-phase terminals of aggregates of single-phase static and linear loads, provide sufficient data for describing such loads in power terms. These crms values enable decomposition of the load current into components, which are associated with distinctive physical phenomena in the load.

The presented decomposition enables us to conclude how these phenomena affect the supply current three-phase rms value and the load power factor. The load current components can be expressed in terms of equivalent parameters of the load, which can be calculated based on measurement of the crms values of the load voltage and current harmonics.

Generalization of results obtained in [20] for ASPSL loads with sinusoidal supply voltage to nonsinusoidal voltage presented in this paper revealed that the sum of harmonic active currents $\boldsymbol{i}_{an}$ could be not equal to the active current $\boldsymbol{i}_a$. A new current component $\boldsymbol{i}_s$, called a scattered current, has to be introduced to the load current decomposition. It increases the load current three-phase rms value, but it does not contribute to energy transfer. Because of that, a new power, called a scattered power D_s, has to be introduced to the power equation of the load.

This paper reveals also a presence of a positive sequence component $\boldsymbol{i}_u^p$ in the unbalanced current, which does not exist at sinusoidal supply voltage. This current is orthogonal to the remaining ones. It contributes, like the currents of the negative and the zero sequence, to the load current three-phase rms value increase. Also, a new unbalanced power D_u^p, which does not exist at sinusoidal voltage, occurs in the load power equation.

The Currents' Physical Components defined in this paper for ASPSL loads supplied with nonsinusoidal voltage can be expressed in terms of the load parameters. The same applies to a reactive compensator parameters. Therefore, although this was beyond of the scope of this paper, this decomposition, as that in [20], provides fundamentals for reactive compensators design.

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Appendix A

Active and scattered currents scalar product

$$(\boldsymbol{i}_\mathrm{a},\boldsymbol{i}_\mathrm{s}) = \mathrm{Re}\sum_{n\in N} I_{\mathrm{a}n}^\mathrm{T} I_{\mathrm{s}n}^* = \mathrm{Re}\sum_{n\in N} G_\mathrm{e}\, U_n^\mathrm{T}(G_{\mathrm{e}n}-G_\mathrm{e})\,U_n^* =$$
$$= G_\mathrm{e}\sum_{n\in N}(G_{\mathrm{e}n}-G_\mathrm{e})\|\boldsymbol{u}_n\|^2 =$$
$$= G_\mathrm{e}\Big(\sum_{n\in N} G_{\mathrm{e}n}\|\boldsymbol{u}_n\|^2 - G_\mathrm{e}\sum_{n\in N}\|\boldsymbol{u}_n\|^2\Big) = G_\mathrm{e}\,(P-P) = 0.$$

Appendix B

Active and reactive currents scalar product

$$(\boldsymbol{i}_\mathrm{a},\boldsymbol{i}_\mathrm{r}) = \mathrm{Re}\sum_{n\in N} I_{\mathrm{a}n}^\mathrm{T} I_{\mathrm{r}n}^* = \mathrm{Re}\sum_{n\in N} G_\mathrm{e}\, U_n^\mathrm{T}(-jB_{\mathrm{e}n})\,U_n^* =$$
$$= \mathrm{Re}\sum_{n\in N} G_\mathrm{e}(-jB_{\mathrm{e}n})\|\boldsymbol{u}_n\|^2 = 0.$$

Appendix C

Active and unbalanced currents scalar product

$$(\boldsymbol{i}_\mathrm{a},\boldsymbol{i}_\mathrm{u}) = (\boldsymbol{i}_\mathrm{u},\boldsymbol{i}_\mathrm{a}) = \mathrm{Re}\sum_{n\in N} I_{\mathrm{u}n}^\mathrm{T} I_{\mathrm{a}n}^* =$$
$$= \mathrm{Re}\sum_{n\in N}(\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{p}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{n}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{z})^\mathrm{T}\boldsymbol{U}_{\mathrm{R}n}G_\mathrm{e}\,U_n^*.$$

Let us check the values of terms under the Σ symbol for harmonics of particular sequences.

For harmonics of the positive sequence, $\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p} = 0$, hence

$$(\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{p}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{n}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{z})^\mathrm{T}\boldsymbol{U}_{\mathrm{R}n}U_n^* =$$
$$= (\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{nT}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{zT})\boldsymbol{U}_{\mathrm{R}n}(1^{\mathrm{p}*}\boldsymbol{U}_{\mathrm{R}n}^*) =$$
$$= (\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{nT}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{zT})1^\mathrm{n}U_{\mathrm{R}n}^2 = 0$$

since

$$1^\mathrm{nT}1^\mathrm{n} = 1+\alpha^*+\alpha = 0 \quad\text{and}\quad 1^\mathrm{zT}1^\mathrm{n} = 1+\alpha+\alpha^* = 0.$$

For harmonics of the negative sequence, $\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n} = 0$, hence

$$(\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{p}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{n}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{z})^\mathrm{T}\boldsymbol{U}_{\mathrm{R}n}U_n^* =$$
$$= (\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{pT}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{zT})\boldsymbol{U}_{\mathrm{R}n}(1^{\mathrm{n}*}\boldsymbol{U}_{\mathrm{R}n}^*) =$$
$$= (\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{pT}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{zT})1^\mathrm{p}U_{\mathrm{R}n}^2 = 0$$

since

$$1^\mathrm{pT}1^\mathrm{p} = 1+\alpha+\alpha^* = 0 \quad\text{and}\quad 1^\mathrm{zT}1^\mathrm{p} = 1+\alpha^*+\alpha = 0.$$

For harmonics of the zero sequence, $\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z} = 0$, hence

$$(\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{p}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{n}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{z})^\mathrm{T}U_{\mathrm{R}n}U_n^* =$$
$$= (\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{pT}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{nT})\boldsymbol{U}_{\mathrm{R}n}(1^{\mathrm{z}*}\boldsymbol{U}_{\mathrm{R}n}^*) =$$
$$= (\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{pT}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{nT})1^\mathrm{z}U_{\mathrm{R}n}^2 = 0$$

since

$$1^\mathrm{pT}1^\mathrm{z} = 1+\alpha^*+\alpha = 0 \quad\text{and}\quad 1^\mathrm{nT}1^\mathrm{z} = 1+\alpha+\alpha^* = 0.$$

Thus the values of terms under the Σ symbol for harmonics of any sequences are zero and consequently,

$$(\boldsymbol{i}_\mathrm{a},\boldsymbol{i}_\mathrm{u}) = 0\,.$$

Appendix D

Scattered and reactive currents scalar product

$$(\boldsymbol{i}_\mathrm{s},\boldsymbol{i}_\mathrm{r}) = \mathrm{Re}\sum_{n\in N} I_{\mathrm{s}n}^\mathrm{T} I_{\mathrm{r}n}^* =$$
$$= \mathrm{Re}\sum_{n\in N}(G_{\mathrm{e}n}-G_\mathrm{e})\,U_n^\mathrm{T}(-jB_{\mathrm{e}n})\,U_n^* =$$
$$= \mathrm{Re}\sum_{n\in N}(G_{\mathrm{e}n}-G_\mathrm{e})(-jB_{\mathrm{e}n})\|\boldsymbol{u}_n\|^2 = 0.$$

Appendix E

Scattered and unbalanced currents scalar product

$$(\boldsymbol{i}_\mathrm{s},\boldsymbol{i}_\mathrm{u}) = (\boldsymbol{i}_\mathrm{u},\boldsymbol{i}_\mathrm{s}) = \mathrm{Re}\sum_{n\in N} I_{\mathrm{u}n}^\mathrm{T} I_{\mathrm{s}n}^* =$$
$$= \mathrm{Re}\sum_{n\in N}(\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{p}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{n}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{z})^\mathrm{T}\boldsymbol{U}_{\mathrm{R}n}(G_{\mathrm{e}n}-G_\mathrm{e})\,U_n^*$$

Similarly as in Appendix C, for harmonic of each sequence

$$(\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{p}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{n}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{z})^\mathrm{T}\boldsymbol{U}_{\mathrm{R}n}U_n^* = 0$$

and hence

$$(\boldsymbol{i}_\mathrm{s},\boldsymbol{i}_\mathrm{u}) = 0\,.$$

Appendix F

Reactive and unbalanced currents scalar product

$$(\boldsymbol{i}_\mathrm{r},\boldsymbol{i}_\mathrm{u}) = (\boldsymbol{i}_\mathrm{u},\boldsymbol{i}_\mathrm{r}) = \mathrm{Re}\sum_{n\in N} I_{\mathrm{u}n}^\mathrm{T} I_{\mathrm{r}n}^* =$$
$$= \mathrm{Re}\sum_{n\in N}(\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{p}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{n}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{z})^\mathrm{T}\boldsymbol{U}_{\mathrm{R}n}(-jB_{\mathrm{e}n})\,U_n^*.$$

Similarly as in Appendix C, for harmonic of each sequence

$$(\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{p}1^\mathrm{p}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{n}1^\mathrm{n}+\boldsymbol{Y}_{\mathrm{u}n}^\mathrm{z}1^\mathrm{z})^\mathrm{T}\boldsymbol{U}_{\mathrm{R}n}U_n^* = 0$$

and hence

$$(\boldsymbol{i}_\mathrm{r},\boldsymbol{i}_\mathrm{u}) = 0\,.$$

BIOGRAPHIES

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