

Adaptive Balancing of Three-Phase Loads at Four-Wire Supply with Reactive Compensators and Nonsinusoidal Voltage

Leszek S. Czarnecki, IEEE Life Fellow
School of Electrical Engineering and Computer Science,
Louisiana State University, LSU
Baton Rouge
lsczar@cox.net; Internet Page: czarnecki.study;

Motab Almousa
School of Electrical Engineering and Computer Science,
Louisiana State University, LSU
Baton Rouge
malmou2@lsu.edu;

Abstract – The paper presents fundamentals of an adaptive balancing of three-phase loads supplied from a four-wire line with a nonsinusoidal voltage, with the use of reactive compensators. Parameters of the compensator are calculated using the Currents' Physical Components (CPC) – based power theory. Thyristor Switched Inductors (TSIs) are used to convert a fixed-parameters compensator into an adaptive one. The adaptive compensator is composed of two sub-compensators, one in the Y structure and the other in the Δ structure. The method is illustrated in the paper with a computer model.

Keywords— *asymmetrical systems, CPC, Currents' Symmetrical Components, unbalanced loads, power definitions, power theory.*

I. INTRODUCTION

Due to the presence of single-phase loads in manufacturing plants, there is usually in such plants some level of the load imbalance. Due to such imbalance, an unbalanced current occurs in the supply lines of such plants. This current, along with the reactive current, causes a degradation of the power factor at the supply terminals.

In systems with the power in a range of a few MVA, the load imbalance, along with the reactive and harmonic currents, can be reduced by switching compensators, commonly known as “active power filters”. When this power is in the range of hundreds MVA, as this is common for major manufacturing plants, in particular, in metallurgic ones, the power of switching compensators could not be sufficient and reactive balancing compensators have to be used instead.

Unlike switching compensators, reactive compensators are essentially fixed parameters devices. To convert them to adaptive devices, thyristor switched inductors (TSIs) are commonly used. With the switching capability of thyristors now on the level of 50 kA, there are technical tools needed for the development of the adaptive reactive compensators. Unfortunately, such TSIs are sources of harmonic currents, which can substantially disturb the compensator and the system performance.

This paper is focussed on the development of adaptive reactive compensators of very high power and on how to handle harmonic currents in such devices.

To this moment, the major obstacle in the development of the reactive compensators was a theoretical [5, 11] The first balancing compensator was developed by Steinmetz [1], and this compensator is known [8] as a Steinmetz circuit.

Because of controversy [3, 5, 11, 12, 15] on power proper-

ties and power definitions in systems with nonsinusoidal voltages and currents, only optimization methods were capable [10] of providing LC parameters of the balancing reactive compensators. Unfortunately, a lot of computation is needed at optimization methods, so that they could be too slow to be used for an adaptive compensator control in real-time. Formulae that enable a direct calculation of these parameters are needed for that and there were a number of published reports on it [9, 13, 14, 18]. These formulae were developed [5, 17] in a frame of the CPC – based power theory [19] of electrical systems.

This paper will draft the procedure needed for calculating the parameters of a reactive balancing compensator for an unbalanced load of the fixed structure and power. Details of this procedure can be found in the referenced literature. Conversion of the fixed-parameters reactive compensator into an adaptive one is the main subject of this paper.

II. A DRAFT OF A BALANCING REACTIVE COMPENSATOR DESIGN PROCEDURE

Distribution systems in manufacturing plants supply not only balanced three-phase loads, but also aggregates of single-phase loads such as lightning, instrumentation and control systems, and electrical transportation. Electrical systems that supply such unbalanced loads are built as three-phase systems with neutral, i.e., as four-wire systems, shown in Fig. 1.

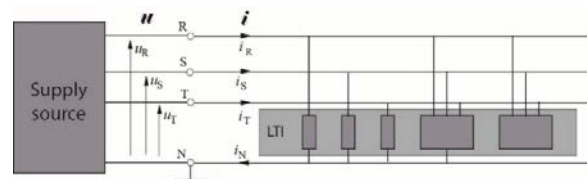


Fig.1. A structure of a three-phase, four-wire system.

Some of these loads, due to non-linearity or periodic time variance can generate current harmonics. Reactive compensators do not have the capability of compensating them, however. Therefore, from the perspective of a reactive compensator design, the load has to be approximated by a linear, time-invariant (LTI) load. After the major harmful current components, meaning the reactive and the unbalanced currents are compensated by a reactive compensator, harmonic currents can be reduced by a switching compensator of a substantially reduced power.

The supply voltage in such systems is usually distorted and the compensator has to be able to operate in the presence of the voltage harmonics. This voltage can be presented in the form

$$\mathbf{u}(t) = \sum_{n \in N} \mathbf{u}_n(t) = \sqrt{2} \operatorname{Re} \sum_{n \in N} \begin{bmatrix} U_{Rn} \\ U_{Sn} \\ U_{Tn} \end{bmatrix} e^{jn\omega t} = \sqrt{2} \operatorname{Re} \sum_{n \in N} U_n e^{jn\omega t}. \quad (1)$$

In response to this voltage, the load current is

$$\mathbf{i}(t) = \sum_{n \in N} \mathbf{i}_n(t) = \sqrt{2} \operatorname{Re} \sum_{n \in N} \begin{bmatrix} I_{Rn} \\ I_{Sn} \\ I_{Tn} \end{bmatrix} e^{jn\omega t} = \sqrt{2} \operatorname{Re} \sum_{n \in N} I_n e^{jn\omega t}. \quad (2)$$

Symbol N in these formulae denotes the set of orders n of the supply voltage dominating harmonics.

According to the Currents' Physical Components (CPC) – based power theory [19], the load current of such a load can be decomposed into six physical components

$$\mathbf{i} = \mathbf{i}_a + \mathbf{i}_s + \mathbf{i}_r + \mathbf{i}_u^p + \mathbf{i}_u^n + \mathbf{i}_u^z. \quad (3)$$

In this decomposition only the active current

$$\mathbf{i}_a \stackrel{\text{df}}{=} \begin{bmatrix} i_{Ra} \\ i_{Sa} \\ i_{Ta} \end{bmatrix} = G_e \mathbf{u} = \sqrt{2} \operatorname{Re} \sum_{n \in N} G_e U_n e^{jn\omega t} \quad (4)$$

where

$$G_e = \frac{P}{\|\mathbf{u}\|^2} \quad (5)$$

is useful. In (5) the symbol P denotes the load active power; $\|\mathbf{u}\|$ denotes the three-phase rms value of the supply voltage. All remaining components in (3) are harmful and contribute to an increase of the supply current three-phase rms value

$$\|\mathbf{i}\|^2 = \|\mathbf{i}_a\|^2 + \|\mathbf{i}_s\|^2 + \|\mathbf{i}_r\|^2 + \|\mathbf{i}_u^p\|^2 + \|\mathbf{i}_u^n\|^2 + \|\mathbf{i}_u^z\|^2. \quad (6)$$

Apart from the scattered current \mathbf{i}_s , all remaining current components in (3), defined in [3], can be modified by a reactive compensator [17]. This applies to the reactive current \mathbf{i}_r and the unbalanced current \mathbf{i}_u , which has three symmetrical components of the positive, negative and the zero sequence, namely

$$\mathbf{i}_u = \mathbf{i}_u^p + \mathbf{i}_u^n + \mathbf{i}_u^z. \quad (7)$$

All these current components can be specified having an equivalent circuit of the load, shown in Fig. 2

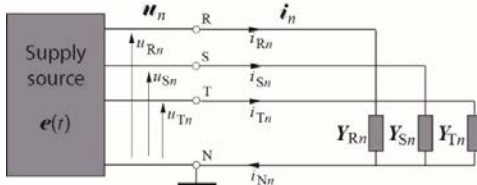


Fig. 2. An equivalent circuit of the LTI load.

with line-to-neutral admittances for harmonic frequencies Y_{Rn} , Y_{Sn} and Y_{Tn} , which can be calculated based on a measurement of the complex rms (crms) values of the load voltage and current harmonics at the load terminals R, S, and T

$$Y_{Ln} = G_{Ln} + jB_{Ln} = \frac{I_{Ln}}{U_{Ln}}, \quad L = R, S \text{ or } T. \quad (8)$$

Having these admittances, the unbalanced admittances of the load for the positive sequence harmonics

$$Y_{un}^p = \frac{1}{3}[(Y_{Rn} + \alpha\beta Y_{Sn} + \alpha^*\beta^* Y_{Tn}) - Y_{en}(1 + \alpha\beta + \alpha^*\beta^*)] \quad (9)$$

where

$$Y_{en} = G_{en} + jB_{en} = \frac{1}{3}(Y_{Rn} + Y_{Sn} + Y_{Tn}) \quad (10)$$

$$\beta \stackrel{\text{df}}{=} (\alpha^*)^n = \begin{cases} 1, & \text{for } n = 3k \\ \alpha^*, & \text{for } n = 3k+1, \\ \alpha, & \text{for } n = 3k-1 \end{cases} \quad \alpha = 1e^{j2\pi/3} \quad (11)$$

for the negative sequence harmonics

$$Y_{un}^n = \frac{1}{3}[(Y_{Rn} + \alpha^*\beta Y_{Sn} + \alpha\beta^* Y_{Tn}) - Y_{en}(1 + \alpha^*\beta + \alpha\beta^*)] \quad (12)$$

and for the zero sequence harmonics

$$Y_{un}^z = \frac{1}{3}[(Y_{Rn} + \beta Y_{Sn} + \beta^* Y_{Tn}) - Y_{en}(1 + \beta + \beta^*)] \quad (13)$$

can be calculated.

As it was discussed in [17], a balancing reactive compensator of a load supplied by a four-wire line has to be built of two compensators, one of a Y-structure, and the second of the Δ -structure, as it is shown in Fig. 3. Symbols T_{XYn} in this figure stand for the susceptances of the compensator branches for the n^{th} order harmonic.

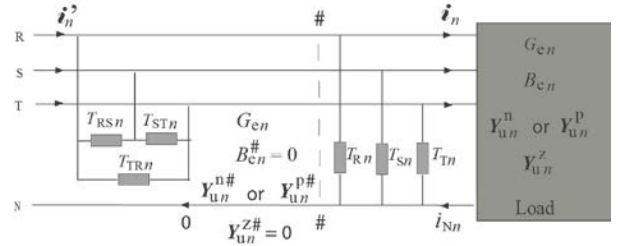


Fig. 3. Load with a reactive compensator for the n^{th} order harmonic of the positive or negative sequence.

When the supply voltage is nonsinusoidal, while the load, as assumed, is LTI, then the supply current of the load is a sum of current responses to individual voltage harmonics $\mathbf{u}_n(t)$. Consequently, compensation of such a load can be studied with a harmonic-by-harmonic approach.

The Y structure compensator is needed for the reduction the zero sequence symmetrical component of the unbalanced current. It can also compensate the reactive current. To do this, its susceptances has to satisfy for each order n from the set N the conditions

$$\frac{1}{3}j(T_{Rn} + \alpha^* T_{Sn} + \alpha T_{Tn}) + Y_{un}^z = 0 \quad (14)$$

$$\frac{1}{3}(T_{Rn} + T_{Sn} + T_{Tn}) + B_{en} = 0. \quad (15)$$

These equations result in the Y-structure compensator susceptances

$$\begin{aligned} T_{Rn} &= -2\operatorname{Im}Y_{un}^z - B_{en} \\ T_{Sn} &= -\sqrt{3}\operatorname{Re}Y_{un}^z + \operatorname{Im}Y_{un}^z - B_{en}. \\ T_{Tn} &= \sqrt{3}\operatorname{Re}Y_{un}^z + \operatorname{Im}Y_{un}^z - B_{en} \end{aligned} \quad (16)$$

The sub-compensator of the Y structure with such susceptances compensates entirely the reactive current and the unbalanced current of the zero sequence. The positive and negative sequence components of the unbalanced current are not yet compensated. The sub-compensator of the Δ structure can compensate only one of them. Usually, the negative sequence component of the unbalanced current is higher than the positive sequence component. So that let us assume that this higher value component is to be compensated by the Δ sub-compensator.

The Y structure sub-compensator changes [17] the unbalanced admittance of the n^{th} order harmonic of the negative sequence in the cross-section #-# to

$$\mathbf{Y}_{un}^{n\#} = \mathbf{Y}_{Cun}^n + \mathbf{Y}_{un}^n = \mathbf{Y}_{un}^{z*} + \mathbf{Y}_{un}^n. \quad (17)$$

The unbalanced current of the negative sequence can be compensated entirely by a Δ -structure compensator with line-to-line susceptances that satisfy for each harmonic of the order n from the set N the condition

$$j(T_{STn} + \alpha T_{TRn} + \alpha^* T_{RSn}) + \mathbf{Y}_{un}^{n\#} = 0. \quad (18)$$

Since the reactive component of the current n^{th} harmonic in this cross-section is compensated entirely by the Y-structure compensator, thus susceptances of the Δ -structure compensator should satisfy the condition

$$T_{STn} + T_{TRn} + T_{RSn} = 0. \quad (19)$$

Equations (18) and (19) result in the compensator susceptances for harmonic frequencies

$$\begin{aligned} T_{RSn} &= \frac{1}{3}(\sqrt{3} \operatorname{Re} \mathbf{Y}_{un}^{n\#} - \operatorname{Im} \mathbf{Y}_{un}^{n\#}) \\ T_{STn} &= \frac{1}{3}(2 \operatorname{Im} \mathbf{Y}_{un}^{n\#}) \\ T_{TRn} &= \frac{1}{3}(-\sqrt{3} \operatorname{Re} \mathbf{Y}_{un}^{n\#} - \operatorname{Im} \mathbf{Y}_{un}^{n\#}) \end{aligned} \quad (20)$$

Equations (16) and (20) specify susceptances T_{XYn} of the compensator branches for each harmonic of the order n from the set N . Methods of synthesis of such reactive branches are well developed and can be found, for example, in [2]. At this point, we encounter, however, a major obstacle in the compensator design process, namely, the compensator complexity. Two reactive elements are needed on average to fix the branch susceptance at a required value T_{Xn} . If $N = \{1, 3, 5, 7\}$ then, up to eight reactive elements might be needed for each branch of the Y-structure compensator and up to six of them for the Δ -structure compensator. Such a compensator would be too complex to have a technical value. The reduction of this complexity is necessary.

The compensator's complexity can be reduced if the goal of whole compensation of the unbalanced and reactive currents by sort of an ideal compensator, is abandoned for minimization of these currents by a compensator built of a lower number of reactive devices. It should minimize the three-phase rms value of the supply current.

The most simple branches of a compensator are purely inductive or purely capacitive. Since a purely capacitive branch in the compensator could cause a resonance for harmonic frequencies with an inductive impedance of the supply source, purely capacitive branches are not acceptable, however. The

compensator can be built [4, 6] exclusively of L or LC branches shown in Fig. 4.



Fig. 4. Acceptable branches of a reduced complexity compensator.

To avoid confusion, susceptances of a compensator of the reduced complexity are denoted by D_n . These susceptances, depending on the branch structure, could have values

$$D_n = -\frac{1}{n\omega_1 L} \quad \text{or} \quad D_n = \frac{n\omega_1 C}{1 - n^2\omega_1^2 LC}. \quad (21)$$

A compensator built of such branches minimizes the supply current three-phase rms value on the condition [17], that for each branch

$$\sum_{n \in N} (T_{kn} - D_{kn})^2 U_{kn}^2 = \text{Min}. \quad (22)$$

where susceptances T_{kn} in this formula are known from formulae (16), and (20).

The supply voltage rms value of the fundamental harmonic U_{k1} in common distribution systems is much higher than the rms value of other harmonics. Therefore, the component of (57) with U_{k1} is the dominating one. Therefore, the term $(T_{k1} - D_{k1})$ should be as close to zero as possible. Thus, when $T_{k1} < 0$, the compensator branch should be chosen such that $D_{k1} < 0$, i.e., the inductive branch. When $T_{k1} > 0$, the branch should be chosen such that $D_{k1} > 0$, i.e., the LC branch. Consequently, for purely inductive branches, the inductance L_k should be chosen such that

$$\sum_{n \in N} (T_{kn} + \frac{1}{n\omega_1 L_k})^2 U_{kn}^2 = \text{Min}. \quad (23)$$

For LC branches, the inductance L_k and capacitance C_k should be chosen such that

$$\sum_{n \in N} (T_{kn} - \frac{n\omega_1 C_k}{1 - n^2\omega_1^2 L_k C_k})^2 U_{kn}^2 = \text{Min}. \quad (24)$$

Condition (23) is satisfied when its derivative with respect to L_k is zero, i.e.,

$$\frac{d}{dL_k} \{ \sum_{n \in N} (T_{kn} + \frac{1}{n\omega_1 L_k})^2 U_{kn}^2 \} = 0 \quad (25)$$

and this condition results in the optimum value of the branch inductance

$$L_{k, \text{opt}} = -\frac{1}{\omega_1} \frac{\sum_{n \in N} \frac{1}{n^2} U_{kn}^2}{\sum_{n \in N} T_{kn} \frac{1}{n} U_{kn}^2}. \quad (26)$$

The form on the left side of (25) is a function of two variables, the inductance L_k and capacitance C_k . With respect to L_k it is continuously declining function, meaning it does not have a minimum for any finite value of inductance L_k . It has to be selected at a designer's discretion [4, 6]. When it is selected, the capacitance C_k can be calculated such that (24) is minimum. When it is minimum, its derivative with respect to C_k is zero, i.e.,

$$\frac{d}{dC_k} \{ \sum_{n \in N} (T_{kn} - \frac{n\omega_1 C_k}{1 - n^2\omega_1^2 L_k C_k})^2 U_{kn}^2 \} = 0. \quad (27)$$

It results in the equation

$$\sum_{n \in N} \frac{T_{kn} n U_{kn}^2}{1 - n^2 \omega_1^2 L_k C_k} - \sum_{n \in N} \frac{n^2 \omega_1 C_k U_{kn}^2}{(1 - n^2 \omega_1^2 L_k C_k)^2} = 0 \quad (28)$$

which cannot be solved directly with respect to the optimum value of the capacitance C_k . Numerical methods are needed for that. In particular, it can be solved in an iterative process

$$C_{k,s+1} = \frac{\sum_{n \in N} \frac{T_{kn} n U_{kn}^2}{1 - n^2 \omega_1^2 L_k C_{k,s}}}{\omega_1 \sum_{n \in N} \frac{n^2 U_{kn}^2}{(1 - n^2 \omega_1^2 L_k C_{k,s})^2}} \quad (29)$$

which results in a sequence of capacitances usually convergent to the optimum capacitance $C_{k,opt}$ which minimizes (24).

Numerical illustration Let us assume that the load shown in Fig. 5, with $\omega_1 L = R = 0.5 \Omega$, is supplied with symmetrical voltage of the fundamental harmonic rms value $U_1 = 240 \text{ V}$, distorted by the 3rd, 5th, and 7th order harmonics of relative rms value $U_3 = 2\% U_1$, $U_5 = 3\% U_1$ and $U_7 = 1.5\% U_1$.

As shown in [17] the supply source with such a load has the power factor $\lambda = P/S = 0.408$. Three-phase rms values of particular physical components of the load are shown in Fig. 5.

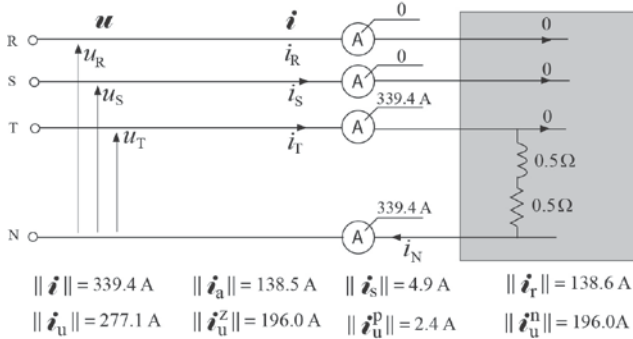


Fig. 5. An example of an unbalanced load and its analysis.

The load can be compensated by a reduced complexity compensator shown in Fig. 6. Assuming that the frequency is normalized to $\omega_1 = 1 \text{ rad/s}$, and the resonance frequency of the LC branches is approximately equal to $\omega_r = 2.5 \text{ rad/s}$, the LC parameters have values compiled in Table 1.

Table 1. LC parameters of a reduced complexity compensator

	Line:	R	S	T	RS	ST	TR
L	mH	1730	770	444	0	2600	1155
C	mF	0	399	691	0	0	266

The results of compensation are shown in Fig. 6. The power factor is improved by the compensator of the reduced complexity to $\lambda = 0.994$.

III. ADAPTIVE COMPENSATOR

The compensation method drafted above results in a reactive compensator with fixed parameters. It improves the power factor λ of the supply source when its load has also fixed parameters, or they change insignificantly. Otherwise, such compensators are losing their effectiveness. Adaptive compensators are needed [18] instead.

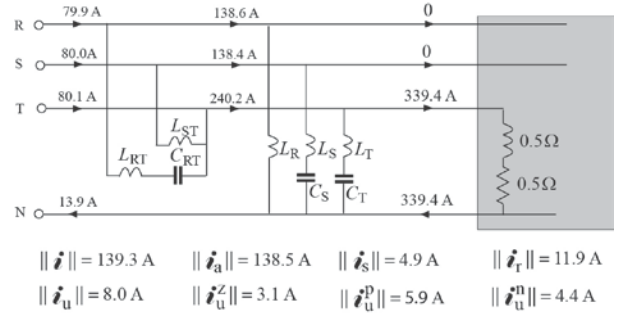


Fig. 6. Results of compensation with a reduced complexity compensator.

One-ports with thyristor switched inductors (TSR), and a capacitor connected as shown in Fig. 7, are commonly used in adaptive compensators of reactive power. This is an example of a branch that provides a thyristor controlled susceptance (TCS). Several other structures of TCS were investigated in [7].

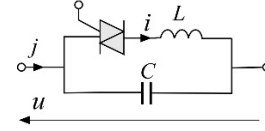


Fig. 7. A thyristor switched inductor (TSI) with a shunt capacitor.

The current of a thyristor switched inductor depends on the firing angle α of thyristors. It changes for some angle as shown in Fig. 8. Symbol i_0 denotes current at $\alpha = 0$.

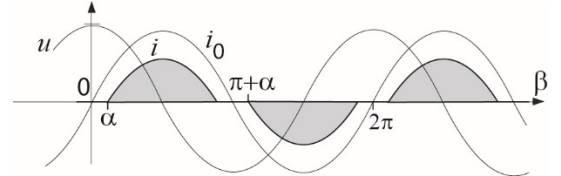


Fig. 8. The currents waveform of a thyristor switched inductor (TSI).

Assuming that the voltage $u(t)$ on the TSI is sinusoidal, then the ratio of the crms value of the current $i(t)$ fundamental harmonic $i_1(t)$ and the voltage crms value

$$Y_1 = \frac{I_1}{U} = jT = -j \left(1 - \frac{2\alpha + \sin 2\alpha}{\pi} \right) \frac{1}{\omega_1 L} \quad (30)$$

specifies the branch susceptance T for the fundamental frequency. It changes with the change of the firing angle α .

The TSI branch is also a source of current harmonics, in particular, the 3rd order. When compensation is confined, as it is common, only to the reactive power, the compensator is built as a balanced device, meaning, it is configured in Δ and thyristors are fired, with a shift of $T/6$, at the same firing angle. Since the 3rd order harmonics generated in compensator branches are in-phase and mutually equal, they do not leave the compensator, they are confined to its Δ loop.

When TSIs are used in a balancing compensator, it operates as an unbalanced device, thyristors are switched at different angles and the 3rd order harmonic generated in particular branches have different values. Consequently, the 3rd order harmonic currents are injected into the supply lines. This causes waveform distortion and the resonance of the compensator

capacitance with the supply system inductance can occur. To avoid it, the capacitor should be replaced by a filter, as shown in Fig. 9, tuned to the frequency of the 3rd order harmonic.

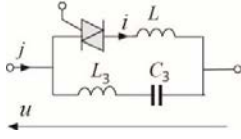


Fig. 9. A TSI with a filter of the 3rd order harmonic.

Although the above-mentioned structure effectively reduces harmonics generated by the switching device, the filter as seen from the supply acts as a short circuit. Therefore, the structure presented in Fig. 9 should be modified to that shown in Fig. 10.

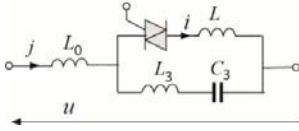


Fig. 10. TSI with a filter and series inductor.

With the change of the firing angle from zero to $\pi/2$, the susceptance of the whole branch changes from its minimum value

$$T_{\min} = \frac{9}{8} \omega_1 C_3 - \frac{1}{\omega_1 L} \quad (31)$$

to its maximum value

$$T_{\max} = \frac{9}{8} \omega_1 C_3. \quad (32)$$

Thus, if the required range of change of the branch susceptance T is known, the parameters of the branch can be calculated, namely

$$L = \frac{8}{9\omega_1} T_{\max}, \quad C_3 = \frac{1}{\omega_1 (T_{\max} - T_{\min})}. \quad (33)$$

With the adequate parameter selection, the series inductor L_0 will increase the impedance for the 3rd order harmonic as seen from the distribution system. It was assumed, to keep the susceptance of the branch in the thyristor's ON state unchanged, that the inductance L of the original circuit is divided by half, so that in the circuit shown in Fig. 10, L_0 is equal to $L/2$.

The parameters for the branches of the Δ and Y sub-compensators of the structure shown in Fig. 10, calculated from (33), are compiled in Table 2.

Table 2. Compensator's branch parameters.

	L	L_3	C_3	L_0
Δ	0.65 H	0.40 H	0.27 F	0.65 H
Y	0.30 H	0.16 H	0.68 F	0.20 H

To calculate the firing angle α of thyristors, equation (30) has to be solved. It was solved using a look-up table. The angles are compiled in Table 3.

Table 3. Firing angles of the compensator's branches.

		Δ			Y		
Line:		RS	ST	TR	R	S	T
α	deg.	133.5	112.0	180.0	117.6	145.8	180.0

The results of compensation are shown in Fig. 11.

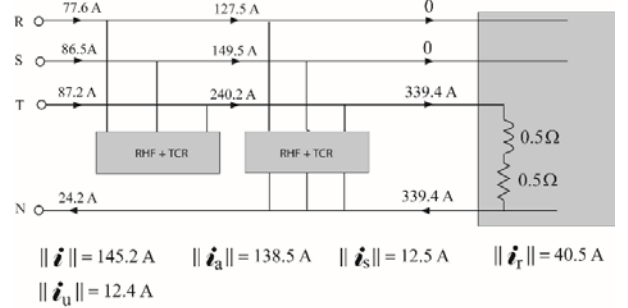


Fig. 11. Results of adaptive compensation.

These results differ from those shown in Fig. 6 with a fixed-parameters compensator, however. This is because the formula (30) is valid only for the TCS shown in Fig. 7, and Fig. 9 at sinusoidal voltage $u(t)$, on the branch with TSI. This voltage in branch with inductor L_0 is not sinusoidal, however.

In the presence of the series inductor L_0 , when the TSI voltage $u(t)$ is nonsinusoidal, the relation between the branch susceptance for the fundamental harmonic T_1 and the firing angle α is too complex to be written explicitly. Iterative methods are needed to find the right values of the thyristors' firing angles. The corrected firing angles obtain in this iteration are compiled in Table 4.

Table 4. Corrected firing angles.

		Δ			Y		
Line:		RS	ST	TR	R	S	T
α	deg.	132.1	110.6	180.0	115.9	144.6	180.0

Results of the compensation with such firing angles are shown in Fig. 12

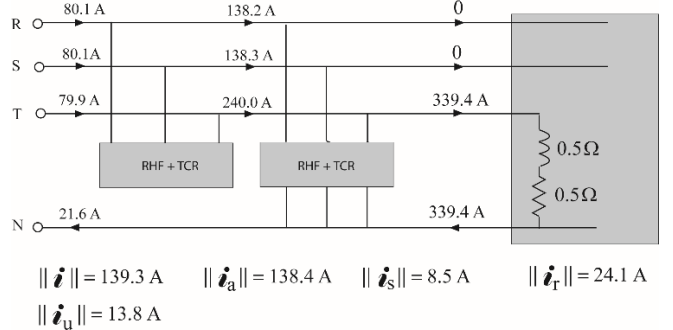


Fig.12. Results of compensation corrected firing angles.

IV. CONCLUSIONS

The paper demonstrates that unbalanced LTI loads supplied with a nonsinusoidal voltage by a four-wire line can be almost ideally balanced, and their power factor can be improved to almost unity value, using reactive compensator with substantially reduced complexity.

The results presented in this paper can provide a starting platform for the development of an adaptive reactive compensator of varying loads.

The results obtained from the computer model of an adaptive balancing compensator with thyristor switched inductor are promising, but they revealed that transients can degrade the effectiveness of the compensator. At rapid changes in the load parameters, because of these transients, the compensator can be not fast enough to handle these transients. Harmonic distortion remains also an issue. Hybrid compensator composed a reactive and a switching one could be a remedy for it.

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