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# Physical Phenomena that Affect the Effectiveness of the Energy Transfer in Electrical Systems

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Cost of the electric energy production and delivery
is related
to the apparent power *S.*

$$S = U \times I$$

A merit for customers has only
the energy delivered

$$W = \int_0^{\tau} P\,dt$$

**The inequality**

$$S > P$$

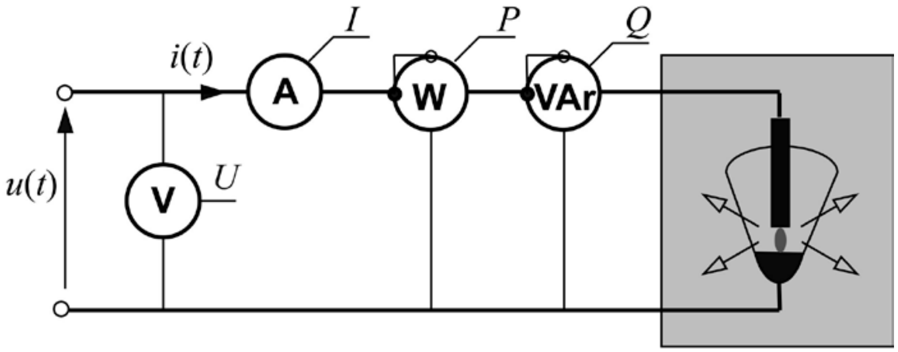
**is of the major importance for electrical systems economy**

By the end of XIX century
it was concluded
that

$$S^2 - P^2 = Q^2$$

where $Q$ denotes the reactive power of the load

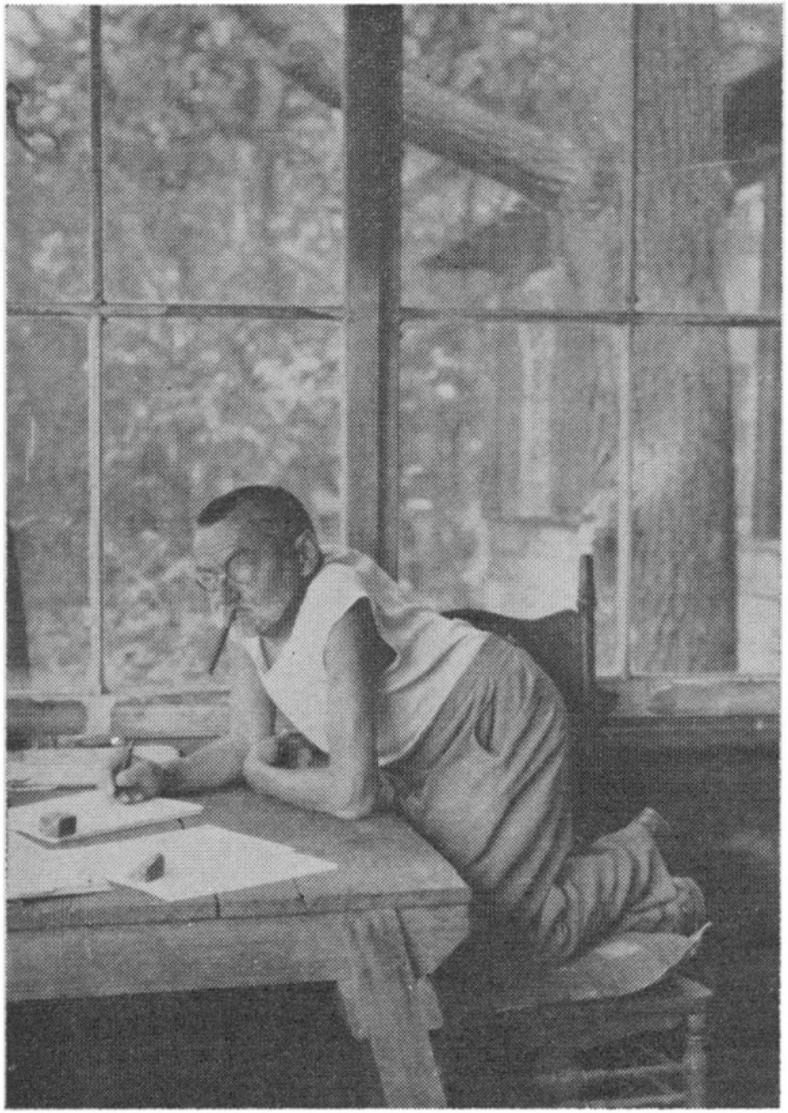
# Steinmetz Experiment: 1892



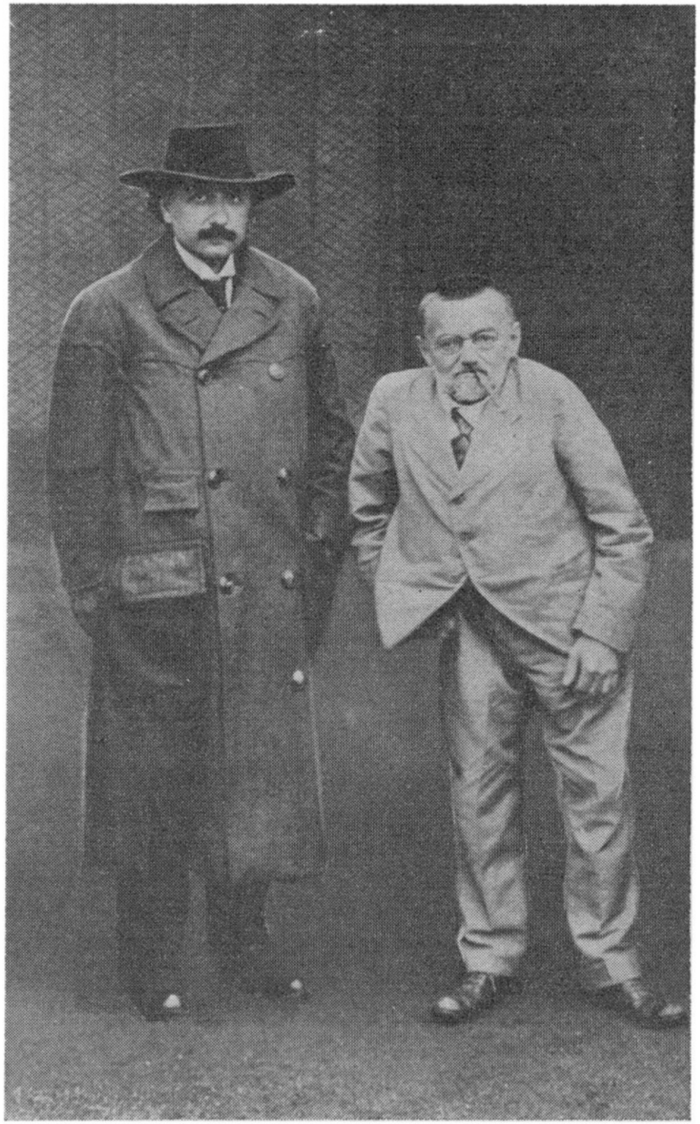

$$P^2 + Q^2 < S^2, \qquad Q = 0$$

Why the apparent power *S* is higher than the active power *P* ?

How the difference between *S* and *P* can be reduced ?



**Charles Proteus Steinmetz**

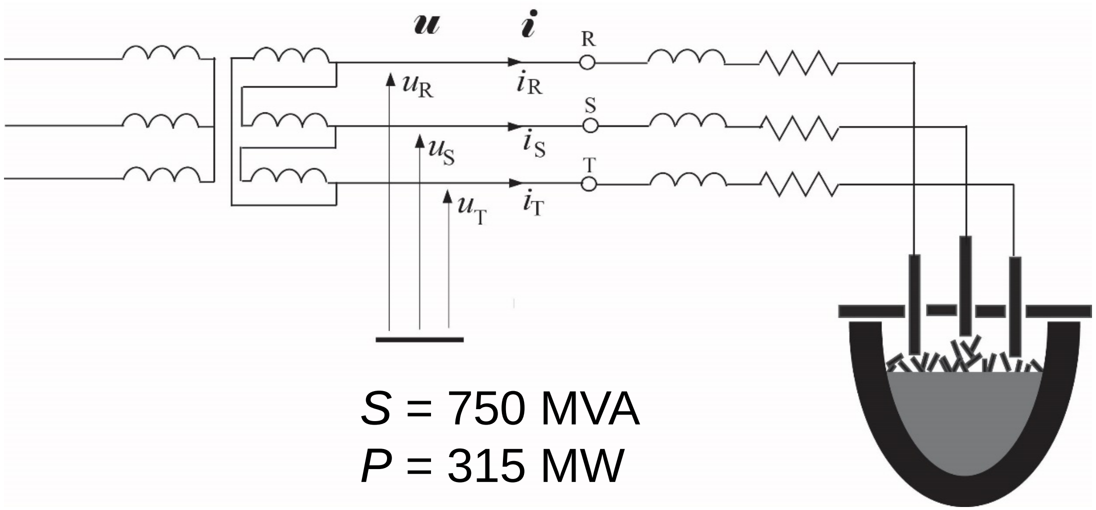


Einstein and Steinmetz.

**In Einstain's company…**

Present day “Steinmetz Experiment”
with line currents up to

625 kA




$S$ = 750 MVA
$P$ = 315 MW

Current not only distorted, but also asymmetrical and random
Power factor: $\lambda$ ~ 0.42

Annual bill for energy ~ 500 Million $

A number of different answers to the Steinmetz's question have been suggested:

1927: Budeanu: $S^2 - P^2 = Q_B^2 + D^2$ $\quad Q_B = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$

Endorsed by the IEEE Standard Dictionery of Electrical and Electronics Terms in 1992 and German Standards DIN in 1972

1931: Fryze: $S^2 - P^2 = Q_F^2$ $\quad Q_F = ||u||\,||i_{rF}||$

Endorsed by German Standards DIN in 1972

1971: Shepherd: $S^2 - S_R^2 = Q_S^2$ $\quad Q_S = ||u||\,||i_{rS}||$

1975: Kusters: $S^2 - P^2 = Q_K^2 + Q_r^2$ $\quad Q_K = ||u||\,||i_{rC}||$

Endorsed by the International Electrotechnical Commission in 1980

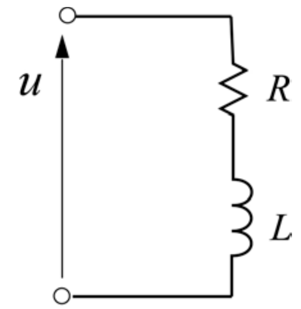
1979: Depenbrock: $S^2 - P^2 = Q_1^2 + V^2 + N^2$ $\quad Q_1 = U_1 I_1 \sin\varphi_1$

2003: Tenti: $S^2 - P^2 = Q_T^2 + D_T^2$ $\quad Q_T = ||u||\,||i_{rT}||$

Since the Steinmetz's experiment
in 1892, after 91 years
of Power Theory development for single-phase, LTI loads,
such as

$$u = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

$$i = G_0 U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{Y}_n \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

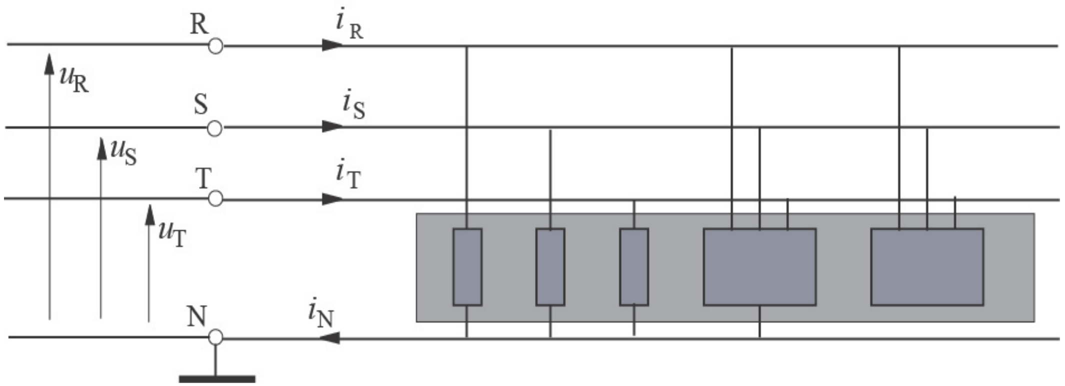



we have had five different power equations and five different reactive powers

Compensation problem was not solved

The problem was eventually solved
in frame of the
Currents' Physical Components (CPC) – based power theory
by
L.S. Czarnecki in 1983

# A draft of Currents' Physical Components (CPC) – based Power Theory




$$\boldsymbol{u} = \begin{bmatrix} u_{\mathrm{R}} \\ u_{\mathrm{S}} \\ u_{\mathrm{T}} \end{bmatrix} = \sum_{n\in N} \boldsymbol{u}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t}$$

$$\boldsymbol{i} = \begin{bmatrix} i_{\mathrm{R}} \\ i_{\mathrm{S}} \\ i_{\mathrm{T}} \end{bmatrix} = \sum_{n\in N} \boldsymbol{i}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \begin{bmatrix} \boldsymbol{I}_{\mathrm{R}n} \\ \boldsymbol{I}_{\mathrm{S}n} \\ \boldsymbol{I}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t}$$

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{Ca}} + \boldsymbol{i}_{\mathrm{Cs}} + \boldsymbol{i}_{\mathrm{Cr}} + \boldsymbol{i}_{\mathrm{Cu}} + \boldsymbol{i}_{\mathrm{G}}$$

**Scalar product:**

$$(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\,\boldsymbol{y}(t)\,dt = 0$$

**Three-phase RMS value:**

$$||\boldsymbol{x}|| = \sqrt{(\boldsymbol{x}, \boldsymbol{x})} = \sqrt{||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2}$$

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{Ca}}||^2 + ||\boldsymbol{i}_{\mathrm{Cs}}||^2 + ||\boldsymbol{i}_{\mathrm{Cr}}||^2 + ||\boldsymbol{i}_{\mathrm{Cu}}||^2 + ||\boldsymbol{i}_{\mathrm{G}}||^2$$

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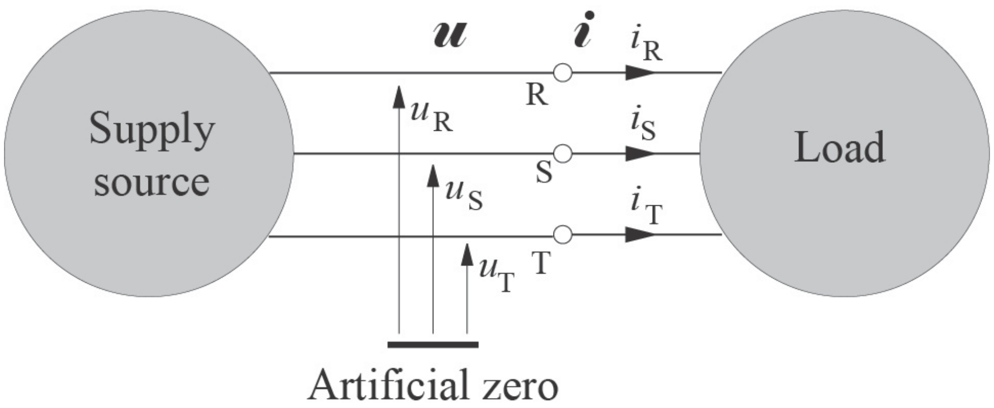
# Instantaneous power: $p(t)$

$$p(t) = \frac{d}{dt} W(t) = u(t)\, i(t)$$




$$p(t) = u_R i_R + u_S i_S + u_T i_T = \boldsymbol{u}^T \boldsymbol{i}$$

There is not any power
with so clear physical interpretation
as the instantaneous power
$p(t)$
It is the rate of energy flow from the source to the load

There are opinions,
that it should be regarded
**as the major power in the power theory**

The major question of the power theory:

*Why*

*the apparent power $S$ can be higher than the active power $P$?*

*cannot be explained*

*in terms of the instantaneous power $p(t)$*

Nonetheless,

there is a power theory,

that has the instantaneous power $p(t)$

as one of two major powers.

It is

the Instantaneous Reactive Power (IRP) p-q

Power Theory

## Clarke's Transform to $\alpha$ and $\beta$ coordinates for three-wire systems:

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} x_R \\ x_S \end{bmatrix} = \mathsf{C} \begin{bmatrix} x_R \\ x_S \end{bmatrix}$$

## Instantaneous active and reactive powers, *p* and *q* definitions:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} u_\alpha, & u_\beta \\ -u_\beta, & u_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathsf{U}_C \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

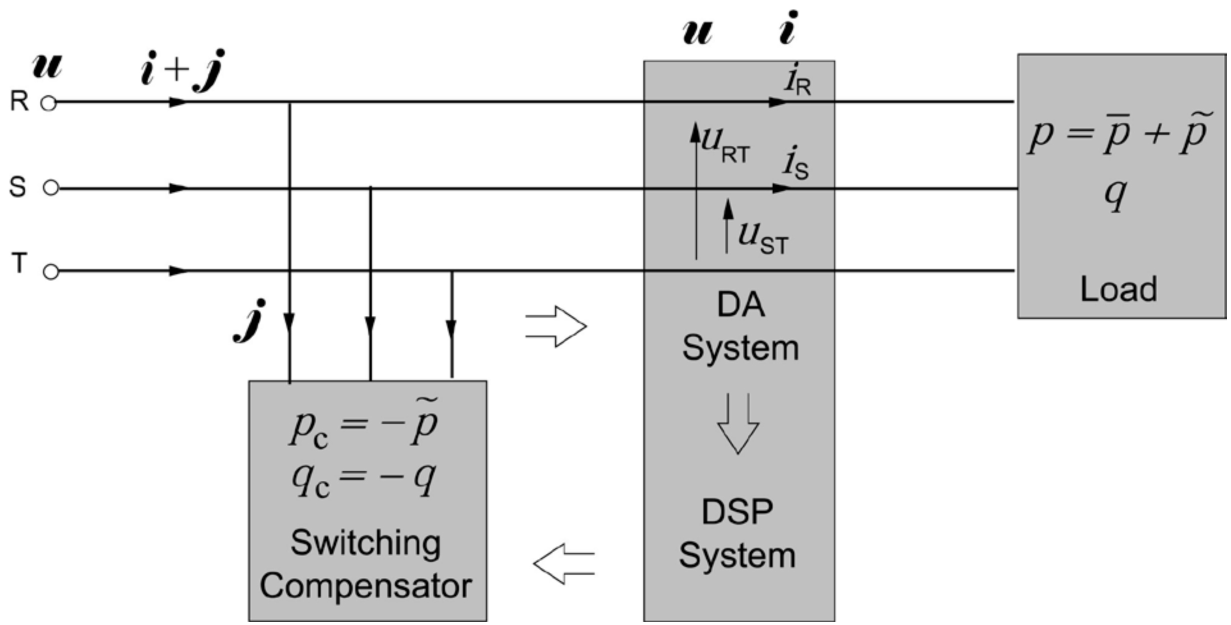
According to IRP p-q Power Theory:

If the load has the instantaneous active power

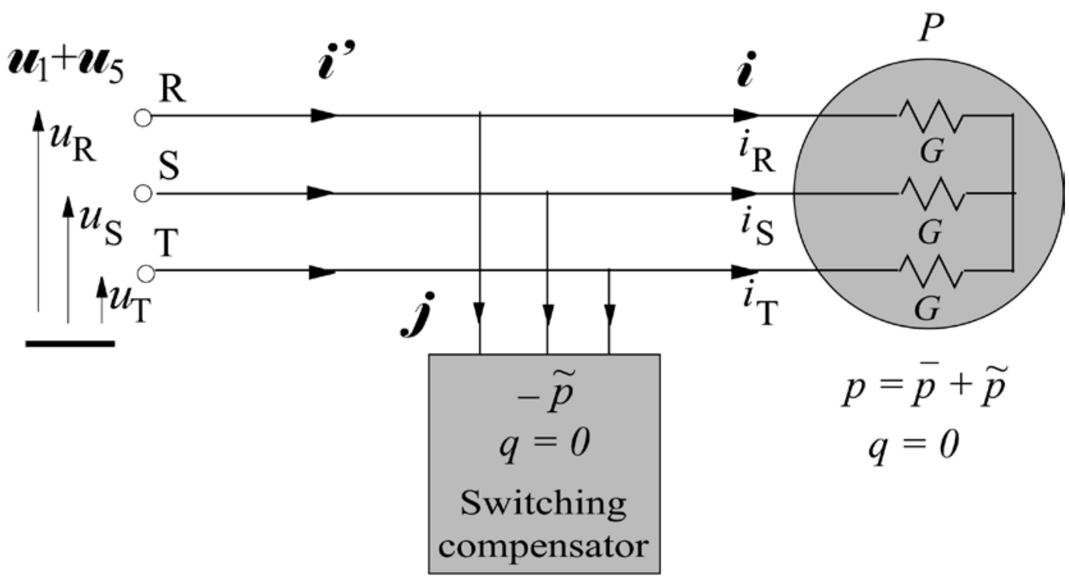
$$p = \bar{p} + \tilde{p}$$

Compensator should be controlled
in such a way that
Its instantaneous active power

$$p_C = -\tilde{p}$$



u
i + j
R
S
T
j
u
i
i_R
u_RT
i_S
u_ST
DA
System
DSP
System
p = p̄ + p̃
q
Load
p_c = −p̃
q_c = −q
Switching
Compensator



$\boldsymbol{u}_1+\boldsymbol{u}_5$
R
S
T
$u_R$
$u_S$
$u_T$
$\boldsymbol{i}'$
$\boldsymbol{i}$
$\boldsymbol{j}$
$i_R$
$i_S$
$i_T$
$P$
$G$
$G$
$G$
$-\tilde{p}$
$q = 0$
Switching compensator
$p = \bar{p} + \tilde{p}$
$q = 0$

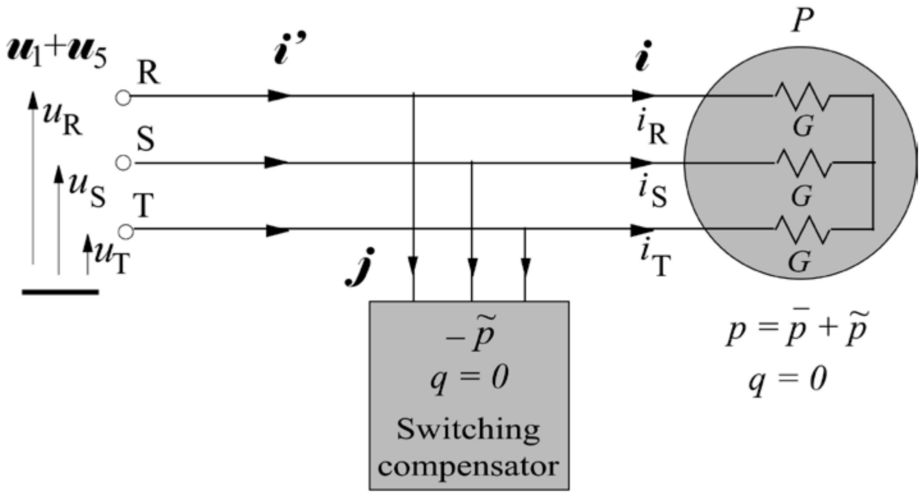
# IRP p-q based control at distorted supply voltage

$$\boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_5$$

$$u_{R1} \triangleq \sqrt{2}\,U_1 \cos \omega_1 t, \qquad u_{R5} \triangleq \sqrt{2}\,U_5 \cos 5\omega_1 t$$

L.S. Czarnecki:
"Effect of supply voltage harmonics on IRP-based switching compensator control,"
*IEEE Transactions on Power Electronics*, Vol. 24, No. 2, Feb. 2009.

$$p = P_1 + P_5 + 6GU_1U_5 \cos 6\omega_1 t = \bar{p} + \tilde{p}$$



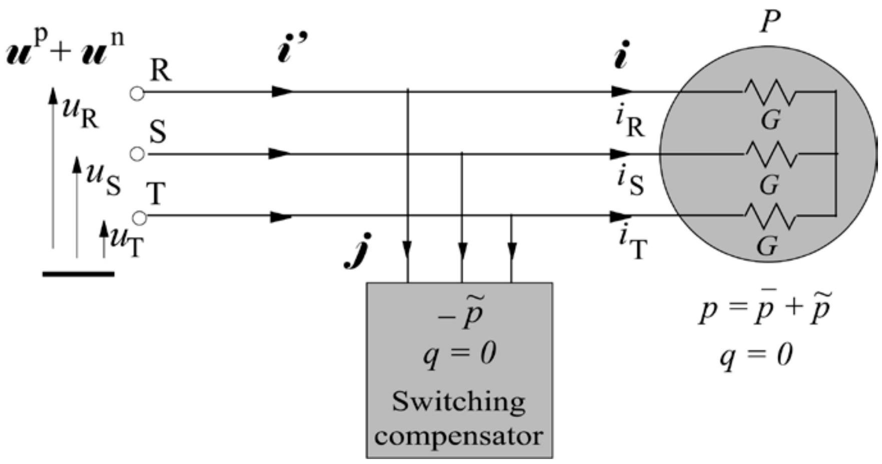

$$\boldsymbol{j} = \begin{bmatrix} j_R \\ j_S \end{bmatrix} = \frac{-2\sqrt{2}\,GU_1U_5 \cos 6\omega_1 t}{U_1^2 + U_5^2 + 2U_1U_5 \cos 6\omega_1 t} \begin{bmatrix} U_1 \cos \omega_1 t + U_5 \cos 5\omega_1 t \\ U_1 \cos(\omega_1 t - 120^0) + U_5 \cos(5\omega_1 t + 120^0) \end{bmatrix}.$$

# IRP p-q based control at asymmetrical  supply voltage

$$\boldsymbol{u} = \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}$$

L.S. Czarnecki:
“Effect of supply voltage asymmetry on IRP p-q - based switching compensator control”,
*IET Proc. on Power Electronics*, 2010, Vol. 3, No. 1

$$p(t) = \frac{dW}{dt} = P^{\mathrm{p}} + P^{\mathrm{n}} + 6GU^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega_1 t = \bar{p} + \tilde{p}$$




$$j_{\mathrm{R}} = \sqrt{\frac{2}{3}}\, j_{\alpha} = \frac{-2\sqrt{2}\,G(U^{\mathrm{p}} + U^{\mathrm{n}})U^{\mathrm{p}}U^{\mathrm{n}}\cos\omega_1 t\cos 2\omega_1 t}{U^{\mathrm{p}2} + U^{\mathrm{n}2} + 2U^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega_1 t}.$$

## Conclusions

Energy oscillations between the supply source and the load
can occur
when the load is purely resistive.

Such oscillations do not degrade the power factor
from the unity value

We can have a reactive power $Q$ in the circuit

Is this reactive power $Q$
caused
by the energy oscillations
?

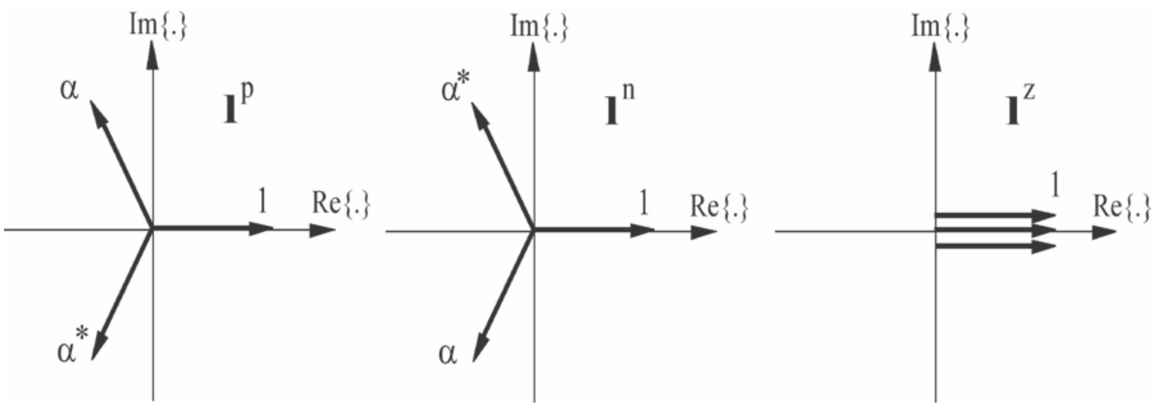
## Currents' Physical Components (CPC) in linear circuits with sinusoidal supply voltage

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}$$

Active current: $\boldsymbol{i}_{\mathrm{a}} = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}}\, \mathbf{1}^{\mathrm{p}} \boldsymbol{U}\, e^{j\omega t}\}$

Reactive current: $\boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re}\{\, jB_{\mathrm{e}}\, \mathbf{1}^{\mathrm{p}} \boldsymbol{U}\, e^{j\omega t}\}$

Unbalanced current: $\boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}_{\mathrm{u}}\, \mathbf{1}^{\mathrm{n}} \boldsymbol{U}\, e^{j\omega t}\}$



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# Instantaneous power

$$p(t) = \frac{d}{dt}W(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}) =$$

$$= p_{\mathrm{a}}(t) + p_{\mathrm{r}}(t) + p_{\mathrm{u}}(t)$$

$$p_{\mathrm{a}}(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{a}} = \boldsymbol{u}^{\mathrm{T}}G_{\mathrm{e}}\,\boldsymbol{u} = P$$

$$p_{\mathrm{r}}(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{r}} = 0$$

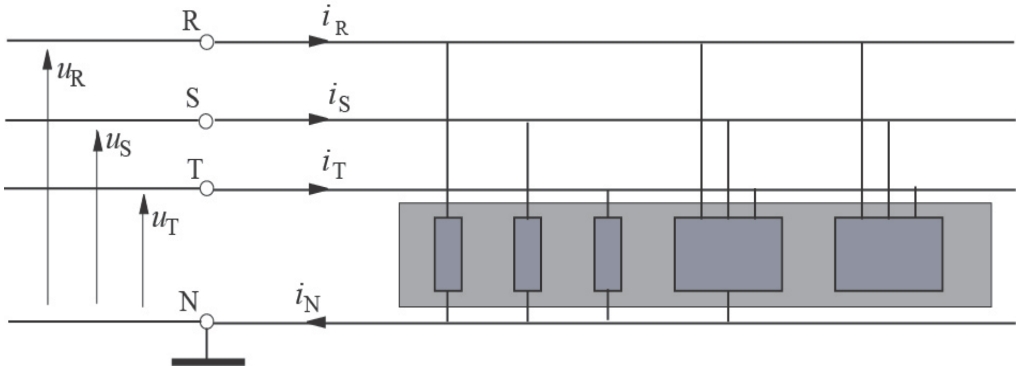
$$p_{\mathrm{u}}(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{u}} = 3Y_{\mathrm{u}}\,U_{\mathrm{R}}^{2}\cos(2\omega t + \Psi)$$

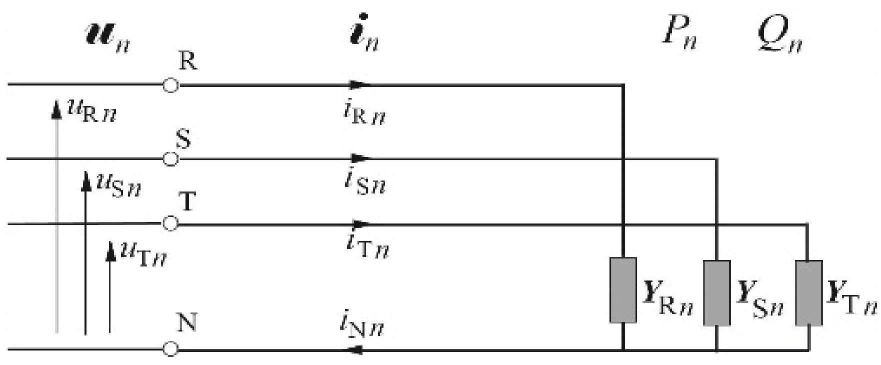
The reactive current
and consequently, the reactive power

$$Q = \pm\,||\boldsymbol{u}||\,||\boldsymbol{i}_{\mathrm{r}}||$$

are not caused by the energy oscillations

# Physical phenomena that affect the energy transfer




$$\boldsymbol{Y}_{Rn} = \frac{\boldsymbol{I}_{Rn}}{\boldsymbol{U}_{Rn}}, \qquad \boldsymbol{Y}_{Sn} = \frac{\boldsymbol{I}_{Sn}}{\boldsymbol{U}_{Sn}}, \qquad \boldsymbol{Y}_{Tn} = \frac{\boldsymbol{I}_{Tn}}{\boldsymbol{U}_{Tn}}.$$

Active power of an individual harmonic:

$$P_n = \mathrm{Re}\{\boldsymbol{Y}_{\mathrm{R}n} + \boldsymbol{Y}_{\mathrm{S}n} + \boldsymbol{Y}_{\mathrm{T}n}\}\,||\boldsymbol{u}_n||^2$$

When the active power of the *n*-th order harmonic
is negative
the energy is transferred from the load buck to the supply source

$$\text{if } P_n \geq 0, \text{ i.e., } |\varphi_n| \leq \pi/2, \text{ then } n \in N_{\mathrm{C}}$$

$$\text{if } P_n < 0, \text{ i.e., } |\varphi_n| > \pi/2, \text{ then } n \in N_{\mathrm{G}}\,.$$

The load generated current

$$\boldsymbol{i}_{\mathrm{G}} = \sum_{n \in N_{\mathrm{G}}} \boldsymbol{i}_n$$

is associated
with the phenomenon of the permanent energy transfer
from the load to the supply source

The current, voltage and the active power decomposition:

$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \sum_{n\in N_\mathrm{C}} \boldsymbol{i}_n + \sum_{n\in N_\mathrm{G}} \boldsymbol{i}_n = \boldsymbol{i}_\mathrm{C} + \boldsymbol{i}_\mathrm{G}$$

$$\boldsymbol{u} = \sum_{n\in N} \boldsymbol{u}_n = \sum_{n\in N_\mathrm{C}} \boldsymbol{u}_n + \sum_{n\in N_\mathrm{G}} \boldsymbol{u}_n = \boldsymbol{u}_\mathrm{C} - \boldsymbol{u}_\mathrm{G}$$

$$P = \sum_{n\in N} P_n = \sum_{n\in N_\mathrm{C}} P_n + \sum_{n\in N_\mathrm{G}} P_n = P_\mathrm{C} - P_\mathrm{G}$$

The active current:

$$\boldsymbol{i}_\mathrm{Ca} = \frac{P_\mathrm{C}}{||\boldsymbol{u}_\mathrm{C}||^2}\boldsymbol{u}_\mathrm{C} = G_\mathrm{Ce}\,\boldsymbol{u}_\mathrm{C}$$

is associated
with the phenomenon of the permanent energy transfer
from the supply source to the load

## Equivalent admittance for *n*-th order harmonic

$$\boldsymbol{Y}_{\mathrm{e}n} = G_{\mathrm{e}n} + jB_{en} = \frac{P_n - jQ_n}{\|\boldsymbol{u}_n\|^2} = \frac{1}{3}(\boldsymbol{Y}_{\mathrm{R}n} + \boldsymbol{Y}_{\mathrm{S}n} + \boldsymbol{Y}_{\mathrm{T}n})$$

## Equivalent conductance:

$$G_{\mathrm{Ce}} = \frac{P_{\mathrm{C}}}{\|\boldsymbol{u}_{\mathrm{C}}\|^2}$$

The scattered current

$$\boldsymbol{i}_{\mathrm{Cs}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N_{\mathrm{C}}} (G_{\mathrm{e}n} - G_{\mathrm{Ce}})\,\mathbf{1}_n\,\boldsymbol{U}_{\mathrm{R}n}\,e^{jn\omega_1 t}$$

is associated
with the phenomenon of the equivalent conductance change
with the harmonic order

The reactive current

$$\boldsymbol{i}_{\mathrm{Cr}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N_{\mathrm{C}}} jB_{\mathrm{e}n}\,\mathbf{1}_n\,\boldsymbol{U}_{\mathrm{R}\,n}\,e^{jn\omega_1 t}$$

is associated
with the phenomenon of the phase-shift
between the supply voltage and the load current harmonics

The unbalanced current

$$\boldsymbol{i}_{\mathrm{Cu}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N_{\mathrm{C}}}(\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}\,\mathbf{1}^{\mathrm{p}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}\,\mathbf{1}^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}}\,\mathbf{1}^{\mathrm{z}})\boldsymbol{U}_{\mathrm{R}n}\,e^{jn\omega_1 t}$$

is associated

with the phenomenon of the line currents asymmetry

caused by the load imbalance

There are five and only five physical phenomena
which affect
the energy transfer between the supply source and the load

1. Permanent transfer of the energy from the supply source to the load
2. Permanent transfer of the energy from the load to the supply source
3. Change of the load conductance with the harmonic order
4. Phase-shift between the supply voltage and the load current harmonics
5. Line currents asymmetry

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{Ca}} + \boldsymbol{i}_{\mathrm{Cs}} + \boldsymbol{i}_{\mathrm{Cr}} + \boldsymbol{i}_{\mathrm{Cu}} + \boldsymbol{i}_{\mathrm{G}}$$

Components of this decomposition are referred to as
Currents' Physical Components (CPC)

# Currents' Physical Components (CPC)

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{Ca}} + \boldsymbol{i}_{\mathrm{Cs}} + \boldsymbol{i}_{\mathrm{Cr}} + \boldsymbol{i}_{\mathrm{Cu}} + \boldsymbol{i}_{\mathrm{G}}$$

These components are mutually orthogonal, meaning their scalar products

$$(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\,\boldsymbol{y}(t)\,dt = 0$$

Three-phase RMS values

$$||\boldsymbol{x}|| = \sqrt{(\boldsymbol{x}, \boldsymbol{x})} = \sqrt{||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2}$$

satisfy the relationship:

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{Ca}}||^2 + ||\boldsymbol{i}_{\mathrm{Cs}}||^2 + ||\boldsymbol{i}_{\mathrm{Cr}}||^2 + ||\boldsymbol{i}_{\mathrm{Cu}}||^2 + ||\boldsymbol{i}_{\mathrm{G}}||^2$$

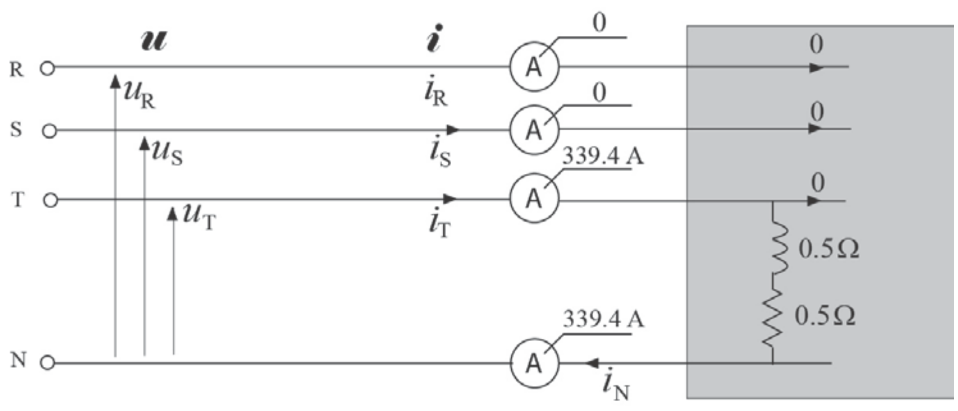
Conclusions:

CPC – based Power Theory
provides explanation of all the energy flow-related
physical phenomena in electrical systems

It also provide fundamentals
for synthesis of reactive compensators

# Numerical illustration

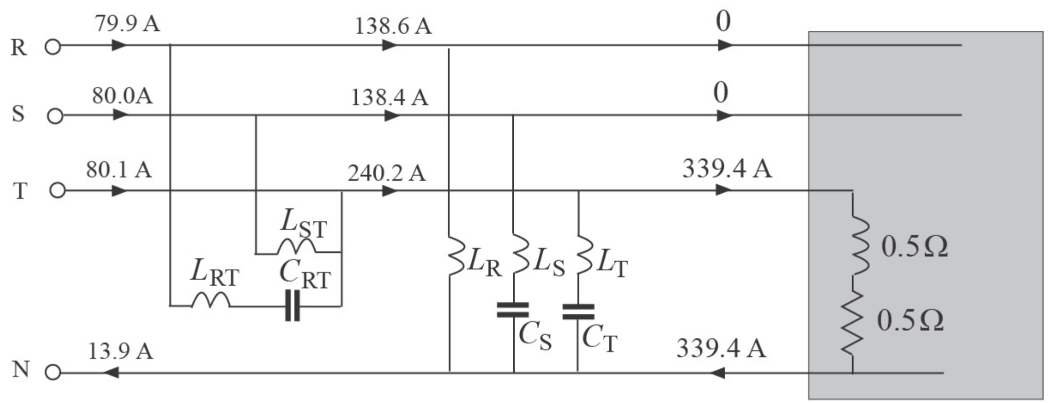
$$U_3 = 2\%U_1, \quad U_5 = 3\%U_1 \text{ and } U_7 = 1.5\%U_1.$$




$\|\boldsymbol{i}\| = 339.4$ A $\quad \|\boldsymbol{i}_a\| = 138.5$ A $\quad \|\boldsymbol{i}_s\| = 4.9$ A $\quad \|\boldsymbol{i}_r\| = 138.6$ A

$\|\boldsymbol{i}_u\| = 277.1$ A $\quad \|\boldsymbol{i}_u^z\| = 196.0$ A $\quad \|\boldsymbol{i}_u^p\| = 2.4$ A $\quad \|\boldsymbol{i}_u^n\| = 196.0$A

**$\lambda$ = 0.408**




$\|\boldsymbol{i}\| = 139.3$ A $\quad \|\boldsymbol{i}_a\| = 138.5$ A $\quad \|\boldsymbol{i}_s\| = 4.9$ A $\quad \|\boldsymbol{i}_r\| = 11.9$ A

$\|\boldsymbol{i}_u\| = 8.0$ A $\quad \|\boldsymbol{i}_u^z\| = 3.1$ A $\quad \|\boldsymbol{i}_u^p\| = 5.9$ A $\quad \|\boldsymbol{i}_u^n\| = 4.4$ A

$\lambda = 0.994$

**Thank you for attention!**