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Is Reactive Power a Physical Quantity?

Czy moc bierna jest wielkością fizyczną?

Abstract: *Reactive power is regarded in the power engineering community as a physical quantity, the article shows, however, that there is no physical phenomenon in electrical circuits that might be described in terms of reactive power. This power satisfies the Power Balance Principle, but this cannot bolster the opinion that it is a physical quantity because this Principle for reactive power cannot be derived from the Energy Conservation Principle but from the Tellegen Theorem which is only a mathematical not physical property of electrical circuits. The article shows that the meaning of reactive power Q is similar to the meaning of apparent power, which is just an arbitrary quantity characterizing supply sources, it is only a product of rms values of the load voltage and reactive current. Reactive power Q is a product of rms values of the load voltage and reactive current, on the condition, however, that this current correctly defined.*

Streszczenie: Moc bierna Q odbiorników elektrycznych jest w środowisku inżynierów uważana zwykle za wielkość fizyczną. Artykuł ten pokazuje jednak, że nie istnieje w obwodach elektrycznych żadne zjawisko fizyczne, które można by było wyjaśnić i opisać z pomocą mocy biernej. Moc ta spełnia zasadę bilansu mocy, lecz ta cecha nie wzmacnia opinii o fizycznym charakterze tej mocy, gdyż zasady tej dla mocy biernej Q nie da się wyprowadzić, tak jak dla mocy czynnej P, z zasady zachowania energii. Można ją wyprowadzić z twierdzenia Tellegena, które jednak nie opisuje fizycznej właściwości obwodów elektrycznych lecz tylko ich pewne właściwości matematyczne. Artykuł ten pokazuje, że natura mocy biernej Q jest podobna do natury mocy pozornej S, która nie jest wielkością fizyczną, lecz pewną umowną wielkością charakteryzującą źródła zasilania, mianowicie jest iloczynem wartości skutecznej napięcia zasilania i wartości skutecznej prądu zasilania. Moc bierna Q jest iloczynem wartości skutecznej napięcia zasilania i wartości skutecznej prądu biernego odbiornika, pod warunkiem jednak, że prąd ten jest poprawnie zdefiniowany.

Keywords: *reactive power, definition, mathematical property*
Słowa kluczowe: *moc bierna, definicje, właściwości matematyczne*

1. INTRODUCTION

Apart from active power P, reactive power Q is the second most important load power in the power systems. Its value is indispensable when systems are designed or modernized, when we want to assess the power factor of loads or when we want to design compensators aimed at improving the said power factor. It may also be measured with analog or digital instruments or else its value is predicted in the systems' modeling processes.

The physical interpretation of this power by the engineers is based upon the teaching of fundamentals of electric circuit theory and power engineering in the electrical engineering faculties of technical universities. The author of this paper is more conversant with US teaching than with teaching in Poland; however, from numerous discussions during seminars in Poland he may conclude, that education is not that dissimilar as far as interpretation of reactive power is concerned. If we examine American university textbook [1] on electrical circuit theory, we find that **"The reactive power represents the oscillatory energy exchange…"**, while in textbook [2] on fundamentals of power engineering we may read that **"The reactive power Q is, by definition, equal to the peak value of the power component travelling in the line back and forth"**. Therefore, both explanations interpret the existence of load reactive power Q by the effect of electrical energy oscillation between the power source and the load. Actually, in the single-phase circuit with sinusoidal voltage and current and reactive-resistive load, this oscillatory transfer of energy is really present. This explanation also suggests that energy loads with non-zero reactive power Q must be able to accumulate energy so that the energy transfer may indeed oscillate, that means that energy might flow not only from the power source to the load, but also in the opposite direction, from the load to the source.

However, the interpretation associating reactive power Q with energy oscillation and ability of the load to store energy is wrong. To arrive at this conclusion, it is enough to analyze the flow of energy in a simple circuit (see Fig.1) composed of a resistor and TRIAC connected in series, with TRIAC triggered at a specific angle **α**, instruments measuring rms values of voltage U/current I and active (P)/reactive power (Q) meters. When supply voltage is sinusoidal with rms valu $U = 220$ V, kącie zapłonu **α** = 135° trigger angle $R = 1\ \Omega$, reactive power meter indicates $Q = 7{,}7$ kVar.

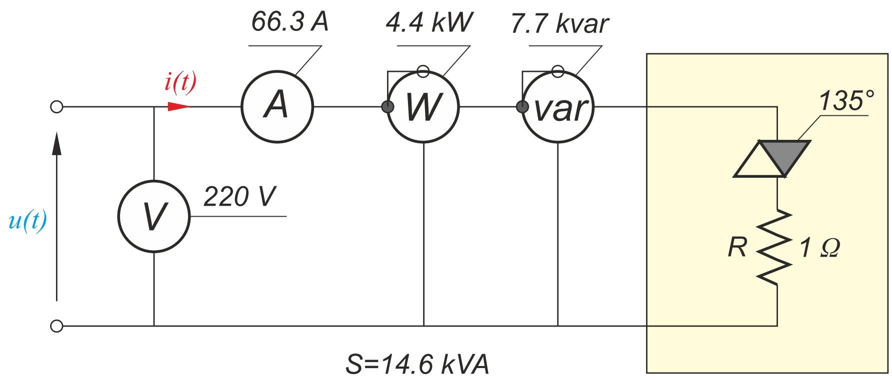

Fig. 1. Resistive circuit with a TRIAC

The load current waveform, at a specific value of the trigger angle **α,** is shown in Fig. 2

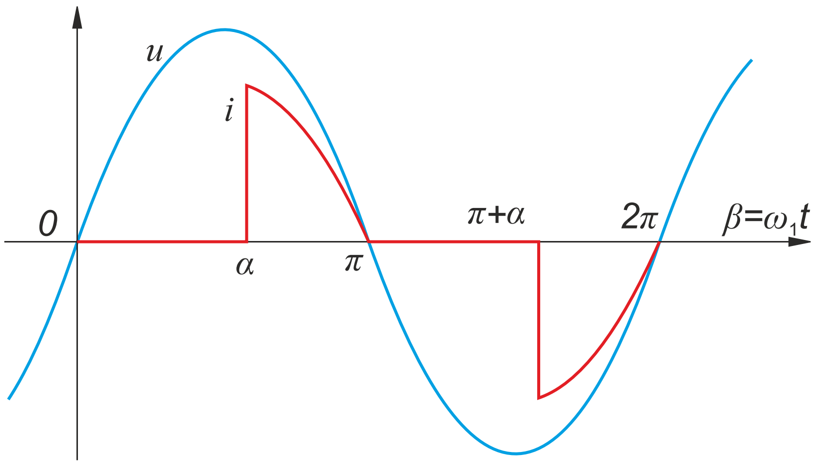

Fig. 2. Current waveform in a circuit containing a TRIAC

Since the current in this circuit cannot be polarized in opposition to the supply voltage, energy can flow only from the source to the load. Moreover, energy oscillations are not possible, since the load does not possess the ability to store energy. In spite of this, the load may exhibit non-zero reactive power Q, which may be measured with a popular varmeter. Incidentally, it may also be pointed out that measurement results for this circuit do not satisfy commonly used power equation:

$$P^2 + Q^2 = S^2\ , \tag{1}$$

since

$$\sqrt{P^2 + Q^2} = \sqrt{4{,}4^2 + 7{,}7^2} = 8{,}9\ \text{kVA} < S = U \times I = 220 \times 66{,}3 = 14{,}6\ \text{kVA}. \tag{2}$$

However, the issue of incorrectness of this equation does not lie within the scope of the current article.

Analysis of the circuit presented above shows that reactive power Q in circuits containing non-linear loads or loads with periodically-varying parameters cannot be interpreted in the way given in textbooks [1, 2]. That interpretation is flawed.

Such interpretation is not true in linear circuits either, if these are three-phase circuits. This is illustrated by a three-phase circuit shown in Fig. 3, where a balanced resistive-inductive load *RL* is supplied from a symmetrical three-phase voltage source.

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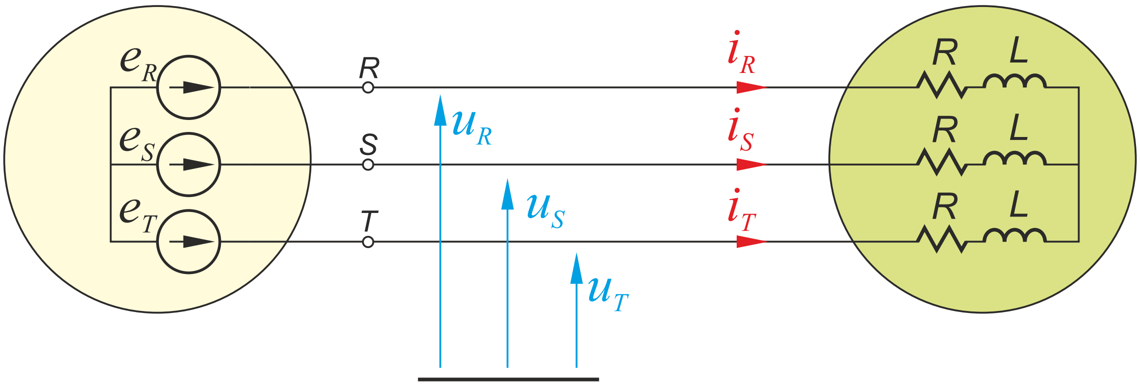

Fig. 3. Three-phase circuit with a balanced RL load

If the supply voltage is symmetric and positive-sequence, voltage at terminal *R* is expressed as

$$u_R(t) = \sqrt{2}U\cos\omega_1 t\,, \tag{3}$$

and the load is balanced, with current in *R* phase expressed as

$$i_R(t) = \sqrt{2}I\cos(\omega_1 t - \varphi)\,, \tag{4}$$

then reactive power of this load is equal to

$$Q = 3UI\sin\varphi\,. \tag{5}$$

The instantaneous power $p(t)$ at load terminals, i.e. the speed of energy $W(t)$ flow from a source to the load is expressed as

$$p(t) = \frac{\mathrm{d}W(t)}{\mathrm{d}t} = u_R(t)i_R(t) + u_S(t)i_S(t) + u_T(t)i_T(t) = 3UI\cos\varphi = P = \mathrm{const}\,. \tag{6}$$

Thus , the speed of energy flow is constant, which means that in spite of non-zero value of reactive power Q, the energy flows without any oscillations. The existence of the power does not result from oscillations of energy between the supply source and the load. But, if we reject the universally adopted interpretation of the reactive power, what is the true physical nature of this power or, to put it more bluntly, is this power a physical quantity?

The question whether reactive power Q is a physical quantity implies an even more fundamental question, i.e. what exactly is "physical quantity"?

2. PHYSICAL QUANTITIES

It seems that the answer to this question is not clear even beyond quantum physics. The author is not aware whether a definition of physical quantity exists. It seems that it does not. In physics, there are many quantities, which are considered to be physical quantities; these have been introduced into physics not through definitions, but through investigation and description of their properties. One and often recalled feature of physical property is its measurability. This does not, however, mean that a measurable quantity must be a physical quantity. We can measure numerous quantities which are not physical quantities. For instance, rms value of current may be measured with an ammeter; it is not a physical quantity. It is only current which is a physical quantity, **and rms value is just one of many, mathematically defined, properties of this current**. A similar case occurs with symmetrical components of currents in three-phase circuits. They may be directly measured with symmetrical component filters, but they do not exist physically. For instance, when one supply line does not conduct current, the symmetrical component filter indicates non-zero value of zero-sequence component. The fact that measurement is possible might suggest a physical existence of this component, while it is a sole result of mathematical transformation of three-phase linear currents, when one of these currents is equal to zero.

The same applies to harmonics. For instance, current in the line with periodically closing and opening switch may be represented as a sum of the harmonics, same as in circuit shown in Fig.1. These harmonics are measurable, but physically they do not exist, since the current harmonic is a continuous quantity, so it cannot be present physically when the line is open. So, the harmonics, even though they are measurable quantities, are just mathematical objects and not physical ones.

Hence , measurability is a necessary condition, if we want to regard a said quantity as a physical one, but it is not a sufficient condition. Measurability of the reactive power Q does not suffice to consider it a physical quantity.

Thus, if there is no definition of the term **"physical quantity",** then in order to judge whether a specifical quantity exists physically or not, we must rely on intuition. It might also be helpful to be aware of how different quantities emerged in physics and in particular, in electric circuit theory. Some comparisons might support our intuition with respect to the assessment whether some quantity is indeed a physical quantity or not.

3. REACTIVE POWER IN CIRCUITS WITH NON-SINUSOIDAL CURRENT AND VOLTAGE WAVEFORMS

The operating conditions when currents/voltages are sinusoidal are just special cases in distribution systems (even though they are common) or they just approximate real conditions, where waveforms are really non-sinusoidal. The answer to the question "what quantity should be considered as reactive power in the circuits with non-sinusoidal current and voltage waveforms" is one of the most controversial issues in the electric circuit theory in the last century.

At first, when loads were linear, this was just an academic issue and not a practical one, since currents/voltages in distribution networks were, to a high level of accuracy, sinusoidal. This started to change with the development of electrochemistry which needed direct currents, i.e. high-power rectifiers, for electrolytical processes, with the replacement of light bulbs by discharge tubes and then LEDs, with the arrival of video apparatus, PCs and computer-like equipment in homes, offices and commercial facilities, and, lastly, with the influx of power electronics devices. Each such device, even if it is supplied with sinusoidal voltage, causes a deformation of the supply current. It turned out that electric circuit theory was no longer able to describe the flow of energy with such energy quantities as powers.

The community of electrical engineers and researchers rose to this challenge by conducting intensive investigations into the nature of power in circuits with non-sinusoidal current/voltage waveforms; the research was initiated by Steinmetz in 1892 [3]. The number of scientific papers devoted to this issue may be assessed at several thousand. Also, **IEEE Working Group on Power Definitions in Systems Under Nonsinusoidal Conditions** was established, it was chaired by Prof. A. Emanuel. A conference called **International Workshop on Power Definitions and Measurement Under Nonsinusoidal Conditions** and directed by Prof. A. Ferrero (Mediolan University of Technology) was convened biennially. Also, moreover, **International School on Nonsinusoidal Currents and Compensation (ISNCC)** was held four times in the University of Zielona Góra, it was chaired by the author of this article.

The selection of reactive power definition was one of the central topics of the research and discussion on the development of electric circuit **Power Theory**, i.e. interpretation and description of phenomena accompanying the flow of electrical energy. Numerous different electrical quantities were propounded as reactive power, this made the titular issue of this article still more difficult. A list of different concepts of reactive power definition in the circuits with non-sinusoidal voltage and current waveforms is provided below.

C.I. Budeanu [4], assumed that supply voltage and load current are approximated with Fourier series

$$u(t) = \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n), \tag{7}$$

$$u(t) = \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \alpha_n + \varphi_n), \tag{8}$$

and he defined (1927) the reactive power with a formula:

$$Q_B \overset{\mathrm{df}}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n \,. \tag{9}$$

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S. Fryze [5] introduced (1931) the conception of the reactive current

$$i_{rF}(t) \overset{\mathrm{df}}{=} i(t) - \frac{P}{\|u\|^2}u(t), \tag{10}$$

where $\|u\|$ is the rms value of supply voltage, that is

$$\|u\|^2 \overset{\mathrm{df}}{=} \sqrt{\frac{1}{T}\int_0^T u^2(t)\mathrm{d}t}. \tag{11}$$

This current was used by Fryze to define the reactive power

$$Q_F \overset{\mathrm{df}}{=} \|u\|\,\|i_{rS}\|^2. \tag{12}$$

W. Shepherd oraz **P. Zakikhani** [6] introduced (1972) another definition of reactive current, that is

$$i_{rS}(t) \overset{\mathrm{df}}{=} \sqrt{2}\sum_{n=1}^{\infty} I_n \sin\varphi_n \sin(n\omega_1 t + \alpha_n) \tag{13}$$

which helped to define the reactive power as

$$Q_S \overset{\mathrm{df}}{=} \|u\|\,\|i_{rS}\|. \tag{14}$$

N.L. Kusters and **W.J.M. Moore** [7] introduced (1980) the conception of capacitative reactive current

$$i_{qC}(t) \overset{\mathrm{df}}{=} \frac{(\dot{u},i)}{\|\dot{u}\|^2}\dot{u}(t), \text{ gdzie } \dot{u} = \frac{\mathrm{d}u}{\mathrm{d}t} \tag{15}$$

and

$$(\dot{u},i) \overset{\mathrm{df}}{=} \frac{1}{T}\int_0^T \dot{u}(t)i(t)\mathrm{d}t \tag{16}$$

Using this current they defined capacitative reactive power as

$$Q_C \overset{\mathrm{df}}{=} \|u\|\,\|i_{qC}\|. \tag{17}$$

L.S. Czarnecki [8] defined (1983) the reactive current within the framework of **Currents' Physical Components (CPC) Theory** as

$$i_r(t) \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n U_n e^{jn\omega_1 t}, \tag{18}$$

where B_n is load susceptance for n^{th} order harmonic of the supply voltage, while U_n is complex rms value (crms) of this harmonic. Using this current, load's reactive power was defined as

$$Q \overset{\mathrm{df}}{=} \|u\|\,\|i_r\|. \tag{19}$$

P. Tenti [9] defined (2003) the reactive current within the framework of **Conservative Power Theory (CPT)** with the formula

$$i_{rT}(t) \overset{\mathrm{df}}{=} \frac{(u,i)}{\|\hat{u}\|^2}\hat{u}(t), \tag{20}$$

where

$$(\hat{u},i) \overset{\mathrm{df}}{=} \frac{1}{T}\int_0^T \hat{u}(t)i(t)\mathrm{d}t, \tag{21}$$

and $\hat{u}(t)$ designs the oscillating component of supply voltage integral. Using this current, load's reactive power was defined as:

$$Q_T \overset{\mathrm{df}}{=} \|u\|\,\|i_{rT}\|. \tag{22}$$

H. Akagi, A. Nabae, Y. Kazanawa [10] worked out (1984) the theory of **Instantaneous Reactive Power,** pq; reactive power Q was replaced with instantaneous reactive power q. It was defined by instantaneous values of current and voltage of three-phase load, transformed with the help pf Clarke Transform to new quantities in the orthogonal coordinate system α, β. In three-phase and three-wire circuit, Clarke Transform of supply voltages u_R, u_S and line currents i_R, i_S, (in general, x_R, x_S, quantities) may be expressed as

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} \overset{\text{df}}{=} \begin{bmatrix} \sqrt{3-2}, & 0 \\ 1-\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} x_R \\ x_S \end{bmatrix}. \tag{23}$$

Instantaneous reactive power q was defined as:

$$q \overset{\text{df}}{=} u_\alpha i_\beta - u_\beta i_\alpha . \tag{24}$$

Since power theories of electric circuits do not differ as to the definition and interpretation of the active power P, the differences between these theories are due mainly to the way of selecting a quantity recognized as passive power, its definition and interpretation. Since different reactive power theories have had their proponents and opponents, several schools of power theory emerged.

The choice of reactive power definition impacts the feasibility of power compensation and therefore, apart from Budeanu and Fryze concepts, the specific selection depended on the research on possibility of improving power factor in circuits with non-sinusoidal voltage/current waveforms.

4. COMMENTS ON THE PROPOSED DEFINITIONS OF REACTIVE POWER

The proposed definitions of reactive power were subjected to critical research, undertaken – among others – by the author of current article. Budeanu's theory was analyzed in publications [11, 12], Fryze's theory was discussed in [12], Shepherd and Zakikhani's work in [13], Kusters and Moore's definition in [14], Akagi and Nabae's theory in [15], and, finally, Tenti's CPT theory in [16].

The research has proven that in electric circuits there is no physical phenomenon which might be explained and described with any of the proposed reactive powers. The reactive currents alone, defined by Shepherd [6] and Czarnecki [8], might be associated with a specific physical phenomenon. Actually, these currents are physically identical, while their mathematical definitions differ, so that

$$i_{rS}(t) \equiv i_r(t). \tag{24}$$

This is a sum of harmonics of the load current shifted by the angle π/2 in relation to the harmonics of the supply voltage. Even though this is a result of mathematical decomposition of load current, we can still visualize its physical existence. Within the framework of **Currents' Physical Components (CPC)** [17, 20], this is one of physical components of load current. It is the only current component, which may be completely reduced [18] with a reactance compensator.

Let us put together the conclusions on different reactive power definitions provided earlier, the interpretations associated with them and the question why these definitions are not acceptable.

Budeanu interpreted the reactive power Q_B proposed in his definition as a specific measure of energy oscillations between load and supply source. However, it has been demonstrated in [11] that such oscillations may exist even if Budeanu's reactive power Q_B, is equal to zero, so it cannot be a measure of energy oscillation.

Fryze introduced reactive power as a secondary quantity in relation to the reactive current $i_{rF(t)}$, defined by him. He did not, however, explain the physical nature of this current. The discussion in publications [8] and [12] has shown that this current is a complex quantity. Even in linear circuits it may contain scattered current $i_{r(t)}$. This means that Fryze's reactive current is associated with two different phenomena in electric circuits. It is only partially reducible with a reactance compensator.

The reactive power defined by Kustres and Moore may be compensated by a capacitative compensator connected in parallel to the load. However, it has been shown in [13] that load's power factor can still be improved with a reactance compensator connected in parallel.

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Tenti's definition of reactive power Q_T was based upon the conception of a new quantity termed "reactive energy". However, it has been demonstrated in [16] that this quantity may be negative, so that it cannot be energy. This is not a physical quantity, and therefore reactive power Q_T derived from it is not a physical quantity either.

When Akagi et al. introduced the definition of instantaneous reactive power q physical interpretation did not accompany the introduction. Moreover, all the attempts conducted since 1982 and aimed at finding some acceptable interpretation of this power were unsuccessful. The interpretation is hindered by the fact, that it has been shown in [15] that instantaneous power q may appear in three-phase circuits with purely resistive load.

5. REACTIVE POWER IN CIRCUITS WITH SINUSOIDAL VOLTAGE AND CURRENT

Physical interpretation of passive power cannot depend on linearity or non-linearity of the load or whether the supply voltage is sinusoidal or otherwise. This means that if reactive power does not possess physical interpretation when voltage/current waveforms are non-sinusoidal, then there is no physical interpretation when these waveforms are sinusoidal. The set of sinusoidal waveforms is just a subset of non-sinusoidal waveforms. Someone might say that the harmonics prohibit the physical interpretation of physical power. Moreover, situations when currents/ voltages in distribution systems are sinusoidal are so common (or current/voltage waveforms may be approximated by sine functions) and so important, that it is worthwhile to propound a question if – under such sinusoidal conditions – reactive power may be interpreted as a physical quantity?

Under sinusoidal conditions active power P and reactive power Q are defined with similarly looking formulas

$$P \stackrel{\text{df}}{=} \frac{1}{T}\int_0^T u(t)i(t)\mathrm{d}t\,, \tag{26}$$

$$Q \stackrel{\text{df}}{=} \frac{1}{T}\int_0^T u(t)i(t-T/4)\mathrm{d}t\,, \tag{27}$$

which might suggest a similarity of both quantities. However, the differences between the two powers are enormous. Active power P characterizes a physical phenomenon, that is energy flow $W(t)$. This is an average value of energy flow speed, since $u(t)i(t) = \mathrm{d}W(t)/\mathrm{d}t$. Quantity $u(t)i(t-T/4)$ present under the integral defining reactive power Q does not characterize a physical phenomenon, since load current is considered at a different time instant from the supply voltage.

The opinion that reactive power is a physical quantity is sometimes supported by the observation that this power satisfies (same as active power) the Power Balance Principle. However, this observation does not bolster the opinion, since this principle results from different causes for these two powers. In the case of active power P, the Power Balance Principle is drawn as a conclusion from Energy Conservation Principle, which is a fundamental law of physics. If the circuit is electrically isolated, the sum of energies of all K circuit elements is constant

$$\sum_{k=1}^{K} W_k(t) \equiv \text{const.}\,, \tag{28}$$

so that from the identity

$$\frac{\mathrm{d}}{\mathrm{d}t}\sum_{k=1}^{K} W_k(t) \equiv \sum_{k=1}^{K}\frac{\mathrm{d}}{\mathrm{d}t}W_k(t) \equiv \sum_{k=1}^{K} p_k(t) \equiv \sum_{k=1}^{K} u_k(t)i_k(t) \equiv 0\,, \tag{29}$$

the principle of active power conservation is derived

$$\frac{1}{T}\int_0^T \sum_{k=1}^{K} u_k(t)i_k(t)\mathrm{d}t = \sum_{k=1}^{K}\frac{1}{T}\int_0^T u_k(t)i_k(t)\mathrm{d}t = \sum_{k=1}^{K} P_k = 0\,. \tag{30}$$

However, the principle of reactive power balance cannot be derived from the Energy Conservation Principle. It is a deduction from Tellegen's Theorem[19] and that is a mathematical and not a physical property of electric circuits. Namely, if two different circuits are topologically identical, i.e. they possess the same number of nodes connected in identical way by K branches (see Fig.4), but apart from this, the branches are totally different, then Tellegen's Theorem states, that the sum of products of all branch voltages from one circuit and branch currents from the other for all K circuit elements is equal to zero at each time instant.

$$\sum_{k=1}^{K} u_k^a(t) i_k^b(t) \equiv 0 . \tag{31}$$

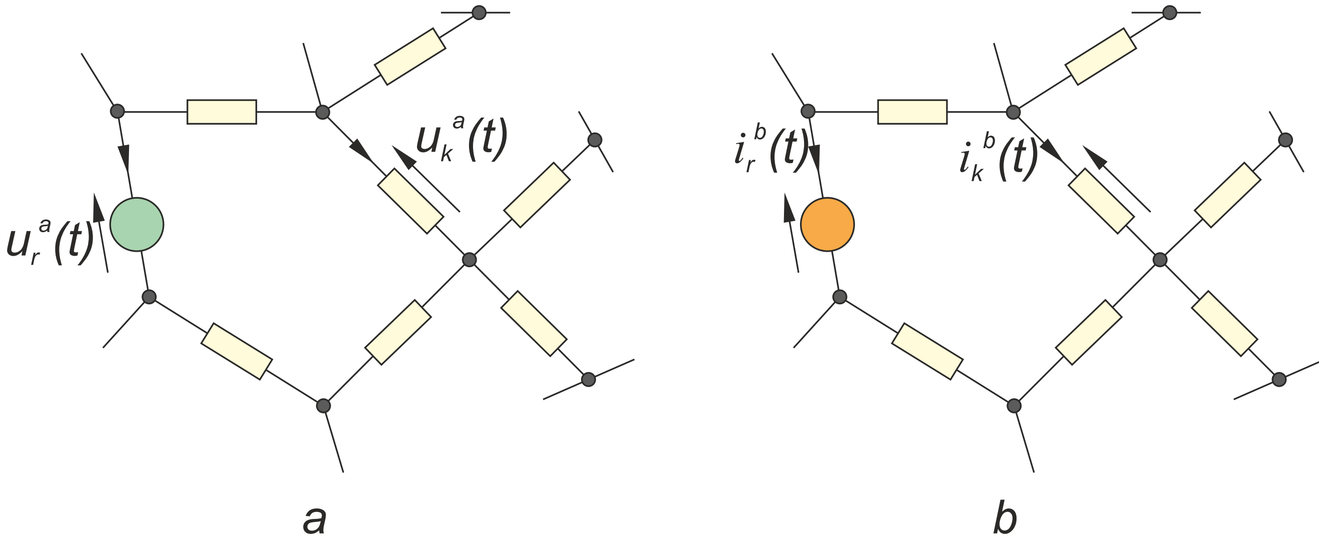

Fig. 4. Two circuits with identical topologies

Let us assume that all voltages and currents of energy sources in circuit (*b*) are shifted in time in relation to all voltages and currents of energy sources in circuit (*a*) by one quarter of the period *T*/4. Then, all branch currents in circuit (*b*) will be shifted in relation to branch currents in circuit (*a*) by *T*/4

$$i_k^b(t) \equiv i_k^a(t - \frac{T}{4}) . \tag{32}$$

So, it may be derived from **Tellegen's Theorem**, that

$$\sum_{k=1}^{K} u_k^a(t) i_k^b(t) \equiv \sum_{k=1}^{K} u_k^a(t) i_k^b(t - \frac{T}{4}) \equiv \sum_{k=1}^{K} u_k(t) i_k(t - \frac{T}{4}) \equiv 0 \tag{33}$$

and therefore

$$\frac{1}{T}\int_0^T \sum_{k=1}^{K} u_k(t) i_k(t - \frac{T}{4}) \mathrm{d}t \equiv \sum_{k=1}^{K} Q_k(t) \equiv 0 . \tag{34}$$

and this means that reactive power must satisfy the balance principle. This does not however result from the physical characteristics of this power.

6. REACTIVE POWER IN CIRCUITS WITH SINUSOIDAL VOLTAGE AND NON-SINUSOIDAL CURRENT

Non-linear loads and periodically operating switches cause current distortions, even if voltage remains almost sinusoidal. That is why approximation of current and voltage waveforms in distribution systems, where voltage is sinusoidal and current is non-sinusoidal, is justified and reflects reality closely. With this approximation, the reactive power is associated with supply voltage and fundamental harmonic of the load current $i_{1(t)}$, and, to be more precise, with the reactive component of this current $i_{1r(t)}$, which is shifted in relation to the supply voltage by a quarter of a period, *T*/4. This is equal to

$$Q = Q_1 = U \cdot \mathrm{Im}\{\boldsymbol{I}_1\} = U I_1 \sin\varphi_1 . \tag{1}$$

This explains existence of non-zero reactive power Q in a purely resistive circuit containing a TRIAC (see Fig.1). This is also explained in Fig. 5.

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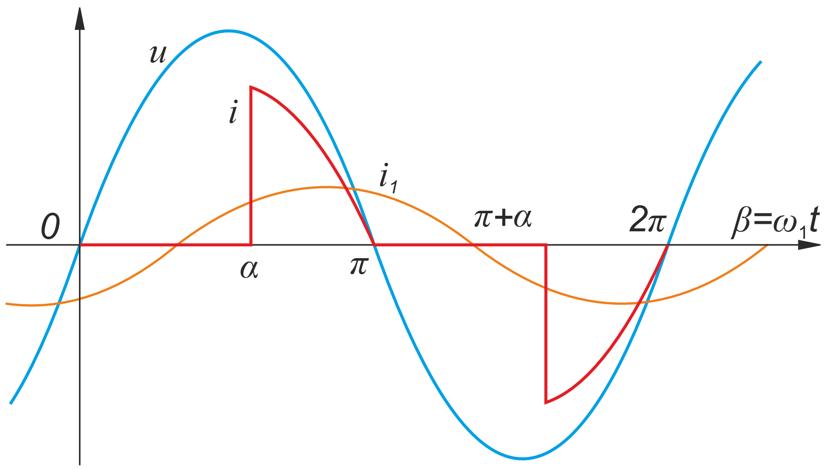

Fig. 5. Waveforms of supply voltage, current and current's fundamental harmonic in a circuit containing TRIAC

Fourier analysis of the load current shows that the waveform of the fundamental harmonic may be expressed as:

$$i_1(t) = \sqrt{2}I_1\sin(\omega_1 t - \varphi_1) = 40.3\sqrt{2}\,\sin(\omega_1 t - 60.3^{\circ})\ \text{A}\,, \tag{35}$$

so that load's reactive power is equal to

$$Q = UI_1\sin\varphi_1 = 220\cdot 40.3\sin(60.3^{\circ}) = 7.7\ \text{kvar}\,. \tag{36}$$

7. CONCLUSIONS

This article demonstrates that reactive power Q, regarded as a physical quantity in the community of electrical engineers and researchers, is not a physical quantity at all. There is no phenomenon in the electric circuits, which might be explained or described with the help of reactive power; conversely, no physical phenomenon in the circuit can explain the existence of reactive power. It is only reactive current (provided that it is correctly defined), which is associated with a specific physical phenomenon, i.e. with the shift of current harmonics by the angle $\pi/2$ in relation to the supply voltage harmonics. The reactive power is similar to apparent power S. The apparent power is not a physical quantity either, it is just the product of rms value of supply voltage $||u||$ and rms value of load current $||i||$, namely

$$S \overset{\text{df}}{=} ||u||\,||i||\,. \tag{37}$$

Reactive power is a similar product of rms value of supply voltage $||u||$ and rms value of load reactive current $||i_r||$

$$Q \overset{\text{df}}{=} ||u||\,||i_r||\,. \tag{38}$$

Let us point out that the reactive power defined in this way is non-negative, and therefore it cannot satisfy the Power Balance Principle. This principle is fulfilled by reactive power in circuits with sinusoidal voltage/current waveforms only. There, the reactive power is calculated as follows:

$$Q = UI\sin\varphi\,. \tag{39}$$

In such circuits, it is equal to the amplitude of alternating component of instantaneous power. **However, this interpretation is not true for other circuits.**

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