Key Note
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From Steinmetz to CPC: History of the Power Theory Development

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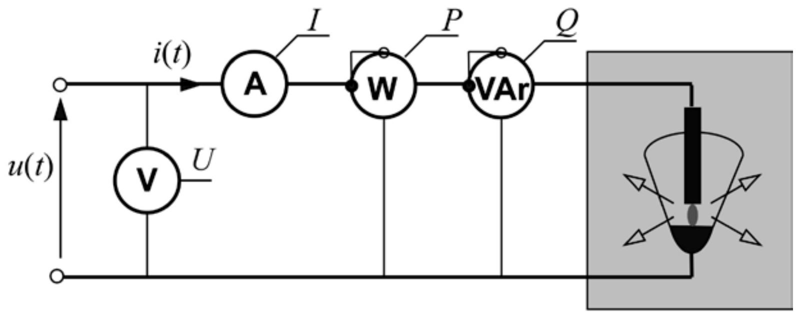
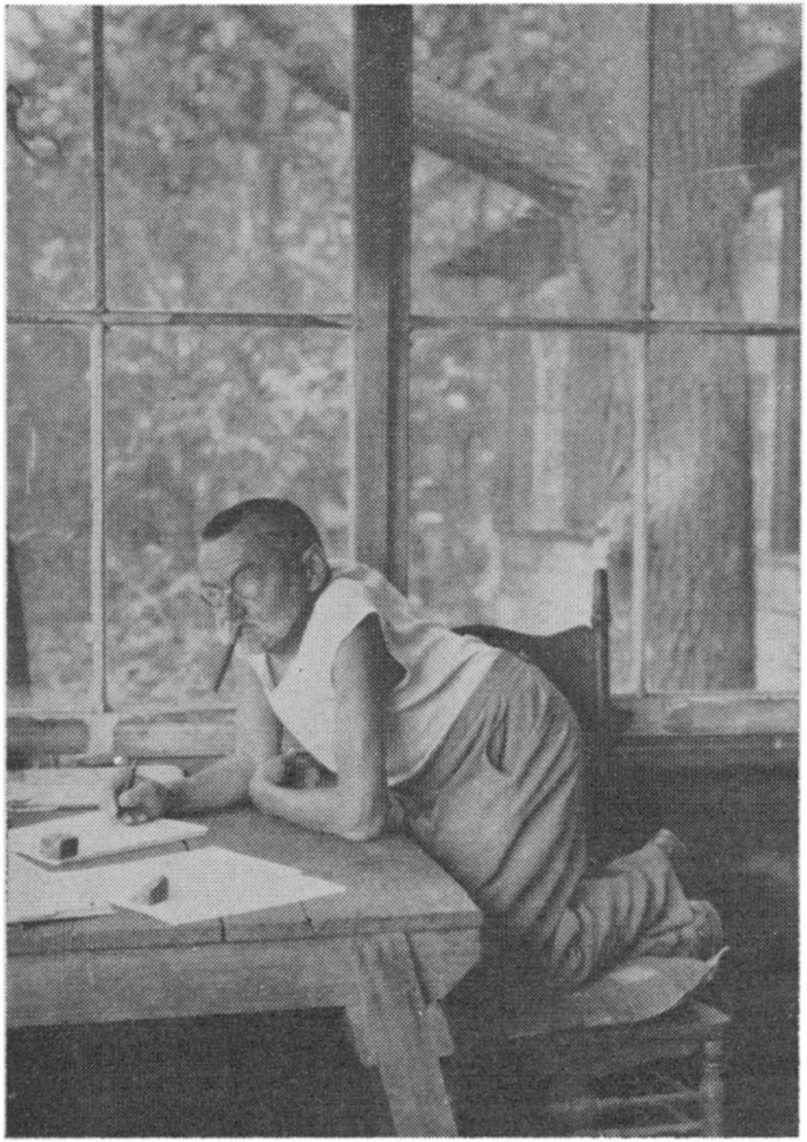
Steinmetz Experiment: 1892

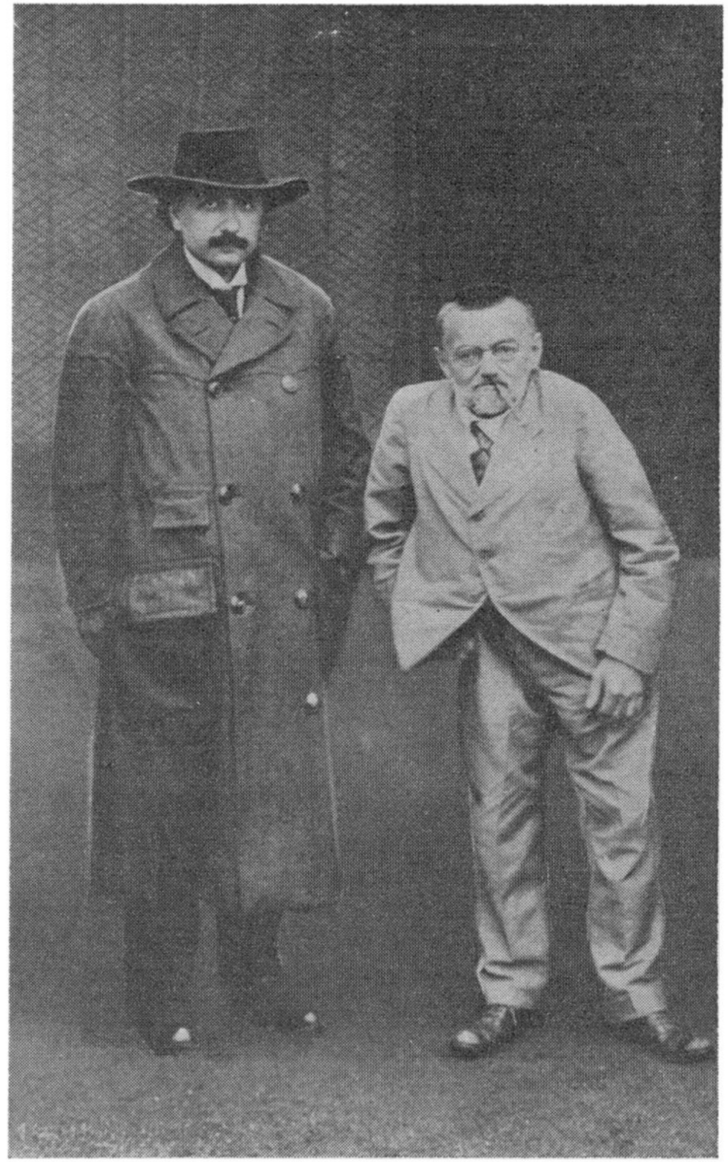


$$P^2 + Q^2 < S^2, \qquad Q = 0$$

?

$$S = U\,I > P$$

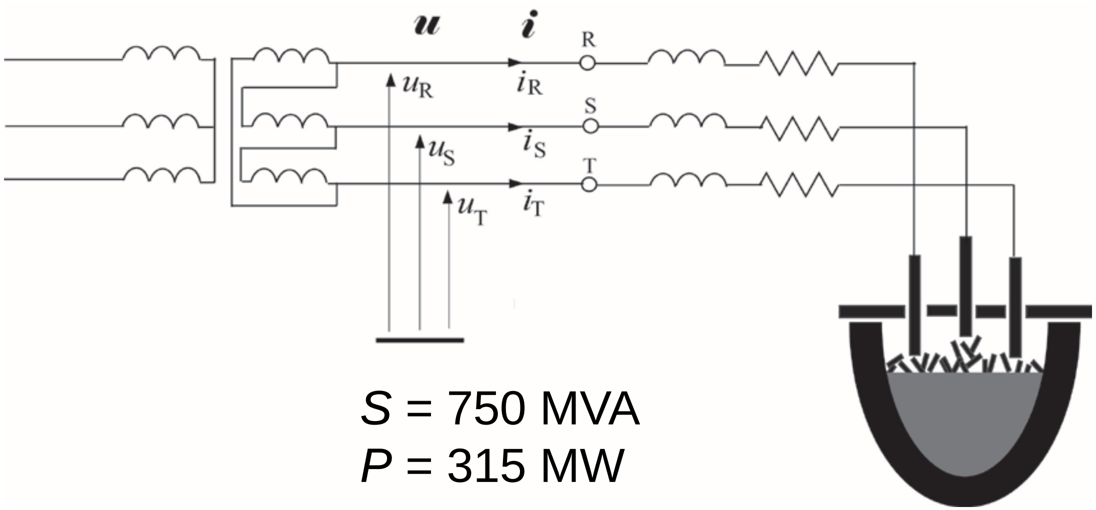




Einstein and Steinmetz.

Present day “Steinmetz Experiment”
with line currents up to

625 kA



S = 750 MVA
P = 315 MW

Current not only distorted, but also asymmetrical and random
Power factor: λ ~ 0.42

Annual bill for energy ~ 500 Million $

$$P^2 + Q^2 < S^2, \quad \text{even if} \quad Q = 0$$

$$P < S$$

?

**Explanation of this inequality
is the main subject
of the power theory of electrical circuits**

Inequality

$$P < S$$

is fundamental for power systems economy

since

the cost of the electric energy production and delivery is mainly dependent on the apparent power *S*

while

only the integral of the active power *P* has the value for the power system customers,

but

apparent power S is specified by the load properties

1927: Budeanu: $$S^2 = P^2 + Q_B^2 + D^2 \qquad Q_B = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$$

Endorsed by the IEEE Standard Dictionery of Electrical and Electronics Terms in 1992 and German Standards DIN in 1972

1931: Fryze: $$S^2 = P^2 + Q_F^2 \qquad Q_F = \|u\| \|i_{rF}\|$$

Endorsed by German Standards DIN in 1972

1971: Shepherd: $$S^2 = S_R^2 + Q_S^2 \qquad Q_S = \|u\| \|i_{rS}\|$$

1975: Kusters: $$S^2 = P^2 + Q_K^2 + Q_r^2 \qquad Q_K = \|u\| \|i_{rC}\|$$

Endorsed by the International Electrotechnical Commission in 1980

1979: Depenbrock: $$S^2 = P^2 + Q_1^2 + V^2 + N^2$$

2003: Tenti: $$S^2 = P^2 + Q_T^2 + D_T^2 \qquad Q_T = \|u\| \|i_{rT}\|$$

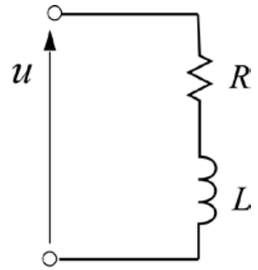
Major disscussion forums**:**

International Workshop on Reactive Power Definition and Measurements in Nonsinusoidal Systems,
Bi-annual meetings in Italy,
Chaired by A. Ferrero

International School on Nonsinusoidal Currents and Compensation (ISNCC)
Bi-annual meetings in Poland
Chaired by L.S. Czarnecki

Since the Steinmetz's experiment in 1892,
by 1983, after 91 years of Power Theory development
for single-phase, LTI loads, such as below

we had five different power equations and five different reactive powers



Compensation problem was not solved

1927: Budeanu: $S^2 = P^2 + Q_B^2 + D^2$ $\qquad Q_B = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$

1931: Fryze: $S^2 = P^2 + Q_F^2$ $\qquad Q_F = ||u|| \, ||i_{rF}||$

1971: Shepherd: $S^2 = S_R^2 + Q_S^2$ $\qquad Q_S = ||u|| \, ||i_{rS}||$

1975: Kusters: $S^2 = P^2 + Q_K^2 + Q_r^2$ $\qquad Q_K = ||u|| \, ||i_{rC}||$

1979: Depenbrock: $S^2 = P^2 + Q_1^2 + V^2 + N^2$

The problem was solved:

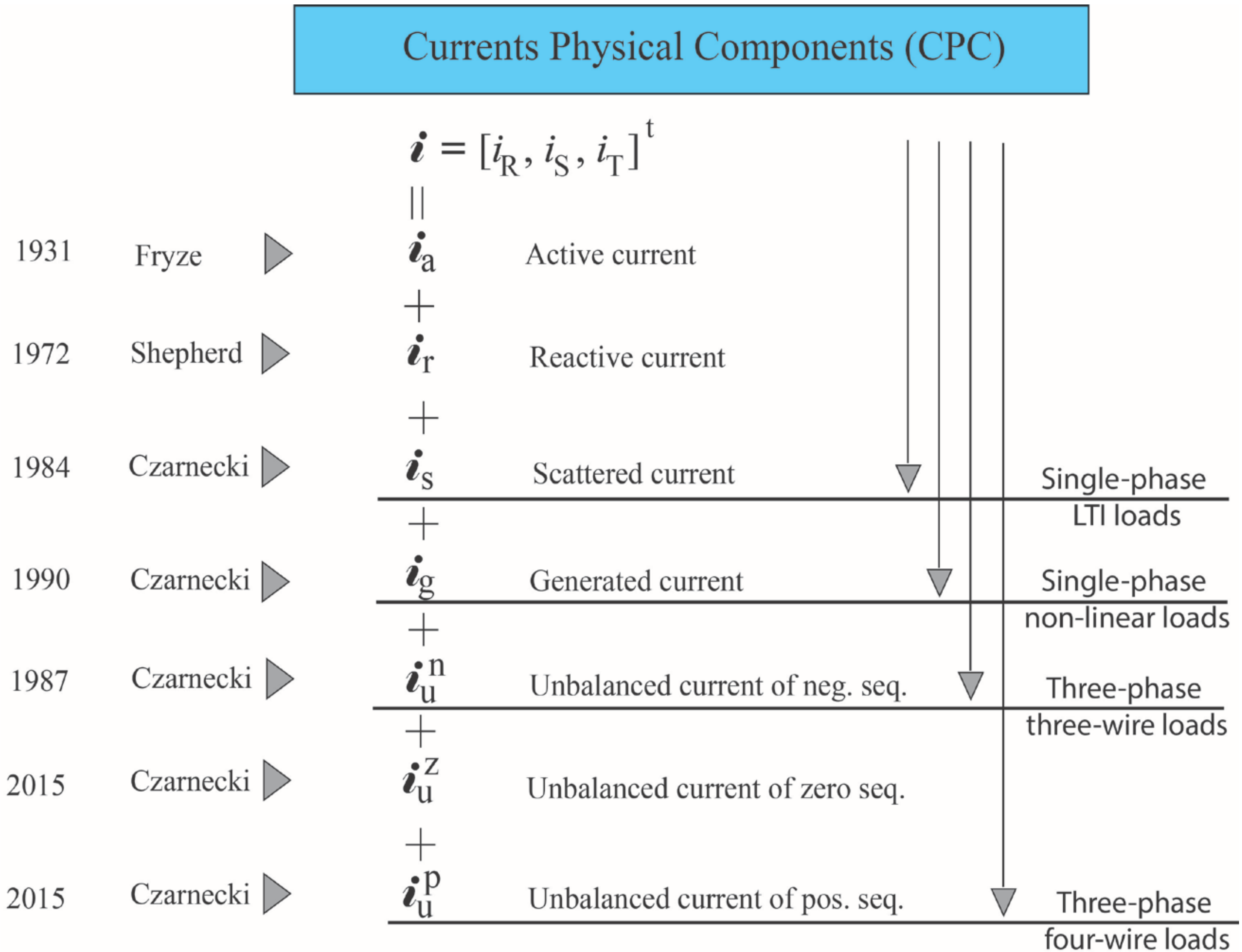
1984 r. Czarnecki: $S^2 = P^2 + Q^2 + D_s^2$

This, eventually a positive result
was a conclusion
of a specific approach to the power theory development.

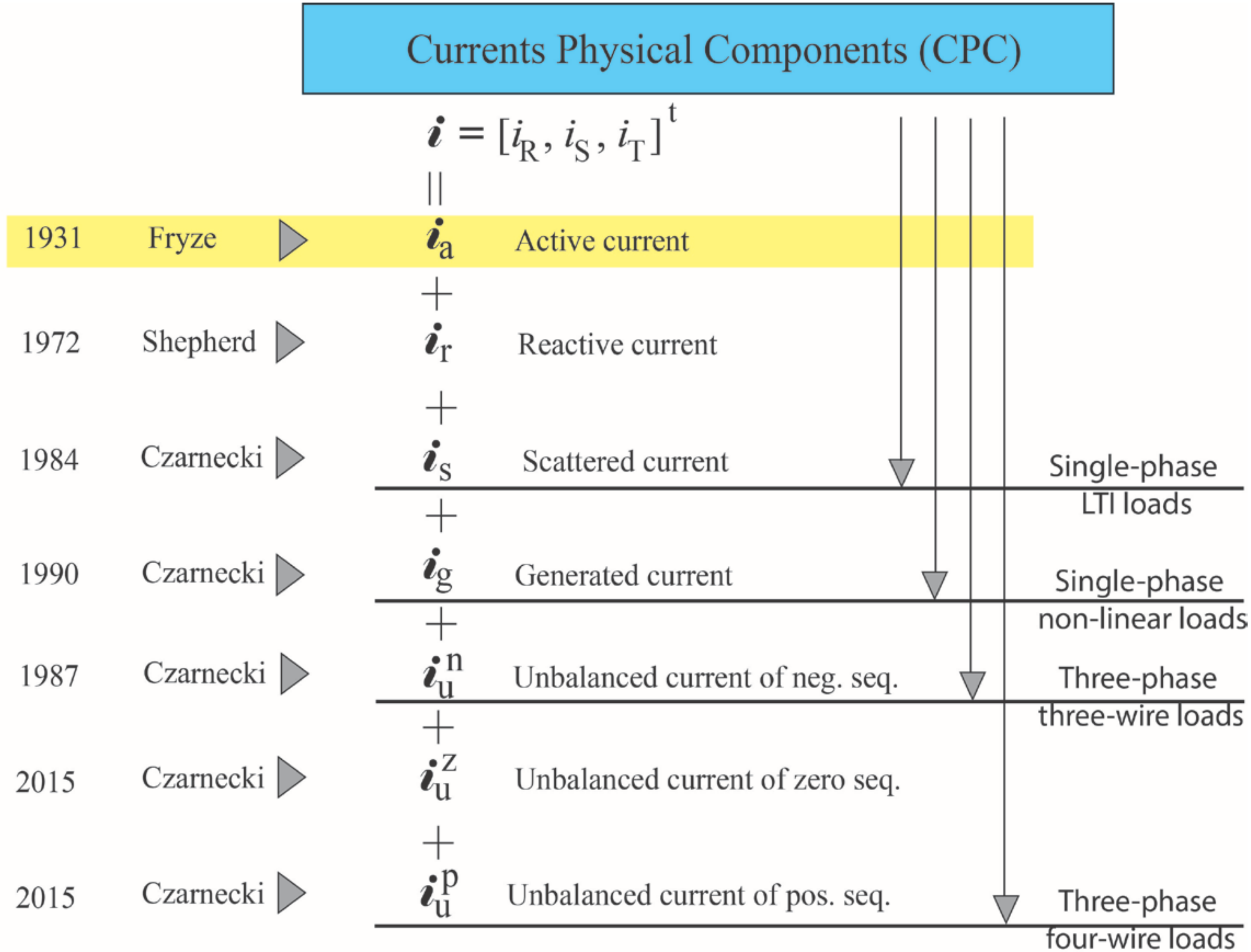
Its very core was decomposition of the load current

into

Currents' Physical Components (CPC)



Currents Physical Components (CPC)
$\boldsymbol{i} = [i_R, i_S, i_T]^t$
1931
Fryze
$\boldsymbol{i}_a$
Active current
1972
Shepherd
$\boldsymbol{i}_r$
Reactive current
1984
Czarnecki
$\boldsymbol{i}_s$
Scattered current
Single-phase LTI loads
1990
Czarnecki
$\boldsymbol{i}_g$
Generated current
Single-phase non-linear loads
1987
Czarnecki
$\boldsymbol{i}_u^n$
Unbalanced current of neg. seq.
Three-phase three-wire loads
2015
Czarnecki
$\boldsymbol{i}_u^z$
Unbalanced current of zero seq.
2015
Czarnecki
$\boldsymbol{i}_u^p$
Unbalanced current of pos. seq.
Three-phase four-wire loads



The active current is associated
with the phenomenon

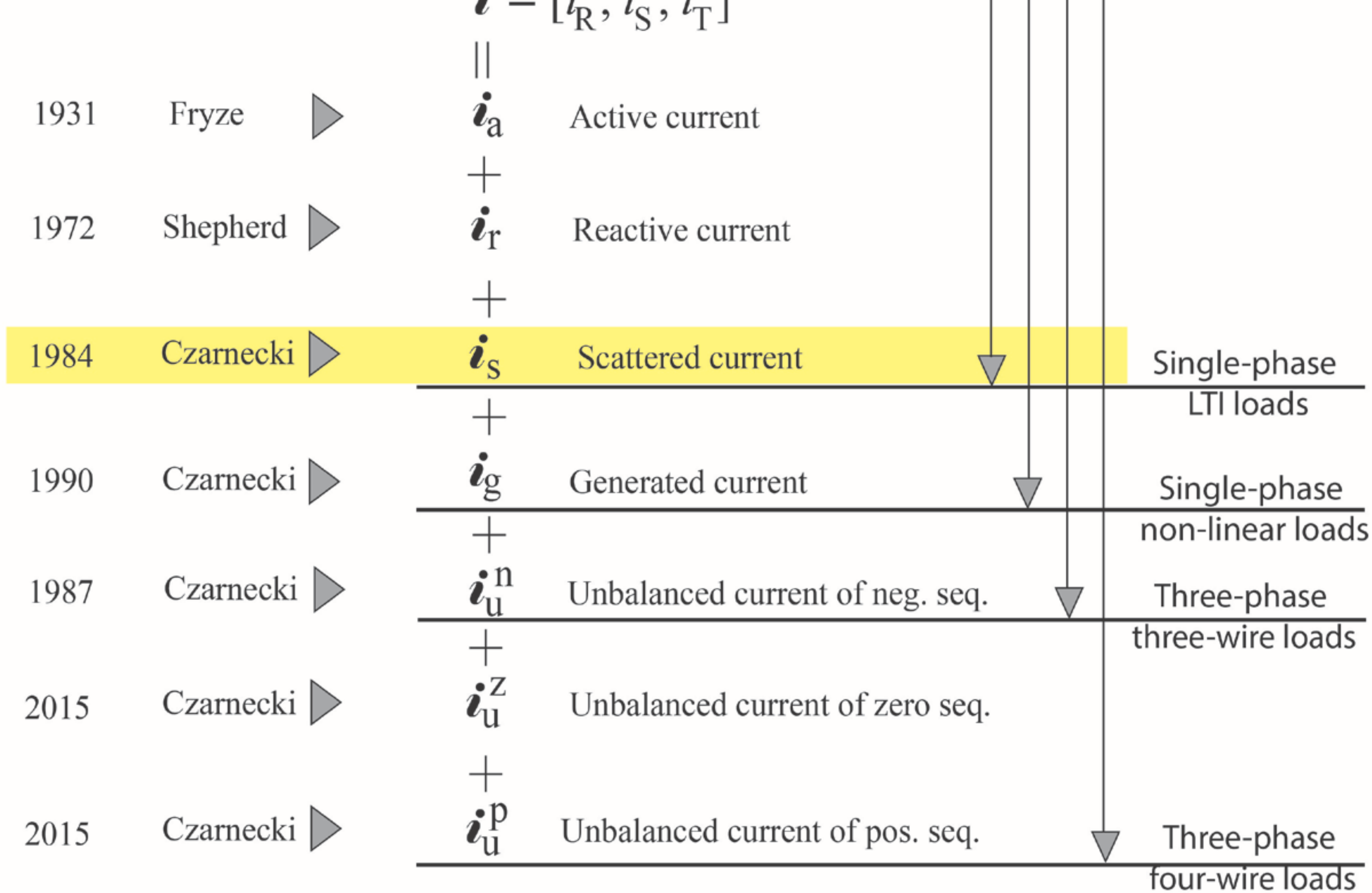
Currents Physical Components (CPC)

$$\boldsymbol{i} = [i_{\mathrm{R}}, i_{\mathrm{S}}, i_{\mathrm{T}}]^{\mathrm{t}}$$

$$||$$

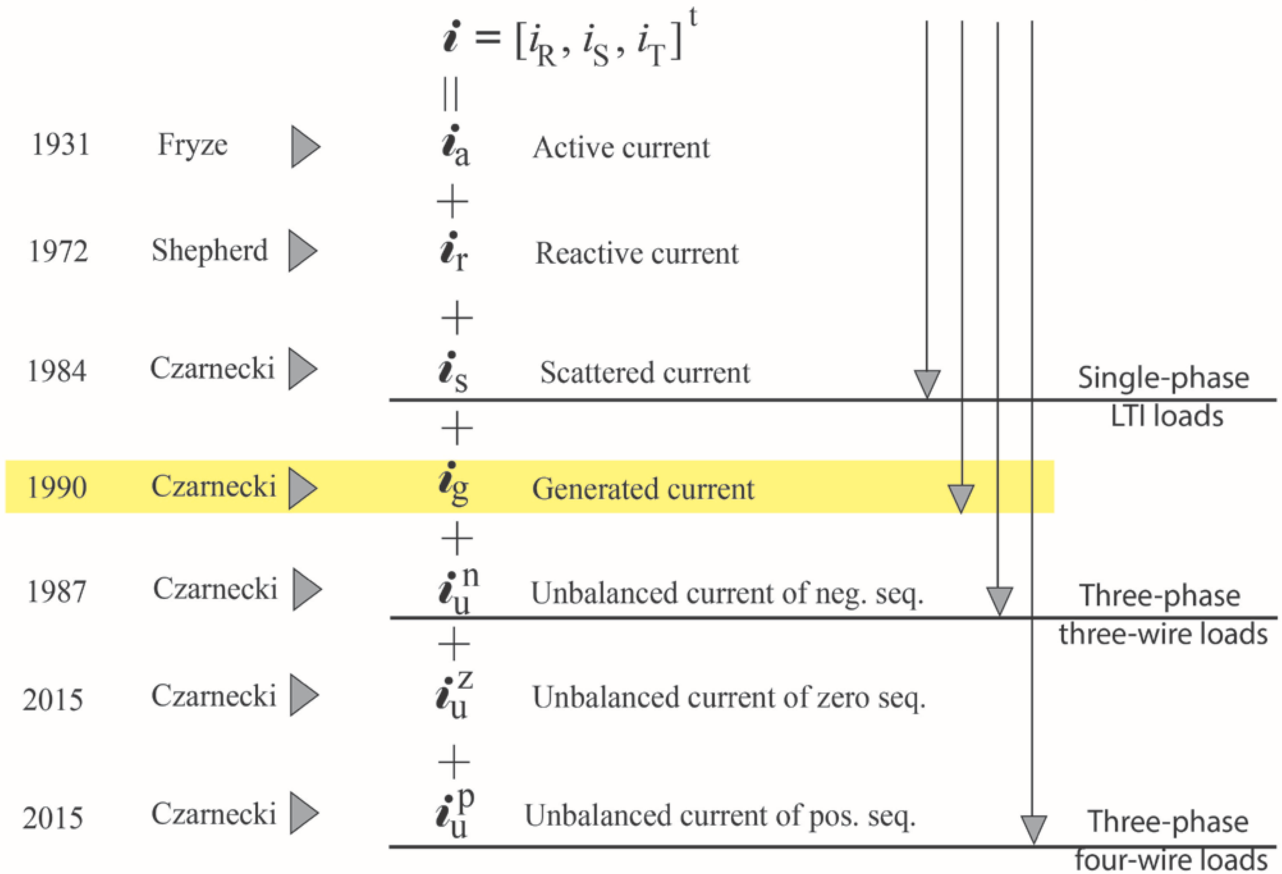
1931	Fryze	$\boldsymbol{i}_{\mathrm{a}}$	Active current	
		$+$		
1972	Shepherd	$\boldsymbol{i}_{\mathrm{r}}$	Reactive current	
		$+$		
1984	Czarnecki	$\boldsymbol{i}_{\mathrm{s}}$	Scattered current	Single-phase LTI loads
		$+$		
1990	Czarnecki	$\boldsymbol{i}_{\mathrm{g}}$	Generated current	Single-phase non-linear loads
		$+$		
1987	Czarnecki	$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}$	Unbalanced current of neg. seq.	Three-phase three-wire loads
		$+$		
2015	Czarnecki	$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$	Unbalanced current of zero seq.	
		$+$		
2015	Czarnecki	$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}$	Unbalanced current of pos. seq.	Three-phase four-wire loads

The reactive current is associated
with the phenomenon
of the phase-shift between voltage and current harmonics



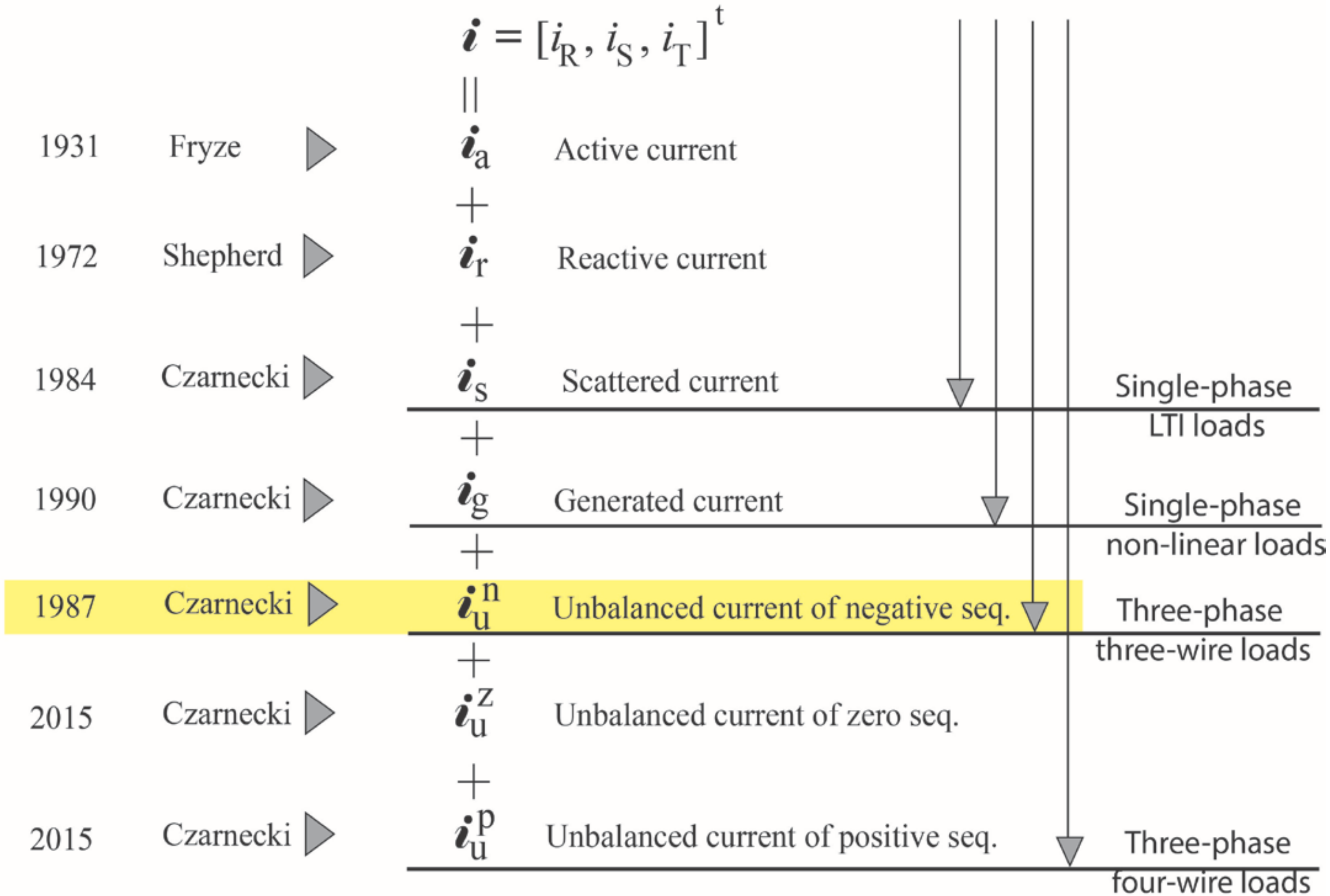
The scattered current is associated
with the phenomenon
of the change of the load conductance with harmonic order

Currents Physical Components (CPC)



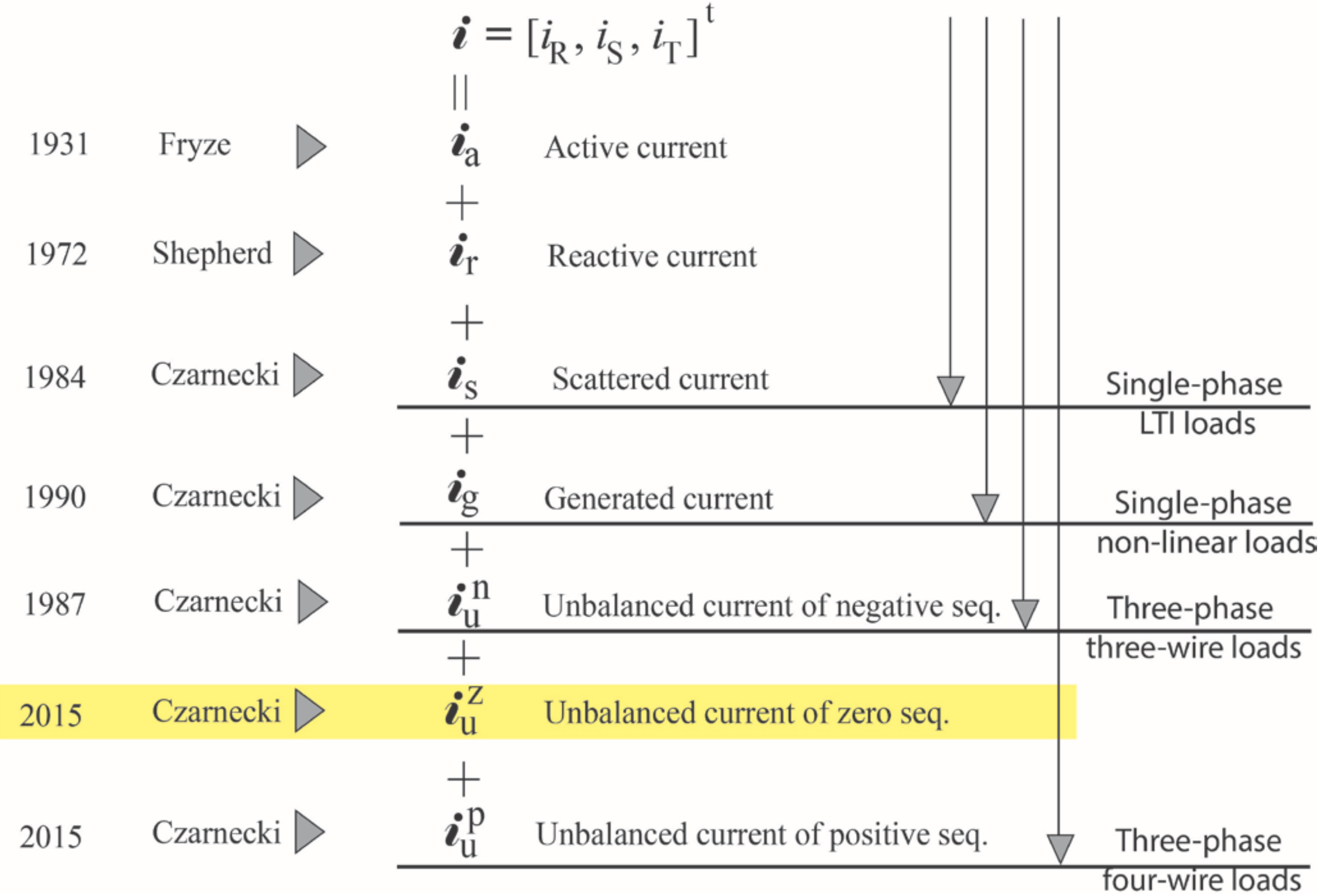
The generated current is associated
with the phenomenon
of reversed direction of energy flow due to current harmonics
generated in the load

Currents Physical Components (CPC)



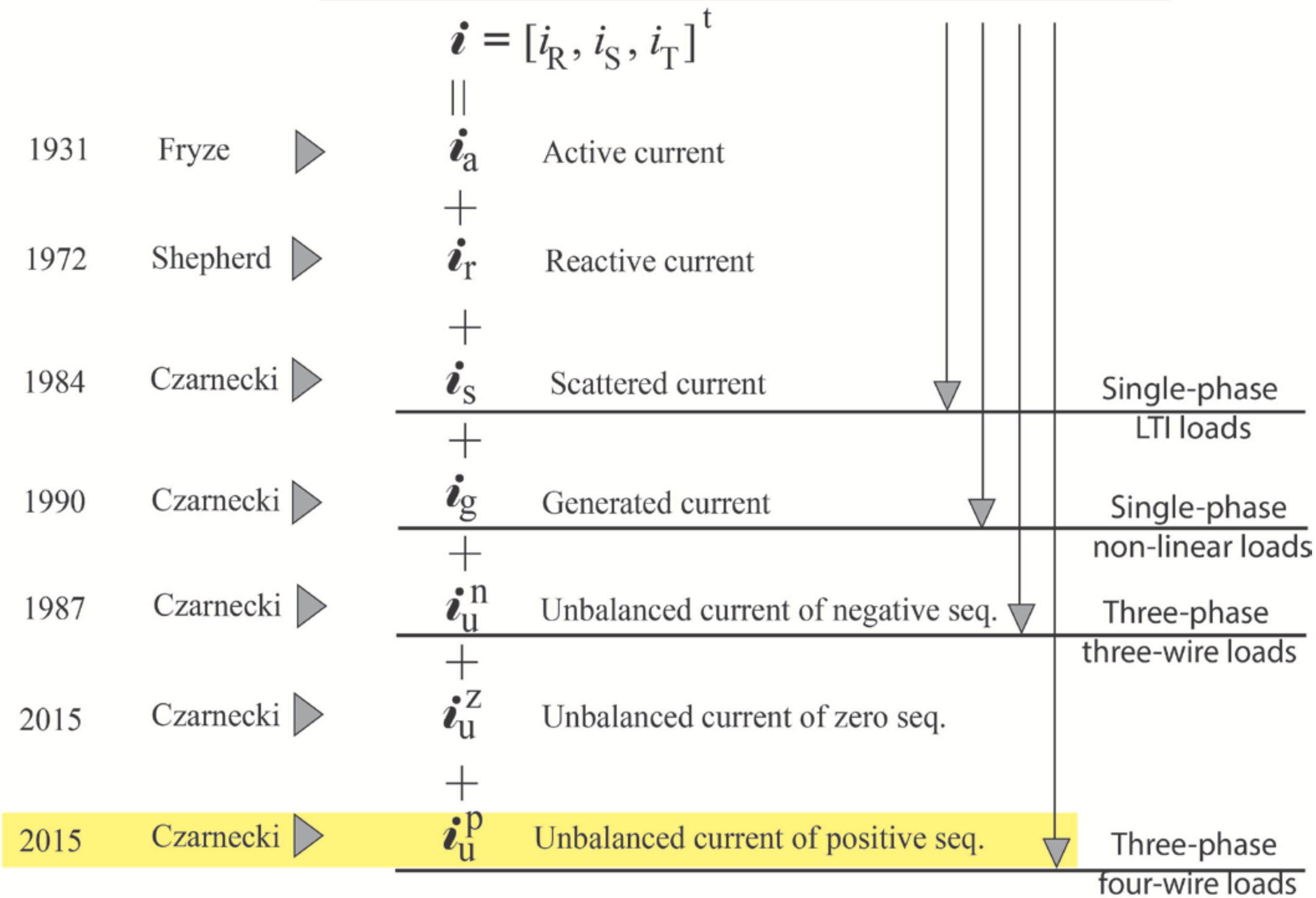
The unbalanced current of negative sequence is associated
with a phenomenon
of generation of negative sequence current by the load imbalance

Currents Physical Components (CPC)

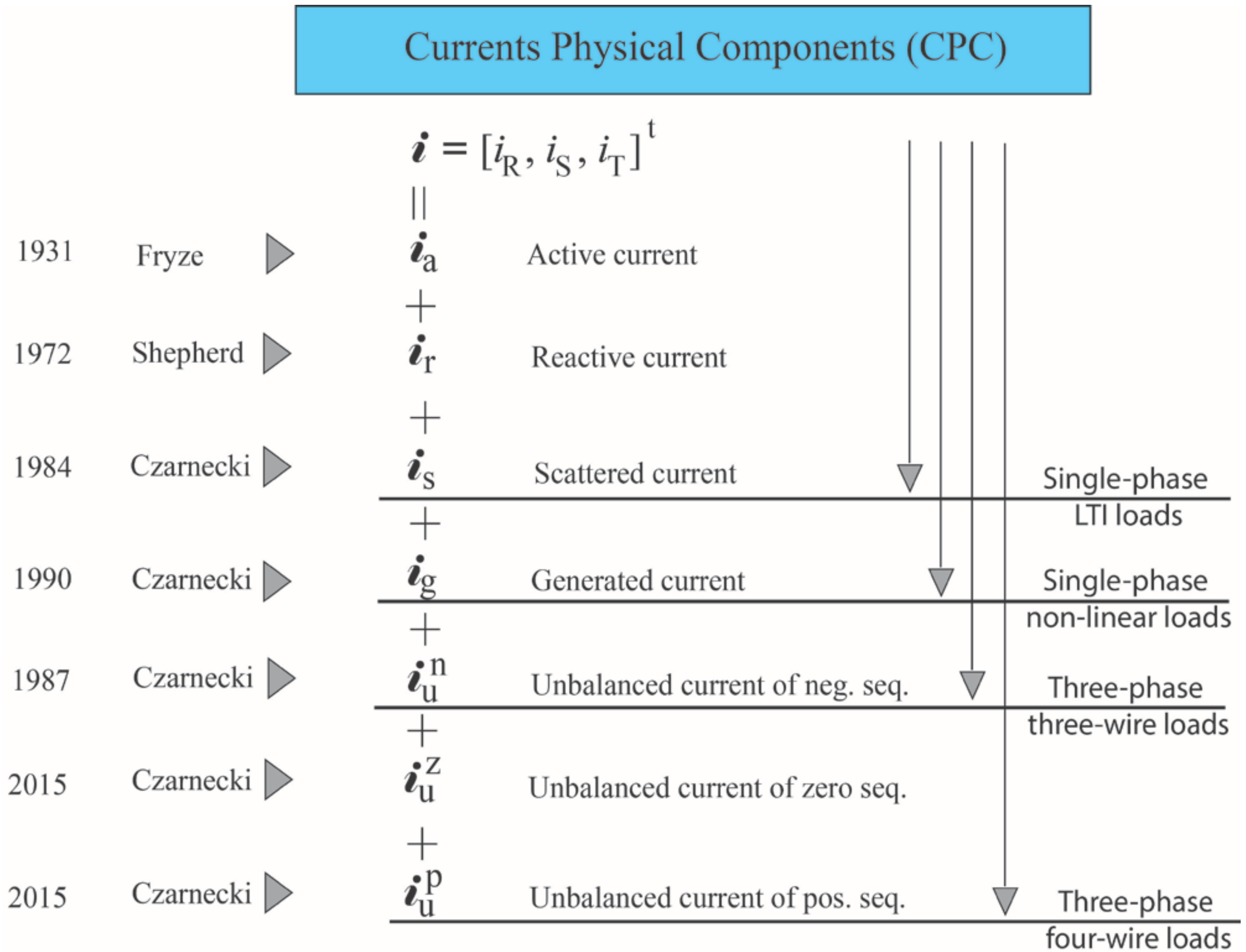


The unbalanced current of zero sequence is associated
with a phenomenon
of generation of zero sequence current by the load imbalance

Currents Physical Components (CPC)



The unbalanced current of positive sequence is associated
with a phenomenon
of generation of a positive sequence current by the load imbalance



Seven different phenomena contribute to the load current independently of each other

Each of these phenomena
contribute to the load current three-phase rms value

$$||\boldsymbol{i}|| = \sqrt{||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2}$$

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{s}}||^2 + ||\boldsymbol{i}_{\mathrm{g}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}||^2$$

When this equation is multiplied by the square of the voltage three-phase rms

$$||\boldsymbol{u}|| = \sqrt{||u_{\mathrm{R}}||^2 + ||u_{\mathrm{S}}||^2 + ||u_{\mathrm{T}}||^2}$$

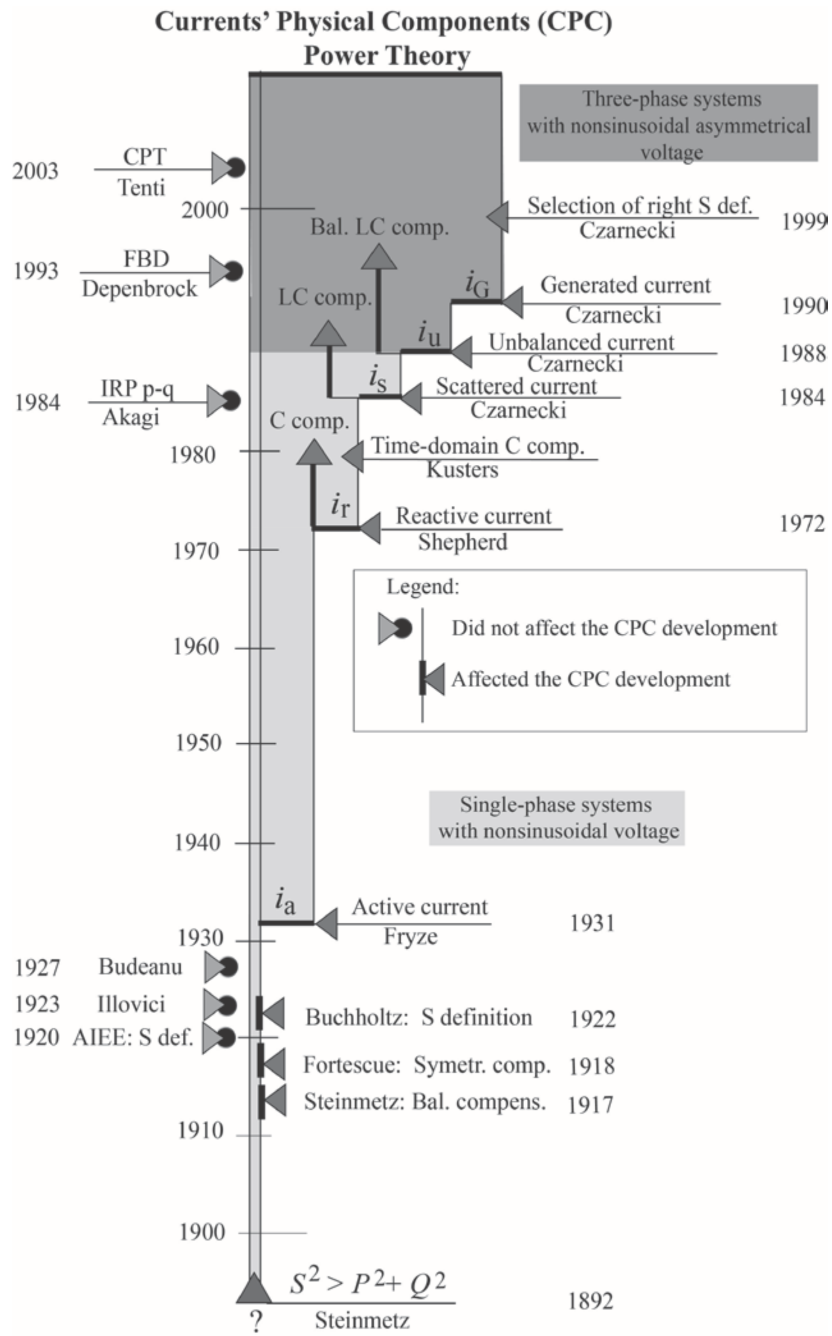
the power equation is obtained

$$S^2 = P^2 + Q^2 + D_{\mathrm{s}}^2 + D_{\mathrm{g}}^2 + D_{\mathrm{u}}^{\mathrm{n}\,2} + D_{\mathrm{u}}^{\mathrm{z}\,2} + D_{\mathrm{u}}^{\mathrm{p}\,2}$$

Development
of the Currents' Physical Components (CPC) – based Power Theory

was strongly correlated with evaluation of results

of other approaches
to this Theory development



Currents' Physical Components (CPC)
Power Theory
Three-phase systems with nonsinusoidal asymmetrical voltage
2003
CPT
Tenti
2000
Bal. LC comp.
Selection of right S def.
Czarnecki
1999
1993
FBD
Depenbrock
LC comp.
i_G
Generated current
Czarnecki
1990
i_u
Unbalanced current
Czarnecki
1988
i_s
Scattered current
Czarnecki
1984
1984
IRP p-q
Akagi
C comp.
1980
Time-domain C comp.
Kusters
i_r
Reactive current
Shepherd
1972
1970
Legend:
Did not affect the CPC development
Affected the CPC development
1960
1950
Single-phase systems with nonsinusoidal voltage
1940
i_a
Active current
Fryze
1931
1930
1927 Budeanu
1923 Illovici
1920 AIEE: S def.
Buchholtz: S definition 1922
Fortescue: Symetr. comp. 1918
Steinmetz: Bal. compens. 1917
1910
1900
$S^2 > P^2 + Q^2$
Steinmetz
1892
?

Two main approaches to the power theory development:

- Frequency-domain

- Time-domain

have competed for the whole period of its development

Prof. Budeanu (1927),

suggesting the reactive power definition:

$$Q_{\mathrm{B}} \stackrel{\mathrm{df}}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$$

attampted to develop the power theory

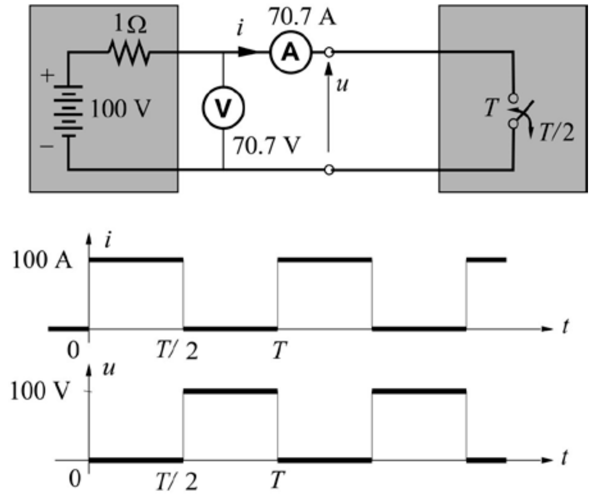
in the frequency-domain

Prof. Fryze (1931)

suggested that power theory
should be developed

without use of the concept of harmonics

Fryze's circuit:



Frequency-domain:

$$u(t) = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n) = \sum_{n=0}^{\infty} u_n$$

$$i(t) = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \beta_n) = \sum_{n=0}^{\infty} i_n.$$

$$p(t) = \frac{dW}{dt} = u(t)\, i(t) = \sum_{r=0}^{\infty} u_r \sum_{s=0}^{\infty} i_s = \sum_{n=0}^{\infty} S_n \cos(n\omega_1 t + \psi_n) = ?$$

Time-domain:

$$p(t) = \frac{dW}{dt} = u(t)\, i(t) = 0$$

was the main Fryze's argument against the frequency-domain

The Currents' Physical Components (CPC) – Power Theory,
which is currently

the most advanced concept of the power theory of electrical systems

was formulated

in the frequency-domain

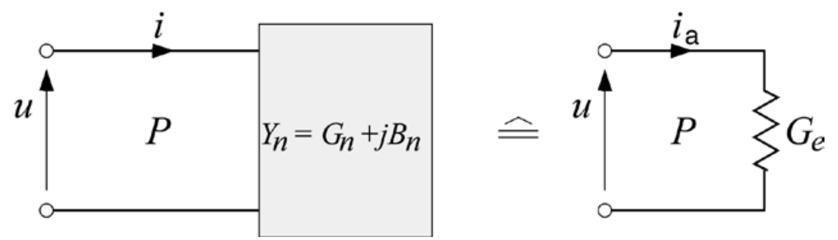
1984

L.S. Czarnecki: Considerations on the Reactive Power Under Nonsinusoidal Conditions

IEEE Transactions on Instrumentation and Measurements,

$$u = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

$$i = G_0 U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{Y}_n \boldsymbol{U}_n\, e^{jn\omega_1 t}$$



$$i = i_\mathrm{a} + i_\mathrm{s} + i_\mathrm{r}$$

$$i_\mathrm{a} \overset{\mathrm{df}}{=} G_\mathrm{e}\, u, \qquad G_\mathrm{e} \overset{\mathrm{df}}{=} \frac{P}{\|u\|^2}$$

Active current (Fryze)

$$i_\mathrm{r} \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

Reactive current (Shepherd)

$$i_\mathrm{s} \overset{\mathrm{df}}{=} (G_0 - G_\mathrm{e})U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} (G_n - G_\mathrm{e})\boldsymbol{U}_n\, e^{jn\omega_1 t}$$

Scattered current (Czarnecki)

The scattered and reactive currents are defined in the frequency-domain.

Equivalent definitions of these currents in the time-domain are not known.

There is also a dabate on,
should the power theory be developed
based on

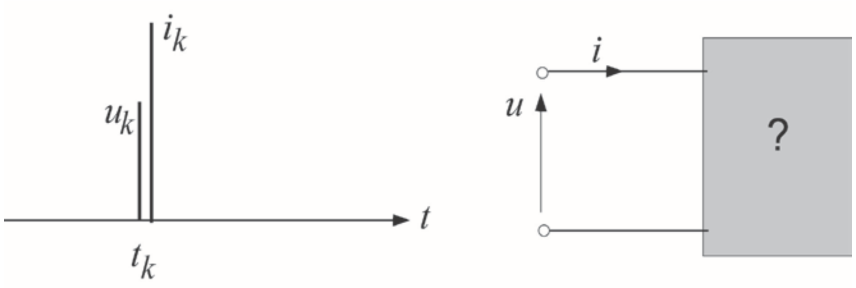
- instantaneous approach
or
- averaged approach

?

The Instantaneous Reactive Power p-q Theory,
developed
by Akagi, Nabae, Kanazawa

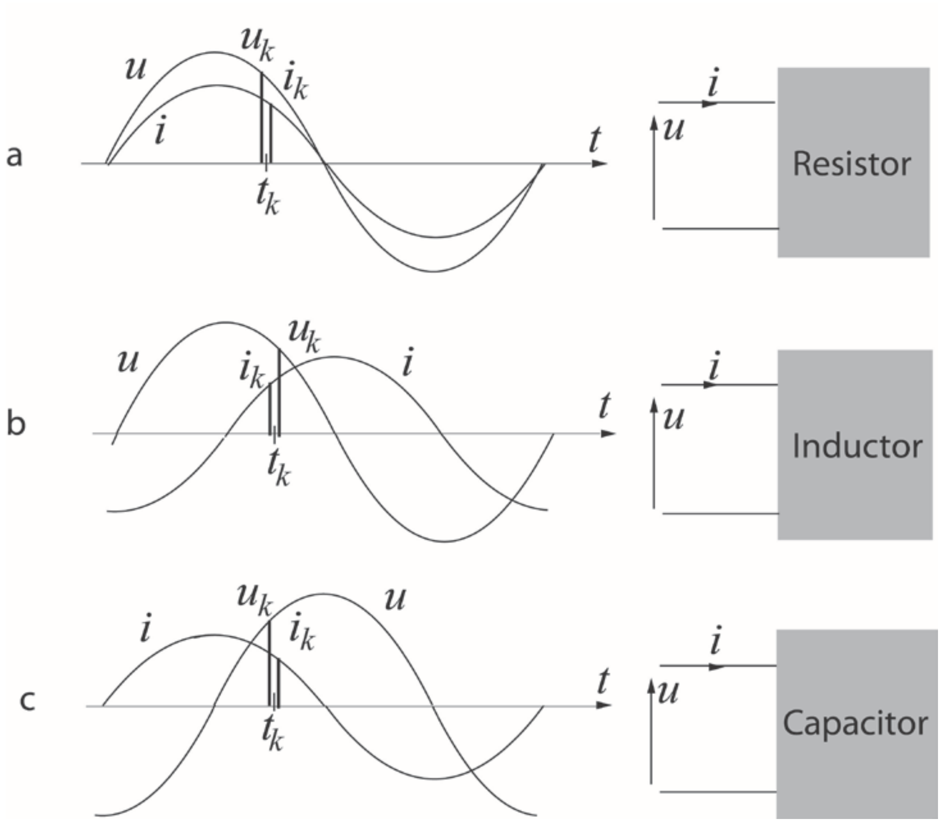
is the main example of the instantaneous approach

Let us have a pair of instantaneous values
of voltages and currents



Question:

What it is in the box: resistor, inductor or capacitor?



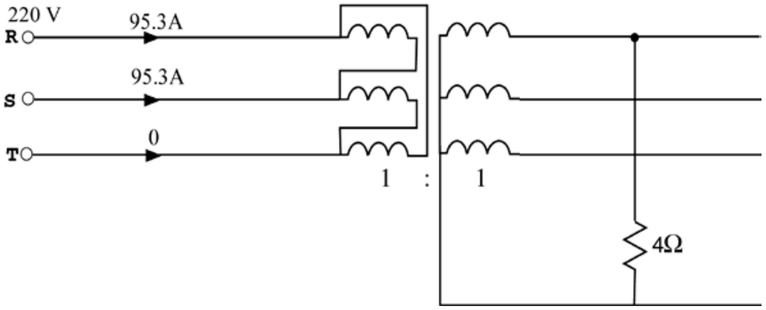
A pair of instantaneous values of voltages and current does not enable us to determine the load

To determine the load properties, these pairs have to be observed for the whole period *T*

Numerical illustration 1

$$u_R = \sqrt{2}\,U\cos\omega t, \quad U = 220\ \text{V}$$

$$i_R = \sqrt{2}\,I\cos(\omega t + 30^0), \quad I = 95.3\ \text{A}$$



$Q = 0$

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = C\begin{bmatrix} u_R \\ u_S \end{bmatrix} = \begin{bmatrix} \sqrt{3}\,U\cos\omega t \\ \sqrt{3}\,U\sin\omega t \end{bmatrix}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = C\begin{bmatrix} i_R \\ -i_R \end{bmatrix} = \begin{bmatrix} \sqrt{3}\,I\cos(\omega t + 30^o) \\ -I\cos(\omega t + 30^o) \end{bmatrix}$$

Instantaneous powers,

Active: $p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}\,U\,I\,[1 + \cos 2(\omega t + 30^0)]$

Reactive: $q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}\,U\,I\sin 2(\omega t + 30^0)$

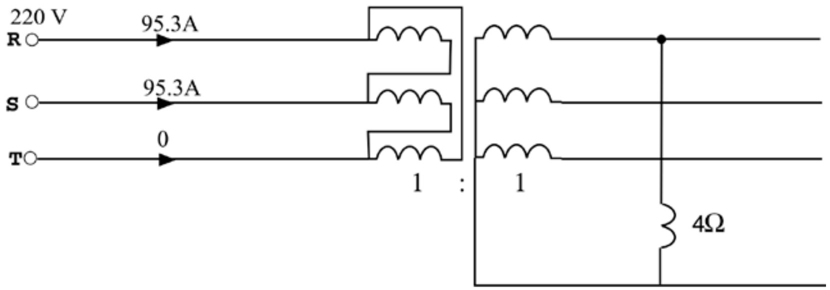
For $2(\omega t + 30^0) = 90^0, \quad p = -q$

Instantaneous currents,

Active: $$\begin{bmatrix} i_{Rp} \\ i_{Sp} \end{bmatrix} = \sqrt{\frac{2}{3}}\,I\,[1 + \cos 2(\omega t + 30^0)]\begin{bmatrix} \cos\omega t \\ \cos(\omega t - 120^0) \end{bmatrix}$$

Reactive: $$\begin{bmatrix} i_{Rq} \\ i_{Sq} \end{bmatrix} = \sqrt{\frac{2}{3}}\,I\sin 2(\omega t + 30^0)\begin{bmatrix} \sin\omega t \\ \sin(\omega t - 120^0) \end{bmatrix}$$

Numerical illustration 2



$$u_R = \sqrt{2}\,U\cos\omega t, \quad U = 220\ \text{V}$$

$$i_R = \sqrt{2}\,I\cos(\omega t - 60^0), \quad I = 95.3\ \text{A}$$

$P = 0$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = C\begin{bmatrix} i_R \\ -i_R \end{bmatrix} = \begin{bmatrix} \sqrt{3}\,I\cos(\omega t - 60^0) \\ -I\cos(\omega t - 60^0) \end{bmatrix}$$

Instantaneous powers,

Active: $p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}\,U\,I\,[\cos(2\omega t - 30^0)]$

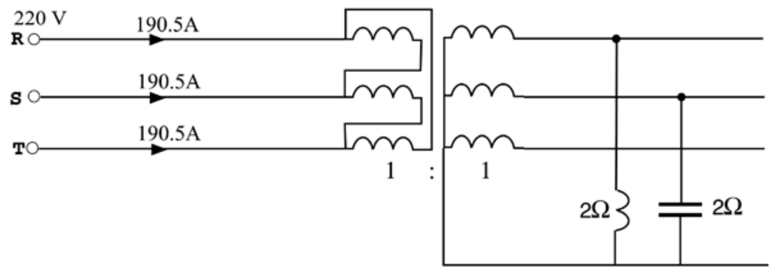
Reactive: $q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}\,U\,I\,[1 + \sin(2\omega t - 30^0)]$

For $2\omega t - 30^0 = 0, \quad p = -q$

Instantaneous active current in R line:

$$i_{Rp} = \frac{I}{\sqrt{6}}[\cos(\omega t - 30^0) + \cos(3\omega t - 30^0)]$$

Numerical illustration 3



$$u_R = \sqrt{2}\,U\cos\omega t, \quad U = 220 \text{ V}$$

$$i_R = \sqrt{2}\,I\cos(\omega t - 60^0), \quad I = 190.5 \text{ A}$$

$$i_S = \sqrt{2}\,I\cos(\omega t + 60^0)$$

$P = 0, \quad Q = 0$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = C\begin{bmatrix} i_R \\ i_S \end{bmatrix} = \begin{bmatrix} \sqrt{3}\,I\cos(\omega t - 60^0) \\ -\sqrt{3}\,I\sin(\omega t - 60^0) \end{bmatrix}$$

Instantaneous powers,

Active: $p = u_\alpha i_\alpha + u_\beta i_\beta = 3UI\cos(2\omega t - 60^0)$

Reactive: $q = u_\alpha i_\beta - u_\beta i_\alpha = -3UI\sin(2\omega t - 60^0)$

For $2\omega t - 60^0 = 45^0$, $p = -q$

Instantaneous currents,

Active:

$$i_{\alpha p} = \frac{\sqrt{3}}{2} I[\cos(\omega t - 60^0) + \cos(3\omega t - 60^0)]$$

$$i_{\beta p} = \frac{\sqrt{3}}{2} I[-\sin(\omega t - 60^0) + \sin(3\omega t - 60^0)]$$

Reactive:

$$i_{\alpha q} = \frac{\sqrt{3}}{2} I[\cos(\omega t - 60^0) - \cos(3\omega t - 60^0)]$$

$$i_{\beta q} = \frac{\sqrt{3}}{2} I[\sin(\omega t - 60^0) + \sin(3\omega t - 60^0)]$$

All measures that determine power system performance, such us:

- rms values of voltages and currents,
- active power *P*
- reactive power *Q*
- apparent power *S*
- power factor $\lambda = P/S$
- level of harmonic distortion
- level of the voltage and current asymmetry

are functionals
calculated over the whole period *T*

Instantaneous power theories such as, for example, the Instantaneous Reactive Power p-q Theory, do not provide such measures.

The Currents' Physical Components (CPC) – Power Theory,
which is
was formulated
based on the averaging approach

Short overview of results of various approaches to power theory development

Budeanu (1927) Power Theory:
(In the frequency-domain)

Endorsed by the IEEE Standard Dictionery of Electrical and Electronics Terms in 1992 and German Standards DIN in 1972

- Reactive and distortion powers *Q* and *D* are not associated with any power phenomena

- It does not provide fundamentals for compensation

1987

L.S. Czarnecki: **What is Wrong with the Budeanu's Concept of Reactive and Distortion Powers and why it Should be Abandoned**,
IEEE Trans. on Instrumentation and Measurements

Fryze (1931) Power Theory

(In the time-domain)

Endorsed by German Standards DIN in 1972

- It introduced the concept of the active current
- It introduced the concept of the current orthogonal decomposion
- It does not provide physical interpretation of power phenomena
- It does not provide fundamentals for reactive compensation

L.S. Czarnecki, “Budeanu and Fryze: Two frameworks for interpreting power properties of circuits with nonsinusoidal voltages and currents,” *Archiv fur Elektrotechnik*, (81), N. 2, pp. 5-15, 1997.

Shepherd and Zakikhani (1972) Power Theory
(In the frequency-domain)

- It introduced the concept of the reactive current

- It solved the problem of optimal capacitive compensation at nonsinusoidal supply voltage

Kusters and Moore (1980) power theory

(In the time-domain)

Endorsed by the International Electrotechnical Commission in 1980

- It solved the problem of optimal capacitive compensation at nonsinusoidal supply voltage in time-domain

L.S. Czarnecki, "Additional discussion to "Reactive power under nonsinusoidal conditions," *IEEE Trans. on Power and Systems,* Vol. PAS-102, No. 4, April 1983.

L.S. Czarnecki, "Comments on reactive powers as defined by Kusters and Moore for nonsinusoidal systems," *Rozprawy Elektrotechniczne,* Tom XXX, Z. 3-4, pp. 1089-1099, 1984.

Nabae and Akagi (1984): Instantaneous Reactive Power p-q Theory

(in the time-domain)

- It solved the problem of compensation in three-phase of unbalanced, harmonics generating loads supplied with sinusoidal and symmetrical supply voltage
- It does not describe power properties of systems with nonsinusoidal supply voltage
- It misinterprates power phenomena in three-phase systems

L.S. Czarnecki, "On some misinterpretations of the Instantaneous Reactive Power p-q Theory," IEEE Trans. on Power Electronics, Vol. 19, No.3, pp. 828-836, 2004.

L.S. Czarnecki, "Comparison of the Instantaneous Reactive Power, p-q, Theory with Theory of Current's Physical Components," *Archiv fur Elektrotechnik*, Vol. 85, No. 1, Feb. 2004, pp. 21-28.

L.S. Czarnecki, "Effect of supply voltage harmonics on IRP-based switching compensator control," *IEEE Trans. on Power Electronics*, Vol. 24, No. 2, Feb. 2009. pp. 483-488.

L.S. Czarnecki, "Effect of supply voltage asymmetry on IRP p-q - based switching compensator control," *IET Proc. on Power Electronics*, 2010, Vol. 3, No. 1, pp. 11-17.

L.S. Czarnecki, "Constraints of the Instantaneous Reactive Power p-q Theory", IET Power Electronics, Vol. 7, No. 9, pp.2201-2208, 2014.

Depenbrock (1993) the FBD Method

(In the time-domain)

- It generalizes Fryze's power theory to three-phase systems

- It correctly defines the apparent power S

- It does not provide interpretation of power phenomena

- It does not provide fundamentals for reactive compensation

Tenti (2003): The Conservative Power Theory (CPT)
(In the time-domain)

- It misinterprates power phenomena

- It does not provide right fundamentals for compensation

L.S.Czarnecki: What is Wrong with the Conservative Power Theory (CPT), Int. Conference on Applied and Theoretical Electrical Eng. (ICATE) Romania, 2016

Conclusions

Power properties of electrical circuits,
along with fundamentals of their compensation
are now
entirely explaind
in terms of
Currents' Physical Componets (CPC) – based power theory

There is now
a lot of challanges
related
to its technical implementations, especially for compensation

Thanks for your attantion!!