A Method of Calculating LC Parameters of Balancing Compensators for AC Arc Furnaces

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***Abstract*—A method of calculating LC parameters of areactive balancing compensator for ac arc furnaces at the supply-voltage asymmetry is the subject of this paper. The developed method is illustrated numerically with the calculation of LC parameters of a balancing compensator for an ac arc furnace approximated by a linear model with fixed parameters. The presented method can be regarded as an initial step toward developing balancing compensation for ultra-high-power ac arc furnaces. The method of calculation of LC parameters of balancing compensators is based in this paper on the currents' physical component (CPC) power theory.**

***Index Terms*—AC arc furnaces, currents' physical components, electric arc furnace (EAF), unbalanced loads, unbalanced power.**

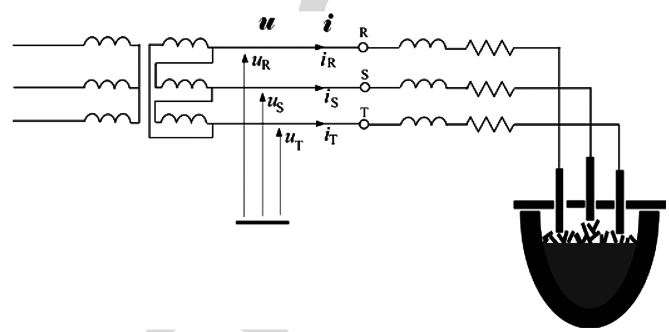

Fig. 1. Structure of an arc furnace with a supply transformer.

I. Introduction

AC ARC furnaces seem to be the highest power individual loads in distribution systems now. There are currently installed units of the power above 300 MVA, and in the future even 1GVA units are not beyond the scope of imagination. Such a furnace is a single load that has the power equivalent to a city with approximately half a million population. Annual energy consumption of a single arc furnace could be [4] in the range of 1000 GWh, meaning that the energy bill could be in the range of a hundred million dollars.

A structure of an arc furnace with the supply transformer is shown in Fig. 1.

The furnace shell is of course grounded, but its interior is coated with relatively high resistance ceramic. Consequently, at least two arcs have to be ignited. The voltage-current relation of electric arc furnaces is strongly nonlinear and time-variant. Only the last period of the furnace operation cycle, the *refining* period, is relatively quiet. In the first two cycles of furnace operation, namely, in the *boring* period and in the *melting* period [1], the arc ignition is strongly random. In effect, the supply currents of arc furnaces are not only strongly asymmetrical, but also distorted and random. The same applies to the furnace supply voltage. This is because the power rating of the furnace transformer is usually no more than two times higher than the furnace power. Power of distribution transformers in common systems is at least twenty times higher than the load power. Consequently, because of a very high relative impedance of the furnace transformer, the furnace current asymmetry and distortion causes much higher voltage asymmetry and distortion as compared to their levels in common distribution systems.

To stabilize the arc, the supply has to have sufficiently large inductance which requires that series inductors be connected into supply lines [2]–[4]. In effect, arc furnaces operate at relatively low power factor, which usually is in the range of 0.7. Consequently, the distribution system that supplies an arc furnace is affected not only by the furnace current asymmetry and its distortion, but also by a reactive current of the rms value comparable to the active one.

Compensation of the reactive and unbalanced currents of the ultra-high power arc furnaces is currently beyond the capability of the PWM-inverter-based switching compensators (known commonly as "active power filters") due to limited switching power of transistors. Resonant harmonic filters tuned to individual harmonics are used instead. The switching power of transistors might be sufficient if switching compensation is confined only to selected harmonics. The reactive and unbalanced currents have to be compensated by reactive compensators [3], [4]. Unfortunately, the power theory of electrical systems at the present state of development does not provide fundamentals for design of reactive balancing compensators operated in situations created by ultra-high power arc furnaces, mainly due to the supply voltage asymmetry. The term "power theory" used above means a set of true statements on power properties of electrical systems and on possibility of their improvement through compensation.

There are a lot of research reports [3]–[5] where authors attempt to describe the electric arc mathematically and find a relation between the arc voltage and the current. They suggest a variety of arc models. All of them depend on the arc geometry and local temperature, which are fast varying agents and are unknown for an external observer. Consequently, the arc furnace for such an external observer is a black box with only measurable currents and voltages at the furnace terminals as well as a few measurable parameters of the melting process such as the internal temperature and the state of melting.

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There were a number of attempts [9]–[13] of describing the arc furnace as an electric load in terms of the voltages and currents at its terminals. Of all these attempts, according to [12], [13], only the Currents Physical Components (CPC)-based power theory provides some tools that could be useful for identification of processes inside of the arc furnace shell. It is because the CPC decomposes the load current into components that are associated with distinctive physical phenomena in the load.

This paper is focused on only a single issue related to reactive compensation of the reactive and unbalanced currents produced by an AC arc furnace. This issue is: how to calculate compensator LC parameters in a situation where the voltages at the compensator terminals are asymmetrical, assuming linear approximation of the arc furnace. Such an approximation is still very distant from the arc furnace properties, but unfortunately, the power theory at its present state of development does not provide any answer to this question, even if voltages and currents are sinusoidal. This question should be answered, however, before a similar question is asked, when the voltages and currents are not only asymmetrical, but also nonsinusoidal and random.

The situation for which LC parameters of a reactive balancing compensator are looked for in this paper is very distant from the real situation, however. Results of studies presented in this paper can be regarded as a contribution to a balancing compensator design in general terms. Nonetheless, these results may provide a starting point for developing reactive balancing compensators for ultra-high power arc furnaces.

This paper presents a Currents' Physical Components (CPC)-based approach to reactive compensation which has a strong analogy to that discussed in [25], and most symbols, originally introduced in [18], have the same meanings. Therefore, it is recommended that a reader is acquainted with [25].

II. Currents' Physical Components (CPC) at Asymmetrical Voltage

Studies on methods of designing reactive balancing compensators for three-phase unbalanced systems have a long history. These studies were initiated by Steinmetz [14] in 1917, who developed the first such compensator, known as a Steinmetz compensator. Some results of studies on such compensators' design in sinusoidal situations and their performance can be found in [15]–[21]. This problem for systems with nonsinusoidal, but symmetrical supply voltage was solved in [18].

The design method of a reactive balancing compensator of linear unbalanced loads supplied with asymmetrical voltage presented in this paper is based on the load current decomposition into the physical components, presented in [26]. Fundamentals of this decomposition without details and proofs are drafted below.

Any unbalanced but linear three-phase load with a three-wire supply as shown in Fig. 2 has an equivalent circuit as shown in Fig. 3.

There are an infinite number of equivalent loads, i.e., sets of line-to-line admittances $\boldsymbol{Y}_{\rm RS}$, $\boldsymbol{Y}_{\rm ST}$, and $\boldsymbol{Y}_{\rm TR}$, that at the same supply voltages have the same active and reactive power P and

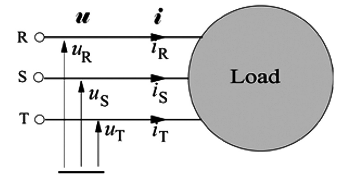

Fig. 2. Three-phase linear load with three-wire supply.

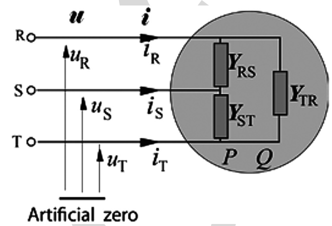

Fig. 3. Equivalent circuit of a three-phase load.

Q. Consequently, one of these admittances can have any value. In particular, we can choose zero.In these figures

$$\boldsymbol{u} \stackrel{\rm df}{=} \left[u_R,\ u_S,\ u_T \right]^T, \qquad \boldsymbol{i} \stackrel{\rm df}{=} \left[i_R,\ i_S,\ i_T \right]^T. \quad (1)$$

are three-phase vectors of the supply voltage and the load current. The complex rms (crms) values of line voltages and currents can be arranged into vectors

$$\boldsymbol{U} \stackrel{\rm df}{=} \left[\boldsymbol{U}_R\ \boldsymbol{U}_S\ \boldsymbol{U}_T \right]^T, \qquad \boldsymbol{I} \stackrel{\rm df}{=} \left[\boldsymbol{I}_R\ \boldsymbol{I}_S\ \boldsymbol{I}_T \right]^T. \quad (2)$$

The complex power of the load is equal to

$$\boldsymbol{U}^T \boldsymbol{I}^* = P + jQ \stackrel{\rm df}{=} \boldsymbol{C} = Ce^{j\varphi}. \quad (3)$$

Observe that $\boldsymbol{C}$ is not referred to as the "complex apparent power" which is a common custom in electrical engineering, since the apparent power S is defined in this paper according to [23] as

$$S \stackrel{\rm df}{=} \| \boldsymbol{u} \| \| \boldsymbol{i} \| = \sqrt{U_R^2 + U_S^2 + U_T^2}\sqrt{I_R^2 + I_S^2 + I_T^2}. \quad (4)$$

A justification for this selection can be found in [24]. At such definition of the apparent power,

$$S \geq \sqrt{P^2 + Q^2}. \quad (5)$$

Thus the magnitude of the complex power $\boldsymbol{C}$ is not equal to the apparent power S of the load. This issue was discussed in detail in [25].

Let us introduce three-phase unit symmetrical vectors of the positive sequence $\mathbf{1}^p$ and the negative sequence, $\mathbf{1}^n$, defined as

$$\mathbf{1}^p \stackrel{\rm df}{=} \begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{-j2\pi/3} \\ 1e^{j2\pi/3} \end{bmatrix}, \quad \mathbf{1}^n \stackrel{\rm df}{=} \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{j2\pi/3} \\ 1e^{-j2\pi/3} \end{bmatrix} \quad (6)$$

and shown in Fig. 4.

The supply voltage can be decomposed into the positive and the negative sequence symmetrical components of the crms

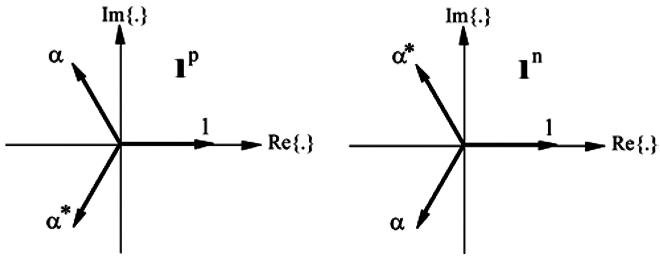

Fig. 4. Three-phase symmetrical unit vectors $\mathbf{1}^p$ and $\mathbf{1}^n$.

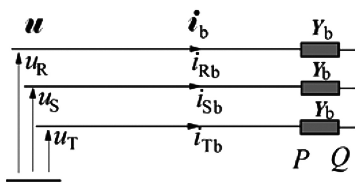

Fig. 5. Balanced load equivalent to original load with respect to the active and reactive powers P and Q.

values $\boldsymbol{U}^p$ and $\boldsymbol{U}^n$ equal to

$$\begin{bmatrix}\boldsymbol{U}^p\\ \boldsymbol{U}^n\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1, & \alpha, & \alpha^*\\ 1, & \alpha^*, & \alpha\end{bmatrix}\begin{bmatrix}\boldsymbol{U}_R\\ \boldsymbol{U}_S\\ \boldsymbol{U}_T\end{bmatrix}. \tag{7}$$

With respect to the active and reactive powers P and Q at the supply voltage $\boldsymbol{u}$, the unbalanced load shown in Fig. 2, is equivalent to a balanced load shown in Fig. 5 on the condition that its phase admittance is equal to

$$\begin{aligned}\boldsymbol{Y}_b &\stackrel{\text{df}}{=} G_b + jB_b \stackrel{\text{df}}{=} \frac{P-jQ}{\|\boldsymbol{u}\|^2}=\frac{\boldsymbol{C}^*}{\|\boldsymbol{u}\|^2}\\ &= \frac{\boldsymbol{C}^*_{\text{RS}}+\boldsymbol{C}^*_{\text{ST}}+\boldsymbol{C}^*_{\text{TR}}}{\|\boldsymbol{u}\|^2}=\frac{\boldsymbol{Y}_{\text{RS}}\,U^2_{\text{RS}}+\boldsymbol{Y}_{\text{ST}}\,U^2_{\text{ST}}+\boldsymbol{Y}_{\text{TR}}\,U^2_{\text{TR}}}{\|\boldsymbol{u}\|^2}.\end{aligned} \tag{8}$$

Since $\boldsymbol{Y}_b$ is the admittance of a balanced load, which is equivalent to the original load with respect to the active and reactive powers, it will be referred to as the equivalent balanced admittance. The equivalent balanced load draws the current

$$\boldsymbol{i}_b=\boldsymbol{i}_a+\boldsymbol{i}_r=\sqrt{2}\,\text{Re}\{\boldsymbol{I}_b\,\text{e}^{j\omega t}\}=\sqrt{2}\,\text{Re}\{\boldsymbol{Y}_b\boldsymbol{U}\,\text{e}^{j\omega t}\} \tag{9}$$

composed of the active current

$$\begin{aligned}\boldsymbol{i}_a &\stackrel{\text{df}}{=} G_b\,\boldsymbol{u}=\sqrt{2}\,\text{Re}\{G_b\,(\boldsymbol{U}^p+\boldsymbol{U}^n)\,e^{j\omega t}\}\\ &= \sqrt{2}\,\text{Re}\{G_b\,(\mathbf{1}^p\,\boldsymbol{U}^p+\mathbf{1}^n\boldsymbol{U}^n)\,e^{j\omega t}\}\end{aligned} \tag{10}$$

and the reactive current

$$\begin{aligned}\boldsymbol{i}_r\,(t) &\stackrel{\text{df}}{=} B_b\,\boldsymbol{u}\,(t+T/4)=\sqrt{2}\,\text{Re}\{jB_b\,(\boldsymbol{U}^p+\boldsymbol{U}^n)\,e^{j\omega t}\}\\ &= \sqrt{2}\,\text{Re}\{jB_b\,(\mathbf{1}^p\,\boldsymbol{U}^p+\mathbf{1}^n\boldsymbol{U}^n)\,e^{j\omega t}\}.\end{aligned} \tag{11}$$

The remaining current of the load after the current of the balanced load is subtracted, namely

$$\boldsymbol{i}-\boldsymbol{i}_b=\sqrt{2}\,\text{Re}\{(\boldsymbol{I}-\boldsymbol{I}_b)\,\text{e}^{j\omega t}\}\stackrel{\text{df}}{=}\boldsymbol{i}_u=\sqrt{2}\,\text{Re}\{\boldsymbol{I}_u\text{e}^{j\omega t}\} \tag{12}$$

is caused by the load imbalance. At symmetrical voltage, the active and reactive currents can be expressed in terms of equivalent admittance of the load, $\boldsymbol{Y}_e$, defined in [22] as

$$\boldsymbol{Y}_e \stackrel{\text{df}}{=} G_e + jB_e = \boldsymbol{Y}_{\text{ST}}+\boldsymbol{Y}_{\text{TR}}+\boldsymbol{Y}_{\text{RS}}. \tag{13}$$

The equivalent balanced admittance of the load $\boldsymbol{Y}_b$ in the presence of the supply voltage asymmetry differs from the admittance $\boldsymbol{Y}_e$ by an admittance $\boldsymbol{Y}_d$ referred to as the voltage asymmetry dependent admittance $\boldsymbol{Y}_d$, since the balanced admittance $\boldsymbol{Y}_b$ can be rearranged as follows

$$\begin{aligned}\boldsymbol{Y}_b &= \frac{\boldsymbol{C}^*}{\|\boldsymbol{u}\|^2}=\frac{\boldsymbol{C}^*_{\text{RS}}+\boldsymbol{C}^*_{\text{ST}}+\boldsymbol{C}^*_{\text{TR}}}{\|\boldsymbol{u}\|^2}\\ &=2\boldsymbol{Y}_e-\frac{3}{\|\boldsymbol{u}\|^2}\left(\boldsymbol{Y}_{\text{ST}}U_R^2+\boldsymbol{Y}_{\text{TR}}U_S^2+\boldsymbol{Y}_{\text{RS}}U_T^2\right)\stackrel{\text{df}}{=}\boldsymbol{Y}_e-\boldsymbol{Y}_d\end{aligned} \tag{14}$$

meaning

$$\boldsymbol{Y}_d=\frac{3}{\|\boldsymbol{u}\|^2}\left(\boldsymbol{Y}_{\text{ST}}U_R^2+\boldsymbol{Y}_{\text{TR}}U_S^2+\boldsymbol{Y}_{\text{RS}}U_T^2\right)-\boldsymbol{Y}_e. \tag{15}$$

The crms value of the current in line R is equal to

$$\boldsymbol{I}_R=\boldsymbol{Y}_{\text{RS}}\,(\boldsymbol{U}_R-\boldsymbol{U}_S)-\boldsymbol{Y}_{\text{TR}}\,(\boldsymbol{U}_T-\boldsymbol{U}_R) \tag{16}$$

and can be rearranged to the form

$$\boldsymbol{I}_R=\boldsymbol{Y}_e\boldsymbol{U}_R-(\boldsymbol{Y}_{\text{ST}}\boldsymbol{U}_R+\boldsymbol{Y}_{\text{TR}}\boldsymbol{U}_T+\boldsymbol{Y}_{\text{RS}}\boldsymbol{U}_S). \tag{17}$$

The crms values of the supply voltage can be expressed in terms of crms values of the voltage positive and negative sequence components $\boldsymbol{U}^p$ and $\boldsymbol{U}^n$, namely

$$\boldsymbol{U}_R=\boldsymbol{U}^p+\boldsymbol{U}^n,\ \ \boldsymbol{U}_S=\alpha^*\boldsymbol{U}^p+\alpha\boldsymbol{U}^n,\ \ \boldsymbol{U}_T=\alpha\boldsymbol{U}^p+\alpha^*\boldsymbol{U}^n \tag{18}$$

and formula (17) for the crms value of the current in line R can be rearranged to the form

$$\boldsymbol{I}_R=\boldsymbol{Y}_e\boldsymbol{U}_R+\boldsymbol{Y}^{\text{pn}}\boldsymbol{U}_R^p+\boldsymbol{Y}^{\text{np}}\boldsymbol{U}_R^n \tag{19}$$

where

$$\boldsymbol{Y}^{\text{pn}} \stackrel{\text{df}}{=} -(\boldsymbol{Y}_{\text{ST}}+\alpha\boldsymbol{Y}_{\text{TR}}+\alpha^*\boldsymbol{Y}_{\text{RS}}) \tag{20}$$

$$\boldsymbol{Y}^{\text{np}} \stackrel{\text{df}}{=} -(\boldsymbol{Y}_{\text{ST}}+\alpha^*\boldsymbol{Y}_{\text{TR}}+\alpha\boldsymbol{Y}_{\text{RS}}). \tag{21}$$

are the load unbalanced admittances for the voltages of the positive and the negative sequences, respectively.

Similarly, the crms values of the currents in lines S and T can be presented in the form

$$\boldsymbol{I}_S = \boldsymbol{Y}_e\boldsymbol{U}_S+\boldsymbol{Y}^{\text{pn}}\boldsymbol{U}_T^p+\boldsymbol{Y}^{\text{np}}\boldsymbol{U}_T^n \tag{22}$$

$$\boldsymbol{I}_T = \boldsymbol{Y}_e\boldsymbol{U}_T+\boldsymbol{Y}^{\text{pn}}\boldsymbol{U}_S^p+\boldsymbol{Y}^{\text{np}}\boldsymbol{U}_S^n. \tag{23}$$

These three crms values of the load currents can be expressed in the vector form

$$\boldsymbol{I} = \begin{bmatrix} \boldsymbol{I}_R \\ \boldsymbol{I}_S \\ \boldsymbol{I}_T \end{bmatrix} = \boldsymbol{Y}_e \boldsymbol{U} + \boldsymbol{1}^n \boldsymbol{Y}^{\text{pn}} \boldsymbol{U}^p + \boldsymbol{1}^p \boldsymbol{Y}^{\text{np}} \boldsymbol{U}^n \tag{24}$$

so that the vector of the crms values of the unbalanced currents is equal to

$$\boldsymbol{I}_u = \boldsymbol{I} - \boldsymbol{I}_b = (\boldsymbol{Y}_e - \boldsymbol{Y}_b)\,\boldsymbol{U} + \boldsymbol{1}^n \boldsymbol{Y}^{\text{pn}} \boldsymbol{U}^p + \boldsymbol{1}^p \boldsymbol{Y}^{\text{np}} \boldsymbol{U}^n, \tag{25}$$

or it can be rearranged as follows

$$\boldsymbol{I}_u = \boldsymbol{Y}_d \boldsymbol{U} + \boldsymbol{J}^n + \boldsymbol{J}^p \tag{26}$$

where

$$\boldsymbol{J}^n = \boldsymbol{1}^n\, \boldsymbol{Y}^{\text{pn}} \boldsymbol{U}^p, \qquad \boldsymbol{J}^p = \boldsymbol{1}^p\, \boldsymbol{Y}^{\text{np}} \boldsymbol{U}^n. \tag{27}$$

Thus, in the presence of asymmetry in the supply voltage $\boldsymbol{u}$ a three-phase unbalanced load draws the unbalanced current of the value:

$$\boldsymbol{i}_u = \sqrt{2}\,\text{Re}\left\{\boldsymbol{I}_u\, \text{e}^{j\omega t}\right\} = \sqrt{2}\,\text{Re}\left\{[\boldsymbol{Y}_d\, \boldsymbol{U} + \boldsymbol{J}^n + \boldsymbol{J}^p]\; \text{e}^{j\omega t}\right\}. \tag{28}$$

At given voltage $\boldsymbol{u}$ this current is dependent on three admittances of the load, namely: $\boldsymbol{Y}_d$, $\boldsymbol{Y}^{\text{pn}}$ and $\boldsymbol{Y}^{\text{np}}$.

Formula (26) shows that the voltage asymmetry dependent admittance $\boldsymbol{Y}_d$ affects the load unbalanced current, but the dependence of this admittance on this voltage asymmetry in formula (15) remains unclear. To find this dependence, observe that

$$\begin{aligned} U_R^2 &= \boldsymbol{U}_R \boldsymbol{U}_R^* = (\boldsymbol{U}^p + \boldsymbol{U}^n)\,(\boldsymbol{U}^p + \boldsymbol{U}^n)^* \\ &= U^{p2} + U^{n2} + 2\text{Re}\{\boldsymbol{U}^{p*}\text{U}^n\}. \end{aligned} \tag{29}$$

Similarly

$$U_S^2 = U^{p2} + U^{n2} + 2\text{Re}\{\alpha^* \boldsymbol{U}^{p*} \boldsymbol{U}^n\} \tag{30}$$

$$U_T^2 = U^{p2} + U^{n2} + 2\text{Re}\{\alpha \boldsymbol{U}^{p*} \boldsymbol{U}^n\}. \tag{31}$$

The crms values of the supply voltage symmetrical components $\boldsymbol{U}^p$ and $\boldsymbol{U}^n$ have in general the form

$$\boldsymbol{U}^p = U^p e^{j\phi}, \qquad \boldsymbol{U}^n = U^n e^{j\varphi}. \tag{32}$$

Therefore, if we denote

$$\boldsymbol{U}^{p*} \boldsymbol{U}^n = U^p U^n e^{j(\varphi - \phi)} \stackrel{\text{df}}{=} \boldsymbol{W} = W e^{j\psi} \tag{33}$$

admittance $\boldsymbol{Y}_d$, given by (15), can be expressed as

$$\boldsymbol{Y}_d = 2\,\frac{\boldsymbol{Y}_{\text{ST}}\text{Re}\{\boldsymbol{W}\} + \boldsymbol{Y}_{\text{TR}}\text{Re}\{\alpha^* \boldsymbol{W}\} + \boldsymbol{Y}_{\text{RS}}\text{Re}\{\alpha \boldsymbol{W}\}}{U^{p2} + U^{n2}}. \tag{34}$$

When the supply voltage asymmetry is specified by a complex asymmetry coefficient $\boldsymbol{a}$,

$$\frac{\boldsymbol{U}^n}{\boldsymbol{U}^p} = \frac{U^n e^{j\varphi}}{U^p e^{j\phi}} = \frac{U^n}{U^p} e^{j(\varphi - \phi)} \stackrel{\text{df}}{=} \boldsymbol{a} = a e^{j\psi} \tag{35}$$

then

$$\frac{\text{Re}\{\boldsymbol{W}\}}{U^{p2} + U^{n2}} = \frac{U^p U^n}{U^{p2} + U^{n2}}\,\text{Re}\left\{e^{j(\varphi - \phi)}\right\} = \frac{a}{1 + a^2}\cos\psi, \tag{36}$$

and consequently, the asymmetry dependent unbalanced admittance $\boldsymbol{Y}_d$ can be rearranged to the form

$$\begin{aligned} \boldsymbol{Y}_d = \;& \frac{2a}{1 + a^2}\Bigg[\boldsymbol{Y}_{\text{ST}}\cos\psi + \boldsymbol{Y}_{\text{TR}}\cos\left(\psi - \frac{2\pi}{3}\right) \\ & + \boldsymbol{Y}_{\text{RS}}\cos\left(\psi + \frac{2\pi}{3}\right)\Bigg]. \end{aligned} \tag{37}$$

which reveals its dependence on the supply voltage asymmetry. In particular, when the supply voltage is symmetrical and of the positive sequence then $a = 0$, and consequently, this admittance is zero. It is also zero for balanced loads.

Equation (12) combined with (9) results in the load current decomposition into the active, reactive and unbalanced components, such that

$$\boldsymbol{i}(t) = \boldsymbol{i}_a(t) + \boldsymbol{i}_r(t) + \boldsymbol{i}_u(t). \tag{38}$$

They are associated with distinctive physical phenomena in the circuit. Active current $\boldsymbol{i}_a(t)$ is associated with permanent transfer of energy to the load. Reactive current $\boldsymbol{i}_r(t)$ is associated with the phenomenon of the phase-shift between the load voltage and the current. Unbalanced current $\boldsymbol{i}_u(t)$ is associated with the load imbalance. Therefore, these three currents are regarded as the load Currents' Physical Components (CPC).

It was proven in [26] that the active, reactive and the unbalanced currents are mutually orthogonal and consequently, their three-phase rms values $\|\,.\,\|$ satisfy the relationship

$$\|\,\boldsymbol{i}\,\|^2 = \|\,\boldsymbol{i}_a\|^2 + \|\,\boldsymbol{i}_r\|^2 + \|\,\boldsymbol{i}_u\,\|^2. \tag{39}$$

with

$$\|\,\boldsymbol{i}_a\,\| = G_b\,\|\,\boldsymbol{u}\,\|, \quad \|\,\boldsymbol{i}_r\,\| = |B_b|\,\|\,\boldsymbol{u}\,\|. \tag{40}$$

$$\|\,\boldsymbol{i}_u\,\| = \sqrt{\|\,\boldsymbol{i}\,\|^2 - \|\,\boldsymbol{i}_a\|^2 - \|\,\boldsymbol{i}_r\,\|^2}. \tag{41}$$

Multiplying (39) by the square of the three-phase rms value of the load voltage $\|\,\boldsymbol{u}\,\|$ the power equation of an unbalanced load with asymmetrical voltage is obtained,

$$S^2 = P^2 + Q^2 + D_u^2 \tag{42}$$

with

$$Q \stackrel{\text{df}}{=} \pm\,\|\,\boldsymbol{u}\,\| \times \|\,\boldsymbol{i}_r\,\| = -B_b\|\,\boldsymbol{u}\,\|^2 \tag{43}$$

$$D_u \stackrel{\text{df}}{=} \|\,\boldsymbol{u}\,\| \times \|\,\boldsymbol{i}_u\,\|. \tag{44}$$

Power equation (42), when compared with the power equation for systems with symmetrical supply voltage developed in [22], shows that the structure of this equation is not affected by the voltage asymmetry. The same is with definitions of all powers. The difference is in definitions of the Currents' Physical Components. The voltage asymmetry affects their dependence on the load parameters.

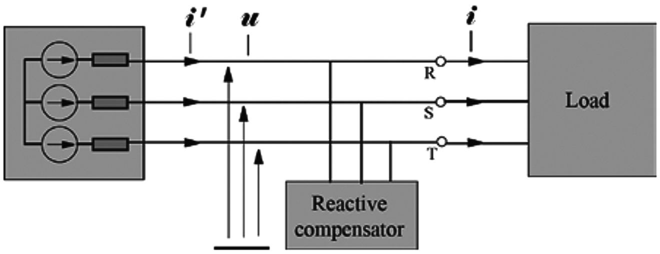

Fig. 6. Three-phase system with reactive compensator.

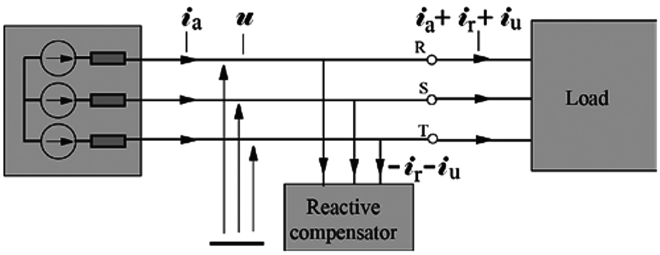

Fig. 7. Three-phase system with reactive balancing compensator, which draws the negative of reactive und unbalanced currents.

III. Parameters of Balancing Compensator

Presence of the reactive and unbalanced currents in the supply current $\boldsymbol{i}$ reduces the power factor λ since its value is equal to

$$\lambda \stackrel{\mathrm{df}}{=} \frac{P}{S} = \frac{\| \boldsymbol{i}_a \|}{\| \boldsymbol{i} \|} = \frac{1}{\sqrt{1 + \left(\frac{\|\boldsymbol{i}_r\|}{\|\boldsymbol{i}_a\|}\right)^2 + \left(\frac{\|\boldsymbol{i}_u\|}{\|\boldsymbol{i}_a\|}\right)^2}}. \tag{45}$$

Reduction of their three-phase rms values enables the power factor improvement.

In the case of ultra-high power arc furnaces supplied from transformers of relatively low power, reduction of the reactive and unbalanced currents can substantially reduce the voltage asymmetry and increase its rms value at the arc furnace terminals. Consequently, the active power P and active current three-phase rms value $\| \boldsymbol{i}_a \|$ can change as well.

Parameters of a reactive compensator, connected as shown in Fig. 6, can be found in an optimization process which is aimed at minimization of the supply current three-phase rms value $\| \boldsymbol{i}' \|$.

Power properties of the load are not needed for such optimization. Instead, the effect of the voltage change upon the load current, i.e., the load model is needed. In the case of an arc furnace, unknown and fast varying arc geometry and physical conditions in the furnace shell make such optimization questionable.

When the reactive and unbalanced currents of the load are known, no optimization procedure is needed, however. Parameters of a compensator can be calculated from the condition that it will draw the negative reactive and unbalanced currents, as shown in Fig. 7.

Observe that the compensator also changes the load and the compensator voltage, so that recalculation of the compensator

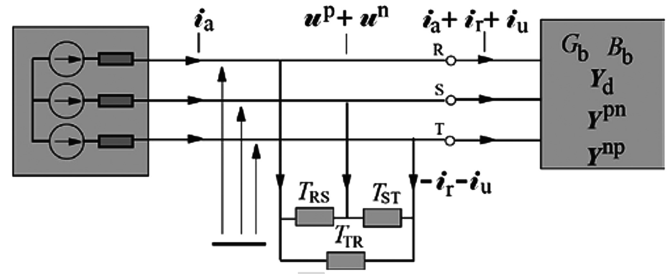

Fig. 8. Three-phase system with a compensator of Δ structure.

parameters might be needed. It means that calculation of the compensator parameters is an iterative process.

We can assume that a compensator in Δ structure is built of three lossless reactive elements of susceptance T_{RS}, T_{ST}, and T_{TR} as shown in Fig. 8.

Current decomposition into CPC, dependence of these currents on the load parameters G_b, B_b, $\boldsymbol{Y}_d$, $\boldsymbol{Y}^{\mathrm{pn}}$ and $\boldsymbol{Y}^{\mathrm{np}}$ and the voltage asymmetry coefficient $\boldsymbol{a}$, as presented above, can be reversed to calculate parameters of a compensator that will draw the current $-\boldsymbol{i}_r - \boldsymbol{i}_u$.

Resulting from (8) the compensator balanced susceptance, denoted by B_{Cb}, is equal to

$$B_{\mathrm{Cb}} = \frac{T_{\mathrm{RS}} U_{\mathrm{RS}}^2 + T_{\mathrm{ST}} U_{\mathrm{ST}}^2 + T_{\mathrm{TR}} U_{\mathrm{TR}}^2}{\| \boldsymbol{u} \|^2} \tag{46}$$

while unbalanced admittances of the compensator

$$\boldsymbol{Y}_C^{\mathrm{pn}} = -j\,(T_{\mathrm{ST}} + \alpha T_{\mathrm{TR}} + \alpha^* T_{\mathrm{RS}}) \tag{47}$$

$$\boldsymbol{Y}_C^{\mathrm{np}} = -j\,(T_{\mathrm{ST}} + \alpha^* T_{\mathrm{TR}} + \alpha T_{\mathrm{RS}}), \tag{48}$$

and the asymmetry dependent unbalanced admittance

$$\boldsymbol{Y}_{\mathrm{Cd}} = j\frac{2a}{1+a^2}\left[T_{\mathrm{ST}}\cos\psi + T_{\mathrm{TR}}\cos\left(\psi - \frac{2\pi}{3}\right) + T_{\mathrm{RS}}\cos\left(\psi + \frac{2\pi}{3}\right)\right]. \tag{49}$$

The compensator reduces the reactive current to zero at the condition that

$$B_{\mathrm{Cb}} + B_b = 0, \tag{50}$$

and it reduces the unbalanced current to zero on the condition that the sum of the unbalanced current of the load, specified by (26), and the compensator are equal to zero. i.e.,

$$(\boldsymbol{Y}_{\mathrm{Cd}} + \boldsymbol{Y}_d)\,\boldsymbol{U} + \boldsymbol{1}^n\,(\boldsymbol{Y}_C^{\mathrm{pn}} + \boldsymbol{Y}^{\mathrm{pn}})\,\boldsymbol{U}^p + \boldsymbol{1}^p(\boldsymbol{Y}_C^{\mathrm{np}} + \boldsymbol{Y}^{\mathrm{np}})\boldsymbol{U}^n = \boldsymbol{0}. \tag{51}$$

This equation has to be satisfied by each line, in particular, for line R it has the form

$$(\boldsymbol{Y}_{\mathrm{Cd}} + \boldsymbol{Y}_d)\,\boldsymbol{U}_R + (\boldsymbol{Y}_C^{\mathrm{pn}} + \boldsymbol{Y}^{\mathrm{pn}})\,\boldsymbol{U}^p + (\boldsymbol{Y}_C^{\mathrm{np}} + \boldsymbol{Y}^{\mathrm{np}})\,\boldsymbol{U}^n = 0 \tag{52}$$

Since coefficients and variables in this equation are complex numbers, it has to be satisfied for both the real and the imaginary

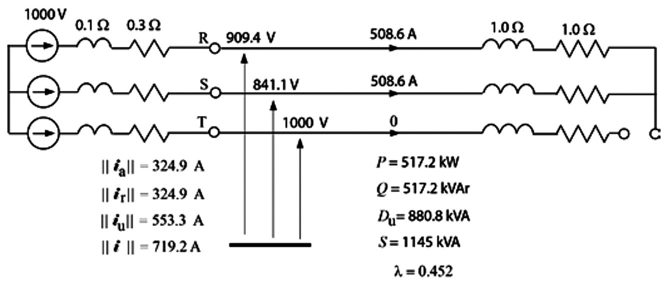

Fig. 9. Results of the circuit analysis.

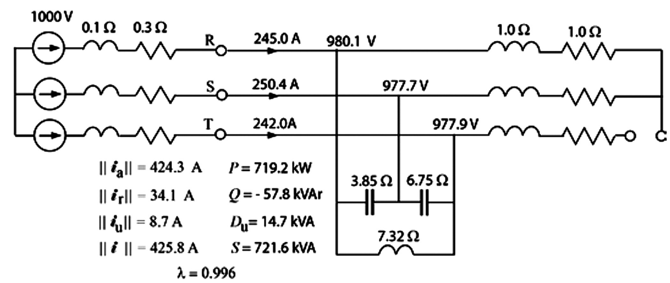

Fig. 10. System with compensator and compensation results.

parts. Thus (52) represents two equations. Consequently, (50) and (52) provide three equations needed for calculating three unknown susceptances $T_{\rm RS}$, $T_{\rm ST}$, and $T_{\rm TR}$ of the balancing compensator. Unfortunately, since these unknown susceptances are hidden, according to (47), (48) and (49), inside admittances $\boldsymbol{Y}_C^{\rm pn}, \boldsymbol{Y}_C^{\rm np}, \boldsymbol{Y}_{\rm Cd}$, these equations with respect to unknown parameters $T_{\rm RS}, T_{\rm ST}$, and $T_{\rm TR}$ are very complex. Arranging them into explicit form with respect to unknown parameters requires much more space than available in this paper. Therefore, only final equations are provided below. Assuming that $T_{\rm RS} = x$, $T_{\rm ST} = y$, $T_{\rm TR} = z$, these equations can be written in the form

$$U_{\rm RS}^2 x + U_{\rm ST}^2 y + U_{\rm TR}^2 z = -B_b \| \boldsymbol{u} \|^2 \tag{53}$$

$$\operatorname{Re}\{\boldsymbol{F}_1\} x + \operatorname{Re}\{\boldsymbol{F}_2\} y + \operatorname{Re}\{\boldsymbol{F}_3\} z = -\operatorname{Re}\{\boldsymbol{F}_4\} \tag{54}$$

$$\operatorname{Im}\{\boldsymbol{F}_1\} x + \operatorname{Im}\{\boldsymbol{F}_2\} y + \operatorname{Im}\{\boldsymbol{F}_3\} z = -\operatorname{Im}\{\boldsymbol{F}_4\} \tag{55}$$

where

$$F_1 \stackrel{\rm df}{=} c_3 \left(1 + a e^{j\psi}\right) - j\left(\alpha^* + \alpha a e^{j\psi}\right) \tag{56}$$

$$F_2 \stackrel{\rm df}{=} c_1 \left(1 + a e^{j\psi}\right) - j\left(1 + a e^{j\psi}\right) \tag{57}$$

$$F_3 \stackrel{\rm df}{=} c_2 \left(1 + a e^{j\psi}\right) - j\left(\alpha + \alpha^* a e^{j\psi}\right) \tag{58}$$

$$F_4 \stackrel{\rm df}{=} \left(1 + a e^{j\psi}\right) \boldsymbol{Y}_d + \boldsymbol{Y}^{\rm pn} + \left(1 + a e^{j\psi}\right) \boldsymbol{Y}^{\rm np} \tag{59}$$

while

$$c_1 \stackrel{\rm df}{=} j\frac{2a\cos\psi}{1 + a^2} \tag{60}$$

$$c_2 \stackrel{\rm df}{=} j\frac{2a\cos(\psi - 120^\circ)}{1 + a^2} \tag{61}$$

$$c_3 \stackrel{\rm df}{=} j\frac{2a\cos(\psi - 240^\circ)}{1 + a^2}. \tag{62}$$

Numerical Illustration: Let us calculate parameters of a balancing compensator for an arc furnace approximated by a linear load, shown in Fig. 9, with extinguished arc in line T. The furnace is supplied from a transformer with relatively low power. Its short-circuit parameters, recalculated to the secondary side, are shown in Fig. 9. It is assumed that the internal voltage of the supply $\boldsymbol{e}$ is symmetrical and for line R its crms value is equal to $\boldsymbol{E}_R = 1000 \exp\{j0^\circ\}$ V. At such assumptions the arc furnace voltage asymmetry is caused only by asymmetry of the furnace current.

When the arc furnace under consideration is balanced, i.e., all three arcs are ignited, then the furnace operates at voltage rms value $U_R = 830.4$ V with the active power $P = 1.03$ MW and the power factor $\lambda = 0.707$.

When the arc in phase T is not ignited, as assumed in this illustration, then

$$\boldsymbol{Y}_{\rm RS} = (0.25 - {\rm j}0.25)\ {\rm S}; \quad \boldsymbol{Y}_{\rm ST} = 0; \quad \boldsymbol{Y}_{\rm TR} = 0$$

$$\boldsymbol{U}_R = 909.4 e^{-j7.9^\circ} \quad \boldsymbol{U}_S = 841.1 e^{-j118.4^\circ}\ {\rm V}, \quad \boldsymbol{U}_T = 1000 e^{j120^\circ}\ {\rm V}.$$

Powers and three-phase rms values || . || of the Currents' Physical Components of the furnace current are shown in Fig. 9. The load power factor is $\lambda = 0.45$.

The crms values of the positive and the negative sequence voltages at the load terminals are

$$\boldsymbol{U}^p = 914.5 e^{-j2.2^\circ}\ {\rm V}, \quad \boldsymbol{U}^n = 92.85 e^{-j98.2^\circ}\ {\rm V},$$

and consequently, the coefficient of the load voltage asymmetry

$$\boldsymbol{a} = a e^{j\psi} = 0.102 e^{-j96.1^\circ}.$$

The load admittances needed for the compensator design are equal to

$$\boldsymbol{Y}_b = (0.204 + j0.204)\ {\rm S}, \quad \boldsymbol{Y}_d = 0.0649 e^{-j45.0^\circ}\ {\rm S},$$

$$\boldsymbol{Y}^{\rm pn} = 0.354 e^{j15.0^\circ}\ {\rm S}, \quad \boldsymbol{Y}^{\rm np} = 0.354 e^{-j105.0^\circ}\ {\rm S}.$$

For such load admittances, the compensator equations (53)–(55) result in the compensator line-to-line susceptances:

$$T_{\rm RS} = 0.259\ {\rm S}, \quad T_{\rm ST} = 0.148\ {\rm S}, \quad T_{\rm TR} = -0.138\ {\rm S}.$$

The system with the compensator and compensation results are shown in Fig. 10.

These results show that in spite of the fact that the reactive and unbalanced currents were substantially reduced, full compensation was not achieved. This was because the compensator changed the voltage at its terminals, so that the compensator susceptances do not satisfy (53)–(55), i.e., these susceptances do not have the right values. Their calculation has to be repeated for voltages in the compensator's presence. The compensator with modified susceptances will again change the voltage, however.

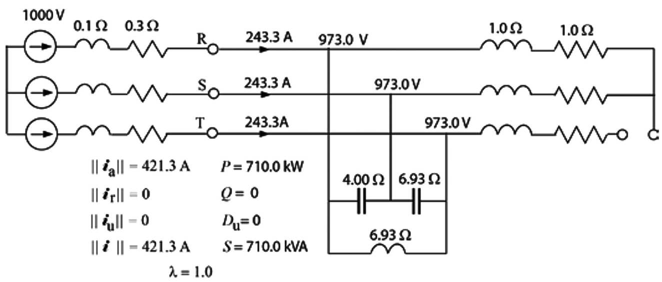

Fig. 11. Final results of compensation.

This leads to iterative calculations. In the situation as assumed in this illustration, the third iteration results in the compensator susceptances

$$T_{\rm RS} = 0.250\ {\rm S}, \quad T_{\rm ST} = 0.144\ {\rm S}, \quad T_{\rm TR} = -0.144\ {\rm S}.$$

The effects of compensation are shown in Fig. 11.

This illustration validates the suggested method and the compensator equations (53)–(55). It confirms that compensator LC parameters can be calculated with the method presented even in the situation where the supply voltage is asymmetrical. Such a compensator is capable of eliminating entirely both the reactive and unbalanced currents, thus improving the power factor to unity.

Performance of such a balancing compensator in a real environment is affected by harmonics, however. Moreover, the compensator has to have an adaptive property. These issues cannot be covered in the frame of a single paper, and consequently, they are beyond the scope of this paper.

IV. Conclusion

The presented method of calculation of LC parameters of a reactive balancing compensator in the presence of supply voltage asymmetry fills a theoretical gap of the power theory. It can be applied for design of a balancing compensator in unbalanced systems composed of large aggregates of single-phase loads or loads that by nature are unbalanced, such as traction systems. Since AC arc furnaces are fast varying loads with high current distortion, the presented method cannot be used directly for compensators for such furnaces' design, without further studies on the effect of harmonics upon the compensator parameters and without integrating the method with adaptive compensation. Nonetheless, the method developed in this paper could be regarded as an initial step towards developing compensators for balancing and reducing the reactive power of ultra-high power AC arc furnaces.

References

[1] G. Mazzanti, L. Lusetti, and A. Fragiacomo, "The state of the art about electric arc furnaces for steel use and the compensation of their perturbing effects on the grid," in *Proc. Int. Symp. Power Electron., Elect. Drives, Autom. Motion*, 2012, pp. 1277–1282.

[2] H. Samet, T. Ghanbari, and J. Ghaisari, "Maximum performance of electric arc furnace by optimal setting of the series reactor and transformer taps using a nonlinear model," *IEEE Trans. Power Del.*, vol. 30, no. 2, pp. 764–772, Apr. 2015.

[3] R. Grünbaum, P. Ekström, and A.-A. Hellström, "Powerful reactive power compensation of a very large electric arc furnace," *Proc. of the 4th Int. Conf. Power Eng., Energy Elect. Drives*, Istanbul, Turkey, 2013.

[4] A. A. Nikolaev, G. P. Kornilov, T. R. Khramshin, I. Akcay, and Y. Gok, "Application of static Var compensator of ultra-high power electric arc furnace for voltage drops compensation in factory power supply system of metallurgical enterprise," in *Proc. Elect. Power Energy Conf.*, 2014, pp. 235–241.

[5] T. Zheng, and E. B. Makram, "An adaptive arc furnace model," *IEEE Trans. Power Del.*, vol. 15, no. 3, pp. 931–939, Jul. 2000.

[6] I. Vervenne, K. Van Reusel, and R. Belmans, "Electric arc furnace modelling from a "Power Quality" point of view," *Proc. of the 9th Int. Conf. Elect. Power Qual. Utilisation*, Barcelona, Spain, 2007.

[7] M. T. Esfahani and B. Vahidi, "A new stochastic model of electric arc furnace based on hidden Markov Model: a study of its effects on the power system," *IEEE Trans. Power Del.*, vol. 27, no. 4, pp. 1893–1901, Oct. 2012.

[8] S. M. M. Agah, S. H. Hosseinian, H. A. Abyaneh, and N. Moaddabi, "Parameter identification of arc furnace based on stochastic nature of arc length using two-step optimization technique," *IEEE Trans. Power Del.*, vol. 25, no. 4, pp. 2859–2867, Oct. 2010.

[9] T.-H. Fu, and C.-J. Wu, "Load characteristics analysis of ac and dc arc furnaces using various power definitions and statistic method," *IEEE Trans. Power Del.*, vol. 17, no. 4, pp. 1099–1105, Oct. 2002.

[10] I. Masoudipour and H. Samet, "Comparison of various reactive power definitions in non sinusoidal networks with the practical data of electrical arc furnace," *Proc. of the 22nd Int. Conf. Elect. Distrib. CIRED*, Stockholm, Sweden, 2013.

[11] K. Jagiela, J. Rak, M. Gala, and M. Kepinski, "Identification of electric power parameters of ac arc furnace low voltage system," in *Proc. 14th Int. Conf. Harmon. Qual. Power*, 2010, pp. 1–7.

[12] F. Martell, A. R. Izaguirre, and M. E. Macias, "CPC power theory for analysis of arc furnaces," *Proc. of the XII Int. School Nonsinusoidal Currents and Compensation, ISNCC*, Lagow, Poland, 2015.

[13] A. R. Izaguirre, M. E. Macias, and F. Martell, "Accurate CPC power analysis under extreme EAF's distortion conditions," *Proc. of the XII Int. School Nonsinusoidal Currents and Compensation*, Lagow, Poland, 2015.

[14] Ch. P. Steinmetz, *Theory and Calculations of Electrical Apparatus*, New York, USA: McGraw-Hill, 1917.

[15] M. Grandpierre, and B. Trannoy, "A static power device to rebalance and com-pensate reactive power in three-phase network: Design and control," in *Proc. IEEE Ind. Appl. Soc. Annu. Meeting*, 1977, pp. 127–135.

[16] G. Klinger, "LC Kompensation und symmetrierung für Mehrphasensysteme mit beliebigen Spannungsverlauf," *ETZ Archiv*, no. 2, pp. 57–61, 1979.

[17] T. J. E. Miller, *Reactive Power Control in Electric Systems*, New York, USA: Wiley, 1982.

[18] L. S. Czarnecki, "Reactive and unbalanced currents compensation in three-phase asymmetrical circuits under nonsinusoidal conditions," *IEEE Trans. Instrum. Meas.*, vol. IM-38, no. 3, pp. 754–759, Jun. 1989.

[19] S.-Y. Lee, and C.-J. Wu, "On-line reactive power compensation schemes for unbalanced three-phase four-wire distribution feeders," *IEEE Trans. Power Del.*, vol. 8, no. 4, pp. 1958–1965, Oct. 1993.

[20] M. A. S. Masoum, P. S. Moses, and A. S. Masoum, "Derating of asymmetric three-phase transformers serving unbalanced nonlinear loads," *IEEE Trans. Power Del.*, vol. 23, no. 4, pp. 2033–2041, Oct. 2008.

[21] D. Mayer, and P. Kropik, "New approach to symmetrization of three-phase network," *J. Elect. Eng.*, vol. 56, no. 5–6, pp. 156–161, 2005.

[22] L. S. Czarnecki, "Orthogonal decomposition of the currents in a three-phase non-linear asymmetrical circuit with a nonsinusoidal voltage source," *IEEE Trans. Instrum. Meas.*, vol. IM-37, no. 1, pp. 30–34, Mar. 1988.

[23] F. Buchholz, "Die drehstrom-scheinleistung bei unglaichmäßiger Belastung der drei Zweige," *Licht Kraft, Zeitschrift Elektr. Energie-Nutzung*, no. 2, pp. 9–11, Jan. 1922.

[24] L. S. Czarnecki, "Energy flow and power phenomena in electrical circuits: illusions and reality," *Archiv Elektrot.*, vol. 82, no. 3–4, pp. 119–126, Mar. 2000.

[25] L. S. Czarnecki, and P. M. Haley, "Unbalanced power in four-wire systems and its reactive compensation," *IEEE Trans. Power Del.*, vol. 30, no. 1, pp. 53–63, Feb. 2015.

[26] L. S. Czarnecki, and P. D. Bhattarai, "Currents' physical components (CPC) in three-phase systems with asymmetrical voltage," *Przegląd Elektrotech-Niczny*, no. 6, pp. 40–47, 2015.

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