PRZEGLĄD ELEKTROTECHNICZNY

Ukazuje się od 1919 roku 7'20

Organ Stowarzyszenia Elektryków Polskich • **Wydawnictwo SIGMA-NOT Sp. z o.o.**

Leszek S. CZARNECKI, Motab ALMOUSA

School of Electrical Engineering and Computer Science, Louisiana State University, Baton Rouge, USA

doi:10.15199/48.2020.07.01

What is Wrong with the Paper ”The IEEE Standard 1459, the CPC Power Theory and Geometric Algebra in Circuits with Nonsinusoidal Sources and Linear Loads”?

Abstract. *There are published opinions that the complex number algebra as used for circuits analysis, power theory of electrical circuits and methods of reactive compensators design should be superseded by geometric algebra. Such opinions were presented in the paper ”The IEEE Standard 1459, the CPC-based Power Theory, and Geometric Algebra in Circuits with Nonsinusoidal Sources and Linear Loads”, and “Advantages of Geometric Algebra Over Complex Numbers in the Analysis of Linear Networks with Nonsinusoidal Sources and Linear Loads”, published in IEEE Transactions on Circuits and Systems-I, in 2012. These opinions were supported in these papers by results obtained using the geometric algebra for describing power properties of electrical circuits and reactive compensator design.*
This paper presents critical comments to these papers. It shows that their authors were able to apply the geometric algebra to only single-phase circuits with linear loads but the current decomposition they obtained was known 40 years earlier. The same applies to reactive compensators design, thus the results obtained in the commented papers are dramatically obsolete with regard to the current state the power theory development and state of the knowledge on compensation. The suggestion that the algebra of complex numbers, successfully used now for linear circuits analysis, should be superseded by the geometric algebra was not supported by any credible argument. It was not shown that geometric algebra is superior over the circuit analysis based on the algebra of the complex numbers. Quite opposite, it makes this analysis dramatically much more complex, without any benefits for this complexity. It does not contribute to our comprehension of power-related phenomena in electrical circuits.

Streszczenie. *Istnieją opublikowane opinie sugerujące, że algebra liczb zespolonych, tak jak jest ona używana w analizie obwodów, teorii mocy i syntezie kompensatorów reaktancyjnych, powinna być zastąpiona algebrą geometryczną. Takie opinie zostały przedstawione w artykułach “The IEEE Standard 1459, the CPC-based Power Theory, and Geometric Algebra in Circuits with Nonsinusoidal Sources and Linear Loads”, oraz “Advantages of Geometric Algebra Over Complex Numbers in the Analysis of Linear Networks with Nonsinusoidal Sources and Linear Loads”, opublikowanych w IEEE Transactions on Circuits and Systems-I, w 2012 roku. Opinie te zostały poparte wynikami ilustującymi zastosowanie algebry geometrycznej w teorii mocy oraz w metodach syntezy kompensatorów reaktancyjnych.*
Niniejszy artykuł przedstawia krytyczną ocenę tych artykułów. Pokazuje, że ich autorzy byli w stanie zastosować algebrę geometryczną wyłącznie do obwodów jedno-fazowych z odbiornikami liniowymi, jednak otrzymali jedynie ortogonalny rozkład prądu odbionika, znany już od lat 40-tu. To samo dotyczy syntezy kompensatorów reaktancyjnych. Wyniki te są dramatycznie opóźnione w stosunku do obecnego stanu teorii mocy i wiedzy o syntezie kompensatorów. Opinia o tym, że algebra liczb zespolonych, która jest obecnie skutecznym i całkowicie wystarczającym narzędziem analizy obwodów liniowych, powinna być zastąpina algebrą geometryczną, nie została poparta żadnym przekonywującym argumentem. Przeciwnie, komplikuje ona dramatycznie tę analizę, bez jakichkolwiek korzyści. Nie przyczynia się też do pogłębienia interpretacji zjawisk fizycznych w obwodach elektrycznych. ***(Co jest nie tak z artykułem ”The IEEE Standard 1459, the CPC Power Theory and Geometric Algebra in Circuits with Nonsinusoidal Sources and Linear Loads”?)***

Keywords: Power definitions, power equation, Tellegen Theorem, Power Balance Principle.
Słowa kluczowe: Definicje mocy, równanie mocy, Prawo Tellegena, Zasada Bilansu Mocy.

Introduction

The power theory (PT) and methods of compensation of electrical loads aimed at an improvement of the effective-ness of the energy transfer are crucially important for the power engineering technology and power systems economy. Therefore, hundreds of scientists have been involved in rese-arch on PT development, initiated by Steinmetz [3], Budeanu [4], Fryze [5], and on methods of compensation. Hundreds of papers that report the results of this research were published.

The papers “The IEEE Standard 1459, the CPC Power Theory, and Geometric Algebra in Circuits with Nonsinusoidal Sources and Linear Loads” [1] and “Advantages of Geometric Algebra Over Complex Numbers in the Analysis of Linear Networks with Nonsinusoidal Sources and Linear Loads” [2] are ones of them.

The authors of [1] claim that due to “the limitations of the algebra of complex numbers…” an alternative circuit analysis technique is needed. Unfortunately, they do not specify these limitations before suggesting that the algebra of complex numbers should be superseded by the geometric algebra $\mathcal{G}_N$. That claim is wrong. In the case of the power theory of elec-trical systems and their reactive compensation, the algebra of complex numbers has occurred to be sufficient to provide the solutions needed in the power systems engineering. That claim looks as credible only because the authors of [1] do not provide a fair account of the present state of research on power theory and compensation. The commented paper is confined to power theory and compensation of only single-phase linear loads, while at the moment papers [1] and [2] were published, these problems were solved even for three-phase, nonlinear loads. A reader of these papers will not be able to find any information on it. He might be convinced that single-phase liner loads are indeed the very center of studies on power theory and compensation. These issues for single-phase linear loads were solved [6, 7] a few decades earlier, in 1981 and 1984, respectively.

Numerous misleading opinions expressed in these papers, when they will be read by an unprepared young engineer or a scientist, could have an adversarial effect on his research on the circuits analysis, power theories, power properties of electrical systems, and on compensation in such systems.

Therefore, it is important to turn the reader's attention to the fact that the power theory of electrical systems is not a piece of mathematics for itself, but a tool for power systems (PE) engineers, who have to handle a variety of technological and economic problems they encounter in these systems. New mathematical methods are usually needed if the existing ones are not sufficient for handling such problems. This necessity should be somehow demonstrated when a new mathematical tool was suggested to be used. Moreover, power theory should be formulated in the mathematical language the PE engineers are prepared at universities to comprehend and use. The commented paper does not satisfy these expectations, however.

Circuits analysis, power theory, and methods of reactive compensation differ substantially as to goals and methods. Their studies should take, therefore, these differences into account. Unfortunately, they are mixed in the commented paper to such a degree, that it can cause a substantial confu-sion of an unexperienced reader.

Let us illustrate this. A reader should be aware that only voltages and currents at the supply terminals of the load have to be known for a compensator design. They can be known, for example, by a measurement. Circuits analysis is not needed for that. We do not need to know the load topology and its parameters, which are necessary for the circuit ana-lysis, to design a compensator. The commented paper does not convey, however, this very basic information to a reader.

$\mathcal{G}_N$ geometric algebra versus algebra of complex numbers

The use of geometric algebra in the commented papers is motivated [2] by "The limitations of the algebra of complex numbers…". According to the authors of [1], the algebra of complex numbers should be replaced in circuit analysis by the $\mathcal{G}_N$ geometric algebra. This is an extremely strong statement, that if it is true would overturn the whole approach to the analysis of electrical circuits. Therefore, before we abandon the complex numbers in circuits analysis, power theory, and compensation for the $\mathcal{G}_N$ geometric algebra, let us remain how the complex numbers are used in electrical engineering in circuits analysis in the presence of harmonics.

A periodic quantity $x(t)$ with a period T and limited norm

$$\sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt} < \infty \tag{1}$$

can be expressed by the Fourier series

$$x(t) = X_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_1 t + \sum_{n=1}^{\infty} b_n \sin n\omega_1 t\ . \tag{2}$$

This traditional form can be converted to the complex form

$$x(t) = X_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{X}_n\, e^{jn\omega_1 t} \tag{3}$$

where

$$\boldsymbol{X}_n = \frac{a_n - jb_n}{\sqrt{2}} = X_n\, e^{j\alpha} \tag{4}$$

is the complex rms (crms) value of the n^{th} order harmonic. At linear circuits analysis only some number of dominating har-monics of the order n from the set N, are usually taken into account. In such a case the Fourier series can be written in the complex form, namely

$$x(t) = X_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{X}_n\, e^{jn\omega_1 t}\ . \tag{5}$$

A linear circuit with sources voltages specified by the Fourier series of the form (5), can be described by a set of linear equations for the circuit loops or nodes, separately for each harmonic of the order n from the set N. After this set is solved, the crms values $\boldsymbol{I}_n$ and $\boldsymbol{U}_n$ of the current and voltage harmonics at the circuit R, L, and C elements can be calcu-lated.

A substantial part of the commented paper is devoted to applications of the $\mathcal{G}_N$ geometric algebra to linear, single-phase circuits analysis.

Due to the development in electronics, circuits in a grow-ing degree are composed now of nonlinear and time-variant components. They are described for their analysis by sets of differential equations, which can be integrated numerically using computer programs. Programs such as Simulink or PSpice or are often used for that. When the load is linear, then the sets of these differential equations can be converted to sets of algebraic equations and solved using the complex numbers algebra. The MatLab is just a program, written to just handle the complex numbers algebra.

The authors of [1] support their opinion on the necessity of replacing the algebra of complex numbers by algebra also by "…impossibility to apply the principle of conservation of apparent power…". This is simply an erroneous statement. The principle of conservation, as used in electrical engine-ering, applies only to the energy, but there is no relationship between the energy and the apparent power. The principle of the energy conservation principle (ECP) is commonly used to develop the balance principle for the instantaneous and the active powers. Even the balance principle for the reactive power Q in sinusoidal systems cannot be derived from the ECP.

The values of voltages, currents, and powers of the circuit elements have to be identical, independently of the method of their calculation. It can be based on complex numbers alge-bra, on $\mathcal{G}_N$ geometric algebra or obtained by integration of differential equations of the circuit. Results have to be identical. Therefore, the apparent power S does not satisfy the balance principle not because of the method of this power calculation, but because of this power definition. The appa-rent power S is defined as the product of the voltage and current rms values, hence it cannot be negative. Their sum over all elements of a circuit cannot be equal to zero. Thus, there is a lack in [1] and in [2] of any credible justification for replacing the complex numbers algebra by the $\mathcal{G}_N$ geometric algebra.

In the section on the $\mathcal{G}_N$ geometric algebra, the authors of the commented paper [1] provide four main features of it when used for electrical circuits analysis. These features are ques-tionable, however. Namely:

1. The $\mathcal{G}_N$ domain is not the frequency domain. This statement is not true, however. Even symbol "N" in that domain is specified by a harmonic order. Frequency and harmonics, which are the very essence of the frequency-domain approach, are visible in most of the formulas written in [1] in terms of the $\mathcal{G}_N$ algebra. For example, in (48), section IV. Also, the phrase "*cross-frequency products*" below (35) confirm that this statement is simply not true.
2. "The method allows the application of Kirchoff's laws…. and the principle of conservation of energy". This is

a sort of misinformation, which seems to suggest that other methods do not allow for that. Kirchoff's laws and the energy conservation principle have to be satisfied independently on the method of the circuit analysis.

3. "…the definition of the apparent power and...are different… from the traditional definition". This statement could be heavy in consequences for the power systems engineering. More than a hundred years ago PEs concluded that transformers that supply the loads are heated by the voltage $u(t)$ they provide for customers and by the customers' current $i(t)$. They agreed that the product of these two quantities rms value specifies the apparent power, $S = ||u||\ ||i||$. The highest value of the apparent power specifies the transformer power ratings, thus its cost. The ratio P/S specifies the power factor, one of the most important factors for financial accounts between the energy providers and customers. Thus, the change of the apparent power definition to that suggested in [1] would affect the PS economy. Fortunately, there is no justifica-tion for replacing the present definition of the apparent power S justified by more than a century-long power engi-neering practice, by that developed in $\mathcal{G}_N$ algebra for only mathematical reasons.

4. "In contrast to the approach of many, the formulation of our power theory is not based on the apparent power concept. Instead,...is based on the power multivector…". This statement does not seem to have any sense, however. The PTs are not "based on apparent power". Scientists involved in PT development attempt only to explain the inequality (16) between the apparent and the active power or equivalent inequalities (17) or (18), and they use different mathematical tools for that.

A reader accustomed to mathematical strictness can find that the paper [1] mathematics is not easy to follow, mainly because of the lack of strictness and precision in the paper. First of all, symbols are not defined or used in a few different and not specified meanings. For example, the capital italic, such as V is used in circuits analysis to denote the rms value of the sinusoidal voltage

(6) $$v(t) = \sqrt{2}\, V \cos(\omega_1 t + \alpha)$$

meaning, the value

(7) $$V = \sqrt{\frac{1}{T}\int_0^T v^2(t)\ dt}\ .$$

Thus, what is the meaning of the symbol $||V||$ in formula (15) in [1] or used in apparent power definition above (22), namely $S = ||V||\ ||I||$? Later, in (20) and below, the symbol V does not denote the rms value, but a "multivector". Even worse, the term "multivector V" is not defined in the commen-ted paper. A reader is sent to referenced papers, where this definition also does not exist. It seems that a reader deserves a bit of mercy, but he will not find it in [1] or [2].

The authors say in [1] below (18) "…multiplying any term in (18) by….performs 90^{o} rotation…". What does it mean, however, "rotation" the function of time: $x_{c1}(t) = X\sqrt{2}\ \cos\omega t$ " by 90^{o}"? What does it mean "rotation" of the rms value X_{c1}?

The next issue is the meaning of symbols s_1 and s_2. These symbols are not defined in the paper [1]. Moreover, looking on the inductor impedance in (23) one can conclude that the bivector

(8) $$\sigma_1\sigma_2 = -j.$$

This bivector is used next in formula (24) for the admittance. Since the admittance for the h order harmonic is

(9) $$\boldsymbol{Y}_h = \boldsymbol{Z}_h^{-1} = G_h + jB_h$$

thus, the bivector

(10) $$\sigma_1\sigma_2 = +j.$$

Which of these two values is right? This is confusing also because the authors do not distinguish, as it can be seen in (23) and (24) of [1], symbols of complex numbers from sym-bols of rms values and instantaneous values. Two lines below (24) authors write: "…the total current is given by

(11) $$I = \sum\nolimits_{h=1}^{n} I_h\ \text{''}.$$

This formula is deeply wrong, because neither the rms values of different harmonics I_h nor their complex rms values $\boldsymbol{I}_h$, can be added. Such a sum, for example in (64), has no sense. The total current is commonly denoted by $i(t)$ and can be presented as a sum of current harmonics $i_h(t)$, namely,

(12) $$i(t) = \sum\nolimits_{h=0}^{n} i_h(t)$$

but not their rms values I_h.

Let us suppose that the crms value of the 13th order voltage harmonic on some element of the circuit is

(13) $$\boldsymbol{U}_{13} = 100\, e^{j45^{\rm o}}\ [\rm V]$$

thus, the voltage harmonic of the 13th order at that element is

(14) $$u_{13}(t) = \sqrt{2}{\rm Re}\{\boldsymbol{U}_{13} e^{j13\omega_1 t}\} = 100\sqrt{2}\cos(13\omega_1 t + 45^{\rm o})\ [\rm V]\ .$$

Now, let us express the same voltage harmonic in terms of symbols suggested to be used in [1] and [2]

$$\begin{aligned} u_{13}(t) &= 100\sqrt{2}\cos(13\omega_1 t + 45^{\rm o}) = \\ &= 100\sqrt{2}\cos 45^{\rm o}\cos(13\omega_1 t)\ - 100\sqrt{2}\sin 45^{\rm o}\sin(13\omega_1 t) = \\ &= 50\cos(13\omega_1 t) - 50\sin(13\omega_1 t) = 50\wedge_{i=2}^{14}\sigma_i + 50\wedge_{\substack{i\\ i\neq 2}}^{14}\ \sigma_i = \\ &= 50\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6\sigma_7\sigma_8\sigma_9\sigma_{10}\sigma_{11}\sigma_{12}\sigma_{13}\sigma_{14} + \\ &\quad + 50\sigma_1\sigma_3\sigma_4\sigma_5\sigma_6\sigma_7\sigma_8\sigma_9\sigma_{10}\sigma_{11}\sigma_{12}\sigma_{13}\sigma_{14}\ . \end{aligned} \qquad (15)$$

This expression seems to be "a bit" confusing, in particular, that the meaning of the symbols σ_3, σ_4, σ_5, σ_6, σ_7,.. σ_{14}, is not explained, neither in [1] nor in [2]. Thus, how to write a com-puter program for a circuit analysis, based, as suggested in [1] and [2], on the $\mathcal{G}_N$ algebra? Such a program will require to enter the values of these σ_1, σ_2, σ_3, σ_4, σ_5, σ_6,...functions.

Comparing formulas (72)–(79) in [1] with formulas needed for equivalent calculations using complex arithmetic, a reader can see dramatic inferiority of geometric algebra in such applications with regard to the algebra of complex numbers. Therefore, the claim in [2] on the superiority of the $\mathcal{G}_N$ geometric algebra over the algebra of complex numbers looks like a joke.

Despite the lack in [1] of definitions for symbols s_1, and s_2, one could conclude that $\sigma_1 = \cos(\omega t)$, $\sigma_2 = -\sin(\omega t)$, so that, probably also σ_3, σ_4, σ_5, σ_6, σ_7…. are functions of the time and the frequency. Thus, a question arises: whether a computer with such a program will provide the solution of the circuit analysis in the time- or the frequency-domain? Thus, several crucially important questions remain not answered in the commented paper.

Power theory

The commented paper [1], even by its title suggests that it is focused on power theories, but a reader can learn very little on it. In particular, the subject of [1] fits exactly the subject of paper [7], where the power theory of single-phase linear loads with a nonsinusoidal supply voltage, along with a met-hod of reactive compensation was developed. Unfortunately, although the results obtained in [7] go far beyond those in [1], this paper was not even cited.

A reader of [1] cannot learn what the PT is about, for what purpose it has been developed, and on major approaches to its development. A draft of these issues is necessary for the comments presented in this paper.

The power theory of electrical systems was being developed due to the contribution of hundreds of scientists. Some of them, such as Budeanu [4], Fryze [5], Shepherd & Zakikhani (S&Z) [8], Kusters & Moore[11], Depenbrock [15], Czarnecki [7, 12, 13, 14] or Tenti [17] have created a sort of schools of the power theory.

All of them have attempted to provide the answer to the question, traced down to Steinmetz [3], "***why the apparent power S is usually higher than the load active power P***"

(16) $$S \geq P\,.$$

Although some conclusions overlap, essentially each of these scientists has provided a different answer to this question. It refers to powers at the load supply terminals, but not to powers inside of the load.

The powers in (16) can be measured or calculated having available the voltages and currents at the load terminals for a measurement. This could be a single-phase or a three-phase load with three- or four-wire supply line, as shown in Fig. 1.

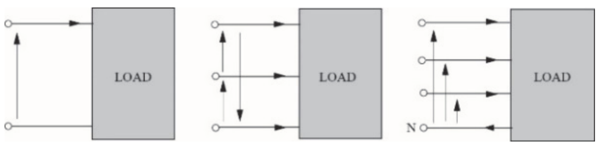

Fig. 1. Single-phase and three-phase loads.

The power theory has to be able to describe the power-related properties of linear, nonlinear and time-varying loads because just such are loads in distribution systems.

Because both powers in the inequality (16) occur in the effect of multiplication of the supply current rms value $||i||$ and then active current rms value $||i_a||$ by the same voltage rms value $||u||$, the original [3] Steinmetz's question was super-seded by Fryze [5] question " ***why the supply current rms value*** $||i||$ ***is higher than the active current rms value*** $||i_a||$***, which is needed for the load supply with the active power P?***" It means that explanation of the inequality

(17) $$||i|| \geq ||i_a||$$

is the major issue of power theory. This inequality for three-phase loads can be modified to the form

(18) $$||\boldsymbol{i}|| \geq ||\boldsymbol{i}_a||$$

where $\boldsymbol{i}$ denotes the three-phase vector of the load line currents,

(19) $$\boldsymbol{i} = [i_R, i_S, i_T]^T$$

and

(20) $$||\boldsymbol{i}|| = \sqrt{||i_R||^2 + ||i_S||^2 + ||i_T||^2}$$

is its three-phase rms value. Its concept was introduced to PE in [12]. Most of the power theories are focused, after Fryze, on explanation inequality (17) and (18) by decomposition of the load current into components, specific for particular power theories. Powers in these theories are regarded usually as secondary to the load current components. Indeed, the energy loss at its delivery is caused by currents, but not by powers.

The studies on the power theory in the commented paper [1] are not separated from studies on the circuit analysis and on compensation, causing substantial confusion as to the stu-dies in this paper's objectives. They are confined to only single-phase linear loads. Power properties of such loads at nonsinusoidal supply voltage, along with its reactive compen-sation, were explained in 1984 in the paper [7]. It means, that a problem which was solved long ago, is again the main subject of the commented paper. Its solution, presented in [6, 7], is not even referenced in the paper [1], however. Conse-quently, it is not clear why the single-phase linear load is the focus of the study again?

The authors of [1] declare that "…noval technique and ….power theory….are used here to illustrate its superiority over the CPC power theory and the IEEE Standard 1459". One can observe, however, that the CPC PT and the Standard 1459 describe power properties not only single-phase linear loads but also three-phase nonlinear loads. Thus, how the $\mathcal{G}_N$ – based PT can be superior over them, even if it does not cover the area of these two concepts of the PT, and does not introduce anything new to the power theory of single-phase linear loads and their reactive compensation as developed in 1984 by Czarnecki in [7]. As it was shown above, it is inferior to the CPC-based method of compensa-tion even in single-phase circuits.

Observations presented in [2], page 2., col. 1, as deficiencies of the CPC, namely, "Unfortunately, the CPC power theory's definition of reactive power is not signed quantity, thus, the balance principle of the reactive power cannot be applied…", and "…the CPC power theory does not allow viewing the flow of the current and energy/powers in each branch of the circuit" demonstrate that authors of [2] confuse the power theory with the circuit analysis. The current and energy flow in individual branches of a circuit and the power balance in such a circuit are not the subjects of the power theory, but the circuit analysis. Thus, these, cited above observations are irrelevant to the CPC power theory.

The paper [1] has in its title "the CPC power theory". It is not acceptable, however, that theory created by another contributor to PT is deformed as can be seen in eqn. (15) of [1]. There are not symbols $||V||$ and V in the CPC-based PT. There is no coefficient "2" in the formulas for the scattered and the reactive currents rms values $||i_s||$ and $||i_r||$. What is the difference between the meaning of the symbols $||V||$ and $||v||$? Why the original paper [7], where the eqn. (15) and (16) in the right form were developed, was not cited in [1]?

Because [7] is not cited, let us draft the CPC-based PT as developed in that paper, because it will be the main reference for the evaluation of the results presented in [1].

Let a linear, time-invariant (LTI) load is supplied with a nonsinusoidal voltage

(21) $$u = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{U}_n e^{jn\omega_1 t}, \quad \boldsymbol{U}_n = U_n e^{j\alpha_n}\,.$$

The load current of such a load can be expressed in the form

(22) $$i = I_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{I}_n e^{jn\omega_1 t} = Y_0 U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{Y}_n \boldsymbol{U}_n e^{jn\omega_1 t}$$

where

(23) $$Y_n = G_n + jB_n$$

is the load admittance for the n^{th} order harmonic.

With regard to the active power P at the voltage $u(t)$, the single-phase LTI load is equivalent to a purely resistive load,
as shown in Fig. 2, on the condition that its conductance is

(24) $$G_\mathrm{e} = \frac{P}{\|u\|^2}\,.$$

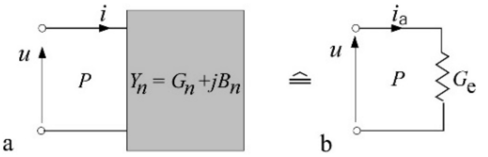

Fig. 2. A linear load (a) and its equivalent load (b) with respect to the active power *P*.

This conductance was called in [5] an ***equivalent conductance*** of the load. It draws the ***active current***,

(25) $$i_\mathrm{a} = G_\mathrm{e}\, u\,.$$

The load current can be decomposed into three components

(26) $$i = i_\mathrm{a} + i_\mathrm{s} + i_\mathrm{r}$$

where

(27) $$i_\mathrm{s} = (G_0 - G_\mathrm{e})U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty}(G_n - G_\mathrm{e})\boldsymbol{U}_n e^{jn\omega_1 t}$$

is the ***scattered current***, and

(28) $$i_\mathrm{r} = \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n \boldsymbol{U}_n e^{jn\omega_1 t}$$

is the ***reactive current***.

These three current components are associated with dis-tinctive physical phenomena. The active current is associated with the phenomenon of the permanent energy transfer to the load; the scattered current is associated with the phenome-non of the load conductance G_N changes with the harmonic order; the reactive current is associated with the phenomenon of the phase-shift of the load current harmonics with regard to the supply voltage harmonics. Because of this association, these currents are referred to as the Current Physical Com-ponents (CPC).

The rms values of the CPC are

(29) $$\|i_\mathrm{a}\| = \frac{P}{\|u\|}$$

(30) $$\|i_\mathrm{s}\| = \sqrt{\sum_{n=0}^{\infty}(G_n - G_\mathrm{e})^2 U_n^2}$$

(31) $$\|i_\mathrm{r}\| = \sqrt{\sum_{n=1}^{\infty} B_n^2 U_n^2}$$

respectively. The currents in decomposition (26) are mutually orthogonal [7], thus, the rms values of CPC satisfy the rela-tionship

(32) $$\|i\|^2 = \|i_\mathrm{a}\|^2 + \|i_\mathrm{s}\|^2 + \|i_\mathrm{r}\|^2\,.$$

This formula shows in a very clear way how distinctive power phenomena in the load contribute to the supply current rms value, thus to the energy loss in the supply source. It is also fundamental for reactive compensation.

Multiplying (32) by the square of the supply voltage rms value, $\|u\|$, the power equation of LTI loads supplied with a nonsinusoidal voltage is obtained, namely

(33) $$S^2 = P^2 + D_\mathrm{s}^2 + Q^2$$

where

(34) $$D_\mathrm{s} = \|u\|\,\|i_\mathrm{s}\|, \qquad Q = \|u\|\,\|i_\mathrm{r}\|.$$

are the scattered and the reactive powers, respectively.

With regard to the load current decomposition, the main result of commented paper [1], after very confusing reasoning, seems to be eqn. (27)

(35) $$I_l = I_g + I_b\,.$$

If we put aside the major confusion as to symbols, equation (27) in [1] looks like the current decomposition into an in-phase component and a component perpendicular to the supply voltage harmonics. Such a decomposition was presented in [8] by S&Z. According to them, at the supply voltage

(36) $$v(t) = V_0 + \sqrt{2}\sum_{n=1}^{\infty} V_n \cos(n\omega_1 t + \alpha_n)$$

the current of a linear load

(37) $$i(t) = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \alpha_n - \varphi_n)$$

the current can be decomposed into an in-phase component

(38) $$i_\mathrm{R}(t) = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos\varphi_n \cos(n\omega_1 t + \alpha_n)$$

and a quadrature component

(39) $$i_\mathrm{r}(t) = \sqrt{2}\sum_{n=1}^{\infty} I_n \sin\varphi_n \sin(n\omega_1 t + \alpha_n)$$

such that

(40) $$i(t) = i_\mathrm{R}(t) + i_\mathrm{r}(t)\,.$$

When symbols used in formula (40) are replaced by symbols used in [1], meaning if $i(t) = I_l$, $i_\mathrm{R}(t) = I_g$, $i_\mathrm{r}(t) = I_b$, then equation (27) in [1] is obtained. Thus (27) is the current decomposition as suggested by S&Z. It means that authors of [1] only repeated, in a confusing way, the S&Z decomposition developed forty years earlier. Unfortunately, although authors of [1] only duplicate the result obtained by S&Z, their paper [8] is not cited in [1]. Moreover, as compared to the result obtained by S&Z, decomposition (27) in [1] is confusing, because the capital italic symbol "I", used in (27), is common-ly used in electrical engineering for denoting the rms value of a sinusoidal current, but not its instantaneous value $i(t)$.

The results obtained in [1] as to powers, expressed in formulas (76)–(79) by mysterious, not defined symbols

(41) $$P^{lcp}, P^{scp}, P^{cp}, P^{zscp}, CN^{lcp}_{r(hi)}, CN^{lcp}_{d}, CN^{scp}_{r(hi)}, CN^{zscp}_{d}, CN^{zscp}_{r(hi)}\,.$$

cannot be even commented. It is not clear what do these symbols mean.

Decomposition (40), when it was suggested by S&Z, it substantially contributed to the PT development, because it revealed the presence of the reactive component $i_\mathrm{r}(t)$ in the load current. At the same time, however, it had a major deficiency, because S&Z were not able to relate the in-line current $i_\mathrm{R}(t)$ to the active power P of the load. It was not pos-sible, because the presence of the scattered component in the load current was not known at that time. It was revealed [7] later, in 1984. Therefore, also decomposition (27) in [1], as identical with (40), is obsolete as compared to the state of the power theory development when the paper [1] was published. By that time not only the PT of linear loads was fully developed in [7], but also the PT of single-phase nonlinear loads [13], and three-phase unbalanced nonlinear loads [12] as well. Unfortunately, none of these papers, closely related to [1], was cited, creating a misleading picture of the state of power theory development.

The central issue for the PT, from Steinmetz observation in [3], was the problem of the inequality between the apparent and the active power, expressed directly by (16) or in terms of current rms values (17) or (18). All main contributors to the PT development have attempted to explain this difference.

This main issue of the power engineering and the PT, meaning the inequalities (16), (17), and (18) even do not exist in papers [1] and [2], however. The very clear concept of the apparent power S, explained in this section, is superseded by a poorly defined "multivector power" entirely stripped of the physical interpretation, technical and economic meanings for the power systems.

The argument in [1] that the apparent power S does not satisfy the balance principle so that it should be replaced by something that obeys this principle, does not have much sense. The apparent power is used for specifying the power ratings of the supply transformers, thus, their cost. This is because the energy loss in their windings is proportional to the square of the current rms value, while the energy loss in the transformer magnetic core is proportional to the square of the voltage rms value. The product of these rms values is positive so that it cannot satisfy the power balance principle. This is entirely irrelevant.

Power balance is usually a tool for verification of the results of the circuit analysis. It is enough for that to check this balance for the active power P, however. If it is satisfied, numerical calculations are right. There are no reasons to look for such a definition of the apparent power that it would satisfy this principle. The sum of apparent powers of all transformers in a power system seems does not have any merit.

Compensation

A substantial part of the commented paper is devoted to studies on reactive compensation, and in particular, to the development of a method needed for calculation of the LC parameters of a reactive compensator in the presence of the supply voltage distortion. They are confined only to single-phase linear loads. Compensation in three-phase systems and in systems with nonlinear loads is not studied in these papers, however.

Since compensation objectives, approaches and the research history are not presented in the commented paper, a short draft of them is needed as a background for the presented comments. A search for methods of compensation in systems with nonsinusoidal voltages and currents is a practical goal of power theory development.

A compensator is a device which, connected at the supply terminals of the load, can reduce its supply current. A com-pensator can be built of reactive elements or its current can be shaped to a required waveform by fast switched transi-stors. Consequently, compensators can be classified as reac-tive, switching, and hybrid compensators, meaning composed of reactive and switching ones. They are installed usually as shunt devices at the supply terminals of single-phase, or three-phase loads, as shown in Fig. 1.

The first results on designing a reactive compensator for single-phase loads in the presence of the supply voltage distortion were obtained in 1972 by Shepherd and Zakikhani [8] and in 1980 by Kusters and Moore [11]. These results were confined to only purely capacitive compensators, however.

The problem of the total compensation of the reactive current of single-phase linear loads by a reactive compens-ator was solved by Czarnecki [6] in 1981. The question: "can a linear load at nonsinusoidal supply voltage be compensated to unity power factor?" was answered [7] in 1984 and [14] in 1991, respectively. The total compensation of the reactive current in the presence of the supply voltage harmonics can require a very complex compensator, however. This comple-xity can be reduced if instead of whole compensation of the reactive current, it is only minimized by a compensator of reduced complexity. This problem for a compensator with only two reactive elements was solved [9] in 1987.

The cited above results of studies on compensation, at the time when papers [1] and [2] were published, show that problems studied in these papers were solved long before. Even more, at that time, also compensation in three-phase systems, which includes also the load reactive balancing, was solved [10]. There were also other studies on methods of compensation, for example, based on the FDB Method, developed by Depenbrock [15, 16]. There was also at that time a remarkable amount of studies on switching compensa-tors, commonly known as active power filters.

The method of calculation of the LC parameters of the reactive compensator as presented in [1], has major deficien-cies. These deficiencies are not visible, however, when the compensator is built of only two reactive elements and its structure, as shown in Fig. 5 of the paper [1], is known.

With the CPC approach, described in [7], the problem of a reactive compensator design is almost trivial. Suscep-tances B_n for harmonic of the load can be specified without any circuit analysis, by measurement of the voltage and current at the load terminals. There is, however, an infinite number of equivalent reactive compensators, i.e., with the same susceptances B_n but different structures and different LC parameters. Such compensators are commonly known as reactive or reactance one-ports. They can be found using methods of network synthesis [18], which is a well-developed branch of the circuit theory.

The method presented in [1] has nothing in common with this known and effective approach. Instead, the LC parame-ters have to be found by solving the set of algebraic equations of the number equal to the compensator complexity, specified by its order N, meaning the total number of inductors and capacitors. This order increases approximately by two with each voltage harmonic. For example, a compensator needed for compensation of the reactive power in the presence of the 3^{rd}, 5^{th}, 7^{th} order harmonics, may need approximately six reactive elements, but the accurate number of them is not known a priori. This number depends on the pattern of signs of the load susceptance B_n for harmonic. With the method suggested in [1] we have four equations but with an unknown number of the compensator parameters, however. One can try to guess this number, but if it is too low, the set of these equations will not have a solution; if it is too high, then the compensator would be too complex.

The next issue is the reactive compensator structure. It can have [18] a Foster, Cauer or a hybrid structure, meaning it can be built of a mixture of the Foster and Cauer links. Depen-ding on the order N of the compensator, it can have S_N differ-rent structures. For example, as proved in [19], the number of structures of equivalent compensators for a few values of their order N are:

$$S_2 = 1;\quad S_3 = 2;\quad S_4 = 4;\quad S_5 = 12;\quad S_6 = 33;\quad S_7 = 120.$$

Such compensators have identical susceptances B_n for harmonic frequencies but can differ substantially as to several other features important from a technical perspective. The situation studied in [1] refers to a compensator operating at the voltage distorted by only one harmonic thus, the compensator complexity $N = 2$.

The numerical illustration of compensation presented in [1], with results in Fig. 5, is a very elementary, almost trivial case of reactive compensation. The authors did not show, how-ever, how to compensate entirely the reactive current of several harmonics, as it can be done [7] using the CPC-based power theory.

Anyway, if the supply voltage, apart from the fundamental, has more than one harmonic, then the reactive current cannot be compensated entirely by any compensator composed of only two reactive elements. The reactive current rms value can be only minimized, as it was demonstrated in [9], by such a compensator. There is, according to [7] and [9], an infinite number of such compensators, with different LC parameters, however. The method presented in [1], has resulted in only one compensator, shown in Fig. 7. It was not proven, moreover, that it minimizes the reactive current rms value. Unfortunately, paper [7] and paper [9], were the problem of minimization of the reactive current was solved, were not referenced in [1].

Thus, the suggested in [1] geometric algebra $\mathcal{G}_N$ - based approach to methods of compensation is inferior to the pre-sently known methods of reactive compensators design.

Conclusions

The commented paper does not contribute to power theory development and the methods of reactive compen-sation. The results obtained are obsolete for decades with regard to the state of power theory and compensation at the time the paper was published.

The paper does not present any credible arguments for the need of replacing the algebra of complex numbers in the circuit analysis by the geometric algebra $\mathcal{G}_N$. There is a sub-stantial lack of physical interpretations and technological perspective in the paper. It is amazing how the relatively simple analysis of linear circuits, based on the algebra of com-plex numbers, handled by undergraduates, becomes compli-cated, as it is demonstrated in [1] when the $\mathcal{G}_N$ algebra is used for the same purpose.

A student after the university course on circuits has sufficient fundamentals to follow the present state of the power theory development and use it for the development of reactive compensators.

The power theory of electrical systems with nonsinusoidal voltages and currents along with reactive compensation is now well developed in the frame of the CPC-based power theory. It covers not only single-phase circuits with linear loads, which are the subject of the commented paper [1] but also three-phase systems with nonlinear, harmonics genera-ting loads supplied by three- and four-wire lines. Details are compiled in [20].

Author: *Prof. dr hab. inż. Leszek S. Czarnecki, IEEE Life Fellow, Distinguished Professor at the School of Electrical Eng. and Comp. Science, Louisiana State Univ., 824 Louray Dr., Baton Rouge, USA, LA 70808, lsczar@cox.net, lczarn1@lsu.edu, Intern.: czarnecki.study*

REFERENCES

[1] Castro-Nunez, M. Castro-Puche, R., The IEEE Standard 1459, the CPC Power Theory, and Geometric Algebra in Circuits with Nonsinusoidal Sources and Linear Loads, *IEEE Trans. on Cir-cuits and Syst.-I*, Vol. 59, No. 12, pp. 2980-2990, 2012.

[2] Castro-Nunez, M. Castro-Puche, R., Advantages of Geometric Algebra Over Complex Numbers in the Analysis of Linear Net-works with Nonsinusoidal Sources and Linear Loads, *IEEE Trans. on Circ. and Syst.-I*, Vol. 59, No. 12, pp. 2056-2064, 2012.

[3] Steinmetz, Ch.P., Does phase displacement occur in the current of electric arcs? (In German), *ETZ*, 587, 1892.

[4] Budeanu, C.I., Puissances reactives et fictives, *Institut Romain de l'Energie,* Bucharest, 1927.

[5] Fryze, S., Active, reactive and apparent power in circuits with nonsinusoidal voltages and currents (in Polish), *Przegląd Elek-trotechniczny,* z. 7, pp. 193-203, z. 8, pp. 225-234, 1931, z. 22, pp. 673-676, 1932.

[6] Czarnecki, L.S., Minimization of distortion power of nonsinusoidal source applied to linear loads, *Proc. IEE,* Vol. 128, Pt. C, No. 4, pp. 208-210, 1981.

[7] Czarnecki, L.S., Considerations on the reactive power in non-sinusoidal situations, *IEEE Trans. on Instr. and Meas.,* Vol. IM-34, No. 3, pp. 399-404, 1984.

[8] Shepherd, W., Zakikhani, P., Suggested definition of reactive power for nonsinusoidal systems, *Proc. IEE,* vol. 119, No. 9, pp. 1361-1362, 1972.

[9] Czarnecki, L.S., Minimization of reactive power in nonsinusoidal situation, *IEEE Trans. on IM,* Vol. IM-36, No. 1, pp. 18-22, 1987.

[10] Czarnecki, L.S., Reactive and unbalanced currents compensa-tion in three-phase circuits under nonsinusoidal conditions, *IEEE Trans. IM,* IM-38, No. 3, pp. 754-459, 1989.

[11] Kusters, N.L., Moore, W.J.M., On the definition of reactive power under nonsinusoidal conditions, *IEEE Trans. Power Appl. Sys-tems,* Vol. PAS-99, No. 3, pp. 1845-1854, 1980.

[12] Czarnecki, L.S., Orthogonal decomposition of the current in a three-phase nonlinear asymmetrical circuit with nonsinusoidal voltage, *IEEE Trans. IM.,* Vol. IM-37, No. 1, pp. 30-34, 1988.

[13] Czarnecki, L.S., Swietlicki, T., Powers in nonsinusoidal networks, their analysis, interpretation and measurement", *IEEE Trans. Instr. Meas.,* Vol. IM-39, No. 2, pp. 340-344, 1990.

[14] Czarnecki, L.S., Scattered and reactive current, voltage, and power in circuits with nonsinusoidal waveforms and their com-pensation, *IEEE Trans. on Instr. Meas.,* Vol. 40, No. 3, pp. 563-567, 1991.

[15] Depenbrock, M., The FDB-method, a generalized tool for analy-zing power relations", *IEEE Trans. on PD*, Vol. 8, No. 2, pp. 381-387, 1993.

[16] Depenbrock, M., Staudt, V., The FDB-Method as a tool for com-pensating total non-active currents, *Proc. of the 8th Int. Conf. on Harmonics and Power Quality (ICHPQ),* Athens, pp. 320-324, 1998.

[17] Tenti, P., Mattavelli, P., A time-domain approach to power terms definitions under nonsinusoidal conditions, *6th Int. Workshop on Power Definitions under Nonsinusoidal Cond.*, Milano, Italy, 2003.

[18] Balabanian, N., Network Synthesis, Prantice-Hall Englewood Cliffs, New York, 1958.

[19] Czarnecki, L.S., The number of different one-ports with the same reactance function, *Scientific Letters ELEKTRYKA,* No. 35, pp. 99-104, Gliwice, Poland, 1972.

[20] Czarnecki, L.S., Currents' Physical Components (CPC) – based Power Theory. A Review, Part I: Power Properties of Electrical Circuits and Systems, *Przegląd Elektrotechniczny,* R. 95, Nr. 95, pp. 1-11, Nr. 10/2019.