What is Wrong with the Budeanu Concept of Reactive and Distortion Power and Why It Should be Abandoned

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***Abstract*—The Budeanu definitions of reactive and distortion power in circuits with nonsinusoidal waveforms have been spread in electrical engineering, despite some objections, for almost 60 years. The objections have concerned mainly with the question of whether these powers should be defined in the frequency domain and whether they can be measured as defined. Unfortunately, the main drawbacks of these definitions have not been noticed. Namely, the Budeanu reactive and distortion powers do not possess any attributes which might be related to the power phenomena in the circuit. Moreover, their values do not provide any information which could enable the design of compensating circuits. Also, the distortion power value does not provide any information about waveform distortion. Therefore, Budeanu's concept has led the power theory of circuits with nonsinusoidal waveforms into a blind alley.**

I. Introduction

Budeanu [1] generalized the power equation of the source in the linear circuit with sinusoidal voltage and current to apply it to circuits with periodical nonsinusoidal waveforms. He defined the reactive power as

$$Q_B \triangleq \sum_{n=1}^{\infty} U_n I_n \sin \phi_n \tag{1}$$

and he introduced a new quantity

$$D_B \triangleq \sqrt{S^2 - P^2 - Q_B^2} \tag{2}$$

called the distortion power.

Despite Fryze's objections [2], which mainly are concerned with the necessity of the voltage and the current harmonic decomposition before the reactive power could be calculated, the Budeanu definitions have spread through publications and academic textbooks in electrical engineering. In 1972 Shepherd and Zakikhani [3] also objected that the Budeanu reactive power is not a real physical quantity. They suggested that another quantity be chosen as the reactive power. This brought about a continuing discussion [3]–[6] on the power properties of nonsinusoidal systems and reactive power. Nonetheless, in 1977 Emanual [6] stated that "Budeanu's model is today universally accepted." Budeanu's model was also supported by Nowomiejski, who expressed [7] the Budeanu reactive power in the time domain and who provided, along with Fisher [8], more sophisticated mathematical foundations for this model.

The Budeanu reactive and distortion power turned out, however, to be very difficult to instrument, and a considerable effort was required [9]–[16], [18] before the first meters for measuring these powers were built, almost 50 years after they were defined.

Unfortunately, the circumstance that the reactive and the distortion powers were not accessible by measurement, and the efforts focused on their instrumentation, have diverted the attention from the quite fundamental issues for power theory. Namely, how the Q_B and D_B quantities are related to the power phenomena in the circuit and whether their values provide the information which would enable the design of compensating circuits, as is the case with reactive power in circuits with sinusoidal waveforms. It will be shown in this paper, that as a matter of fact, the Budeanu reactive and distortion powers do not possess the attributes which could be related to the power phenomena in the circuit, and their values do not provide the information necessary for the design of compensating circuits. It will be shown moreover, that the distortion power value is not related to the waveform distortion. Unfortunately, these essential drawbacks of the Budeanu model were not earlier reported.

The author's research on orthonormal 1-ports and 2-ports synthesis [15], [16] has enabled the construction of both a reactive power Q_B meter [13] and a distortion power D_B meter [14]. He showed also [12] that the reactive power can be measured with the aid of a wideband $\pi/2$ phase-shift circuit. In this way, the author was not only misled by Budeanu's model, but also has contributed to keeping it afloat. Therefore, he feels especially compelled to show the reasons why this model is useless.

II. Relation of the Q_B and D_B Quantities to the Power Properties of the Circuit

Let us suppose that the periodic voltage u of frequency ω_1 contains only the single harmonic u_n of frequency $n\omega_1$ and rms value U_n i.e.,

$$u = u_n \triangleq \sqrt{2} U_n \cos (n\omega_1 t + \alpha_n). \tag{3}$$

If this voltage is applied to a linear load of $Y_n \triangleq Y_n \exp\{-j\phi_n\}$ admittance for frequency $n\omega_1$, then, the load cur-

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rent is

$$i = i_n = \sqrt{2} I_n \cos(n\omega_1 t + \alpha_n - \phi_n), \quad \text{with } I_n = Y_n U_n. \tag{4}$$

The load instantaneous power p_n, i.e., the rate of the energy transmission to the load, can be decomposed as follows:

$$\begin{aligned} p_n &\triangleq \frac{dW}{dt} = u_n i_n \\ &= P_n[1 + \cos 2(n\omega_1 t + \alpha_n)] \\ &\quad + Q_n \sin 2(n\omega_1 t + \alpha_n) \end{aligned}$$

where

$$P_n \triangleq U_n I_n \cos \phi_n \tag{6}$$

$$Q_n \triangleq U_n I_n \sin \phi_n. \tag{7}$$

Formula (5) emphasizes the physical meaning of reactive power Q in circuits with sinusoidal waveforms. It is the "generalized" (in the sense that it can be negative) amplitude of the alternating component of instantaneous power, $Q_n \sin 2(n\omega_1 t + \alpha_n)$. It appears if, due to the phenomenon of energy accumulation in electric or magnetic fields of the reactance components of the load, a part of the energy is transmitted forward and backward between the load and the source.

Defining the reactive power Q_B in circuits with distorted voltage and current, i.e., with

$$u \triangleq \sum_{n=1}^{\infty} u_n \quad i \triangleq \sum_{n=1}^{\infty} i_n \tag{8}$$

as

$$Q_B \triangleq \sum_{n=1}^{\infty} U_n I_n \sin \phi_n = \sum_{n=1}^{\infty} Q_n. \tag{9}$$

Budeanu simply added the generalized amplitudes Q_n of the instantaneous power alternating components of all harmonics. But each of these components has a different frequency and may have different phase angle α_n. Therefore, this sum does not specify the alternating component of the whole instantaneous power $p = ui$. Though each term Q_n has a distinctive physical meaning, their sum Q_B loses it completely. In particular, it can be equal to zero at nonzero values of terms Q_n, i.e., despite the reciprocating energy transmission between the source and the load.

The phenomenon of the reciprocating energy transmission at harmonic frequencies does not affect the source current rms value $\|i\|$ and its apparent power S in the manner suggested by Budeanu's model. Namely, each of the current harmonics i_n can be decomposed into two orthogonal components

$$i_{dn} \triangleq \sqrt{2} I_n \cos \phi_n \cos(n\omega_1 t + \alpha_n) \tag{10}$$

$$i_{rn} \triangleq \sqrt{2} I_n \sin \phi_n \sin(n\omega_1 t + \alpha_n) \tag{11}$$

which rms values, denoted by $\|i_{dn}\|$, $\|i_{rn}\|$, fulfill relation

$$\begin{aligned} \|i_n\|^2 &= \|i_{dn}\|^2 + \|i_{rn}\|^2 \\ &= \left(\frac{P_n}{U_n}\right)^2 + \left(\frac{Q_n}{U_n}\right)^2. \end{aligned} \tag{12}$$

Since all current harmonics are mutually orthogonal, thus the square of the rms value of the current is

$$\begin{aligned} \|i\|^2 &= \sum_{n=1}^{\infty} \|i_n\|^2 \\ &= \sum_{n=1}^{\infty} \left(\frac{P_n}{U_n}\right)^2 + \sum_{n=1}^{\infty} \left(\frac{Q_n}{U_n}\right)^2 \end{aligned} \tag{13}$$

and the square of the source apparent power

$$\begin{aligned} S^2 = \|u\|^2 \|i\|^2 &= \|u\|^2 \sum_{n=1}^{\infty} \left(\frac{P_n}{U_n}\right)^2 \\ &\quad + \|u\|^2 \sum_{n=1}^{\infty} \left(\frac{Q_n}{U_n}\right)^2. \end{aligned} \tag{14}$$

Therefore, if there is the reciprocating energy transmission between the source and the load, then the term $\Sigma (Q_n/U_n)^2$, but not ΣQ_n, is responsible for the source apparent power increase. The apparent power is a minimum, for specified P_n and U_n values, if for each n the harmonic reactive power Q_n is equal to zero, but not if their sum Q_B is equal to zero. This is only a necessary, but not a sufficient condition. The reciprocating energy transmission may exist also when the reactive power Q_B has zero value, therefore, the comparison of (14) with the Budeanu's power equation

$$\|u\|^2 \|i\|^2 = P^2 + Q_B^2 + D_B^2 \tag{15}$$

allows the conclusion to be drawn that not only reactive power Q_B, but also the distortion power D_B is affected by the reciprocating energy transmission. Unfortunately, it means simultaneously that neither of these two power, Q_B, D_B is related distinctively to this phenomenon which is responsible for the source apparent power increase. For just this reason, Budeanu's model is useless for attempts of power factor ($\lambda \triangleq P/S$) improvement. If an ideal reactance 1-port is connected at the source terminals, then, if the source impedance is equal to zero, it does not affect either the P_n *or* U_n value, consequently, it does not affect the source active power P. However, it modifies the Q_n, Q_b, D_B powers. Unfortunately, the Budeanu reactive power compensation does not affect the source apparent power S in an explicit manner, since the change in the Q_B value also changes the distortion power D_B. This is illustrated with the following example.

Example 1: The source voltage of frequency $\omega_1 = 1$ rad/s in the circuit shown in Fig. 1 contains three harmonics of order $n = 1, 5, 7$ with rms value $U_1 = 100$ V, and $U_n = U_1/n$. The source active power $P = 5019$ W. The values of powers S, Q_B, D_B and the power factor λ

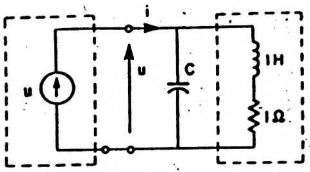

Fig. 1. Circuit example.

TABLE I
REACTIVE POWER COMPENSATION RESULTS

Quantity	Unit	(1)	(2)
C	F	-	0.3802
S	VA	7296	7259
Q_B	VA	5105	0
D_B	VA	1407	5244
λ	-	0.688	0.691

of the circuit without a shunt capacitor are tabulated in column 1 of Table I.

The Budeanu reactive power Q_B can be wholly compensated by the shunt capacitor of $C = 0.3802$ F, but as is shown in column 2, its compensation has almost no effect on the source power factor λ. Thus the compensation of the reactive power Q_B alone may be useless for the power factor improvement. To overcome this drawback of Budeanu's model, one might seek how the distortion power could be compensated simultaneously with the reactive power. However, it does not seem too sensible to have the apparent power expressed in terms of harmonic reactive power Q_n in such an explicit form as in (14). Indeed, as it was shown in paper [17], this equation, expressed in a slightly modified form, elucidates the power properties of the circuits with nonsinusoidal waveforms without Budeanu's Q_B and D_B quantities and provides the fundamentals for the power factor improvement.

The name of another quantity, D_B, introduced by Budeanu, the distortion power, suggests that it is a measure of waveforms distortion in the circuit, though, it has never been said explicitly what this name means. As a matter of fact, it is equal to zero if the waveforms are sinusoidal, but it is equal to zero also if the distorted voltage is applied to a resistive load, i.e., if the current waveform is not distorted with regard to the voltage waveform. This means that distortion power is not a measure of waveform distortion itself, so the next possibility is that it is a measure of the current waveform change with respect to the voltage. Let us check this hypothesis.

The distortion power can be expressed in the form

$$D_B \triangleq \sqrt{S^2 - P^2 - Q_B^2} = \sqrt{\frac{1}{2}\sum_{r=1}^{\infty}\sum_{s=1}^{\infty} A_{rs}} \tag{16}$$

where

$$\begin{aligned} A_{rs} &\triangleq U_r^2 I_s^2 + U_s^2 I_r^2 - 2U_r U_s I_r I_s \cos(\phi_r - \phi_s) \\ &= (U_r I_s - U_s I_r)^2 + 2U_r U_s I_r I_s \\ &\quad \cdot [1 - \cos(\phi_r - \phi_s)] \\ &\geq 0. \end{aligned} \tag{17}$$

Since A_{rs} terms are nonnegative, the distortion power is equal to zero if, and only if, for each r, s, the term $A_{rs} = 0$; i.e., if for each voltage harmonic

$$\frac{U_r}{I_r} = \frac{U_s}{I_s} \quad \text{and} \quad \phi_r = \phi_s = \phi. \tag{18}$$

It means that the distortion power is equal to zero, if for each voltage harmonic the load has the same impedance, $Z = Z \exp\{j\phi\}$, but this condition is not equivalent to the absence of the current distortion with respect to the voltage. If condition (18) is fulfilled with $\phi \neq 0$, then $D_B = 0$ despite the change in the current waveform. This is illustrated with Example 2.

Example 2: The voltage

$$u \triangleq \sqrt{2}U_1 \sin \omega_1 t + \sqrt{2}U_3 \sin 3\omega_1 t$$

is applied to the load shown in Fig. 2. Its admittance for $\omega_1 = 1$ rad/s and $\omega_3 = 3\omega_1$ is

$$Y_1 = Y_3 = Y = j1S.$$

Thus the condition (18) is fulfilled and $D_B = 0$.

Despite this, the source current

$$\begin{aligned} i &= \sqrt{2}YU_1 \sin\left(\omega_1 t + \frac{\pi}{2}\right) \\ &\quad + \sqrt{2}YU_3 \sin\left(3\omega_1 t + \frac{\pi}{2}\right) \\ &= Y(\sqrt{2}U_1 \cos \omega_1 t + \sqrt{2}U_3 \cos 3\omega_1 t) \end{aligned}$$

as it is shown in Fig. 3, is distorted relative to the source voltage.

On the contrary, the current waveform can be only shifted with regard to the voltage without any change in the shape, just as it is in the circuits with sinusoidal waveform, despite the nonzero value of distortion power. This is illustrated with Example 3.

Example 3: The same voltage as in Example 2 is applied to the load shown in Fig. 4.

The load admittance for harmonic frequencies is equal to $Y_1 j1S$, $Y_3 = -j1S$, thus condition (18) is not fulfilled and the distortion power is equal to $D_B = 2YU_1U_3$, where $Y = 1S$. The waveform of the source current

$$\begin{aligned} i &= \sqrt{2}Y_1 U_1 \sin\left(\omega_1 t + \frac{\pi}{2}\right) \\ &\quad + \sqrt{2}Y_3 U_3 \sin\left(3\omega_1 t - \frac{\pi}{2}\right) \\ &= \sqrt{2}I_1 \cos \omega_1 t - \sqrt{2}I_3 \cos 3\omega_1 t \end{aligned}$$

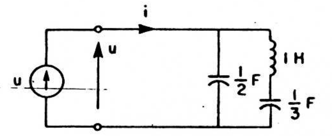

Fig. 2. Circuit with distortion power $D_G = 0$.

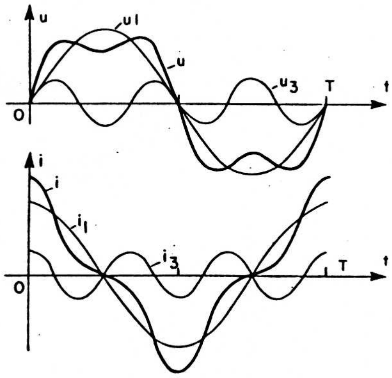

Fig. 3. Voltage and current waveforms in circuit shown in Fig. 2.

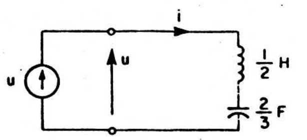

Fig. 4. Circuit with distortion power $D_B \neq 0$.

is shown in Fig. 5. It is only shifted, but not distorted relative to the voltage.

Thus although waveform distortion can affect the value of the distortion power D_B, this power does not provide information as to waveform distortion. Indeed, it was introduced into the power theory of circuits with nonsinusoidal waveforms, since the square of the reactive power Q_B^2 turned out to be less than $S^2 - P^2$. This gap had to be filled with something which was then endowed with delusive name of distortion power.

III. Conclusions

The Budeanu reactive and distortion powers do not possess the attributes which can be related to the power phenomena in circuits with nonsinusoidal waveforms, and their values does not provide the information which could be used to design compensating circuits. Moreover, the distortion power value does not provide information related to waveform distortion.

Budeanu's model has been criticized as not satisfactory since it was first proposed. This criticism brought about the other models that have been developed since. However, the detailed critical analysis of Budeanu's model has not been performed. Moreover, for various reasons, neither of the other models have gained general recognition,

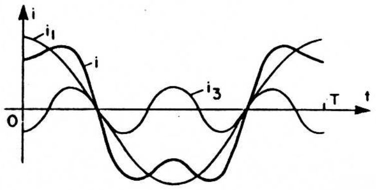

Fig. 5. Voltage and current waveforms in circuit shown in Fig. 4.

and the use of Budeanu's model has become the most widespread. Unfortunately, for the reasons shown in this paper, the Budeanu concept has led the power theory of nonsinusoidal circuits into a blind alley.

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