

Do Energy Oscillations Degrade Energy Transfer in Electrical Systems?

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Abstract – There is the view, widespread in the electrical engineering community, that the effectiveness of energy transfer in electrical systems, thus the power factor, is reduced by energy oscillations which, according to this view, are the cause of the reactive power. This view is challenged in the paper. It demonstrates on a variety of circuits that this view is not right. Power factor can decline without energy oscillations, or energy oscillations do not cause its decline. Also, the reactive power can occur in the absence of energy oscillations or their presence does not cause the reactive power.

This observation has not only cognitive but practical merit as well. According to Instantaneous Reactive Power (IRP) p - q Theory, to improve the power factor, energy oscillations between the supply source and the load should be compensated. Because this conclusion is erroneous, an active power filter, controlled according to this recommendation in the presence of the supply voltage distortion or/and asymmetry, instead of improving the power factor can worsen it.

The physical meaning of the reactive power is also investigated in the paper, with the conclusion that there is no physical phenomenon in electrical systems that could be described in terms of reactive power.

Index Terms – Current’s Physical Components, reactive power, power balance, Tellegen Theorem,

I. INTRODUCTION

The author of this paper had an opportunity to run several seminars and conferences on powers for engineers and scientists involved in power engineering (PE) in various countries. He observed that a great majority of participants of these seminars and conferences used to associate the reactive power and degradation of the power factor with energy oscillation between the supply source and the load. Just this observation has provided a stimulus for writing this paper because such an association is not right.

The concept of the reactive and other powers, particularly in systems with nonsinusoidal voltages and currents, was for decades discussed in the PE community, even in a frame of a few well-established forums, such as:

International Workshop on Power Definitions under Nonsinusoidal Conditions, run bi-annually by Milano University, Italy, in the years 1991-2008.

International School of Nonsinusoidal Currents and Compensation (ISNCC), run by Zielona Góra University, Poland, in the years 1989-2015.

IEEE Power & Energy Society Working Group, which developed a power standard, known as IEEE Standard 1459.

The concept of reactive power and its physical nature were one of the central subjects of discussions at these meetings. Opinions and conclusions, often mutually conflicting, drawn on the nature of the reactive power or energy oscillations, were published in conference proceedings and various journals. These opinions and conclusions, scattered in various papers, usually not as the major issues of those papers, were compiled together in the conference paper [1]. Due to its limited space, some important observations were not reported there, however. They will be covered in this paper.

One of them is the concept of the reactive power as introduced by Budeanu [3] and regarded as a measure of energy oscillations between the supply source and the load. The next ones are some conclusions on the German Standard DIN 40100, [7] which founded power properties of electrical systems on the bi-directional flow of energy. Finally, observations and conclusions as to the physical interpretation of the Instantaneous Reactive Power p - q introduced to PE by Akagi et. al., [11], will be presented in this paper.

The association of the reactive power with energy oscillations in systems with a nonsinusoidal voltage can be attributed mainly to Budeanu who suggested that the following quantity [3]

$$Q = Q_B = \sum_{n=1}^{\infty} U_n I_n \sin \varphi_n . \quad (1)$$

should be regarded as reactive power. This suggestion was supported by the IEEE Standard Dictionary of Electrical and Electronics Terms [16] and the IEEE Standard 1459 [19]. The terms

$$U_n I_n \sin \varphi_n = Q_n \quad (2)$$

in (1) are amplitudes of the sinusoidal oscillating components of the load instantaneous power. Therefore, the reactive power Q_B used to be associated with energy oscillations [14]. Even now, opinions that harmonics contribute to energy oscillations and consequently, to the power factor $\lambda = P/S$ decline, are in the PE community very common [27]. These oscillations are often regarded as the cause for which the reactive power occurs in electrical systems. Such opinions on the effect of energy

oscillations on the reactive power and the power factor value are incorrect, however. As drafted in [1], there is no relation between the value of the apparent power S of the energy provider and energy oscillations.

One could say that the opinion on the relation between the power factor and energy oscillations has a cognitive rather than practical merit. Unfortunately, this could be not true. This relation is fundamental for the compensation based on the Instantaneous Reactive Power (IRP) p-q Theory [11]. According to IRP p-q, Theory, a compensator should eliminate the oscillating component of the instantaneous active power p . Because this is an erroneous conclusion, however, and a switching compensator controlled by an algorithm based on the IRP p-q Theory, instead of improving, can degrade the power factor [24, 25].

II. REACTIVE POWER AND ENERGY OSCILLATIONS

Power engineers when asked for the cause of the reactive power, usually blame energy oscillation between the load and supply source, or energy storage in reactive elements in the loads. The results of energy flow analysis in the following circuit show that these are not the right opinions, however.

Let us consider a three-phase balanced load, composed of only three ideal inductors, as shown in Fig. 1, supplied with symmetrical sinusoidal voltage.

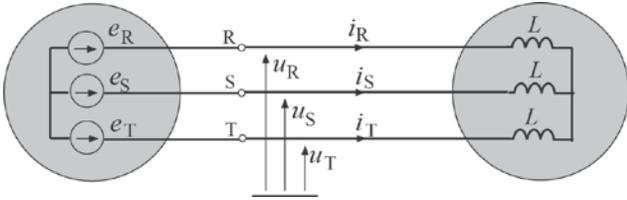


Fig. 1. A three-phase system with a load composed of ideal inductors.

Assuming at the supply voltage at terminal R is

$$u_R(t) = \sqrt{2} U \cos \omega t \quad (3)$$

so that the load current at this terminal

$$i_R(t) = \sqrt{2} I \cos(\omega t - \varphi), \quad \varphi = 90^\circ \quad (4)$$

and due to the system symmetry, the reactive power is equal to

$$Q = 3UI \sin \varphi = 3UI. \quad (5)$$

At the same time, the instantaneous power of the load, $p(t)$, i.e., the rate of energy $W(t)$ flow from the supply source to the load is equal to

$$p(t) = \frac{dW(t)}{dt} = u_R(t)i_R(t) + u_S(t)i_S(t) + u_T(t)i_T(t) \equiv 0. \quad (6)$$

Thus, despite a non-zero reactive power Q , there is no energy flow in this system.

Despite the result expressed by (6), the opinion that the reactive power Q is caused by energy oscillations is defended sometimes by the argument that such oscillations do exist in individual lines but are canceled mutually and consequently, they are not visible in (6). Indeed, the individual product of the voltage and current, say at terminal R, is

$$u_R(t)i_R(t) = 2UI \cos \omega t \cdot \cos(\omega t - 90^\circ) = Q \sin 2\omega t \quad (7)$$

thus, it has an oscillating form.

This argument cannot be defended, however. The product of the voltage and current at one terminal of a three-phase load does not stand for the instantaneous power. There are two reasons for that. First, voltages in three-phase systems are relative quantities, meaning any node in the system can be selected as a reference node, without any effect on the current and energy flow. For example, node R can be selected as a reference node, as shown in Fig. 2, which means that

$$u_R(t) \equiv 0. \quad (8)$$

This is equivalent to grounding that node, on the condition, of course, that there is no other grounded node.

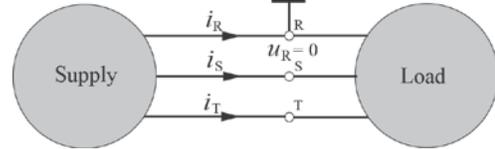


Fig. 2. A three-phase system with grounded terminal R.

The product $u_R(t)i_R(t)$ in (7) in such a case will be zero, but the instantaneous power of the load, defined by formula (6), will remain unchanged because two remaining products will also change with the change of the reference voltage.

This reveals the second reason for which the product of the form (7) does not stand for the instantaneous power. The instantaneous power $p(t)$ is the rate of energy flow, $dW(t)/dt$, to an electrically isolated space, S. Energy can leave such a space in the form of heat, mechanical, chemical, or light energy, but not as electric energy, which is delivered only by the voltage and current at terminals of the supply lines that enter such a space. Such a space for a three-phase, three-wire system is shown in Fig. 3.

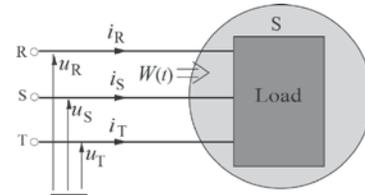


Fig. 3. A three-phase load confined with space S.

Apart from very simple balanced loads, such a space cannot be found for a single supply line in a three-phase system. This is because of possible mutual inductive or capacitive coupling as shown in Fig. 4, or the load imbalance.

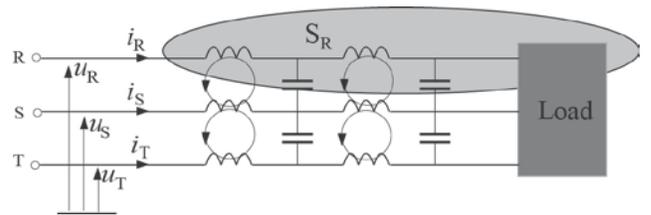


Fig. 4. Illustration for an attempt at separating an electrically isolated space S_R .

If such electrically isolated spaces for individual supply lines cannot be found, then the products $u_R(t)i_R(t)$, $u_S(t)i_S(t)$, and $u_T(t)i_T(t)$ are not the rates of energy flow, thus these products are not instantaneous powers. Only the sum of them is the instantaneous power $p(t)$. This means that there are no energy oscillations between the supply source and the reactive load in

the three-phase system shown in Fig. 1, despite a non-zero reactive power Q .

In systems with a nonsinusoidal supply voltage and the reactive power defined according to formula (1), the opposite situation can occur [12]. Energy can oscillate between the source and the load, despite zero reactive power Q_B .

To demonstrate this, let us consider a single-phase load shown in Fig. 5,

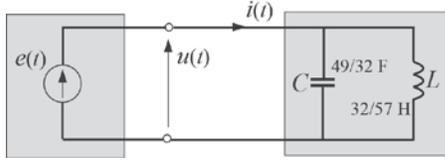


Fig. 5. A single-phase load.

with a nonsinusoidal supply voltage

$$u(t) = \sqrt{2} (100 \sin \omega_1 t + 25 \sin 3\omega_1 t) \text{ V}. \quad (9)$$

Parameters of the load were selected in [12] in such a way that at the fundamental frequency, normalized to $\omega_1 = 1$ rad/s, the load admittance for the fundamental is $Y_1 = -j1/4$ S, while for the third-order harmonic it is $Y_3 = j4$ S. At such a supply voltage and the load parameters, the load current is equal to

$$i(t) = \sqrt{2} [25 \sin(\omega_1 t - 90^\circ) + 100 \sin(3\omega_1 t + 90^\circ)] \text{ A}. \quad (10)$$

The load instantaneous power

$$p(t) = \frac{dW(t)}{dt} = u(t) i(t) \quad (11)$$

has the waveform shown in Fig. 6, thus energy oscillates between the source and the load despite zero reactive power Q_B .

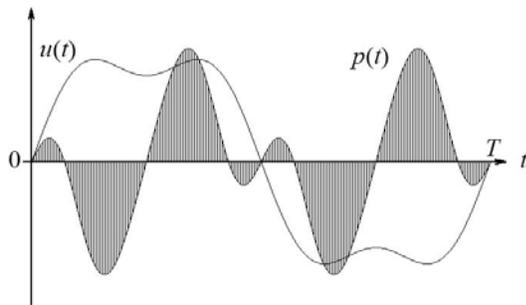


Fig. 6. Variation of the instantaneous power $p(t)$ in the circuit with zero reactive power Q_B .

The reactive power calculated according to Budeanu's definition is equal to zero, however. Indeed

$$Q_B = Q_1 + Q_3 = 2500 - 2500 = 0. \quad (12)$$

This observation, presented originally in [12], shows that the reactive power Q_B is not associated with energy oscillations between the supply source and the load.

The effects of energy oscillations between the supply source and the load upon the power properties were even supported by the German Standard DIN 40100 [7]. It explains these properties in terms of two powers which are the mean values of the positive and the negative pulses of the instantaneous power.

As to the Standard DIN 40100, it specifies the power properties of electrical loads in terms of two powers P^p and P^n calculated for the positive and the negative pulse of energy

flow, as shown in Fig. 7. It emphasizes the oscillating feature of energy flow, but its relation to the reactive power Q and the power factor is very vague.

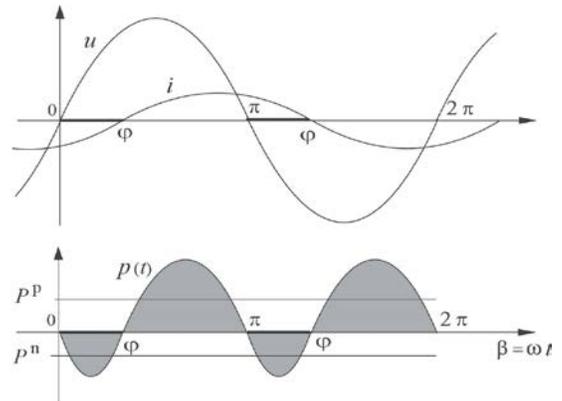


Fig. 7. Time variance of the voltage and current of a linear load and its instantaneous power.

This is because the powers P^p and P^n are intricate functions of the load current phase-shift φ , the reactive power Q , and the active power P . Assuming that the supply voltage is

$$u(t) = \sqrt{2} U \sin \omega_1 t = \sqrt{2} U \sin \beta \quad (13)$$

the powers P^p and P^n are equal, respectively, to

$$P^p = 2 \frac{UI}{\pi} \int_{\varphi}^{\pi} \sin \beta \sin(\beta - \varphi) d\beta = \frac{\pi - \varphi}{\pi} P + \frac{1}{\pi} Q. \quad (14)$$

$$P^n = 2 \frac{UI}{\pi} \int_0^{\varphi} \sin \beta \sin(\beta - \varphi) d\beta = \frac{\varphi}{\pi} P - \frac{1}{\pi} Q. \quad (15)$$

These formulas show that the relationship of the reactive power Q to the bidirectional flow of energy, to powers P^p and P^n , is not distinctive.

The presence of the reactive power Q in the absence of energy oscillations is not only a feature of three-phase systems, as it was demonstrated above. The same is possible also in single-phase circuits with time-variant parameters. A circuit with a TRIAC, shown in Fig. 8, is a very simple example of a situation where the reactive power Q occurs at a unidirectional flow of energy.

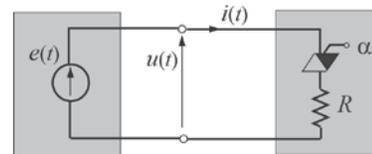


Fig. 8. A purely resistive circuit with a TRIAC.

Assuming that the supply voltage is sinusoidal, the load current changes as shown in Fig. 9.

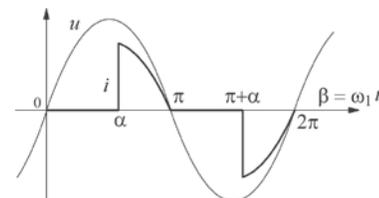


Fig. 9. The voltage and current waveforms in the circuit with TRIAC.

At fixed resistance R , its waveform depends only on the firing angle α of the TRIAC. Because the load is purely resistive, the current has to have the same sign as the supply voltage, so that the instantaneous power $p(t) = u(t) i(t)$ cannot be negative, meaning energy cannot flow back to the supply source. However, it occurs in a lab experiment, with

$$e(t) = u(t) = 220\sqrt{2} \sin \omega_1 t \quad \text{V} \quad (16)$$

the resistance $R = 1 \Omega$, and the TRIAC's firing angle $\alpha = 135^\circ$. A varmeter, connected as shown in Fig. 10, measures in this purely resistive circuit, the reactive power $Q = 7.7 \text{ kVAr}$.

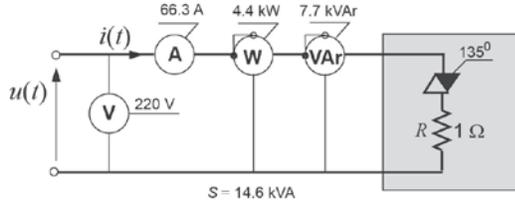


Fig. 10. Meters reading in the circuit with TRIAC.

The explanation for this result is simple. Since the internal voltage $e(t)$ of the source was assumed to be sinusoidal, energy is delivered to the load exclusively by the current fundamental harmonic $i_1(t)$. The Fourier decomposition of the load current, confined to the fundamental harmonic, results in

$$i_1(t) = \sqrt{2} I_1 \sin(\omega_1 t - \phi_1) = 40.3\sqrt{2} \sin(\omega_1 t - 60.3^\circ) \text{ A} \quad (17)$$

Its waveform, along with the voltage and current waveforms, as shown in Fig. 11.

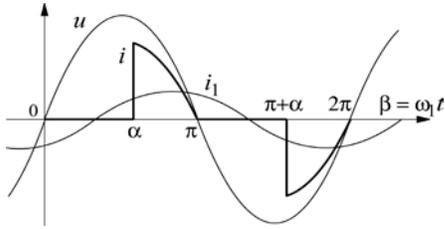


Fig. 11. The TRIAC current and its fundamental harmonic.

Since the load current fundamental harmonic $i_1(t)$ is shifted with regard to the supply voltage $u(t)$, the source is loaded with the reactive power Q equal to

$$Q = Q_1 = UI_1 \sin \phi_1 = 220 \times 40.3 \sin(60.3^\circ) = 7.7 \text{ kVAr} \quad (18)$$

This example shows that even purely resistive loads, i.e., loads without any capability of energy storage, can have a reactive power. Even more, this reactive power of a purely resistive load can be compensated by a capacitor connected at the load terminals as shown in Fig. 12.

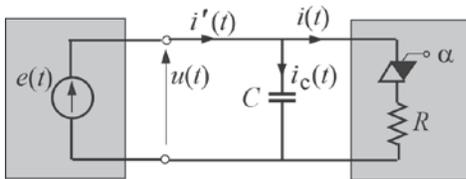


Fig. 12. Capacitive compensation of the load with TRIAC.

Assuming that the capacitor susceptance is

$$\omega_1 C = \frac{Q}{U^2} \quad (19)$$

then such a capacitor compensates the reactive component of the supply current fundamental harmonic, specified by formula (17), to zero, thus it reduces this current to

$$i'_1(t) = \sqrt{2} I_1 \cos \phi_1 \sin \omega_1 t = 20.0\sqrt{2} \sin \omega_1 t \text{ A} \quad (20)$$

and it improves the power factor at the load terminals. Such a capacitor changes the waveform of the supply current as is shown in Fig. 13.

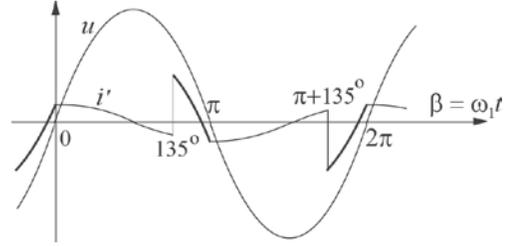


Fig. 13. The supply current waveform after capacitive compensation.

Observe, that after compensation, there are intervals of time where the supply current has the opposite sign to the sign of the supply voltage. The instantaneous power $p(t) = u(t) i'(t)$ in such intervals is negative, thus, energy flows back to the supply source. Compensation of the reactive power to zero and the power factor improvement caused energy oscillations. It is just opposite to what is commonly expected at the power factor improvement.

III. ENERGY FLOW IN TERMS OF CPC

Energy flow in electrical systems can be explained using the concept of the Currents' Physical Components (CPC) – based power theory [13, 30]. This explanation for three-phase systems with sinusoidal and symmetrical supply voltage and linear time-invariant (LTI) loads is presented below.

The supply current $\mathbf{i}(t)$

$$\mathbf{i}(t) = [i_R(t), i_S(t), i_T(t)]^T \quad (21)$$

of any LTI three-phase load supplied from a source of symmetrical sinusoidal voltage $\mathbf{u}(t)$

$$\mathbf{u}(t) = \begin{bmatrix} u_R(t) \\ u_S(t) \\ u_T(t) \end{bmatrix} = \sqrt{2} U_R \begin{bmatrix} \cos \omega t \\ \cos(\omega t - 120^\circ) \\ \cos(\omega t + 120^\circ) \end{bmatrix} \quad (22)$$

can be decomposed into the active, reactive, and unbalanced currents. The load can be specified in terms of line-to-line admittances \mathbf{Y}_{ST} , \mathbf{Y}_{TR} , and \mathbf{Y}_{RS} which can be calculated having measured the complex rms (crms) values of the load voltages and currents at the load terminals.

The active current is

$$\mathbf{i}_a = \sqrt{2} G_e U_R \begin{bmatrix} \cos \omega t \\ \cos(\omega t - 120^\circ) \\ \cos(\omega t + 120^\circ) \end{bmatrix} \quad (23)$$

the reactive current

$$\mathbf{i}_r = -\sqrt{2} B_e U_R \begin{bmatrix} \sin \omega t \\ \sin(\omega t - 120^\circ) \\ \sin(\omega t + 120^\circ) \end{bmatrix} \quad (24)$$

and the unbalanced current

$$\mathbf{i}_u = \sqrt{2}Y_u U_R \begin{bmatrix} \cos(\omega t + \Psi) \\ \cos(\omega t + \Psi + 120^\circ) \\ \cos(\omega t + \Psi - 120^\circ) \end{bmatrix} \quad (25)$$

such that

$$\mathbf{i} = \mathbf{i}_a + \mathbf{i}_r + \mathbf{i}_u. \quad (26)$$

In these formulae

$$G_e = \text{Re}\{Y_e\}, \quad B_e = \text{Im}\{Y_e\} \quad (27)$$

where Y_e is the equivalent admittance of the load, equal to

$$Y_e = Y_{ST} + Y_{TR} + Y_{RS} \quad (28)$$

and

$$Y_u = Y_u e^{j\Psi} = -\{Y_{ST} + \alpha Y_{TR} + \alpha^* Y_{RS}\}, \quad \alpha = 1e^{j120^\circ} \quad (29)$$

is the load unbalanced admittance. Having the current decomposition (26), the instantaneous power $p(t)$ can be decomposed into three instantaneous powers associated with the presence of the active, reactive and unbalanced currents, namely

$$\begin{aligned} p(t) &= \frac{dW(t)}{dt} = \mathbf{u}(t)^T \mathbf{i}(t) = \mathbf{u}^T (\mathbf{i}_a + \mathbf{i}_r + \mathbf{i}_u) = \\ &= p_a(t) + p_r(t) + p_u(t). \end{aligned} \quad (30)$$

These powers, with (23) – (25), are equal to

$$p_a(t) = \mathbf{u}^T \mathbf{i}_a \equiv P \quad (31)$$

$$p_r(t) = \mathbf{u}^T \mathbf{i}_r \equiv 0 \quad (32)$$

$$p_u(t) = \mathbf{u}^T \mathbf{i}_u = 3Y_u U_R^2 \cos(2\omega t + \Psi). \quad (33)$$

The presence of the reactive component $\mathbf{i}_r(t)$ in the load current does not cause energy oscillations. There is an oscillating component in the instantaneous power but caused by the unbalanced current, however, meaning by the load imbalance.

The presence of energy oscillations was concluded by Steinmetz [2] and gave him a starting point for the development of the first compensator for the load balancing, known as the Steinmetz Circuit. Its structure and parameters were selected in such a way that energy oscillations were compensated. It does not mean, however, that these oscillations contribute to the power factor decline. It declines because of the presence of the unbalanced current which increases the supply current three-phase rms value

$$\|\mathbf{i}\| = \sqrt{\|\mathbf{i}_a\|^2 + \|\mathbf{i}_r\|^2 + \|\mathbf{i}_u\|^2}. \quad (34)$$

Energy oscillations only indicate the presence of the unbalanced current.

IV. ROOTS OF POWER PHENOMENA MISINTERPRETATIONS

The association of the reactive power with energy oscillations and energy storage in reactive components of electrical loads is so common in the PE community that the question “*how it has occurred*” seems to be a very natural question. Anyway, comprehension of traits of energy flow in power systems has at least a cognitive merit, while the reactive power is one of the most fundamental power quantities.

As to the author’s opinion, drawn from his teaching experience and textbooks, this erroneous association has occurred because explanations of power phenomena in university courses are usually confined to power-related phenomena in single-phase, linear, time-invariant (LTI) circuits with a sinusoidal supply voltage.

In single-phase circuits with linear, time-invariant load supplied with a sinusoidal voltage the association of the reactive power and energy oscillations apparently exists. To show this, let us consider the instantaneous power $p(t)$ of a single-phase load, shown in Fig. 14.

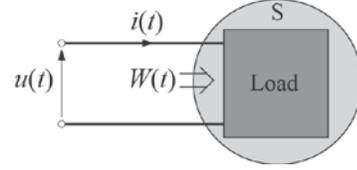


Fig. 14. A single-phase load.

The instantaneous power $p(t)$ is the rate of electric energy $W(t)$ flow to a space S that confines the load in such a way that this energy can flow to that space only through the load terminals with the voltage $u(t)$ and the current $i(t)$, namely

$$p(t) = \frac{d}{dt} W(t) = u(t) i(t). \quad (35)$$

When at the load terminals

$$u(t) = \sqrt{2}U \cos \omega t, \quad i(t) = \sqrt{2}I \cos(\omega t - \varphi)$$

then the instantaneous power is

$$p(t) = u(t) i(t) = 2UI \cos \omega t \cos(\omega t - \varphi) = p_u(t) + p_b(t) \quad (36)$$

where

$$p_u(t) = P(1 + \cos 2\omega t) \quad (37)$$

$$p_b(t) = Q \sin 2\omega t. \quad (38)$$

Decomposition (36) of the instantaneous power into a unidirectional component $p_u(t)$ and a bidirectional component $p_b(t)$, illustrated in Fig. 15, is commonly used for interpretation of the reactive power Q as the amplitude of the oscillating component of the instantaneous power.

Energy can oscillate between the supply source and the LTI load only if the load has a capability of energy storage in the magnetic fields of inductors and/or electric fields of capacitors, however.

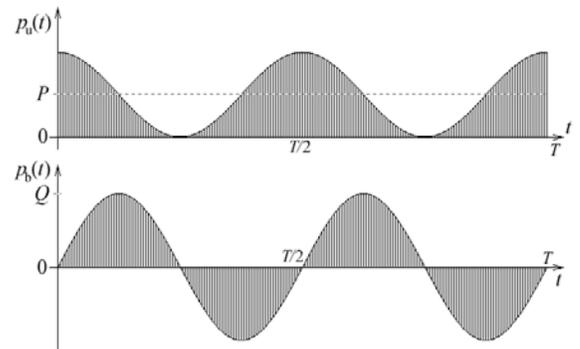


Fig. 15. Components of the instantaneous power.

In common university teaching, these explanations are next extrapolated to three-phase systems, regarded usually as a sort

of aggregates of single-phase loads. It is suggested that interpretations valid in single-phase systems with LTI loads are valid also in three-phase systems with such loads. Unfortunately, as this was shown above, such interpretations are not valid. Moreover, they are also not valid when they are extrapolated to systems with nonsinusoidal voltages and currents.

One could say, however: let us accept the above-presented arguments that there is no relationship between energy oscillations and the reactive power in three-phase systems, but in single-phase linear systems with a sinusoidal supply voltage, the interpretation that this power is the amplitude of such oscillations is true. It would mean that the interpretation of the reactive power Q is dependent on the circuit category or its properties, however. It does not seem to be acceptable. The interpretation should not change with the change of the circuit class.

V. DOES ENERGY OSCILLATIONS DEGRADE THE POWER FACTOR?

A possible relation between energy oscillations and the reactive power was discussed in the previous Sections because the reactive power Q degrades the effectiveness of energy transfer by increasing the supply source apparent power S . The conclusion was negative. Could these oscillations have a direct impact, however, on this transfer effectiveness? The Instantaneous Reactive Power p-q Theory [11] gave a positive answer to this question.

Any three-phase load can be described, according to the IRP p-q Theory, in terms of only two powers, the instantaneous active power p and the instantaneous reactive power q . The instantaneous active power p is identical with the well-known instantaneous power $p(t)$, meaning the rate of energy flow, only it was equipped in the frame of the IRP p-q Theory with an additional adjective “active”. This additional adjective does not have much sense, because, as it was proven in [21], even a purely reactive load can have this instantaneous active power, which can confuse terminology.

The instantaneous active power p is composed of two components, one is constant \bar{p} , the second is oscillating \tilde{p} , i.e.,

$$p = \bar{p} + \tilde{p}. \quad (39)$$

According to the IRP p-q Theory [11], the oscillating component \tilde{p} of the instantaneous active power p degrades the power factor of the load and should be, along with the instantaneous reactive power q , compensated. It means that a switching compensator, commonly known as “an active power filter”, connected as shown in Fig. 16, should be controlled in

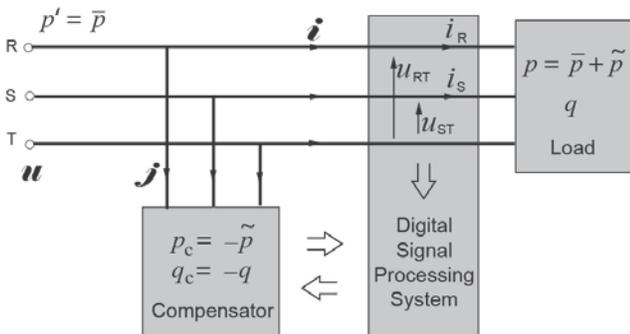


Fig. 16. Compensation approach according to IRP p-q Theory.

such a way that it loads the supply source with the instantaneous active power $p_c = -\tilde{p}$, and the instantaneous reactive power $q_c = -q$.

In effect of such compensation the instantaneous power at the supply source terminals should have a constant value, i.e., it cannot contain any oscillating component. This is an erroneous conclusion, however. It is based on the opinion of the authors of the IRP p-q Theory, that the oscillating component of the instantaneous power degrades the power factor, meaning degrades the effectiveness of energy transfer.

To demonstrate that this opinion is wrong, let us consider an ideal purely resistive balanced load, shown in Fig. 17, supplied from a source of sinusoidal but asymmetrical voltage composed of the positive \mathbf{u}^p and the negative \mathbf{u}^n symmetrical sequence components. The apparent power S of such a load is equal to the active power P , its power factor $\lambda = P/S$ is equal to one, independently on the supply voltage asymmetry, however.

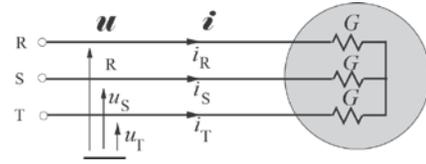


Fig. 17. An ideal purely resistive balanced load.

Let us assume that the voltages of the positive and negative sequences at terminal R are equal, respectively, to

$$u_R^p = \sqrt{2} U^p \cos \omega_1 t, \quad u_R^n = \sqrt{2} U^n \cos \omega_1 t. \quad (40)$$

Thus, the supply voltage can be presented in the form

$$\mathbf{u} = \sqrt{2} U^p \begin{bmatrix} \cos \omega_1 t \\ \cos(\omega_1 t - 2\pi/3) \\ \cos(\omega_1 t + 2\pi/3) \end{bmatrix} + \sqrt{2} U^n \begin{bmatrix} \cos \omega_1 t \\ \cos(\omega_1 t + 2\pi/3) \\ \cos(\omega_1 t - 2\pi/3) \end{bmatrix} = \mathbf{u}^p + \mathbf{u}^n. \quad (41)$$

At such a voltage, the load current is also asymmetrical, namely

$$\mathbf{i} = G \mathbf{u} = G(\mathbf{u}^p + \mathbf{u}^n) = \mathbf{i}^p + \mathbf{i}^n. \quad (42)$$

The instantaneous power $p(t)$ of such a load is equal to

$$p(t) = \frac{d}{dt} W(t) = \mathbf{u}^T(t) \cdot \mathbf{i}(t) = [\mathbf{u}^p + \mathbf{u}^n]^T \cdot [\mathbf{i}^p + \mathbf{i}^n] \quad (43)$$

and it can be presented in the form

$$\begin{aligned} p(t) &= [\mathbf{u}^p + \mathbf{u}^n]^T \cdot [\mathbf{i}^p + \mathbf{i}^n] = \mathbf{u}^{pT} \mathbf{i}^p + \mathbf{u}^{nT} \mathbf{i}^n + \mathbf{u}^{pT} \mathbf{i}^n + \mathbf{u}^{nT} \mathbf{i}^p = \\ &= P^p + P^n + 6GU^p U^n \cos 2\omega_1 t = \bar{p} + \tilde{p} \end{aligned} \quad (44)$$

where P^p and P^n denote the active powers of the positive and the negative sequence symmetrical components of the voltages and currents. This formula shows that, despite the unity power factor λ , the instantaneous power $p(t)$ of such a load at asymmetrical supply voltage has the oscillating component

$$\tilde{p} = 6GU^p U^n \cos 2\omega_1 t. \quad (45)$$

Let us suppose that there is a switching compensator connected at terminals of such a load, as shown in Fig. 18, and it is controlled according to the IRP p-q Theory.

If the oscillating component \tilde{p} is detected, the compensator should load the system with the power \tilde{p} of the opposite sign. The compensator has to draw a compensating current \mathbf{j} for that.

As it was demonstrated in [25], the compensating current in line R, when this condition is satisfied, has the waveform

$$j_R = \frac{-2\sqrt{2}G(U^p + U^n)U^p U^n \cos \omega_1 t \cos 2\omega_1 t}{U^{p2} + U^{n2} + 2U^p U^n \cos 2\omega_1 t}. \quad (46)$$

Observe, that this compensating current increases the supply current rms value. It is distorted, moreover.

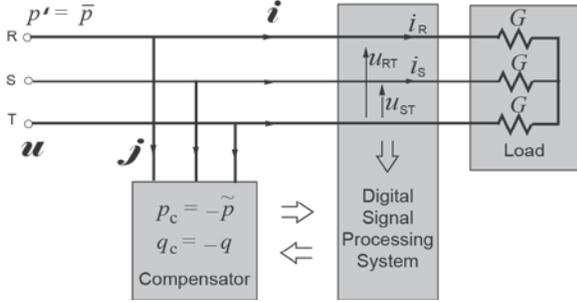


Fig. 18. Compensation approach according to IRP p-q Theory.

In effect of such compensation the instantaneous power will have a constant value but at the cost of an increase of the apparent power and the supply current distortion. This is a result of the erroneous conclusion, presented in [11], that the oscillating component of the instantaneous active power p degrades the power factor and should be compensated.

A similar situation occurs when the supply voltage of an ideal balanced resistive load, as shown in Fig. 17, is symmetrical but distorted. This distortion does not degrade the power factor of such a load because the active power P is equal to the apparent power S .

Let us assume that the supply voltage is distorted by only one harmonic of the 5th order. It can be presented in the form

$$\mathbf{u}(t) = \mathbf{u} = \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix} = \begin{bmatrix} u_{R1} \\ u_{S1} \\ u_{T1} \end{bmatrix} + \begin{bmatrix} u_{R5} \\ u_{S5} \\ u_{T5} \end{bmatrix} = \mathbf{u}_1 + \mathbf{u}_5. \quad (47)$$

Let for the sake of simplicity assume that

$$u_{R1} = \sqrt{2}U_1 \cos \omega_1 t, \quad u_{R5} = \sqrt{2}U_5 \cos 5\omega_1 t. \quad (48)$$

The load current at such a supply contains the 5th order harmonic and can be presented in the form

$$\mathbf{i}(t) = \mathbf{i} = \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} = \begin{bmatrix} i_{R1} \\ i_{S1} \\ i_{T1} \end{bmatrix} + \begin{bmatrix} i_{R5} \\ i_{S5} \\ i_{T5} \end{bmatrix} = \mathbf{i}_1 + \mathbf{i}_5 \quad (49)$$

with

$$i_{R1} = \sqrt{2}GU_1 \cos \omega_1 t, \quad i_{R5} = \sqrt{2}GU_5 \cos 5\omega_1 t. \quad (50)$$

At such a voltage, the instantaneous power $p(t)$ is equal to

$$p(t) = \frac{d}{dt}W(t) = \mathbf{u}^T(t) \cdot \mathbf{i}(t) = [\mathbf{u}_1 + \mathbf{u}_5]^T \cdot [\mathbf{i}_1 + \mathbf{i}_5]. \quad (51)$$

It can be rearranged to the form

$$p(t) = [\mathbf{u}_1 + \mathbf{u}_5]^T \cdot [\mathbf{i}_1 + \mathbf{i}_5] = \mathbf{u}_1^T \mathbf{i}_1 + \mathbf{u}_5^T \mathbf{i}_5 + \mathbf{u}_5^T \mathbf{i}_1 + \mathbf{u}_1^T \mathbf{i}_5 = P_1 + P_5 + 6GU_1U_5 \cos 6\omega_1 t = \bar{p} + \tilde{p} \quad (52)$$

where P_1 and P_5 denote the active power of the 1st and the 5th

order harmonics. This formula shows that, despite the unity power factor, the instantaneous power $p(t)$ of such a load has the oscillating component

$$\tilde{p} = 6GU_1U_5 \cos 6\omega_1 t. \quad (53)$$

Compensation of this component according to IRP p-q Theory requires [24] that the compensator draws the current

$$\mathbf{j}(t) = \begin{bmatrix} j_R(t) \\ j_S(t) \end{bmatrix} = \frac{-2\sqrt{2}GU_1U_5 \cos(6\omega_1 t)}{U_1^2 + U_5^2 + 2U_1U_5 \cos(6\omega_1 t)} \begin{bmatrix} u_R(t) \\ u_S(t) \end{bmatrix}. \quad (54)$$

It increases the apparent power thus it reduces the power factor and causes an additional distortion of the supply current.

One could have an opinion that the effect of energy oscillations on the reactive power and the power factor has only cognitive merit; an opinion, that this has nothing in common with practical issues in the power systems and compensation. The conclusion of the IRP p-q Theory as to the compensation goal, shows that such an opinion is not right. The conclusion as to the effect of energy oscillations on the power factor is crucial for methods of compensation in electrical systems.

VI. HAS THE REACTIVE POWER ANY PHYSICAL INTERPRETATION?

With the demonstration in the previous Sections that the reactive power cannot be interpreted as the amplitude of energy oscillations, it loses its physical interpretation which is common in the PE community. So, a very natural question occurs: **“is there any physical phenomenon in electrical circuits that can be described using the reactive power?”**

The reactive power in single-phase circuits with sinusoidal voltages and currents is sometimes defined with the formula

$$Q = \frac{1}{T} \int_0^T u(t) i(t - \frac{T}{4}) dt \quad (55)$$

i.e., as a mean value of a quantity that resembles the active power. The similarity of this definition to the definition of the active power P might suggest that the reactive power Q is also a physical quantity. This is a wrong conclusion, however. This formula defines the reactive power Q throughout a current shifted with regard to the voltage, i.e., by the quantity $i(t - T/4)$. Such a quantity does not exist in the circuit at the instant t when the value of the voltage $u(t)$ is specified, however. It is only a mathematical, but not a physical quantity. Thus, the formula (55) does not describe any physical phenomenon.

Formula (55) for the reactive power is valid only when the voltage and current are sinusoidal. When they are nonsinusoidal, then the formula (55) can be generalized [10] to the form

$$Q = \frac{1}{T} \int_0^T u(t) \mathcal{H}\{i(t)\} dt = Q_B \quad (56)$$

where $\mathcal{H}\{\cdot\}$ denotes the Hilbert Transform, defined for quantity $x(t)$ as

$$\mathcal{H}\{x(t)\} = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{x(\tau)}{\tau - t} d\tau \quad (57)$$

and the symbol PV denotes the principal value of the integral. Unfortunately, this formula does not bolster the argument that the reactive power is a physical quantity because the Hilbert Transform is only a mathematical but not a physical entity.

Some confusion as to the physical meaning of the reactive power was enhanced by the IRP p-q Theory. This theory is based on two instantaneous powers, and the instantaneous reactive power q is one of them. The reactive power Q in systems with sinusoidal voltages and currents is equal to the negative mean value of the instantaneous reactive power q [21].

$$Q = -\frac{1}{T} \int_0^T q(t) dt. \quad (58)$$

The instantaneous reactive power q was introduced to PE in [11], using Clark's Transform of three-phase voltages and currents into orthogonal α and β coordinates. As it was shown in [26], for loads supplied from a three-wire line, it can be calculated directly, without that Transform, as

$$q = q(t) = \sqrt{3} [u_R(t)i_S(t) - u_S(t)i_T(t)]. \quad (59)$$

Unfortunately, although the adjective “reactive” associates this power with reactive loads, it was shown in [21, 26], the instantaneous reactive power q can have a non-zero value in circuits with purely resistive loads. Indeed, in the circuit in Fig. 19, assuming that the supply voltage is sinusoidal, symmetrical of the positive sequence, and

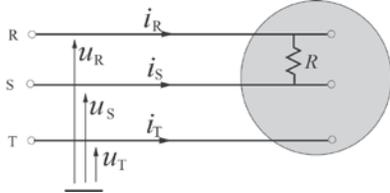
$$u_R = \sqrt{2} U \cos \omega_1 t \quad (60)$$


Fig. 19. A purely resistive three-phase load with non-zero instantaneous reactive power q .

then the formula (59) for the instantaneous reactive power q results in

$$q = -\sqrt{3} UI \sin 2(\omega_1 t + 30^\circ). \quad (61)$$

This result shows that there is a major problem with the physical interpretation of the instantaneous reactive power q . To clarify this interpretation, Akagi, one of the authors of the IRP p-q Theory, et.al., stated in [22], that

“...the imaginary power q is proportional to the quantity of energy that is being exchanged between the phases of the system...” “Figure...” summarizes the above explanations about the real and imaginary powers.” The figure, with the original caption, copied from [22], is shown in Fig. 20.

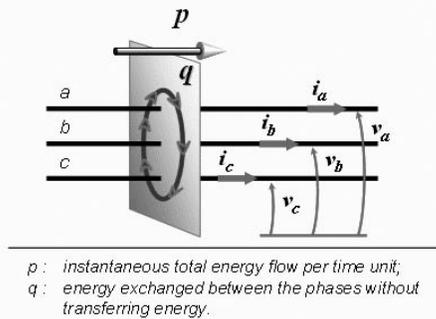


Fig. 20. Physical meaning of the instantaneous active and reactive powers.

The adjective “imaginary” is used in the IRP p-q Theory as a synonym to the adjectives “instantaneous reactive”.

One could observe that Akagi's explanation does not fit the figure which was to illustrate this explanation, because the power q marks energy that rotates around the supply lines of a load. Such a rotation is not possible, however.

A flow of energy in electromagnetic fields was described by Poynting in 1884. Energy flows along with the Poynting Vector \vec{P} , which is a vector product of the electric and magnetic field intensities, \vec{E} and \vec{H} , namely

$$\vec{P} = \vec{E} \times \vec{H}. \quad (62)$$

It is perpendicular to each of them, as shown in Fig. 21.

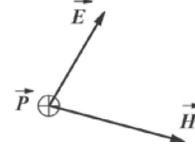


Fig. 21. The orientation of the electric and magnetic field intensities and the Poynting Vector.

The q -power cannot represent energy rotation as it is suggested in Fig. 20, since the Poynting Vector cannot be parallel to the magnetic field intensity \vec{H} , which rotates around supply lines currents, as shown in Fig. 22.

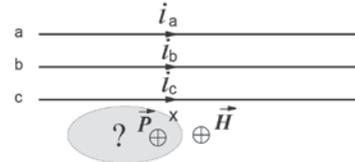


Fig. 22. The orientation of the magnetic field intensity at point x in conductors' plane.

It means that the physical interpretation of the instantaneous reactive power q as suggested by Akagi is not right.

VI. REACTIVE POWER BALANCE

The reactive power satisfies, similarly to the active power, the Balance Principle (BP). This fact is sometimes used in the PE community to support the opinion that the reactive power, similarly to the active power, is a physical quantity.

There is a substantial difference between the active power and the reactive power as to the roots on which the BP for these two powers is founded. The BP for active powers is an immediate conclusion from the Energy Balance Principle (EBP), one of the fundamentals of physics. The BP for the reactive power cannot be concluded from the EBP, however. It is a conclusion from the Tellegen Theorem, instead [5].

The Tellegen Theorem describes the following property of electrical circuits shown in Fig. 23.

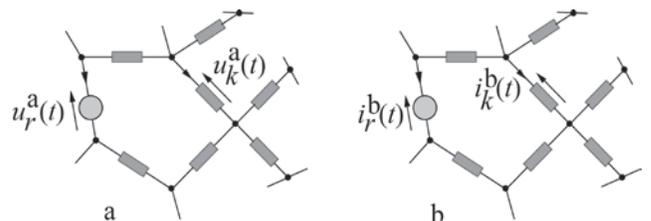


Fig. 23. Two circuits with the same topology.

Let us suppose that there are two entirely different circuits of the same topology, i.e., these circuits have the same number of nodes that are connected in the same way by K branches. Branches can be entirely different.

Let us multiply the k -branch voltage from the first circuit 23(a) by the k -branch current from the second circuit 23(b). According to the Tellegen Theorem, a sum of such products for all K branches for any instant of time is equal to zero, i.e.,

$$\sum_{k=1}^K u_k^a(t) i_k^b(t) \equiv 0. \quad (63)$$

The BP for reactive powers can be concluded from the Tellegen Theorem as follows. Let both circuits in Fig. 23 have identical branches, but source voltages and source currents in circuit 23(b) are shifted with regard to source voltages and source currents in circuit 23(a) by the quarter of the period T . Consequently, all branch currents in the circuit 23(b) are identical to that 23(a), but shifted by $T/4$ with regard to branch currents in the circuit 23(a), i.e.,

$$i_k^b(t) \equiv i_k^a(t - \frac{T}{4}). \quad (64)$$

Hence, according to the Tellegen Theorem

$$\sum_{k=1}^K u_k^a(t) i_k^b(t) = \sum_{k=1}^K u_k^a(t) i_k^a(t - \frac{T}{4}) = \sum_{k=1}^K u_k(t) i_k(t - \frac{T}{4}) = 0 \quad (65)$$

and the mean value of (65) over period T

$$\frac{1}{T} \int_0^T \sum_{k=1}^K u_k(t) i_k(t - \frac{T}{4}) dt = \sum_{k=1}^K \frac{1}{T} \int_0^T u_k(t) i_k(t - \frac{T}{4}) dt = \sum_{k=1}^K Q_k = 0. \quad (66)$$

Thus, the reactive power, defined by (55), satisfies the BP.

The Tellegen Theorem does not describe any physical phenomena in the circuit, however, because the voltages and currents are taken in the Tellegen Theorem from different circuits. The Balance Principle for the reactive power Q does not have any physical, but only mathematical roots. In this regard, the active and reactive powers are substantially different. The opinion suggesting that the satisfaction of the BP for the reactive power indicates its physical nature is not convincing.

Thus, it seems that to the present time, a physical phenomenon that could be described by the reactive power Q was not identified.

VII. REACTIVE POWER IN THE FRAME OF CPC

A debate in the PE community on the definition of the reactive power has a long history, with hundreds of papers on it. Several different quantities were suggested to be regarded as reactive power. Some of them were supported by national and international standards [7, 8, 16, 19]. The most known are those suggested by Budeanu [3], Fryze [4], Shepherd and Zakikhani [6], Kusters and Moore [9], Depenbrock [17], and Tenti [20]. An overview of these definitions with a discussion on their deficiencies can be found in [30]. Opinions on power definitions and compensation can be found also in [15, 18, 23, 27, 28, 29, 30, 31, 32].

According to the Currents' Physical Components – based power theory of electrical systems [13, 30], the reactive power Q is associated exclusively with the phenomenon of the phase-shift of the load current harmonics with regard to the supply voltage harmonics. Due to this phase-shift, a reactive compo-

nent, referred usually as a **reactive current**, occurs in the load current. This current is defined for three-phase systems with nonsinusoidal voltages as a three-phase vector of reactive currents in lines R, S, and T, namely

$$\mathbf{i}_r(t) = \begin{bmatrix} i_{rR}(t) \\ i_{rS}(t) \\ i_{rT}(t) \end{bmatrix} = \sqrt{2} \operatorname{Re} \sum_{n \in N} jB_{en} \begin{bmatrix} U_{Rn} \\ U_{Sn} \\ U_{Tn} \end{bmatrix} e^{jn\omega_1 t}. \quad (67)$$

The symbol N in formula (67) denotes the set of orders n of dominating harmonics, while B_{en} is the equivalent susceptance of the load for the n^{th} order harmonic. This susceptance is the imaginary part of the equivalent admittance

$$B_{en} = \operatorname{Im}\{Y_{RSn} + Y_{STn} + Y_{TRn}\} \quad (68)$$

of an equivalent load in Δ -configuration for the n^{th} -order harmonic, shown in Fig. 24.

The three-phase rms value of the reactive current is

$$\|\mathbf{i}_r\| = \sqrt{\sum_{n \in N} B_{en}^2 \|\mathbf{u}_n\|^2}, \quad \|\mathbf{u}_n\|^2 = U_{Rn}^2 + U_{Sn}^2 + U_{Tn}^2. \quad (69)$$

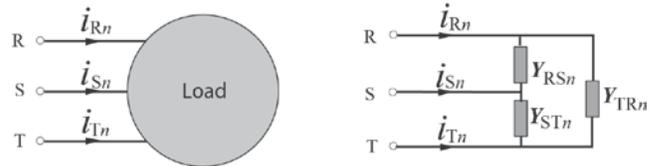


Fig. 24. A load and its equivalent circuit in Δ configuration for the n^{th} -order harmonic.

The reactive power Q is defined as a product of the supply voltage and the reactive current three-phase rms values:

$$Q = \|\mathbf{u}\| \|\mathbf{i}_r\|. \quad (70)$$

This definition has an analogy to the definition of the apparent power S in three-phase systems [13], namely

$$S = \|\mathbf{u}\| \|\mathbf{i}\|. \quad (71)$$

Observe, that the reactive power, similarly to the apparent power S , is a non-negative quantity. It can be “equipped” with a sign only when the voltages and currents are sinusoidal, but not when they are nonsinusoidal, however. The assumption that the reactive power is a non-negative quantity does not create any problems at its compensation, because the reactive current, not the power is the subject of compensation, and reduction of the reactive power is only a by-product of the reactive current rms value reduction.

VIII. CONCLUSIONS

Reactive power is one of the most important of several power quantities while energy oscillation is one of the most important of several physical phenomena in electrical power systems. This paper presents several observations related to this power and this phenomenon and their association. These observations can challenge some opinions in the PE community. The author's observations indicate that there is some level of confusion or even there are misinterpretations in the PE community as to the reactive power, energy oscillations, and their relationship. This applies not only to systems with nonsinusoidal voltages and currents but even to systems where these quantities are

sinusoidal. It is not clear how these confusions or misinterpretations are common, however.

This paper should contribute to a reduction of these confusions and to a correction of misinterpretations on the reactive power and energy oscillations in electrical systems. In such a way the paper would have cognitive value. Some observations on compensation, as presented in this paper, can have a practical value as well.

This paper was focused only on some misconceptions related to alleged energy oscillations and the physical meaning of the reactive power. There could be also other issues in the fundamentals of electrical engineering that might deserve a discussion, however.

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BIOGRAPHY



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