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Do Energy Oscillations Degrade the Energy Transfer in Electrical Systems?

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Abstract: - The views that the energy oscillations between supply sources and loads are responsible for the degradation of the effectiveness of the energy transfer are very common in the power engineering community. A suggestion by the Instantaneous Reactive Power p-q Theory that such oscillations should be removed by a compensator, can result in an erroneous control of switching compensators. The paper shows that energy oscillations do not affect the energy transfer effectiveness, however. The roots of this major misinterpretation are discussed as well.

The physical meaning of the reactive power is also investigated in the paper, with the conclusion that there is no physical phenomenon that could be described in terms of the reactive power.

Key words: ***instantaneous power, CPC, Current's Physical Components, Instantaneous Reactive Power p-q Theory***

I. INTRODUCTION

The energy oscillations between the electric energy providers and consumers of that energy are usually blamed by the power engineering (PE) community for degradation of the effectiveness of the energy transfer. There are also opinions [2, 10, 14, 16, 18, 19] that harmonics contribute to these oscillations. The energy transfer effectiveness is specified by the power factor $\lambda = P/S$. Opinions that the energy oscillations are responsible for its decline are very common. These oscillations are often regarded in the PE community as the cause of reactive power.

Such opinions are incorrect, however. In fact, there is no relation between the value of the apparent power S of the energy provider and the energy oscillations. Because awareness of this in the PE community seems to be not very common, this paper will demonstrate that indeed, the energy oscillations do not contribute to the apparent power S and, in particular, to the reactive power Q increase.

Although the right interpretation by the PE community of the power-related physical phenomena in electrical systems would be, in general, for the state of knowledge beneficial, [12, 14], nonetheless, some misinterpretations may have no negative technical consequences. There are situations, however, in which the opinion that the energy oscillations degrade the power factor λ leads to wrong technical conclusions. For example, the control of an active power filter, used for the power factor improvement, requires according to the Instantaneous Reactive Power (IRP) p-q Theory [8], that the filter compensates the oscillating component of the instantaneous active power p, thus the filter eliminates the energy oscillation. This leads, as demonstrated in [15, 17], to an erroneous control of the device, which instead of improving the power factor, can degrade it.

Because the energy oscillations are commonly associated with the concept of the reactive power Q, which is a misinterpretation of physical phenomena in electrical systems, possible physical interpretations of this power are discussed in the paper as well. Eventually, the paper presents the concept of the reactive power in the frame of the Currents' Physical Components (CPC) – based power theory [9, 20] which is currently the most advanced power theory of electrical systems.

I. ROOTS OF POWER PHENOMENA MISINTERPRETATIONS

Misinterpretations in the PE community of the power-related phenomena with respect to the energy oscillations are so common, that the questions "***why and how they have occurred***" seems to be a very natural question.

As to the author's opinion, drawn from his teaching experience and textbooks on electrical circuits, they have occurred because explanations of power phenomena in university courses are usually confined to power-related phenomena in single-phase, linear, time-invariant (LTI) circuits with a sinusoidal supply voltage. These explanations are next extrapolated to three-phase systems, regarded usually as a sort of aggregates of three single-phase systems. It is suggested that interpretations valid in single-phase LTI systems, are valid also in three-phase systems. But such an extrapolation is not valid, however. To show this, let us consider the instantaneous power $p(t)$ of a single-phase load, shown in Fig. 1.

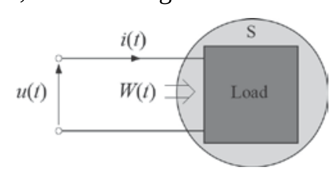

Fig. 1. A single-phase load.

The instantaneous power $p(t)$ is the rate of the electric energy $W(t)$ flow to a space S that confines the load in such a way that this energy can flow to that space only through the load terminals with the voltage $u(t)$ and the current $i(t)$, namely

$$p(t) = \frac{d}{dt}W(t) = u(t)\,i(t)\,. \tag{1}$$

When at the load terminals

$$u(t) = \sqrt{2}\,U\cos\omega t\,, \qquad i(t) = \sqrt{2}\,I\cos(\omega t - \varphi)$$

then the instantaneous power is

$$p(t) = u(t)\,i(t) = 2\,UI\cos\omega t\cos(\omega t - \varphi) = p_{\rm u}(t) + p_{\rm b}(t) \tag{2}$$

where

$$p_{\rm u}(t) = P(1 + \cos 2\omega t) \tag{3}$$

$$p_b(t) = Q \sin 2\omega t\,. \tag{4}$$

Decomposition (2) of the instantaneous power into an unidirectional component $p_u(t)$ and a bi-directional component $p_b(t)$, illustrated in Fig. 2, is commonly used for interpretation of the reactive power Q as the amplitude of the oscillating component of the instantaneous power.

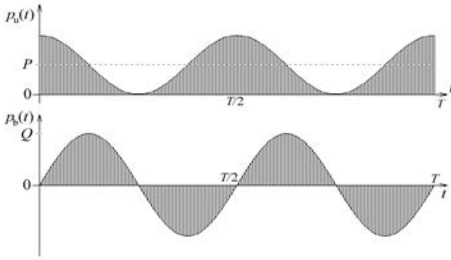

Fig. 2. Components of the instantaneous power.

According to this interpretation, the energy can oscillate between the supply source and the load only because it can have a capability of the energy storage in magnetic fields of inductors and/or electric fields of capacitors.

This, very convincing interpretation cannot be extrapolated to three-phase systems, as shown in Fig. 3, however.

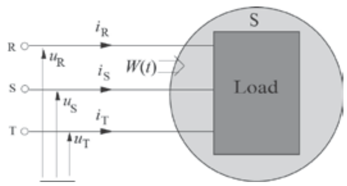

Fig. 3. A three-phase load confined with space S.

The instantaneous power of such a load is

$$p(t) = \frac{d}{dt}W(t) = u_R(t)\,i_R(t) + u_S(t)\,i_S(t) + u_T(t)\,i_T(t)\,. \tag{5}$$

Assuming at the load terminals

$$u_R(t) = \sqrt{2}\,U \cos\omega t\,, \qquad i_R(t) = \sqrt{2}\,I \cos(\omega t - \varphi)$$

and the voltages and currents are symmetrical of the positive sequence, then the instantaneous power $p(t)$ calculated from (5)

$$p(t) = u_R(t)\,i_R(t) + u_S(t)\,i_S(t) + u_T(t)\,i_T(t) = 3UI\cos\varphi = P \tag{6}$$

is constant, meaning, there are no energy oscillations between the supply source and the load, even if the reactive power Q at the load terminals.

$$Q = 3UI\sin\varphi \tag{7}$$

is not equal to zero. Thus, the presence of the reactive power Q cannot be explained by the energy oscillations.

There are arguments that these oscillations do exist, however, but they are only not visible in the instantaneous power $p(t)$, because of mutual cancellation of the oscillating components of the instantaneous powers of individual supply lines. Indeed, each product, $u_R(t)i_R(t)$, $u_S(t)i_S(t)$, $u_T(t)i_T(t)$, can be expressed in form (2), meaning, with the oscillating component. This, apparently convincing argument, is not valid, however. These voltage-current products, calculated for individual R, S, and T terminals, are not instantaneous powers. To be instantaneous powers, these products have to be the rate of the energy flow. The energy is delivered to a whole three-phase load, but not to its sort of single-phase segments. Such segments do not exist. As shown in Fig. 4, it is not possible to separate an isolated space, say S_R, from the three-phase system, to which the energy would be delivered by the voltage and current at only one terminal R.

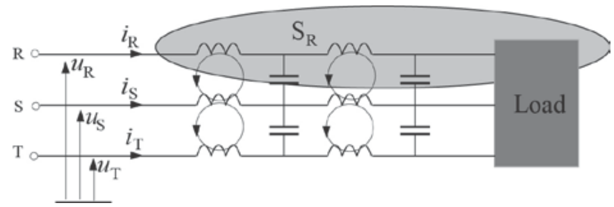

Fig. 4. Separation of a space S_R from a three-phase system.

Moreover, any node can be assumed in a three-phase system as a reference node. If this would be the only ground, it can be grounded, with no effect upon the energy flow in the system. Let the terminal R is grounded, as shown in Fig. 5, so that

$$u_R(t) \equiv 0, \quad u_R(t)\,i_R(t) \equiv 0 \tag{8}$$

but this does not change the instantaneous power $p(t)$ at the load terminals.

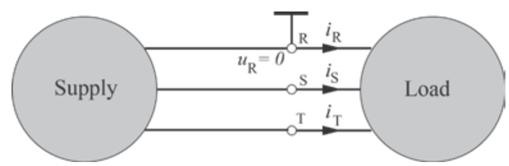

Fig. 5. A three-phase system with grounded terminal R.

Thus, although such individual voltage-current products have oscillating components, these products are not the instantaneous powers. Their oscillating component stands only for a mathematical, but not a physical entity. It does not describe the energy flow. It means that the presence of the reactive power Q in three-phase systems is not caused by the energy oscillations.

The rationale, drafted above, demonstrates that the presence of the reactive power in three-phase systems with sinusoidal voltages and currents does not cause any oscillations of the energy. Such oscillations can occur, however, but due to differ-rent reasons. Let us show this.

The supply current $\boldsymbol{i}(t)$ of any LTI three-phase load supplied from a source of symmetrical sinusoidal voltage $\boldsymbol{u}(t)$ can be decomposed into the active reactive and unbalanced currents. The load can be specified in terms of line-to-line admittances $\boldsymbol{Y}_{ST}$, $\boldsymbol{Y}_{TR}$, and $\boldsymbol{Y}_{RS}$ which can be calculated having measured the complex rms values of the load voltages and currents at the load terminals. Let us assume that

$$\boldsymbol{u} = \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix} = \sqrt{2}\,U_R \begin{bmatrix} \cos\omega t \\ \cos(\omega t - 120^0) \\ \cos(\omega t + 120^0) \end{bmatrix}. \tag{8}$$

Then the active current is

$$\boldsymbol{i}_a = \sqrt{2}\,G_e U_R \begin{bmatrix} \cos\omega t \\ \cos(\omega t - 120^0) \\ \cos(\omega t + 120^0) \end{bmatrix} \tag{9}$$

the reactive current

$$\boldsymbol{i}_{\mathrm{r}} = -\sqrt{2}B_{\mathrm{e}}U_{\mathrm{R}}\begin{bmatrix}\sin\omega t\\ \sin(\omega t-120^0)\\ \sin(\omega t+120^0)\end{bmatrix} \tag{10}$$

and the unbalanced current

$$\boldsymbol{i}_{\mathrm{u}} = \sqrt{2}Y_{\mathrm{u}}U_{\mathrm{R}}\begin{bmatrix}\cos(\omega t+\Psi)\\ \cos(\omega t+\Psi+120^0)\\ \cos(\omega t+\Psi-120^0)\end{bmatrix} \tag{11}$$

such that

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}\,. \tag{12}$$

In these formulae

$$G_{\mathrm{e}} = \mathrm{Re}\{\boldsymbol{Y}_{\mathrm{e}}\},\quad B_{\mathrm{e}} = \mathrm{Im}\{\boldsymbol{Y}_{\mathrm{e}}\} \tag{13}$$

where $\boldsymbol{Y}_{\mathrm{e}}$ is the equivalent admittance of the load, equal to

$$\boldsymbol{Y}_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}} + \boldsymbol{Y}_{\mathrm{RS}} \tag{14}$$

and

$$\boldsymbol{Y}_{\mathrm{u}} = Y_{\mathrm{u}}e^{j\Psi} = -\{\boldsymbol{Y}_{\mathrm{ST}} + \alpha\boldsymbol{Y}_{\mathrm{TR}} + \alpha^*\boldsymbol{Y}_{\mathrm{RS}}\},\ \ \alpha = 1e^{j120^{\mathrm{O}}} \tag{15}$$

is the unbalanced admittance. Having the current decomposetion (12), the instantaneous power $p(t)$ can be decomposed into three instantaneous powers associated with the presence of the active, reactive and unbalanced currents, namely

$$p(t) = \frac{dW(t)}{dt} = \boldsymbol{u}(t)^{\mathrm{T}}\boldsymbol{i}(t) = \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}) =$$
$$= p_{\mathrm{a}}(t) + p_{\mathrm{r}}(t) + p_{\mathrm{u}}(t). \tag{16}$$

These powers, taking into account (8) – (11), are equal to

$$p_{\mathrm{a}}(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{a}} = P \tag{17}$$

$$p_{\mathrm{r}}(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{r}} = 0 \tag{18}$$

$$p_{\mathrm{u}}(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{u}} = 3Y_{\mathrm{u}}\,U_{\mathrm{R}}^2\cos(2\omega t+\Psi)\,. \tag{19}$$

Thus, there is an oscillating component in the instantaneous power, meaning the rate of the energy flow. It is not caused by the reactive, but by the unbalanced current, however, meaning by the load imbalance. The presence of the energy oscillations was concluded by Steinmetz [1] and gave him a starting point for the development of the first compensator for the load balancing, known as the Steinmetz Circuit. Its structure and parameters were selected in such a way that these energy oscillations were compensated. It does not mean, however, that these oscillations contribute to the power factor decline. It declines because of the presence of the unbalanced current which increases the supply current three-phase rms value

$$||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2}\,. \tag{20}$$

Energy oscillations only indicate the presence of the unbalanced current.

II. DOES THE ENERGY OSCILLATIONS DEGRADE THE POWER FACTOR?

It was shown in the previous Section that the power factor can be degraded without the energy oscillations. Now, let us ask a reversed question: can the energy oscillations degrade the power factor? To answer this question, let us consider an ideal purely resistive balanced load, shown in Fig. 6, supplied from a voltage source with symmetrical voltage distorted by the 5th order harmonic.

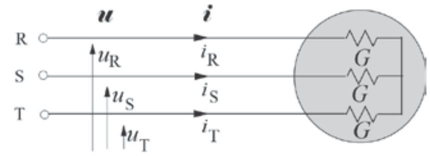

Fig. 6. An ideal purely resistive balanced load.

Since the apparent power S of such a load is equal to the active power P, its power factor $\lambda = P/S$ is, of course, equal to one, independently on the supply voltage.

Let the supply voltage is symmetrical, but distorted, say by the 5th order harmonic.

$$\boldsymbol{u}(t) = \boldsymbol{u} = \begin{bmatrix}u_{\mathrm{R}}\\ u_{\mathrm{S}}\\ u_{\mathrm{T}}\end{bmatrix} = \begin{bmatrix}u_{\mathrm{R1}}\\ u_{\mathrm{S1}}\\ u_{\mathrm{T1}}\end{bmatrix} + \begin{bmatrix}u_{\mathrm{R5}}\\ u_{\mathrm{S5}}\\ u_{\mathrm{T5}}\end{bmatrix} = \boldsymbol{u}_1 + \boldsymbol{u}_5 \tag{20}$$

assuming for the sake of simplicity that

$$u_{\mathrm{R1}} = \sqrt{2}\,U_1\cos\omega_1 t,\qquad u_{\mathrm{R5}} = \sqrt{2}\,U_5\cos 5\omega_1 t\,. \tag{21}$$

The load current at such a supply contains the 5th order harmonic and can be presented in the form

$$\boldsymbol{i}(t) = \boldsymbol{i} = \begin{bmatrix}i_{\mathrm{R}}\\ i_{\mathrm{S}}\\ i_{\mathrm{T}}\end{bmatrix} = \begin{bmatrix}i_{\mathrm{R1}}\\ i_{\mathrm{S1}}\\ i_{\mathrm{T1}}\end{bmatrix} + \begin{bmatrix}i_{\mathrm{R5}}\\ i_{\mathrm{S5}}\\ i_{\mathrm{T5}}\end{bmatrix} = \boldsymbol{i}_1 + \boldsymbol{i}_5 \tag{22}$$

with

$$i_{\mathrm{R1}} = \sqrt{2}\,GU_1\cos\omega_1 t,\qquad i_{\mathrm{R5}} = \sqrt{2}\,GU_5\cos 5\omega_1 t\,. \tag{23}$$

At such a voltage, the instantaneous power $p(t)$ is equal to

$$p(t) = \frac{d}{dt}W(t) = \boldsymbol{u}^{\mathrm{T}}(t)\cdot\boldsymbol{i}(t) = [\boldsymbol{u}_1 + \boldsymbol{u}_5]^{\mathrm{T}}\cdot[\boldsymbol{i}_1 + \boldsymbol{i}_5]\,. \tag{24}$$

It can be rearranged to the form

$$p(t) = [\boldsymbol{u}_1 + \boldsymbol{u}_5]^{\mathrm{T}}\cdot[\boldsymbol{i}_1 + \boldsymbol{i}_5] = \boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{i}_1 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{i}_5 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{i}_1 + \boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{i}_5 =$$
$$= P_1 + P_5 + 6GU_1U_5\cos 6\omega_1 t = \overline{p} + \tilde{p} \tag{25}$$

where P_1 and P_5 denote the active power of the 1st and the 5th order harmonics. This formula shows that, in spite of the unity power factor, the instantaneous power $p(t)$ of such a load has the oscillating component

$$\tilde{p} = 6GU_1U_5\cos 6\omega_1 t\,. \tag{26}$$

Thus, oscillations of the energy delivered to such a load does not degrade the effectiveness of this delivery.

According to the Instantaneous Reactive Power p-q Theory [8] which provides theoretical fundamentals for the control of the switching compensators, commonly known as "active power filters", the instantaneous power $p(t)$ after compensation should be constant, however. It means that the compensator should load the system with the instantaneous reactive power q and the negative value of the oscillating component of the instantaneous active power, which is identical with the instantaneous power $p(t)$, as this is illustrated in Fig. 7. Unfortunately, the compensator that reduces this oscillating component of the instantaneous power has to load the system, as it was demonstrated in [15], with the current

$$j(t) = \begin{bmatrix} j_{\mathrm{R}}(t) \\ j_{\mathrm{S}}(t) \end{bmatrix} = \frac{-2\sqrt{2}\,GU_1U_5\cos(6\omega_1 t)}{U_1^2 + U_5^2 + 2U_1U_5\cos(6\omega_1 t)} \begin{bmatrix} u_{\mathrm{R}}(t) \\ u_{\mathrm{S}}(t) \end{bmatrix} \tag{27}$$

which not only reduces the power factor λ, originally equal to $\lambda = 1$ but also causes the supply current distortion with a non-periodic current.

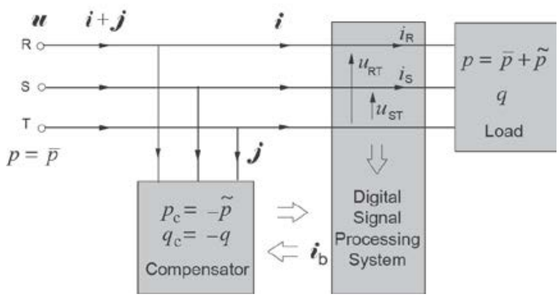

Fig. 7. A system with a switching compensator controlled, according to IRP p-q Theory.

A similar situation occurs when the ideal three-phase load, shown in Fig. 6, is supplied from a source of an asymmetrical voltage. Let the supply voltage is sinusoidal, but asymmetrical due to a negative sequence symmetrical component, namely

$$\boldsymbol{u} = \sqrt{2}U^{\mathrm{p}} \begin{bmatrix} \cos\omega_1 t \\ \cos(\omega_1 t - \frac{2\pi}{3}) \\ \cos(\omega_1 t + \frac{2\pi}{3}) \end{bmatrix} + \sqrt{2}U^{\mathrm{n}} \begin{bmatrix} \cos\omega_1 t \\ \cos(\omega_1 t + \frac{2\pi}{3}) \\ \cos(\omega_1 t - \frac{2\pi}{3}) \end{bmatrix} = \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}. \tag{28}$$

At such a supply voltage, the load current is also asymmetrical, namely

$$\boldsymbol{i} = \boldsymbol{i}^{\mathrm{p}} + \boldsymbol{i}^{\mathrm{n}} = G\,\boldsymbol{u}^{\mathrm{p}} + G\,\boldsymbol{u}^{\mathrm{n}}. \tag{29}$$

The instantaneous power $p(t)$ is equal to

$$p(t) = \frac{d}{dt}W(t) = \boldsymbol{u}^{\mathrm{T}}(t)\cdot\boldsymbol{i}(t) = [\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}]^{\mathrm{T}}\cdot[\boldsymbol{i}^{\mathrm{p}} + \boldsymbol{i}^{\mathrm{n}}] \tag{30}$$

and it can be presented in the form

$$p(t) = [\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}]^{\mathrm{T}}\cdot[\boldsymbol{i}^{\mathrm{p}} + \boldsymbol{i}^{\mathrm{n}}] = \boldsymbol{u}^{\mathrm{pT}}\boldsymbol{i}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{i}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{pT}}\boldsymbol{i}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{i}^{\mathrm{p}} =$$

$$= P^{\mathrm{p}} + P^{\mathrm{n}} + 6GU^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega_1 t = \overline{p} + \tilde{p} \tag{31}$$

where P^{p} and P^{n} denote the active power of the positive and the negative sequence symmetrical components of the voltages and currents. This formula shows that, in spite of the unity power factor, the instantaneous power $p(t)$ of such a load at asymmetrical supply voltage has the oscillating component

$$\tilde{p} = 6GU_1U_5\cos 2\omega_1 t\,. \tag{32}$$

Its compensation, according to the IRP p-q Theory, causes that the system is loaded with the current which, for example, in line R has, as demonstrated in [17] the waveform

$$j_{\mathrm{R}} = \frac{-2\sqrt{2}\,G(U^{\mathrm{p}} + U^{\mathrm{n}})U^{\mathrm{p}}U^{\mathrm{n}}\cos\omega_1 t\cos 2\omega_1 t}{U^{\mathrm{p}2} + U^{\mathrm{n}2} + 2U^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega_1 t}\,. \tag{33}$$

It reduces the power factor and causes a distortion of the supply current.

This section demonstrates that the energy oscillations do not cause degradation of the effectiveness of the energy transfer in electrical systems thus, they do not degrade the load power factor. Just the opposite, attempts of elimination of these oscillations by a compensator controlled according to the IRP p-q Theory, degrades the power factor.

III. Reactive Power in Time-Varying Loads

It was shown in Section II, that there is a relation, expressed in formula (4), between the reactive power Q and the energy oscillations. This power occurs to be the amplitude of the energy oscillations. This relationship could be misleading, however. The reactive power can occur also in circuits with only a unidirectional flow of energy. To show this, let us consider a purely resistive load with a TRIAC, shown in Fig. 8, supplied from an ideal source of a sinusoidal voltage.

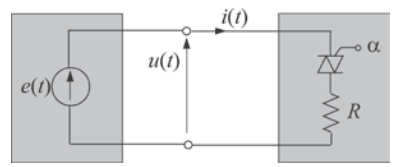

Fig. 8. A purely resistive circuit with a TRIAC.

The supply voltage and the current in this circuit are shown in Fig. 9. Observe that the current has to have the same sign as the supply voltage, so that the instantaneous power $p(t)$ cannot be negative, meaning the energy cannot flow back to the supply source.

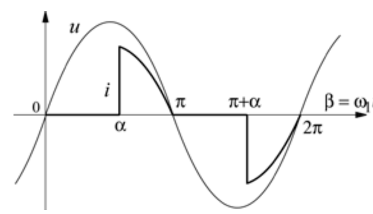

Fig. 9. The voltage and current waveforms in the circuit with TRIAC.

In spite of that, there is the reactive power Q in this circuit. At the assumption that

$$u(t) = 220\sqrt{2}\sin\omega_1 t \quad \mathrm{V}$$

the load resistance is $R = 1\ \Omega$, and the TRIAC's firing angle $\alpha = 135^0$, a varmeter connected as shown in Fig 10 shows the reactive power Q to be equal $Q = 7.7$ kvar.

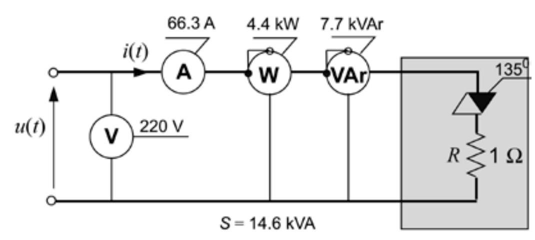

Fig. 10. Meters reding in the circuit with TRIAC.

In the lack of the energy oscillations or the energy storage in the load that would justify the presence of the reactive power, it is present in that circuit because the current fundamental harmonic $i_1(t)$ is shifted with respect to the supply voltage $u(t)$, as it is shown in Fig. 11. Indeed, the current fundamental harmonic is equal to

$$i_1(t) = \sqrt{2}\, I_1 \sin(\omega_1 t - \varphi_1) = 40.32\sqrt{2}\sin(\omega_1 t - 60.28^{\rm o})\ \text{A}$$

and consequently, the reactive power is

$$Q = Q_1 = U_1 I_1 \sin\varphi_1 = 220 \times 40.32 \sin(60.28^{\rm o}) = 7.70\ \text{kvar}\,.$$

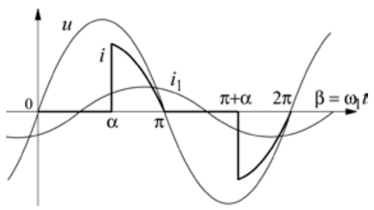

Fig. 11. The TRIAC current and its fundamental harmonic.

This result means that even in single-phase circuits there is no relation between the energy oscillations or the energy storage and the reactive power Q. It can occur even in purely resistive circuits.

IV. Has the Reactive Power Any Physical Interpretation?

With the demonstration in the previous Sections that the reactive power cannot be interpreted as the amplitude of the energy oscillations, it loses its common in the PE community physical interpretation. So that a very natural question occurs: "***is any physical phenomenon in electrical circuits that can be explained and described by means of the reactive power?***"

The reactive power in circuits with sinusoidal voltages and currents is sometimes defined with the formula

$$Q = \frac{1}{T}\int_0^T u(t)\, i(t - \frac{T}{4})\, dt \tag{34}$$

i.e., as a mean value of a quantity that resembles an instantaneous power. The similarity of this definition to the define-tion of the active power P might suggest that the reactive power Q is also a physical quantity. This is a wrong conclusion, however. This formula defines the reactive power Q throughout a current shifted with respect to the voltage, i.e., by the quantity $i(t-T/4)$. Such a quantity does not exist in the circuit, however. It is only a mathematical, but not a physical quantity. Only the current $i(t)$ is a physical quantity. Thus, the formula (24) does not describe any physical phenomenon.

The reactive power Q satisfies, however, similarly as the active power P the Balance Principle (BP). This fact is sometimes used in the PE community to support the opinion that the reactive power, similarly as the active power, is a physical quantity.

There is a substantial difference between the active power and the reactive power as to the roots on which the BP for these two powers is founded. The BP for active powers is an immediate conclusion from the Energy Balance Principle (EBP), one of the fundamentals of physics. The BP for the reactive power cannot be concluded from the EBP, however. It is a conclusion from the Tellegen Theorem [4].

The Tellegen Theorem describes the following property of electrical circuits, as it is shown in Fig. 12.

Let us suppose that there are two entirely different circuits, but of the same topology, i.e., these circuits have the same number of nodes which are connected in the same way by K branches. Branches can be entirely different.

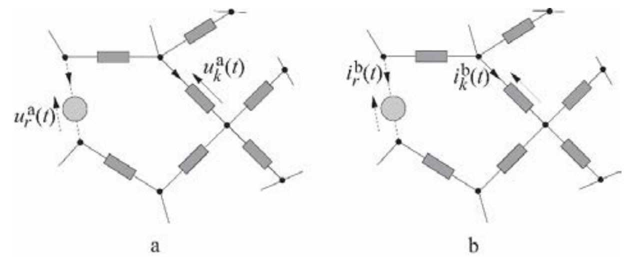

Fig. 12. Two circuits of the same topology.

Let us create a product of the k-branch voltage from the first circuit (a) and the k-branch current from the second circuit (b). According to the Tellegen Theorem, a sum of such products for all K branches is equal to zero, i.e.,

$$\sum_{k=1}^{K} u_k^{\rm a}(t)\, i_k^{\rm b}(t) \equiv 0\,. \tag{35}$$

The Balance Principle for reactive powers can be concluded from the Tellegen Theorem as follows. Let both circuits in Fig. 12 have identical branches, but source voltages and source currents in circuit (b) are shifted with respect to source voltages and source currents in circuit (a) by a quarter of the period T. Consequently, all branch currents in the circuit (b) identical, but shifted by $T/4$ with respect to branch currents in the circuit (a), i.e.,

$$i_k^{\rm b}(t) \equiv i_k^{\rm a}(t - \frac{T}{4})\,. \tag{36}$$

Hence from the Tellegen Theorem

$$\frac{1}{T}\int_0^T \sum_{k=1}^{K} u_k^{\rm a}(t)\, i_k^{\rm b}(t)\, dt = \sum_{k=1}^{K} \frac{1}{T}\int_0^T u_k^{\rm a}(t)\, i_k^{\rm a}(t - \tfrac{T}{4})\, dt = \sum_{k=1}^{K} Q_k^{\rm a} = 0\,. \tag{37}$$

Thus, the reactive power, defined by (34), satisfies the BP. The Tellegen Theorem does not describe any physical phenomena in the circuit, however, because in that the Tellegen Theorem the voltages and currents are taken from different circuits. The Balance Principle for the reactive power Q does not have any physical, but only mathematical roots. In this respect, the active and reactive powers are substantially different. The opinion which suggests that from the fact that the reactive power satisfies the BP, we can draw a conclusion that it is a physical quantity, is not a convincing opinion.

Thus, it seems that to the present time, any physical phenomenon, that could be characterized by the reactive power Q, was not identified.

V Reactive Power In The Frame of CPC

A debate in the PE community on the definition of the reactive power has a long history, with hundreds of papers on it, with the IEEE Comte on Power Definitions, [16], with a biannual Int. Workshop on Power Definitions under Non-sinusoidal Conditions in Italy and the International School on Nonsinusoidal Currents and Compensation (ISNCC), run biannually in Poland. Several different quantities were suggested to be regarded as a reactive power. Some of them were supported by international standards [6, 16]. The most known are those suggested by Budeanu [2], Fryze [3], Shepherd and Zakikhani [5], Kusters and Moore [7], Depenbrock [11] and Tenti [13]. An overview of these definitions with a discussion on their deficiencies can be found in [20].

According to the Currents' Physical Components – based power theory of electrical systems [9, 20], the reactive power Q is associated with the phenomenon of the phase-shift of the load current harmonics with respect to the supply voltage harmonics. Due to this phase-shift, a reactive component, referred usually as a ***reactive current***, occurs in the load current. This current is defined for three-phase systems with nonsinusoidal voltages as a three-phase vector of reactive currents in lines R, S, and T, namely

$$\boldsymbol{i}_{\mathrm{r}}(t)=\begin{bmatrix} i_{\mathrm{rR}}(t) \\ i_{\mathrm{rS}}(t) \\ i_{\mathrm{rT}}(t) \end{bmatrix}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_{\mathrm{e}n}\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t}\,. \tag{38}$$

The symbol N in formula (38) denotes a set of orders n of dominating harmonics, while B_{en} is the equivalent susceptance of the load for the n^{th} order harmonic. This susceptance is the imaginary part of the equivalent admittance

$$B_{\mathrm{e}n}=\mathrm{Im}\{\boldsymbol{Y}_{\mathrm{RS}\,n}+\boldsymbol{Y}_{\mathrm{ST}\,n}+\boldsymbol{Y}_{\mathrm{TR}\,n}\} \tag{39}$$

of an equivalent load in Δ configuration for the n^{th} order harmonic, shown in Fig. 13.

Fig. 13. A load and its equivalent circuit in Δ configuration for the n^{th} order harmonic.

The reactive current three-phase rms value is

$$||\boldsymbol{i}_{\mathrm{r}}||=\sqrt{\sum_{n\in N} B_{\mathrm{e}n}^2||\boldsymbol{u}_n||^2}\,,\qquad ||\boldsymbol{u}_n||^2=U_{\mathrm{R}n}^2+U_{\mathrm{S}n}^2+U_{\mathrm{T}n}^2\,. \tag{40}$$

The reactive power Q is defined as a product of the supply voltage and the reactive current three-phase rms values:

$$Q=||\boldsymbol{u}||\,||\boldsymbol{i}_{\mathrm{r}}||\ . \tag{41}$$

This definition has an analogy to the definition of the apparent power S in three-phase systems [9], namely

$$S=||\boldsymbol{u}||\,||\boldsymbol{i}||\ . \tag{42}$$

Observe, that the reactive power Q, similarly as the apparent power S, are non-negative quantities. It can be "equipped" with a sign only when the voltages and currents are sinusoidal, but not when they are nonsinusoidal, however. The assumption that the reactive power is a non-negative quantity does not create any problems at its compensation, because the reactive current, not the power is the subject of compensation, and reduction of the reactive power is only a by-product of the reactive current rms value reduction.

VI Conclusions

The reactive power is one of a few major power quantities in electrical circuits and systems, unfortunately, often misinterpreted and associated with such physical phenomena as the energy oscillation or the energy storage. A clarification of the misinterpretation of this quantity has mainly cognitive merit. When this misinterpretation is involved in a control algorithm of switching compensators, its clarification can have also practical merit.

References

[1] Ch.P. Steinmetz, Theory and calculation of electrical apparatur, McGraw - Hill Book Comp., New York, 1917.

[2] C.I. Budeanu, "Puissances reactives et fictives," *Institut Romain de l'Energie*, Bucharest, 1927.

[3] S. Fryze, "Active, reactive and apparent power in circuits with nonsinusoidal voltages and currents," *Przegląd Elektrotechniczny*, z. 7, pp. 193-203, z. 8, pp. 225-234, 1931, z. 22, pp. 673-676, 1932.

[4] B.D.H. Tellegen: "A general network theorem with applications," *Philips Research Reports*, (Philips Research Laboratories) 7**,** pp. 259–269, 1952.

[5] W. Shepherd, P. Zakikhani, "Suggested definition of reactive power for nonsinusoidal systems," *Proc. IEE*, vol. 119, no. 9, pp. 1361-1362, 1972.

[6] International Electrotechnical Commission, (1979), Techn. Committee No. 25, Working Group 7, Report: *Reactive power and distortion power*, Doc. No. 25, 113 (Secr.).

[7] N.L. Kusters, W.J.M. Moore, "On the definition of reactive power under nonsinusoidal conditions," *IEEE Trans. Pow. Appl. Syst.*, vol. PAS-99, No. 3, pp. 1845-1854, 1980.

[8] A. Akagi, Y. Kanazawa, A. Nabae, "Instantaneous reactive power compensators comprising switching devices without energy storage components", *IEEE Trans. IA*, Vol. IA-20, No. 3, pp. 625-630, 1984.

[9] L.S. Czarnecki, "Orthogonal decomposition of the currents in a three-phase non-linear asymmetrical circuit with a nonsinusoidal voltage", *IEEE Trans. IM.*, Vol. IM-37, no. 1. pp. 30-34, 1988.

[10] A. E. Emanuel, "Powers in nonsinusoidal situations. A review of definitions and physical meanings", *IEEE Trans. on Power Delivery*, Vol. 5, No. 3, pp. 1377-1389, 1990.

[11] M. Depenbrock, "The FBD-method, a generalized applicable tool for analyzing power relations," *IEEE Trans. on Power Del.*, Vol. 8, No. 2, pp. 381-387, 1993.

[12] G. Superti-Furga, "Searching for generalization of the reactive power – a proposal", *Europ. Trans. on Electric Power, ETEP*, Vol. 4, No. 5, pp. 411-420, 1994.

[13] P. Tenti, P. Mattavelli, "A time-domain approach to power terms definitions under nonsinusoidal conditions", *6th Int. Workshop on Power Definitions under Nonsinusoidal Conditions*, Milano, Italy, 2003.

[14] W.G. Morsi, M.E. El-Hawary, Defining power components in nonsinusoidal unbalanced polyphase systems: the issues, *IEEE Trans. on PD.*, Vol. 22, No. 4, pp. 2428-2438, 2007.

[15] L.S. Czarnecki, "Effect of supply voltage harmonics on IRP-based switching compensator control", *IEEE Trans. on Power Electronics*, Vol. 24, No. 2, pp. 483-488 2009.

[16] IEEE standard definitions for the measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced and unbalanced conditions. *IEEE 1459-2010*.

[17] L.S. Czarnecki, "Effect of supply voltage asymmetry on IRP p-q - based switching compensator control", *IET Proc. on Power Electronics*, Vol. 3, No. 1, pp. 11-17, 2010.

[18] R. Kabri, D.G. Holmes, B.P. McGrath, "Control of active and reactive power ripple to mitigate unbalanced grid voltages", *IEEE Trans. on Ind. Appl.*, Vol. 52, No. 2, pp. 80-87, 2016.

[19] T.D.C. Busarello, A. Morteza, A.Peres, M.G. Simoes, "Application of the Conservative Power Theory current decomposition in load powershering strategy among distributed energy sources", *IEEE Trans. on Ind. Appl.*, Vol. 54, No. 4, pp. 21-27, 2018.

[20] L.S. Czarnecki, "Currents' Physical Components (CPC) – based Power Theory. A Review, Part I: Power Properties of Electrical Circuits and Systems", *Przeglad Elektrotechniczny*, ISSN 0033-2097, R. 95, Nr. 95, pp. 1-11, Nr. 10/2019.