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Effect of Supply Voltage Harmonics on IRP - Based Switching Compensator Control

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***Abstract*—The Instantaneous Reactive Power (IRP) *p-q* Theory is one of the major theoretical tools for the development of algorithms for generating reference signals for control of switching compensators, commonly known as active harmonic or power filters. This paper presents results of study on how the supply voltage harmonics affect the reference signal which is generated using the IRP approach.**

According to this approach, the compensator should compensate, apart from the instantaneous reactive power, also the alternating component of the instantaneous active power of the load. The paper demonstrates, however, that in the presence of the supply voltage harmonics, an ideal, unity power factor load has an instantaneous active power with a non-zero alternating component. According to IRP based approach, it should be compensated and this requires that a distorted current be injected into the distribution system. Thus, the conclusion of the IRP p-q theory, that the instantaneous active power of ideally compensated loads should be constant is not generally true.

The Currents' Physical Components (CPC) Power Theory is the main theoretical tool for the presented analysis.

***Index Terms* - Active harmonic filters, active power filters, harmonic suppression, instantaneous reactive power, currents' physical components; CPC.**

I. INTRODUCTION

The Instantaneous Reactive Power (IRP) p-q Theory, developed by Nabae, Akagi and Kanazawa [1] in 1984, is one of the common theoretical tools [3, 11] for generating reference signals for the control of active harmonic filters.

There is a huge literature, in the order of a few hundred papers, on the IRP p-q theory, its fundamentals, physical interpretations, generalizations, relation to other power theories and implementations for filter control and so on. In this vast area of the p-q related issues, this paper is focused only on a single one, namely, on the question "*how the supply voltage harmonics affect the reference signal for a filter control, when this signal is generated using the p-q theory*." This is an important practical issue because harmonic filters operate always at some level of the supply voltage harmonics contents.

Although this is not the main issue, a Reader should observe, that the name of these devices is not well established. Apart from "active harmonic filters," they are called "active power filters," or "power conditioners." Moreover, the main features of such devices are not characterized by these names. These are not active, but passive devices in the sense, that they are not sources of energy, but dissipate it. Their operation is not based on filtering, but on *compensation* of the undesirable component of the supply current by a current shaped by a fast-switched PWM inverter and injected into the supply system. These are *compensators*, not filters. Because *switching* is their main feature that distinguishes them from reactive compensators, they will be called *switching compensators* in this paper, although, perhaps a better name might be coined. Anyway, a discussion on selection of a proper name for these devices is desirable.

The correctness of the IRP p-q as the power theory was challenged in Refs. [8] and [10]. It was shown there that it does not have properties as claimed in Ref. [3]. In particular, it cannot identify power properties of the load instantaneously. Moreover, any physical phenomenon that can be described in terms of the instantaneous reactive power q is not known. Just opposite, it was proven in Ref. [10] that this power, even in systems with sinusoidal voltages and currents, is an intricate quantity, associated with two power phenomena and consequently, with two different powers, the reactive Q and the unbalanced D powers. It is equal to

$$q = -Q - D\sin(2\omega_1 t + \psi)\,, \qquad (1)$$

where ψ denotes [13] the phase angle of the load unbalanced admittance, $\boldsymbol{A}$.

Thus, a question occurs: *can IRP p-q, being not founded on physical phenomena in electrical systems, provide reliable fundamentals for such systems compensation?*

Implementations of the IRP p-q theory for compensation of three-phase, three-wire systems, stem from the conclusion, repeated in a large number of papers on compensation, to mention a few, such as [15-17], that the compensator should compensate the instantaneous reactive power, q, and the oscillating component of the instantaneous active power, $\tilde{p}$, which can be extracted from the active power p with a high pass filter. Indeed, according to Ref. [16], "*…the original p-q theory authors impose a constant source power as a compensation objective*." the instantaneous active power after compensation should be constant. This common practice in using p-q theory for compensation is illustrated in Fig. 1.

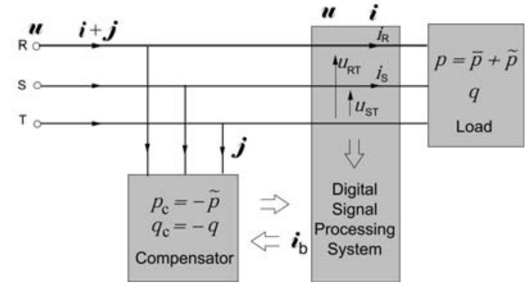

Figure 1. Compensation principle according to IRP p-q theory

It seems that there exists an awareness in the electrical engineering community involved in compensation under nonsinusoidal conditions that the IRP p-q based approach is not fully satisfactory. It was expressed in Ref. [19]: *"…there are recognized limitations to this method including demonstrated poor performance in the presence of unbalance ad voltage distortion…"* Consequently, other approaches have been developed. The Synchronous Reference Frame (SRF) algorithm [2, 9] is one of them. There are also algorithms that stem from the Fryze's power theory [6]. The FDB method [4, 7] is one of such algorithms. Recently, the Currents' Physical Components (CPC) – based power theory is used as a fundamental for switching compensator control [5, 12] These algorithms were developed without clear explanation, however, why the IRP p-q based approach may not fulfill expectations.

The very nature of the instantaneous reactive power q, described by relation (1), was revealed using the Currents' Physical Components (CPC) − based power theory, and that theory, along with symbols used in the CPC provide a theoretical frame for analysis in this paper.

II. Assumptions, Symbols and Compensation Fundamentals

The study in this paper applies to a SSC for a three-phase, three-wire system of the common structure shown in Fig. 2. The SSC should reproduce the reference signal, generated by Digital System Processing (DSP) system, as the compensator line current, $\boldsymbol{j}$. This signal is reproduced, of course, with an error, dependent on the PWM inverter switching strategy and frequency, inverter DC bus voltage variation, the effectiveness of the high frequency noise filtering, the accuracy of the data acquisition as well as the DSP accuracy. This error is irrelevant, however, for the subject of study in this paper.

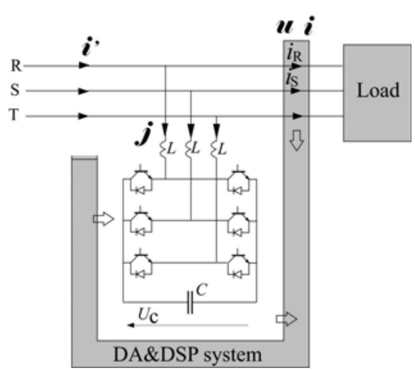

Figure 2. Shunt switching compensator (SSC) structure

Therefore, it is assumed that the compensator is ideal in the sense that it is lossless and reproduces reference signals without error. Because of energy dissipation in the PWM inverter, inductors and the capacitor, the reference signals for the SSC control has to contain an active current. It is not needed, however, at the assumption that the device is lossless. At such an assumption the reference signal becomes independent of the compensator.

Structural complexity of three-phase systems, superimposed on a mathematical complexity of description of nonsinusoidal voltages and currents in terms of Fourier series, can cause the analysis of such systems to lead to mathematically complex results, being difficult for interpretation and comprehension. Therefore, a set of effective symbols that would enable us to present the results of such an analysis in a simple form is needed. Just such a set of symbols was introduced in the CPC power theory [13], but some additional symbols are needed for the study in this paper. They are introduced below.

The line currents i_R, i_S, i_T and line-to-ground voltages u_R, u_S, u_T are arranged into *three-phase vectors*

$$\boldsymbol{i} \triangleq \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix}, \qquad \boldsymbol{u} \triangleq \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix}. \tag{2}$$

Line currents and line-to-ground voltages in three-phase, three-wire system satisfy relations

$$i_R + i_S + i_T \equiv 0, \qquad u_R + u_S + u_T \equiv 0, \tag{3}$$

thus, one line current and one voltage are dependent on the remaining two. Consequently, there is no need for using three line currents and three line-to-ground voltages for three-wire system analysis, but only two, arranged into *reduced three-phase vectors*

$$\mathcal{i} \triangleq \begin{bmatrix} i_R \\ i_S \end{bmatrix}, \qquad \mathcal{u} \triangleq \begin{bmatrix} u_R \\ u_S \end{bmatrix}. \tag{4}$$

Power properties of electric loads are described with the Instantaneous Reactive Power p-q theory in terms of voltages and currents in Clarke coordinates, α, β, 0, meaning in terms of three orthogonal currents i_α, i_β, i_0. Currents in Clarke's coordinates can be expressed in terms of line currents with the Clarke Transform, which has the form

$$\boldsymbol{i}_C \triangleq \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1, & -\frac{1}{2}, & -\frac{1}{2} \\ 0, & \frac{\sqrt{3}}{2}, & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}}, & \frac{1}{\sqrt{2}}, & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} \triangleq \mathbf{C}\,\boldsymbol{i}\,. \tag{5}$$

Current i_0 in three-wire systems has zero value and the current of line T is $i_T = -i_R - i_S$, thus, the system can be described in terms of the reduced vector of Clarke's current as follows

$$\mathcal{i}_C \triangleq \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_R \\ i_S \end{bmatrix} \triangleq \mathbb{C}\,\mathcal{i}\,. \tag{6}$$

Similarly defined is the reduced vector of Clarke's voltage

$$\mathcal{u}_C \triangleq \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_R \\ u_S \end{bmatrix} \triangleq \mathbb{C}\,\mathcal{u}\,. \tag{7}$$

The reduced inverted Clarke Transform matrix is equal to

$$\mathbb{C}^{-1} \triangleq \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix}^{-1} = \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix}. \tag{8}$$

The unwanted component, $\boldsymbol{i}_b$, of the load current can be eliminated by a SSC from the supply current, $\boldsymbol{i}'$, on the condition that the compensator input current, $\boldsymbol{j}$, is equal to $-\boldsymbol{i}_b$, i.e.

$$\boldsymbol{j} = -\,\boldsymbol{i}_b. \tag{9}$$

The switches of the PWM inverter should be controlled such that the SSC input current, $\boldsymbol{j}$, approximates the negative value of the unwanted component of the load current, $-\boldsymbol{i}_b$, as accurately as possible. Thus, the signal proportional to $-\boldsymbol{i}_b$, calculated by the digital signal processing (DSP) system, is the reference signal for the SSC control.

According to the IRP p-q theory [1, 3], the load properties are specified in terms of two instantaneous powers, active p and reactive q, defined as

$$p = u_\alpha i_\alpha + u_\beta i_\beta, \tag{10}$$

$$q = u_\alpha i_\beta - u_\beta i_\alpha, \tag{11}$$

which can be written in the matrix form

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} u_\alpha, & u_\beta \\ -u_\beta, & u_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \triangleq \mathrm{U_C} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}. \tag{12}$$

This equation, when solved with respect to Clarke's currents i_α, i_β, has the form

$$\boldsymbol{i}_\mathrm{C} \triangleq \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_\alpha, & -u_\beta \\ u_\beta, & u_\alpha \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \triangleq \mathrm{U_C^{-1}} \begin{bmatrix} p \\ q \end{bmatrix}. \tag{13}$$

According to numerous papers that implement the IRP p-q theory for compensator control, to mention a few, such as [15-17], the load is entirely compensated, if the alternating compnent, $\tilde{p}$, of the instantaneous power p and the instantaneous reactive power q are compensated, meaning that the instantaneous powers at the compensator terminals should be, as shown in Fig. 1, $-\tilde{p}$ and $-q$. Consequently, the compensator current in Clarke's coordinates should be equal to

$$\boldsymbol{j}_\mathrm{C} \triangleq \begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = \mathrm{U_C^{-1}} \begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix}. \tag{14}$$

Thus, after instantaneous powers $\tilde{p}$ and q of the load are calculated, the reference signal for the compensator current

$$-\boldsymbol{i}_\mathrm{b} = \boldsymbol{j} \triangleq \begin{bmatrix} j_\mathrm{R} \\ j_\mathrm{S} \end{bmatrix} = \mathrm{C}^{-1} \boldsymbol{j}_\mathrm{C} = \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} \tag{15}$$

can be obtained.

III. Reference Signal at Distorted Load Current and Sinusoidal Supply Voltage

Let us start with the calculation of the reference signal for a SSC connected at the terminals of a balanced, purely resistive harmonic generating load (HGL), assuming that the distribution voltage is sinusoidal, of positive sequence, with the line voltage at terminal R equal to

$$u_\mathrm{R} \triangleq \sqrt{2}\, U \cos \omega_1 t\,. \tag{16}$$

The reduced vector of Clarke's voltages is equal to

$$\boldsymbol{u}_\mathrm{C} \triangleq \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \mathrm{C} \begin{bmatrix} u_\mathrm{R} \\ u_\mathrm{S} \end{bmatrix} = \sqrt{2}\,\mathrm{C} \begin{bmatrix} U \cos \omega_1 t \\ U \cos(\omega_1 t - 120^0) \end{bmatrix} = \sqrt{3} \begin{bmatrix} U \cos \omega_1 t \\ U \sin \omega_1 t \end{bmatrix}. \tag{17}$$

Since the Clarke Transform is a linear operation, it is enough to investigate the process of generation of the reference signal for compensation of a single current harmonic, to conclude how the IRP p-q theory-based algorithm handles any distortion of the load current.

Let us assume, that the load current contains, apart from the active current, a 5th order symmetrical harmonic, meaning the harmonic of negative sequence,

$$\boldsymbol{i} = \boldsymbol{i}_\mathrm{a} + \boldsymbol{i}_5, \tag{18}$$

and assume that the line R current is

$$i_\mathrm{R} = \sqrt{2}\, I_\mathrm{a} \cos \omega_1 t + \sqrt{2}\, I_5 \cos 5\omega_1 t\,. \tag{19}$$

The reduced vector of Clarke's currents is

$$\boldsymbol{i}_\mathrm{C} = \mathrm{C} \begin{bmatrix} i_\mathrm{R} \\ i_\mathrm{S} \end{bmatrix} = \mathrm{C}\sqrt{2} \begin{bmatrix} I_\mathrm{a} \cos \omega_1 t + I_5 \cos 5\omega_1 t \\ I_\mathrm{a} \cos(\omega_1 t - 120^0) + I_5 \cos(5\omega_1 t + 120^0) \end{bmatrix} =$$

$$= \sqrt{3} \begin{bmatrix} I_\mathrm{a} \cos \omega_1 t + I_5 \cos 5\omega_1 t \\ I_\mathrm{a} \sin \omega_1 t - I_5 \sin 5\omega_1 t \end{bmatrix}. \tag{20}$$

The instantaneous active and reactive powers p and q are, respectively equal to

$$p = u_\alpha i_\alpha + u_\beta i_\beta = P + 3UI_5 \cos 6\omega_1 t \tag{21}$$

$$q = u_\alpha i_\beta - u_\beta i_\alpha = -3UI_5 \sin 6\omega_1 t\,. \tag{22}$$

The reference signal and consequently, the compensator current in Clarke's coordinates is

$$-\boldsymbol{i}_\mathrm{bC} = \boldsymbol{j}_\mathrm{C} \triangleq \begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = \mathrm{U_C^{-1}} \begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix} = 3UI_5\, \mathrm{U_C^{-1}} \begin{bmatrix} -\cos 6\omega_1 t \\ \sin 6\omega_1 t \end{bmatrix} =$$

$$= \frac{3UI_5}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_\alpha, & -u_\beta \\ u_\beta, & u_\alpha \end{bmatrix} \begin{bmatrix} -\cos 6\omega_1 t \\ \sin 6\omega_1 t \end{bmatrix} = -\sqrt{3} I_5 \begin{bmatrix} \cos 5\omega_1 t \\ -\sin 5\omega_1 t \end{bmatrix}, \tag{23}$$

and in phase coordinates

$$-\boldsymbol{i}_\mathrm{b} = \boldsymbol{j} \triangleq \begin{bmatrix} j_\mathrm{R} \\ j_\mathrm{S} \end{bmatrix} = \mathrm{C}^{-1} \boldsymbol{j}_\mathrm{C} = -\sqrt{3} I_5 \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \cos 5\omega_1 t \\ -\sin 5\omega_1 t \end{bmatrix} =$$

$$= -\sqrt{2}\, I_5 \begin{bmatrix} \cos 5\omega_1 t \\ \cos(5\omega_1 t + 120^0) \end{bmatrix}. \tag{24}$$

Thus, the compensator indeed injects the 5th order harmonic current into the supply lines, the current harmonic which compensates the 5th order harmonic of the load current.

This detailed analysis, performed under the condition that the load is supplied with a sinusoidal symmetrical voltage, shows that the IRP p-q theory-based algorithm of reference signals generation provides correct results. Now, let us repeat this analysis in a situation where the condition of the sinusoidal symmetrical supply voltage is not fulfilled.

IV. Reference Signal at Nonsinusoidal Supply Voltage

Now, let us investigate how the reference signal is affected by supply voltage harmonics. It is reasonable to "clean up" the load for this purpose from all other causes of power factor degradation. Therefore, it is assumed that the load is purely resistive, linear and balanced, as shown in Fig. 3, while the supply voltage is symmetrical, but distorted with the fifth order harmonic

$$\boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_5 , \tag{25}$$

assuming for the sake of simplicity that

$$u_{R1} \triangleq \sqrt{2}\, U_1 \cos \omega_1 t , \qquad u_{R5} \triangleq \sqrt{2}\, U_5 \cos 5\omega_1 t . \tag{26}$$

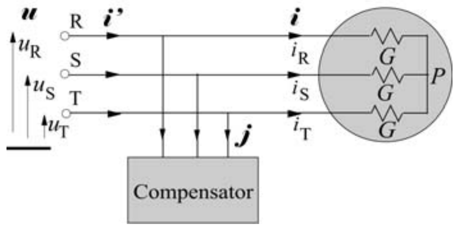

Figure 3. Balanced resistive load with compensator

The 5th order harmonic has the opposite sequence than the sequence of the fundamental harmonic, thus the reduced vector of Clarke's voltages is equal to

$$u_C \triangleq \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = C \begin{bmatrix} u_R \\ u_S \end{bmatrix} = \sqrt{2}\, C \begin{bmatrix} U_1 \cos \omega_1 t + U_5 \cos 5\omega_1 t \\ U_1 \cos(\omega_1 t - 120^0) + U_5 \cos(5\omega_1 t + 120^0) \end{bmatrix} =$$

$$= \sqrt{3} \begin{bmatrix} U_1 \cos \omega_1 t + U_5 \cos 5\omega_1 t \\ U_1 \sin \omega_1 t - U_5 \sin 5\omega_1 t \end{bmatrix} . \tag{27}$$

For the load considered

$$\boldsymbol{i} = G\boldsymbol{u} = G\boldsymbol{u}_1 + G\boldsymbol{u}_5 , \tag{28}$$

thus, the reduced vector of Clarke's load currents is

$$i_C \triangleq \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = C \begin{bmatrix} i_R \\ i_S \end{bmatrix} = \sqrt{3}\, G \begin{bmatrix} U_1 \cos \omega_1 t + U_5 \cos 5\omega_1 t \\ U_1 \sin \omega_1 t - U_5 \sin 5\omega_1 t \end{bmatrix} . \tag{29}$$

The instantaneous active power of the load at such supply is

$$p \;=\; u_\alpha i_\alpha + u_\beta i_\beta =$$

$$= 3G [(U_1 \cos \omega_1 t + U_5 \cos 5\omega_1 t)^2 + (U_1 \sin \omega_1 t - U_5 \sin 6\omega_1 t)^2] =$$

$$= 3G (U_1^2 + U_5^2 + 2U_1 U_5 \cos 6\omega_1 t) . \tag{30}$$

Thus, the instantaneous active power p of balanced resistive loads supplied with a voltage distorted with the 5th – order harmonic is not constant, but changes around its mean value. The alternating component of the instantaneous power is

$$\tilde{p} = 6G U_1 U_5 \cos 6\omega_1 t . \tag{31}$$

The instantaneous reactive power q is equal to zero.

The alternating component of the instantaneous active power p is non-zero, thus, according to the IRP p-q theory-based approach, the reference signal in Clarke's coordinates is given by

$$-i_{bC} = j_C \triangleq \begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = U_C^{-1} \begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix} = U_C^{-1} \begin{bmatrix} -6G U_1 U_5 \cos 6\omega_1 t \\ 0 \end{bmatrix} , \tag{32}$$

hence

$$\begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = \frac{1}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_\alpha , & -u_\beta \\ u_\beta , & u_\alpha \end{bmatrix} \begin{bmatrix} -6G U_1 U_5 \cos 6\omega_1 t \\ 0 \end{bmatrix} =$$

$$= \frac{-6G U_1 U_5 \cos 6\omega_1 t}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} =$$

$$= \frac{-6G U_1 U_5 \cos 6\omega_1 t}{u_\alpha^2 + u_\beta^2} \sqrt{3} \begin{bmatrix} U_1 \cos \omega_1 t + U_5 \cos 5\omega_1 t \\ U_1 \sin \omega_1 t - U_5 \sin 5\omega_1 t \end{bmatrix} . \tag{33}$$

This formula shows that the compensator has to inject a distorted current into the system to compensate the alternating component $\tilde{p}$ of the instantaneous active power. Taking into account that the denominator in formula (33) is equal to

$$u_\alpha^2 + u_\beta^2 = 3(U_1^2 + U_5^2 + 2U_1 U_5 \cos 6\omega_1 t) , \tag{34}$$

meaning, it changes in time, the distortion of the compensator current is very complex. This current in phase coordinates is equal to

$$-i_b = j \triangleq \begin{bmatrix} j_R \\ j_S \end{bmatrix} = C^{-1} j_C =$$

$$= \frac{-6G U_1 U_5 \cos 6\omega_1 t}{u_\alpha^2 + u_\beta^2} \sqrt{3} \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} U_1 \cos \omega_1 t + U_5 \cos 5\omega_1 t \\ U_1 \sin \omega_1 t - U_5 \sin 5\omega_1 t \end{bmatrix} =$$

$$= \frac{-2\sqrt{2}\, G U_1 U_5 \cos 6\omega_1 t}{U_1^2 + U_5^2 + 2U_1 U_5 \cos 6\omega_1 t} \begin{bmatrix} U_1 \cos \omega_1 t + U_5 \cos 5\omega_1 t \\ U_1 \cos(\omega_1 t - 120^0) + U_5 \cos(5\omega_1 t + 120^0) \end{bmatrix} . \tag{35}$$

It means that the compensator, controlled according to the IRP p-q theory, in the presence of supply voltage harmonics "attempts" to compensate even an ideal, resistive, unity power factor load. To do this the compensator injects a distorted current into the supply system.

One can say that installation of a compensator at terminals of such an ideal load has no sense. It is true. The load considered in this Section normally is not a subject of compensation. Operation of a compensator with such a purified load was analyzed in this Section only to avoid situation where the current given by Eqn. (35) is hidden among other components of the compensator current.

This current is injected into the system because the instantaneous active power p of the load in the situation described in this Section has a non-zero alternating component and according to the IRP p-q approach, the compensator should eliminate such a component. One could ask a question, however, "*does this component occur because of the IRP p-q theory properties, or does the instantaneous power p at distorted supply voltage indeed have such a component?*"

To answer this question let us calculate the instantaneous active power, or simply, the instantaneous power $p(t)$, in such a situation without Clarke's Transform. For such a balanced load with the phase conductance G, supplied with a symmetrical voltage with fifth order harmonic, the instantaneous power, i.e., the rate of energy W flow between the load and the supply source is

$$p = p(t) \triangleq \frac{dW}{dt} = \boldsymbol{u}^T \boldsymbol{i} = \boldsymbol{u}^T G \boldsymbol{u} = G[\boldsymbol{u}_1 + \boldsymbol{u}_5]^T [\boldsymbol{u}_1 + \boldsymbol{u}_5] =$$

$$= G\boldsymbol{u}_1^T \boldsymbol{u}_1 + G\boldsymbol{u}_5^T \boldsymbol{u}_5 + G(\boldsymbol{u}_1^T \boldsymbol{u}_5 + \boldsymbol{u}_5^T \boldsymbol{u}_1) . \tag{36}$$

The first two terms are constant components of the instantaneous power

$$G\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_1 = G||\boldsymbol{u}_1||^2 \triangleq P_1, \tag{37}$$

$$G\boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_5 = G||\boldsymbol{u}_5||^2 \triangleq P_5, \tag{38}$$

where P_1 and P_5 are harmonic active powers of the fundamental and the 5th order harmonics. The last term

$$\begin{aligned} G(\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_1) &= G[u_{1R}u_{5R} + u_{1S}u_{5S} + u_{1T}u_{5T}] + \\ &+ G[u_{5R}u_{1R} + u_{5S}u_{1S} + u_{5T}u_{1T}] = \\ &= 2G[u_{1R}u_{5R} + u_{1S}u_{5S} + u_{1T}u_{5T}] = \\ = 4GU_1U_5[\cos\omega_1 t \times \cos 5\omega_1 t &+ \\ &+ \cos(\omega_1 t - 120^0)\times\cos(5\omega_1 t + 120^0) + \\ &+ \cos(\omega_1 t + 120^0)\times\cos(5\omega_1 t - 120^0)] = \\ = 6GU_1U_5\cos 6\omega_1 t. \end{aligned} \tag{39}$$

Consequently, the instantaneous power of the load is

$$p(t) = \frac{dW}{dt} = P_1 + P_5 + \ 6GU_1U_5\cos 6\omega_1 t\ . \tag{40}$$

Thus, the alternating component occurs in the instantaneous active power p not because of some properties of the IRP p-q theory. This is the power property of a load when supplied with distorted voltage. The same was concluded in Ref. [14] using the FDB method. Consequently, only the conclusion, which is very common in papers on compensation namely, that the alternating component of the instantaneous active power should be compensated, is not right when the supply voltage is distorted with harmonics. This alternating component can occur in the instantaneous power even for ideal, resistive, unity power factor loads.

There are reported observations [18, 19] that the p-q theory-based control algorithm works properly only at sinusoidal voltage. Unfortunately, a great majority of papers on the p-q theory-based compensation assumes that the zero reactive and constant instantaneous active power is the goal of compensation.

Even if the condition for sinusoidal supply voltage at p-q approach is articulated in some papers, a due reason is not clearly provided. This paper provides very detailed reasons for that condition.

V. CONCLUSIONS

The Instantaneous Reactive Power p-q theory, used as a fundamental for an algorithm for generating reference signals for switching compensator control, does not provide correct results when the load is supplied from a source of nonsinusoidal voltage. When this voltage is distorted by harmonics, then the reference signal and consequently, the compensator current contain a disturbing component.

This disturbing component is generated by the IRP p-q theory – based control algorithm, because this algorithm relies on a believe that the instantaneous active power p of a load after compensation should be constant, meaning without any alternating component. This conclusion is not true, however, when the supply voltage is distorted. An alternating component can occur in the instantaneous power even for ideal, unity power factor loads.

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