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CPC–Founded Clarification of Decline of the Effectiveness of the Energy Transfer in Power Systems

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***Abstract*—The paper is focused on physical phenomena that degrade the effectiveness of the energy transfer in electrical systems when voltages and currents are nonsinusoidal and asymmetrical. The Currents' Physical Components (CPC) – based power theory provides fundamentals for the analysis of the electric energy flow. This analysis identifies seven distinctive physical phenomena that affect this flow. According to the CPC concept, seven current components are associated with these phenomena. Only one of them contributes to the useful transfer of the energy. Remaining six components only degrade the effectiveness of this transfer. They could be reduced by various compensators and the CPC-based power theory provides fundamentals for their synthesis and control. The results presented in the paper are valid for electrical systems of any complexity, meaning for single- and for three-phase systems and for any loads, meaning for linear and for nonlinear loads.**

Keywords* – *power definitions, scattered current, unbalanced loads, unbalanced power, harmonics generating loads, HGL, asymmetry, power factor.

I. INTRODUCTION

The electric energy providers supply loads with the voltage and current values required by customers. These values specify the apparent power S at the load terminals. To provide this power for all customers the provider has to produce sufficient amount of electric energy, distribute it, and perform a number of very complex tasks needed for a reliable supply, including power system development, staff training, and research. The cost of all of them depends on the apparent power S.

Users of the electric energy are charged financially for the amount of it, which is an integral of the active power P over a billing interval. The active power P is usually lower than the apparent power S, however, i.e.,

$$P < S.$$

The provider of energy provides it because it is paid by customers. Apparently, customers pay only for the received energy. In fact, all costs in power systems, including profits of the energy providers, are paid by their customers. It is done by selection of a fair price of energy unit and extra payments. It means that a customer pays for the difference between the apparent power S and the active power P at the customer's load terminals, even if the cost of this difference is hidden in the price of the electric energy unit. Therefore, the difference between these two powers is of the utmost importance for the power systems economy. Equally important are methods of reduction of this difference by compensation. The ratio of the active and apparent powers

$$\lambda = \frac{P}{S}$$

known as a ***power factor***, specifies the effectiveness of the energy transfer from the electric energy provider to the customer load. Its value is crucially important for the power system economy. It depends on electrical properties of the load, which can degrade its value, but negative effects of this degradation are on the side of the energy provider. These effects can reduce revenues of the provider and consequently, it can have some policies aimed at enforcing the customer for the power factor improvement.

It was concluded by the end of the nineteen century that the difference, in a square, between the apparent S and the active P powers in single-phase systems with sinusoidal voltages and currents can be explained in terms of the reactive power Q, which satisfies the relationship

$$S^2 - P^2 = Q^2$$

and occurs because of the phase-shift between the supply voltage and the load current. In single-phase systems with sinusoidal voltages and currents, only the reactive power Q contributes to the difference between the apparent and the active powers. This difference could be reduced by a capacitor or by an under-excited synchronous machine.

The conclusion that the reactive power Q is responsible for the apparent power S increase was challenged in 1892 by Charles Steinmetz [1] in an experiment shown in Fig. 1. It occurred that the reactive power Q in this experiment had a zero value.

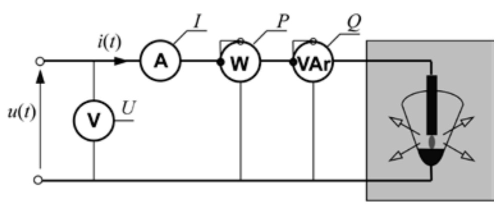

Figure 1. Steinmetz' experiment.

This experiment has demonstrated that in spite of zero reactive power Q, the apparent power S can be higher than the load active power P, thus

$$S^2 \geq P^2 + Q^2\,.$$

This inequality raises two main questions. Namely

- Why can the apparent power S be higher than the active power P? What phenomena in the load are responsible for this inequality?

- How can the difference between the apparent power S and the active power P be reduced by a compensator?

These two questions are crucial for the energy transfer effectiveness in electrical systems. The first question is of a cognitive nature. The second is very practical.

The arc bulb in the Steinmetz's experiment had the power of only a few hundred watts, thus his observation had an academic rather than practical importance. The present-time Steinmetz's experiment can be run with a high power ac three-phase arc furnace, shown in Fig. 2.

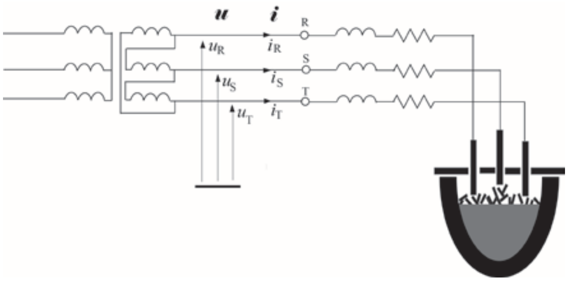

Figure 2. AC arc furnace and its supply structure.

Such an arc furnace can have the power in a range of 750 MVA with line currents in a range of 600 kA. It is equivalent approximately to the power of a million population city. The annual bill for the electric energy of such a furnace could be in the order of 500 million dollars. We should be aware, however, that even if the power of such a furnace is comparable to the power of a one million population city, the voltages and currents at the supply of such a city are almost sinusoidal and symmetrical with a power factor which could be close to unity. In the case of the arc furnace, inductors inserted in the supply lines for the arc stabilization, reduce the power factor to the level of 0.7. When the arc is not ignited, one of the line currents disappears, even for such a short interval as half of the period. Strong asymmetry of the supply current and distortion suddenly occurs. The second harmonic could have a level of one-third of the fundamental harmonic. The power factor can drop even to 0.4.

II. SCHOOLS OF POWER THEORY

Apparently easy questions inspired by the Steinmetz observation have occurred to be some of the most difficult questions of electrical engineering. Hundreds of scientists have attempted to explain and describe power properties of loads with nonsinusoidal voltages and currents and develop methods of compensation. Hundreds of papers were published. A number of "schools" of power theory were established. The most known were power theories suggested by Budeanu in 1927 [5], by Fryze in 1931 [7], by Shepherd and Zakikhani in 1972 [15], by Kusters and Moore in 1980 [16], by Nabae and Akagi in 1984 [19], by Depenbrock in 1993 [25] and by Tenti in 2003 [34]. There are also numerous attempts aimed at development of the power theory that cannot be classified as schools, such as, for example [3, 4, 6, 8, 9, 10, 11, 12, 13, 14, 20, 30, 33, 39, 40, 41, 48]. Some important discussions [9, 17, 18, 20, 21, 24, 27, 28, 30, 31, 33, 35, 36, 38, 40, 41, 42, 43, 47] should be mentioned as well.

Most of the research on the power properties of electrical systems and power theory "schools" were developed at the time when the voltage and current distortions were academic rather than practical issues. It was a time when high power rectifiers needed in chemical industry, relatively low power arc furnaces, and ac/dc converters for HVDC transmission were almost the only sources in power systems of the current distortion.

Now, with the development of the power electronics and consequently, variable speed drives, electronic ballast for fluorescent lamps, microwaves, video equipment, computers and printers, the current and voltage distortion is omnipresent in distribution systems. Most of the home, commercial and industrial equipment generates current harmonics. Not very often the energy is transferred at sinusoidal voltages and currents. This could be even more visible in micro-grids, where even the supply voltage is not provided by synchronous generators, but by PV panels or wind-driven generators. Saturation of such a grid with electronic-based loads could be even higher than in traditional distribution systems

Unfortunately, in the presence of the voltage distortion, all power theories cited above were not capable of describing power properties for even a simple RL load as shown in Fig. 3.

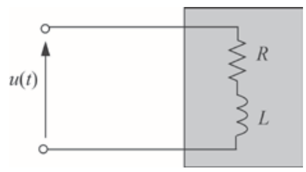

Figure 3. RL load.

Explanation of the energy flow phenomena in single- and in three-phase systems with nonsinusoidal voltages and currents was eventually provided in the frame of the Currents' Physical Components (CPC) – based power theory. It provides the answer to the question on physical reasons of degradation of the effectiveness of the energy transfer in electrical systems.

III. CPC-BASED POWER THEORY DEVELOPMENT

The Currents' Physical Components – based Power Theory is founded on a few major concepts. First of all, for studies on the effectiveness of the energy transfer the supply current of the load is much more fundamental quantity than the load power. The energy loss at delivery is caused by the supply current, but not by powers. Next, decomposition of the supply current into components associated with distinctive physical phenomena is the very core of this decomposition. Just this requirement gave the name of this approach to the power theory development: ***Currents' Physical Components – based Power Theory.*** It is next expected that Currents' Physical Components are mutually orthogonal. When this condition is satisfied, then the physical phenomena in the load contribute to the supply current rms value increase independently of each other. These components should be measured or calculated by measuring voltages and currents at the load terminals.

To make a synthesis of a compensator for the power factor improvement possible, the CPC should be more over related to the compensator parameters.

A diagram which illustrates the CPC-based power theory development, referenced to other research on this theory is shown in Fig. 4.

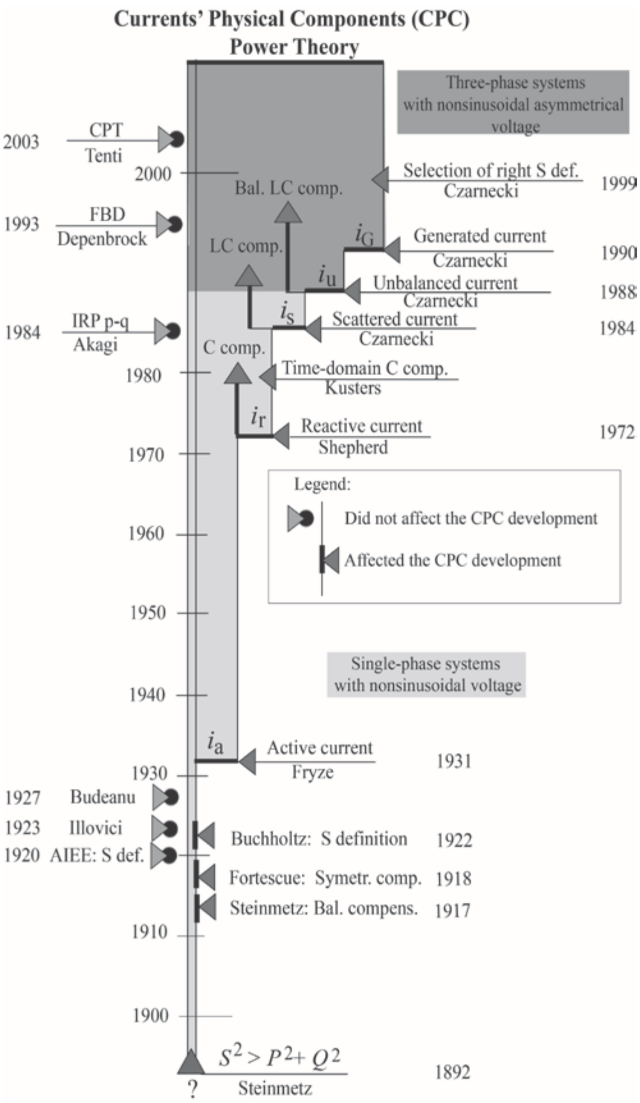

Figure 4. Diagram which illustrates the development of the CPC-based power theory.

IV. MAJOR SYMBOLS

Before a draft of the CPC–based power theory is presented in this paper, the major symbols have to be compiled and explained.

The supply voltage of a single-phase load can be expressed as a Fourier Series in a complex form

$$u(t) = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n\, e^{jn\omega_1 t}, \quad \boldsymbol{U}_n = U_n\, e^{j\alpha_n} \tag{1}$$

where N is the set of orders n of the voltage harmonic and $\boldsymbol{U}_n$ is the complex rms (crms) value of these harmonics.

The load current can be presented in the form

$$i(t) = I_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_n\, e^{jn\omega_1 t}, \quad \boldsymbol{I}_n = I_n\, e^{j\beta_n}\,. \tag{2}$$

Scalar product of two periodic quantities $x(t)$ and $y(t)$ with the same period T is defined as

$$(x, y) = \frac{1}{T}\int_0^T x(t)\, y(t)\, dt = \mathrm{Re}\sum_{n\in N_0} \boldsymbol{X}_n \boldsymbol{Y}_n^* \tag{3}$$

where N_0 is the set N along with the element $n = 0$.

The paper will show how the CPC concept enables to draw conclusions on the physical reasons of degradation the effectiveness of the energy transfer in electrical systems with linear time-invariant (LTI) loads.

The rms value of the voltage and current, denoted in general by $x(t)$, is defined as

$$||x|| = \sqrt{\frac{1}{T}\int_0^T x^2(t)\, dt} = \sqrt{\mathrm{Re}\sum_{n\in N_0} \boldsymbol{X}_n \boldsymbol{X}_n^*} = \sqrt{\sum_{n\in N_0} X_n^2}\,. \tag{4}$$

Two periodic quantities are mutually orthogonal when their scalar product is zero, i.e.,

$$(x, y) = \mathrm{Re}\sum_{n\in N_0} \boldsymbol{X}_n \boldsymbol{Y}_n^* = 0\,. \tag{5}$$

If a quantity $z(t)$ is a sum of components i.e.,

$$z(t) \equiv x(t) + y(t) \tag{6}$$

which are mutually orthogonal, then their rms values satisfy the relationship

$$||z||^2 = ||x||^2 + ||y||^2\,. \tag{7}$$

In the case of three-phase systems, the voltages and currents at the load terminals R, S and T can be arranged in three-phase vectors

$$\boldsymbol{u}(t) \stackrel{\mathrm{df}}{=} [u_{\mathrm{R}}(t),\, u_{\mathrm{S}}(t),\, u_{\mathrm{T}}(t)]^{\mathrm{T}}, \quad \boldsymbol{i}(t) \stackrel{\mathrm{df}}{=} [i_{\mathrm{R}}(t),\, i_{\mathrm{S}}(t),\, i_{\mathrm{T}}(t)]^{\mathrm{T}} \tag{8}$$

and these vectors can be presented in the form of a Fouier Series

$$\boldsymbol{u}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} U_n e^{jn\omega_1 t}\,. \tag{9}$$

Similarly, the load current vector can be expressed in the form

$$\boldsymbol{i}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \begin{bmatrix} \boldsymbol{I}_{\mathrm{R}n} \\ \boldsymbol{I}_{\mathrm{S}n} \\ \boldsymbol{I}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} I_n e^{jn\omega_1 t}\,. \tag{10}$$

It is assumed, for simplicity sake, that voltages and currents in three-phase systems do not have any dc component.

A scalar product of three-phase vectors of quantities $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ with the same period T, is defined as

$$(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\, \boldsymbol{y}(t)\, dt = \mathrm{Re}\sum_{n\in N} X_n^{\mathrm{T}} Y_n^*\,. \tag{11}$$

The three-phase rms value of a three-phase vector is defined as

$$||\boldsymbol{x}|| = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}(t)\, dt} = \sqrt{\mathrm{Re}\sum_{n\in N} X_n^{\mathrm{T}} X_n^*}\,. \tag{12}$$

Two three-phase vectors are mutually orthogonal when their scalar product is zero, i.e.,

$$(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{Re}\sum_{n\in N} X_n^{\mathrm{T}} Y_n^* = 0\,. \tag{13}$$

If a three-phase vector $\boldsymbol{z}(t)$ is a sum of components i.e.,

$$\boldsymbol{z}(t) \equiv \boldsymbol{x}(t) + \boldsymbol{y}(t) \tag{14}$$

which are mutually orthogonal, then their three-phase rms values satisfy the relationship

$$||\boldsymbol{z}||^2 = ||\boldsymbol{x}||^2 + ||\boldsymbol{y}||^2 . \tag{15}$$

V. CPC of Single-Phase LTI Loads

The supply current of single-phase linear time-invariant (LTI) loads with the admittance for harmonic frequencies

$$\frac{\boldsymbol{I}_n}{\boldsymbol{U}_n} = \boldsymbol{Y}_n = Y_n e^{-j\varphi_n} = G_n + jB_n \tag{16}$$

can be decomposed into three physical components

$$i(t) = i_{\mathrm{a}}(t) + i_{\mathrm{s}}(t) + i_{\mathrm{r}}(t) \tag{17}$$

where

$$i_{\mathrm{a}}(t) = \frac{P}{||u||^2} u(t) = G_{\mathrm{e}}\, u(t) \tag{18}$$

is an ***active current.*** This current is associated with a phenomenon of a permanent transfer of the energy from the supply source to the load, with the average rate equal to the active power P. The proportionality coefficient G_{e} is referred to as an ***equivalent conductance*** of the load.

The component

$$i_{\mathrm{r}}(t) \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re} \sum_{n\in N} jB_n \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{19}$$

is a ***reactive current.*** Since the load susceptance is

$$B_n = \mathrm{Im}\{Y_n e^{-j\varphi_n}\} = -Y_n \sin\varphi_n . \tag{20}$$

This current is associated with a phenomenon of a phase-shift between the supply voltage and the load current harmonics.

The component

$$i_{\mathrm{s}}(t) \stackrel{\mathrm{df}}{=} (G_0 - G_{\mathrm{e}})U_0 + \sqrt{2}\,\mathrm{Re} \sum_{n\in N} (G_n - G_{\mathrm{e}})\boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{21}$$

is a ***scattered current***. It is associated with a phenomenon of a scatter of the load conductance for harmonic frequencies G_n around the equivalent conductance G_{e}

Because these three components of the load current are associated with distinctive physical phenomena, they are referred to as the ***Current Physical Components (CPC).***

These three components of the supply currents are mutually orthogonal [22] so that their rms values satisfy the relationship

$$||i||^2 = ||i_{\mathrm{a}}||^2 + ||i_{\mathrm{s}}||^2 + ||i_{\mathrm{r}}||^2 . \tag{22}$$

It means that these three phenomena affect the supply current rms value independently of each other.

Multiplication of this equation by a square of the supply voltage rms value $||u||$ results in a power equation

$$S^2 = P^2 + Q^2 + D_{\mathrm{s}}^2 \tag{23}$$

where

$$Q \stackrel{\mathrm{df}}{=} ||i_{\mathrm{r}}||\,||u|| \tag{24}$$

is the ***reactive power***, while

$$D_{\mathrm{s}} \stackrel{\mathrm{df}}{=} ||i_{\mathrm{s}}||\,||u|| \tag{25}$$

is the ***scattered power***. The power factor can be expressed in a form

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2 + D_{\mathrm{s}}^2}} = \frac{||i_{\mathrm{a}}||}{\sqrt{||i_{\mathrm{a}}||^2 + ||i_{\mathrm{r}}||^2 + ||i_{\mathrm{s}}||^2}} . \tag{26}$$

This formula shows that the effectiveness of the energy transfer to LTI loads in the presence of the voltage harmonics is hindered by two physical phenomena, namely, by a scatter of the load conductance G_n around its equivalent value G_{e} and by the phase-shift φ_n between the supply voltage and the load current harmonics.

VI. CPC of Three-Phase LTI Loads

Since most of the electric energy is transferred by three-phase systems, power properties of such systems are of the crucial importance for the power systems economy. Therefore, there was a lot of research [6, 8, 9, 10, 12, 14, 20, 22, 38, 39, 45, 46, 48] focused on the development of the power theory of three-phase systems in the presence of harmonics and asymmetry.

There were two major obstacles which made these efforts doomed to fail. First, three-phase systems have as their sub-set all single-phase systems. As long as the power properties of single-phase systems were not correctly described and comprehended, it was not possible to correctly describe such properties of three-phase systems. Second, all studies were carried on using a wrong definition of the apparent power S of three-phase systems. In 1920 a committee of AIEE suggested [2] two definitions of the apparent power S, known as the arithmetical definition and the geometrical definition of that power. A debate [6, 8, 9] on these two definitions in the twenties was inconclusive and both of them were eventually adopted as standard definitions. They can be found in the IEEE Standard Dictionary of Electrical and Electronics Terms [29]. There was also a definition of the apparent power suggested in 1922 [3] by Buchholtz, but it was dominated by the arithmetic and the geometric definitions, endorsed by the IEEE, and it was not used.

The supply voltage in three-phase systems can be nonsinusoidal, while the load can be unbalanced. These features can contribute to a reduction in the effectiveness of the energy transfer.

Let the load be linear and time-invariant (LTI). It can be supplied by a four-conductor line, as shown in Fig. 5, i.e., with a neutral conductor.

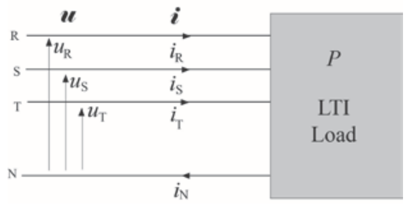

Figure 5. Three-phase LTI load supplied in a system with neutral conductor.

An equivalent circuit, shown in Fig. 6, with line-to-neutral admittances $\boldsymbol{Y}_{\mathrm{R}n}$, $\boldsymbol{Y}_{\mathrm{S}n}$ and $\boldsymbol{Y}_{\mathrm{T}n}$, can be found for each voltage harmonic of the supply voltage.

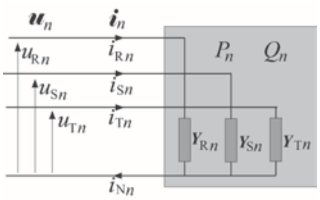

Figure 6. An equivalent circuit of the load for the n^{th} order harmonic.

The equivalent admittance for the n^{th} order harmonic can be defined for such a load, namely

$$\boldsymbol{Y}_{\mathrm{e}n} = G_{\mathrm{e}n} + jB_{\mathrm{e}n} = \frac{P_n - jQ_n}{||\boldsymbol{u}_n||^2} = \frac{1}{3}(\boldsymbol{Y}_{\mathrm{R}n}+\boldsymbol{Y}_{\mathrm{S}n}+\boldsymbol{Y}_{\mathrm{T}n})\,. \tag{27}$$

For such a load the three-phase vector of the supply current can be decomposed into six components, namely

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}\,. \tag{28}$$

In this decomposition

$$\boldsymbol{i}_{\mathrm{a}} \stackrel{\mathrm{df}}{=} G_{\mathrm{e}}\, \boldsymbol{u}(t) = \sqrt{2}\, \mathrm{Re} \sum_{n\in N} G_{\mathrm{e}}\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{29}$$

with

$$G_{\mathrm{e}} = \frac{P}{||\boldsymbol{u}||^2} \tag{30}$$

is an ***active current***. The component

$$\boldsymbol{i}_{\mathrm{r}} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re} \sum_{n\in N} jB_{\mathrm{e}n}\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{31}$$

is a ***reactive current***, and the component

$$\boldsymbol{i}_{\mathrm{s}} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re} \sum_{n\in N} (G_{\mathrm{e}n} - G_{\mathrm{e}})\, \boldsymbol{U}_n e^{jn\omega_1 t} \tag{32}$$

is a ***scattered current***. The remaining three components in this decomposition are unbalanced currents of the positive, negative and the zero sequence. Their calculation requires that unbalanced admittances of the load are calculated. They depend [44] on the sequence of the supply voltage harmonics. For harmonics of the zero sequence, these admittances are equal to

$$\begin{bmatrix} \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} \\ \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}} \\ \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 \\ \boldsymbol{Y}_{\mathrm{R}n} + \alpha \boldsymbol{Y}_{\mathrm{S}n} + \alpha^* \boldsymbol{Y}_{\mathrm{T}n} \\ \boldsymbol{Y}_{\mathrm{R}n} + \alpha^* \boldsymbol{Y}_{\mathrm{S}n} + \alpha \boldsymbol{Y}_{\mathrm{T}n} \end{bmatrix}. \tag{33}$$

For harmonics of the positive sequence

$$\begin{bmatrix} \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} \\ \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}} \\ \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} \boldsymbol{Y}_{\mathrm{R}n} + \alpha^* \boldsymbol{Y}_{\mathrm{S}n} + \alpha \boldsymbol{Y}_{\mathrm{T}n} \\ 0 \\ \boldsymbol{Y}_{\mathrm{R}n} + \alpha \boldsymbol{Y}_{\mathrm{S}n} + \alpha^* \boldsymbol{Y}_{\mathrm{T}n} \end{bmatrix}. \tag{34}$$

For harmonics of the negative sequence

$$\begin{bmatrix} \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} \\ \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}} \\ \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} \boldsymbol{Y}_{\mathrm{R}n} + \alpha \boldsymbol{Y}_{\mathrm{S}n} + \alpha^* \boldsymbol{Y}_{\mathrm{T}n} \\ \boldsymbol{Y}_{\mathrm{R}n} + \alpha^* \boldsymbol{Y}_{\mathrm{S}n} + \alpha \boldsymbol{Y}_{\mathrm{T}n} \\ 0 \end{bmatrix}. \tag{35}$$

Let us define three-phase unite vectors

$$\begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix} \stackrel{\mathrm{df}}{=} 1^{\mathrm{p}}, \qquad \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} \stackrel{\mathrm{df}}{=} 1^{\mathrm{n}}, \qquad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \stackrel{\mathrm{df}}{=} 1^{\mathrm{z}}\,. \tag{36}$$

Having these unite vectors, particular components of the unbalanced current of the load can be defined. The supply current component

$$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re} \sum_{n\in N} \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}\, 1^{\mathrm{p}}\, \boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{37}$$

is an ***unbalanced current of the positive sequence***. It is associated with a phenomenon of the occurrence of the positive sequence unbalanced current in the supply line due to the load imbalance.

The supply current component

$$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re} \sum_{n\in N} \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}\, 1^{\mathrm{n}}\, \boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{38}$$

is an ***unbalanced current of the negative sequence***. It is associated with a phenomenon of the occurrence of the negative sequence unbalanced current in the supply line due to the load imbalance.

The supply current component

$$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re} \sum_{n\in N} \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}}\, 1^{\mathrm{z}}\, \boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{39}$$

is an ***unbalanced current of the zero sequence***. It is associated with a phenomenon of the occurrence of the zero sequence unbalanced current in the supply line due to the load imbalance. Thus these six currents are associated with different phenomena in the system and consequently, can be regarded as the Currents' Physical Components. Their three-phase rms values are equal to, respectively

$$||\boldsymbol{i}_{\mathrm{a}}|| = G_{\mathrm{e}}\, ||\boldsymbol{u}|| = \frac{P}{||\boldsymbol{u}||} \tag{40}$$

$$||\boldsymbol{i}_{\mathrm{s}}|| = \sqrt{3}\sqrt{\sum_{n\in N} (G_{\mathrm{e}n} - G_{\mathrm{e}})^2\, U_{\mathrm{R}n}^2} \tag{41}$$

$$||\boldsymbol{i}_{\mathrm{r}}|| = \sqrt{3}\sqrt{\sum_{n\in N} B_{\mathrm{e}n}^2\, U_{\mathrm{R}n}^2} \tag{42}$$

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}|| = \sqrt{3}\sqrt{\sum_{n\in N} (Y_{\mathrm{u}n}^{\mathrm{p}})^2\, U_{\mathrm{R}n}^2} \tag{43}$$

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}|| = \sqrt{3}\sqrt{\sum_{n\in N} (Y_{\mathrm{u}n}^{\mathrm{n}})^2\, U_{\mathrm{R}n}^2} \tag{44}$$

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}|| = \sqrt{3}\sqrt{\sum_{n\in N} (Y_{\mathrm{u}n}^{\mathrm{z}})^2\, U_{\mathrm{R}n}^2}\,. \tag{45}$$

All these CPC are mutually orthogonal [58] so that their three-phase rms values satisfy the relationship

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{s}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}||^2 \tag{46}$$

thus they affect the supply current three-phase rms value independently of each other.

Before the effect of the CPC on the effectiveness of the energy transfer can be evaluated, the concept of the apparent power S in three-phase systems has to be revised. Definitions of this power were introduced in 1920 by a committee of the American Institute of Electrical Engineers (AIEE) [2]. Two definitions of the apparent power S, namely

$$S = U_R I_R + U_S I_S + U_T I_T = S_A \tag{47}$$

referred to as the ***arithmetical apparent power*** and

$$S = \sqrt{P^2 + Q^2} = S_G \tag{48}$$

referred to as the ***geometrical apparent power,*** were suggested by AIEE and were later supported by the IEEE Standard Dictionary of Electrical and Electronic Terms [29].

Unfortunately, as it was demonstrated in [31], the arithmetical and the geometrical apparent powers, in the presence of the current asymmetry result in a wrong value of the power factor. The right value of this factor is obtained when the apparent power is defined as the product of the three-phase rms values of the supply voltage and the load current, namely

$$S \overset{\text{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}||\,. \tag{49}$$

At such definition of the apparent power S, the power factor can be expressed as

$$\lambda = \frac{P}{S} = \frac{||\boldsymbol{i}_a||}{\sqrt{||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u^p||^2 + ||\boldsymbol{i}_u^n||^2 + ||\boldsymbol{i}_u^z||^2}}\,. \tag{50}$$

This formula explicitly shows how particular phenomena in the system with LTI loads contribute to a degradation of the effectiveness of the energy transfer.

The LTI loads, with the power properties discussed above, cannot be sources of harmonics, however. Generation of harmonics, due to the load nonlinearity or periodic time-variance of the load parameters can change these properties substantially. Such loads are referred to as Harmonics Generating Loads (HGLs)

VII. CPC IN SYSTEMS WITH HGLS

Generation of the current harmonics in the load can affect the power equation of the load. This is illustrated in the example, shown in Fig. 7.

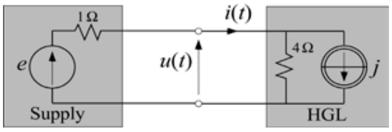

Figure 7. Circuit with harmonic generating load (HGL).

It is assumed that at the supply voltage

$$e = e_1 = 100\sqrt{2}\,\sin\omega_1 t \quad \text{V}$$

a third order current harmonic

$$j = j_3 = 50\sqrt{2}\,\sin 3\omega_1 t \quad \text{A}$$

is generated in the load. The voltage and current at the load terminals are equal to

$$u = u_1 + u_3 = 80\sqrt{2}\sin\omega_1 t - 40\sqrt{2}\sin 3\omega_1 t \quad \text{V}$$

$$i = i_1 + i_3 = 20\sqrt{2}\sin\omega_1 t + 40\sqrt{2}\sin 3\omega_1 t \quad \text{A}$$

thus the active power is zero, since

$$P = \frac{1}{T}\int_0^T u\, i\, dt = \sum_{n=1,3} U_n\, I_n \cos\varphi_n = 1600 - 1600 = 0\,.$$

Observe that there is neither a phase-shift between voltage and current harmonics, nor any change of the load conductance with harmonics order, thus no reactive and scattered currents and powers. The apparent power S at the supply terminals is equal to

$$S = ||u||\,||i|| = 89.44 \times 44.72 = 4000 \text{ VA}$$

but we are not able to write the power equation in the form developed previously for LTI loads. It is caused by the presence of the active power associated with the third order current harmonic j_3 generated in the load, of the negative value, $P_3 = -1600$ W.

When a current harmonic is generated in the load, then there is a source of the energy flow from the load back to the supply source, where this energy is dissipated in the supply source resistance. Thus, there is a current component in the supply current, which cannot be interpreted as a reactive or a scattered current, but it does not contribute to the load active power P. Quite opposite, it reduces that power. This component can be associated with energy flow in the opposite direction to the normal flow, meaning, back from the load to the supply source. Thus, generation of current harmonics in the load, due to its nonlinearity or periodic time-variance, has to be regarded [23] as a phenomenon that affects power properties of electric circuits.

The presence of current harmonics generated in the load can be identified by measuring the phase angle, φ_n, between the voltage and current harmonics, u_n and i_n, at the cross-section between distribution system (C) and a harmonic generating load (G), as shown in Fig. 8. Since the active power of the n^{th} order harmonic is equal to

$$P_n = U_n\, I_n \cos\varphi_n$$

then, if

$$|\varphi_n| < \pi/2$$

there is an average component of energy flow at the n^{th} order harmonic from the supply towards the load, and if

$$|\varphi_n| > \pi/2$$

there is an average component of energy flow at n^{th} order harmonic from the load back the supply source.

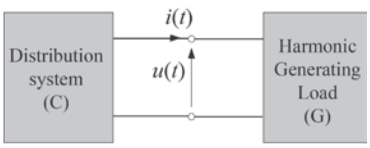

Figure 8. Cross-section between distribution system and harmonic generating load (HGL).

With this observation, the set N of all harmonic orders n can be decomposed into sub-sets N_C and N_G, as follows

$$\begin{aligned} &\text{if} \quad |\varphi_n| \le \pi/2, \ \text{then} \ n \in N_C \\ &\text{if} \quad |\varphi_n| > \pi/2, \ \text{then} \ n \in N_G\,. \end{aligned} \tag{51}$$

It enables the voltage and current decomposition into components with harmonics from sub-sets N_C and N_G,

$$i = \sum_{n\in N} i_n = \sum_{n\in N_C} i_n + \sum_{n\in N_G} i_n = i_C + i_G \tag{52}$$

$$u = \sum_{n\in N} u_n = \sum_{n\in N_C} u_n + \sum_{n\in N_G} u_n = u_C - u_G \,. \tag{53}$$

The voltage u_C is defined as the negative sum of voltage harmonics because, as a supply source response to load generated current, i_C, it has the opposite sign as compared to the sign of the distribution system originated voltage harmonics. The same applies to harmonic active power, thus

$$P = \sum_{n\in N} P_n = \sum_{n\in N_C} P_n + \sum_{n\in N_G} P_n \overset{\text{df}}{=} P_C - P_G \,. \tag{54}$$

Sub-sets N_C and N_G do not contain common harmonic orders n, thus currents i_G and i_C are mutually orthogonal. Hence, their rms values satisfy the relationship

$$\|i\|^2 = \|i_C\|^2 + \|i_G\|^2 \,. \tag{55}$$

The same applies to the voltage rms values, namely

$$\|u\|^2 = \|u_C\|^2 + \|u_G\|^2 \,. \tag{56}$$

Decomposition (51) of harmonic orders and the vol tage and current according to (52) and (53), mean that the system, as presented in Fig. 8, can be regarded as a superposition of two systems. The first, shown in Fig. 9a, has an LTI load and the second, shown in Fig. 9b, has only a current source on the customer side, while the distribution system is a passive energy receiver.

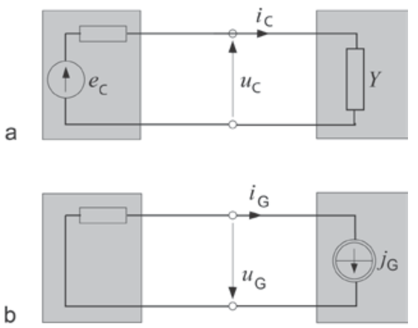

Figure. 9. (a) Equivalent circuit for harmonics $n \in N_C$ (b) equivalent circuit for harmonics $n \in N_G$.

The circuit in Fig. 9a, as a system with an LTI load, can be described according to the CPC approach. Namely, if the equivalent admittance is

$$\boldsymbol{Y}_n = G_n + jB_n = \frac{\boldsymbol{I}_n}{\boldsymbol{U}_n} \tag{57}$$

and the equivalent conductance of the HGL according to the CPC power theory

$$G_{Ce} \overset{\text{df}}{=} \frac{P_C}{\|u_C\|^2} \tag{58}$$

then the current i_C can be decomposed into the active, scattered and reactive components, thus

$$i = i_C + i_G = i_{Ca} + i_{Cs} + i_{Cr} + i_G. \tag{59}$$

with the active current defined as

$$i_{Ca} \overset{\text{df}}{=} G_{Ce}\, u_C \,. \tag{60}$$

These currents are mutually orthogonal, hence

$$\|i\|^2 = \|i_{Ca}\|^2 + \|i_{Cs}\|^2 + \|i_{Cr}\|^2 + \|i_G\|^2 \,. \tag{61}$$

The current i_G is referred to as a ***load generated current.*** It is associated with a phenomenon of the energy flow from the load to the supply source.

This decomposition reveals that only a part of the active current, namely, the current i_{Ca}, contributes to the energy transfer from the energy provider to a customer load. It means, that the concept of the power factor, originally defined as $\lambda = P/S$, should be for systems with HGLs redefined. The effectiveness of the energy transfer in such systems should be specified by a modified power factor, namely

$$\lambda_{HGL} = \frac{\|i_{Ca}\|}{\|i\|} = \frac{\|i_{Ca}\|}{\sqrt{\|i_{Ca}\|^2 + \|i_{Cs}\|^2 + \|i_{Cr}\|^2 + \|i_G\|^2}} \,. \tag{62}$$

These results obtained for single-phase systems with HGLs can be generalized, of course, for three-phase systems with such loads. This generalization does not introduce any new results and it is ignored in this paper, however.

VIII. Conclusions

The paper shows that the Currents' Physical Components – based power theory enables us to identify all physical phenomena in the system that contribute to the degradation of the effectiveness of the energy transfer. It is possible because it enables us to associate the supply current components with distinctive physical phenomena in the load. The Currents' Physical Components were expressed in the paper in terms of the circuit equivalent parameters, which creates fundamentals for a synthesis of compensators that could reduce harmful components of the supply current.

Results presented in this paper can be implemented in systems where the voltage distortion should be taken into account at such systems' economic optimization. These are, first of all, systems with very high power arc furnaces [48]. Islanded micro-grids may also belong to this category, due to high saturation with power electronics equipment and relatively low short-circuit power. Consequently, the voltage distortion in micro-grids could be higher than in common distribution systems.

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