A 8101 F

European Transactions on Electrical Power Engineering

EUREL Publication

ETEP Vol. 3, No. 1, Jan./Feb. 1993

J#48

Reprint from:

vde verlag

Published by VDE in co-operation with AEI, AIM, ASN/CPEF, DIF, KIVI, ODE, ÖVE, SER, SEV/ASE, SRBE/KBVE, with the support of the Commission of the European Communities (CEC)

Power Factor Improvement of Three-Phase Unbalanced Loads with Non-Sinusoidal Supply Voltage

L. S. Czarnecki

Abstract

Three-phase load asymmetry causes a useless current loading of the supply source and this loading increases even more if the supply voltage is non-sinusoidal. This loading can be expressed quantitatively with the unbalanced current i_u of RMS value $\|i_u\|$. A whole compensation of the unbalanced current i_u was proved to be possible with reactive balancing circuits, however, it may be very complex. This paper shows that a substantial minimization of the unbalanced current i_u may be achieved with such a circuit, but of much lower complexity. The method suggested requires that the unbalanced admittances A_n of the load for harmonic frequencies are known. A harmonic analysis that provides the complex RMS values of the phase voltages and currents harmonics in a cross-section between the load and the source enables us to determine these admittances and design the balancing circuit. It was shown, moreover, that the reduction of the unbalanced current i_u can be integrated with the reactive current i_r minimization.

1 Introduction

One of the substantial reasons for measurements of powers, currents or the load parameters in power circuits is providing the input data for compensators design. They fulfill usually two tasks. They should improve the efficiency of the power transmission and improve the power quality. A suitable choice of the powers definition for that purpose has been a subject of concern for a long time.

It is known for a long time [1 – 8] that asymmetry of three-phase loads deteriorates the efficiency of the power transmission, which can be observed in a lowering of the power factor value. Because of the voltage drop on the source impedance, the asymmetrical currents result, moreover, in asymmetry of the voltage being provided to other consumers. This supply asymmetry is considered as one of the important factors affecting unfavorably the power quality. Unfortunately, the technical problems related to the power factor improvement in circuits with non-sinusoidal voltages can not be solved having as a base the power quantities defined in the IEEE Standard Dictionary of Electrical and Electronics Terms [9]. Definitions in that Standard generalize to three-phase circuits *Budeanu*'s definitions of the reactive and distortion powers. They provide, unfortunately [10], a misleading interpretation of energy related phenomena and do not provide any fundamentals for compensators design.

As it was shown in paper [11], the current overloading of the three-phase symmetrical non-sinusoidal voltage source that supplies a linear, but asymmetrical load can be attributed to only three phenomena and the load features:

- The change of the load equivalent conductance G_{en} with the harmonic order n.
- The reciprocating flow of energy between the load and the source, dependent on the load equivalent susceptance B_{en}.
- The electrical asymmetry of the load, which can be specified quantitatively by the unbalanced admittance $\underline{A}_n$.

This overloading can be expressed quantitatively by decomposition of the line currents (**Fig. 1**) vector

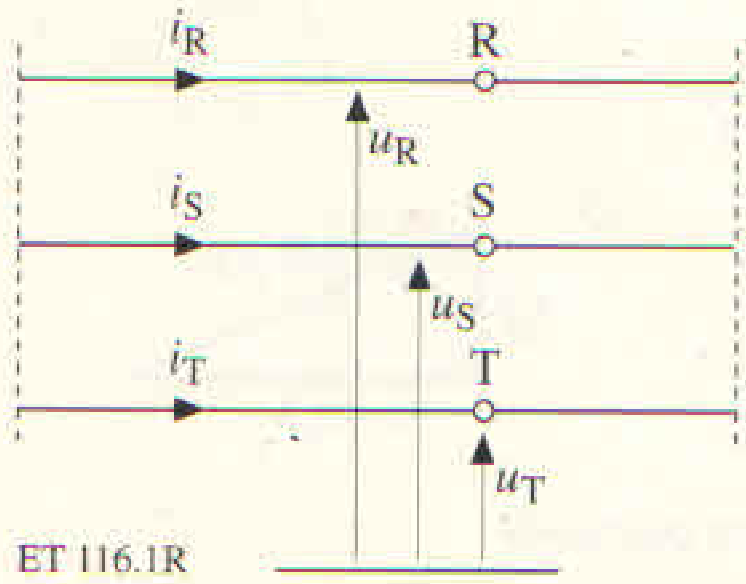
$$\boldsymbol{i} := [\,i_R, i_S, i_T]^T \tag{1}$$

into the active $\boldsymbol{i}_a$, scattered $\boldsymbol{i}_s$, reactive $\boldsymbol{i}_r$, and unbalanced $\boldsymbol{i}_u$ currents

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_s + \boldsymbol{i}_r + \boldsymbol{i}_u. \tag{2}$$

They are mutually orthogonal [11], so that the RMS values of these currents vectors, defined as

$$\|\boldsymbol{i}\| := \sqrt{\frac{1}{T}\int_0^T \boldsymbol{i}^T \boldsymbol{i}\, dt} = \sqrt{\|i_R\|^2 + \|i_S\|^2 + \|i_T\|^2} \tag{3}$$



ET 116.1R

Fig. 1. Cross section of a three-phase circuit

fulfill the relationship

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_s\|^2 + \|\boldsymbol{i}_r\|^2 + \|\boldsymbol{i}_u\|^2. \quad (4)$$

It can be visualized with the currents RMS value diagram, shown in **Fig. 2a**, with the polygon's sides length proportional to the particular currents RMS value, $\|\boldsymbol{i}\|$, $\|\boldsymbol{i}_a\|$, $\|\boldsymbol{i}_s\|$, $\|\boldsymbol{i}_r\|$, $\|\boldsymbol{i}_u\|$.

It was shown in paper [12] that the unbalanced current $\boldsymbol{i}_u$ and the reactive current $\boldsymbol{i}_r$ can be entirely compensated with a shunt linear balancing and compensating (BaCo) circuit. It may reduce (**Fig. 2b**) the square of the source current RMS value $\|\boldsymbol{i}\|$ in an ideal case, if the active power loss in the BaCo circuit and the terminal voltage change are neglected, to the value

$$\|\boldsymbol{i}\|^2_{min} = \|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_s\|^2. \quad (5)$$

If the voltage contains K harmonics, then the required number of reactive elements M in a single phase of the BaCo circuit has to fulfill inequality

$$K \le M \le 2K - 1. \quad (6)$$

Therefore, the paper [12] shows a theoretical possibility of the unbalanced and reactive currents $\boldsymbol{i}_u$ and $\boldsymbol{i}_r$ total compensation, rather than a practical technical solution to this problem. None the less, by showing that they can be wholly compensated with a linear reactive circuit, the paper [12] opened a gateway for the unbalanced and reactive currents minimization in the three-phase circuits with non-sinusoidal voltage using more frugal BaCo circuits, than those described in [12].

As regard to the reactive current, a very efficient minimization of its RMS value $\|\boldsymbol{i}_r\|$ can be achieved with the shunt two elements series LC (TESLC) compensator, applying for its design the method suggested in paper [13] for the single phase circuit.

The goal of this paper is to show that such a BaCo circuit can be designed which minimizes effectively both the unbalanced and reactive current RMS value.

2 Unbalanced Admittance of the Load

The notion of the unbalanced admittances of a load, $\underline{A}_n$, for the quantitative description of the three-phase loads asymmetry was introduced in paper [12]. It occurred to be useful for the balancing circuits design.

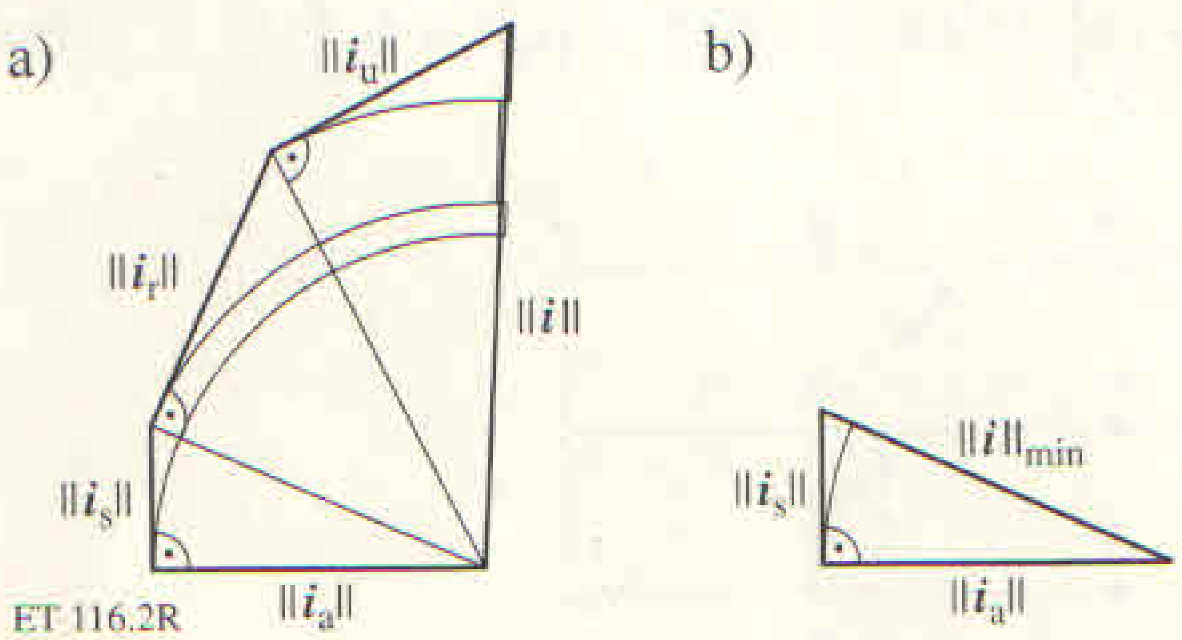


Fig. 2. Current RMS values polygons
a) Not compensated load
b) Load with shunt compensation

Therefore, it deserves more attention, interpretation, analysis of its properties and measurement.

The electrical asymmetry of a three-phase load at a symmetrical supply voltage may result in the source useless loading with an unbalanced current, $\boldsymbol{i}_u := [\,i_{Ru}, i_{Su}, i_{Tu}]^T$. If the source voltage is symmetrical and sinusoidal of frequency $n\omega_1$, then the unbalanced current $\boldsymbol{i}_{un}$ is also symmetrical, but of the opposite sequence to the supply voltage $\boldsymbol{u}_n := [\,u_{Rn}, u_{Sn}, u_{Tn}]$ sequence.

The unbalanced admittance, $\underline{A}_n := A_n e^{j\psi_n}$, can be considered as a complex coefficient of the proportionality between the complex RMS values $\underline{I}_{Rnu}$, $\underline{U}_{Rn}$ of unbalanced current $\boldsymbol{i}_{nu}$ and the supply voltage $\boldsymbol{u}_n$ of the reference phase R, namely

$$\underline{I}_{Rnu} = \underline{A}_{Rn}\underline{U}_{Rn} := \underline{A}_n \underline{U}_n. \quad (7)$$

The phase index R for the reference phase is neglected in eq. (7) for the sake of simplicity. With the symbols

$$\boldsymbol{A}_n := \begin{bmatrix} \underline{A}_{Rn} & 0 & 0 \\ 0 & \underline{A}_{Sn} & 0 \\ 0 & 0 & \underline{A}_{Tn} \end{bmatrix}, \quad \boldsymbol{U}_n := \begin{bmatrix} \underline{U}_{Rn} \\ \underline{U}_{Sn} \\ \underline{U}_{Tn} \end{bmatrix}, \quad (8)$$

the unbalanced current can be expressed in the form

$$\boldsymbol{i}_u = \sqrt{2}\,\mathrm{Re} \sum_{n \in N} \boldsymbol{A}_n \boldsymbol{U}_n e^{jn\omega_1 t}, \quad (9)$$

where N denotes the set of orders n of significant harmonics. The extra current loading of the source caused by the load asymmetry is equal to

$$\|\boldsymbol{i}_u\| = \sqrt{\sum_{n \in N} A_n^2 \|\boldsymbol{u}_n\|^2}. \quad (10)$$

The unbalanced admittance depends only on the load admittances $\underline{Y}_{RSn}$, $\underline{Y}_{STn}$, $\underline{Y}_{TRn}$ and on the supply voltage sequence, since for the positive sequence it is equal to

$$\underline{A}^+_{Rn} := \underline{A}^+_n = -\,[\,\underline{Y}_{STn} + \underline{\alpha}^2 \underline{Y}_{RSn} + \underline{\alpha}\underline{Y}_{TRn}] \quad (11)$$

where $\underline{\alpha} := 1 \cdot e^{j2\pi/3}$, and it is equal to

$$\underline{A}^-_{Rn} := \underline{A}^-_n = -\,[\,\underline{Y}_{STn} + \underline{\alpha}\underline{Y}_{RSn} + \underline{\alpha}^2 \underline{Y}_{TRn}] \quad (12)$$

for the negative voltage sequence. To eliminate this inconvenience in the dependence of the form of eqs. (11) and (12) on the voltage sequence, let us introduce the "sequence index" s defined as

$$s := \begin{cases} +1, \text{ for harmonics of positive sequence, } n = 3k+1, \\ -1, \text{ for harmonics of negative sequence, } n = 3k-1 \end{cases} \quad (13)$$

and the "sequence turn coefficient" as

$$\underline{\beta}(s) := \underline{\beta} := -\frac{1}{2} - js\frac{\sqrt{3}}{2} = e^{-s\frac{j2\pi}{3}} = \begin{cases} \underline{\alpha}^2, \text{ for } s = +1, \\ \underline{\alpha}\;\;, \text{ for } s = -1. \end{cases} \quad (14)$$

This enables us to express eqs. (11) and (12) in a common form

$$\underline{A}_{Rn} = -\,[\,\underline{Y}_{STn} + \underline{\beta}\underline{Y}_{RSn} + \underline{\beta}^* \underline{Y}_{STn}], \quad (15)$$

and for remaining phases, S and T

$$\underline{A}_{Sn} := \underline{I}_{Snu}/\underline{U}_{Sn} = \underline{\beta}\underline{A}_n, \quad \underline{A}_{Tn} := \underline{I}_{Tnu}/\underline{U}_{Tn} = \underline{\beta}^* \underline{A}_n. \quad (16)$$

Unbalanced admittances $\underline{A}_n$ measurement requires that a harmonic analysis of the phase voltages and currents provides us with the complex RMS values $\underline{U}_{Rn}$, $\underline{U}_{Sn}$, $\underline{U}_{Tn}$ and $\underline{I}_{Rn}$, $\underline{I}_{Sn}$, $\underline{I}_{Tn}$ of the voltage and current harmonics. This enables us to calculate the complex apparent power $\underline{S}_n$ of particular harmonics

$$\underline{S}_n := \underline{U}_{Rn}\underline{I}^*_{Rn} + \underline{U}_{Sn}\underline{I}^*_{Sn} + \underline{U}_{Tn}\underline{I}^*_{Tn}. \quad (17)$$

With this power, the equivalent admittance $\underline{Y}_{en}$ of the load can be calculated, namely

$$\underline{Y}_{en} := G_{en} + jB_{en} = \underline{S}^*_n / \|\boldsymbol{u}_n\|^2 \quad (18)$$

where

$$\|\boldsymbol{u}_n\|^2 := U^2_{Rn} + U^2_{Sn} + U^2_{Tn}. \quad (19)$$

The unbalanced admittance $\underline{A}_{Rn}$ can be calculated as

$$\underline{A}_{Rn} = \underline{I}_{Rn}/\underline{U}_{Rn} - \underline{Y}_{en} \quad (20)$$

independently of the harmonic sequence.

The notion of the unbalanced admittance attributes the quantitative measure for the load asymmetry effect on the supply source overloading. In such a way it eliminates an ambiguity of the term 'load asymmetry', since the electrical asymmetry of the load may not result in the asymmetrical loading of the source. Moreover, even at the symmetrical and sinusoidal supply voltage, its sequence may affect the current asymmetry. The circuit, shown in **Fig. 3**, is an example of such a situation. Despite of its asymmetry, it does not load the source of positive sequence voltage of frequency ω_1 with any unbalanced current, so that $\underline{A}^+_{R1} = 0$ and the power factor λ (in the literature the abbreviation PF is often used for the power factor instead of λ) of such a load, $\lambda := P/S = 1$. However, for the same circuit the unbalanced admittance is equal to $\underline{A}^-_{R1} = 2\sqrt{3}\,e^{j\pi}$ S, at the supply voltage of the negative sequence, so that the source current is strongly asymmetrical, which reduces the power factor to $\lambda = 0.447$.

3 Unbalanced and Reactive Currents Minimization

Total compensation of the unbalanced and reactive currents, $\boldsymbol{i}_u$ and $\boldsymbol{i}_r$, requires that a balancing and compensating (BaCo) circuit that draws current $(-\boldsymbol{i}_u - \boldsymbol{i}_r)$ is connected at the source terminals. If the BaCo circuit has a triangle configuration, (**Fig. 4**), then its phase susceptances, denoted by T_{XYn}, have the value [12] generalized with the use of the sequence index s, equal to

$$T_{RSn} = \frac{1}{\sqrt{3}}\, s\,\mathrm{Re}\{\underline{A}_n\} - \frac{1}{3}(\mathrm{Im}\{\underline{A}_n\} + B_{en}),$$
$$T_{STn} = \frac{2}{3}\mathrm{Im}\{\underline{A}_n\} - \frac{1}{3}B_{en}, \quad (21)$$
$$T_{TRn} = \frac{-1}{\sqrt{3}}\, s\,\mathrm{Re}\{\underline{A}_n\} - \frac{1}{3}(\mathrm{Im}\{\underline{A}_n\} + B_{en}).$$

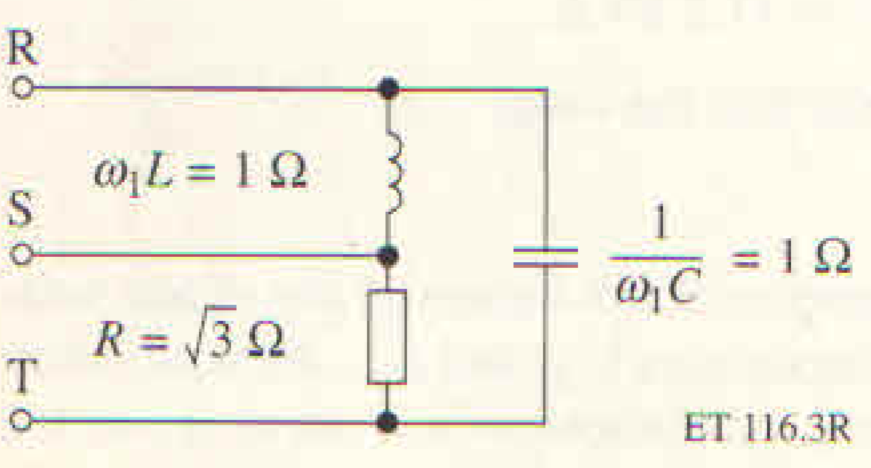


Fig. 3. Example of circuit

If we arrange the phase currents j_{XYn} of the BaCo circuit, its susceptances T_{XYn} and phase to phase voltages $\underline{V}_{XYn}$ into the following vectors

$$\boldsymbol{j}_n = \begin{bmatrix} j_{RSn} \\ j_{STn} \\ j_{TRn} \end{bmatrix}, \quad \boldsymbol{T}_n = \begin{bmatrix} T_{RSn} & 0 & 0 \\ 0 & T_{STn} & 0 \\ 0 & 0 & T_{TRn} \end{bmatrix}, \quad \boldsymbol{V}_n = \begin{bmatrix} \underline{V}_{RSn} \\ \underline{V}_{STn} \\ \underline{V}_{TRn} \end{bmatrix} \quad (22)$$

then the phase currents of the BaCo circuit can be expressed as

$$\boldsymbol{j}_T = \sqrt{2}\,\mathrm{Re} \sum_{n \in N} j\boldsymbol{T}_n \boldsymbol{V}_n e^{jn\omega_1 t}. \quad (23)$$

Current $\boldsymbol{j}_T$ compensates the unbalanced and reactive currents $\boldsymbol{i}_u$ and $\boldsymbol{i}_r$ entirely, however, this requires that the BaCo circuit has the complexity, i. e., the number of reactance elements K in a single phase as specified by the inequality of eq. (6). If it does not have the complexity required, the source loading with the unbalanced and reactive currents cannot be compensated totally, but only minimized.

Let as suppose that a BaCo circuit of a triangle configuration has the phase susceptances D_{RSn}, D_{STn}, D_{TRn} (Fig. 4b), arranged in a diagonal vector $\boldsymbol{D}_n$ similarly as the vector $\boldsymbol{T}_n$. The vector $\boldsymbol{j}_D$ of the BaCo circuit currents, $\boldsymbol{j}_D := [j_{RS}, j_{ST}, j_{TR}]^T$ is equal to

$$\boldsymbol{j}_D = \sqrt{2}\,\mathrm{Re} \sum_{n \in N} j\boldsymbol{D}_n \boldsymbol{V}_n e^{jn\omega_1 t}. \quad (24)$$

The efficiency of the current minimization by this BaCo circuit can be expressed in terms of the distance d of the current $\boldsymbol{j}_D$ from the current $\boldsymbol{j}_T$ of an ideal compensator. The distance d is defined here as the RMS value of their difference, namely

$$d := \|\boldsymbol{j}_T - \boldsymbol{j}_D\|. \quad (25)$$

Since

$$\boldsymbol{j}_T - \boldsymbol{j}_D = \sqrt{2}\,\mathrm{Re} \sum_{n \in N} j[\boldsymbol{T}_n - \boldsymbol{D}_n]\boldsymbol{V}_n e^{jn\omega_1 t}. \quad (26)$$

then, the square of distance d can be expressed as

$$d = \sum \Big[|T_{RSn} - D_{RSn}|^2 V^2_{RSn} + |T_{STn} - D_{STn}|^2 V^2_{STn} + |T_{TRn} - D_{TRn}|^2 V^2_{TRn} \Big]. \quad (27)$$

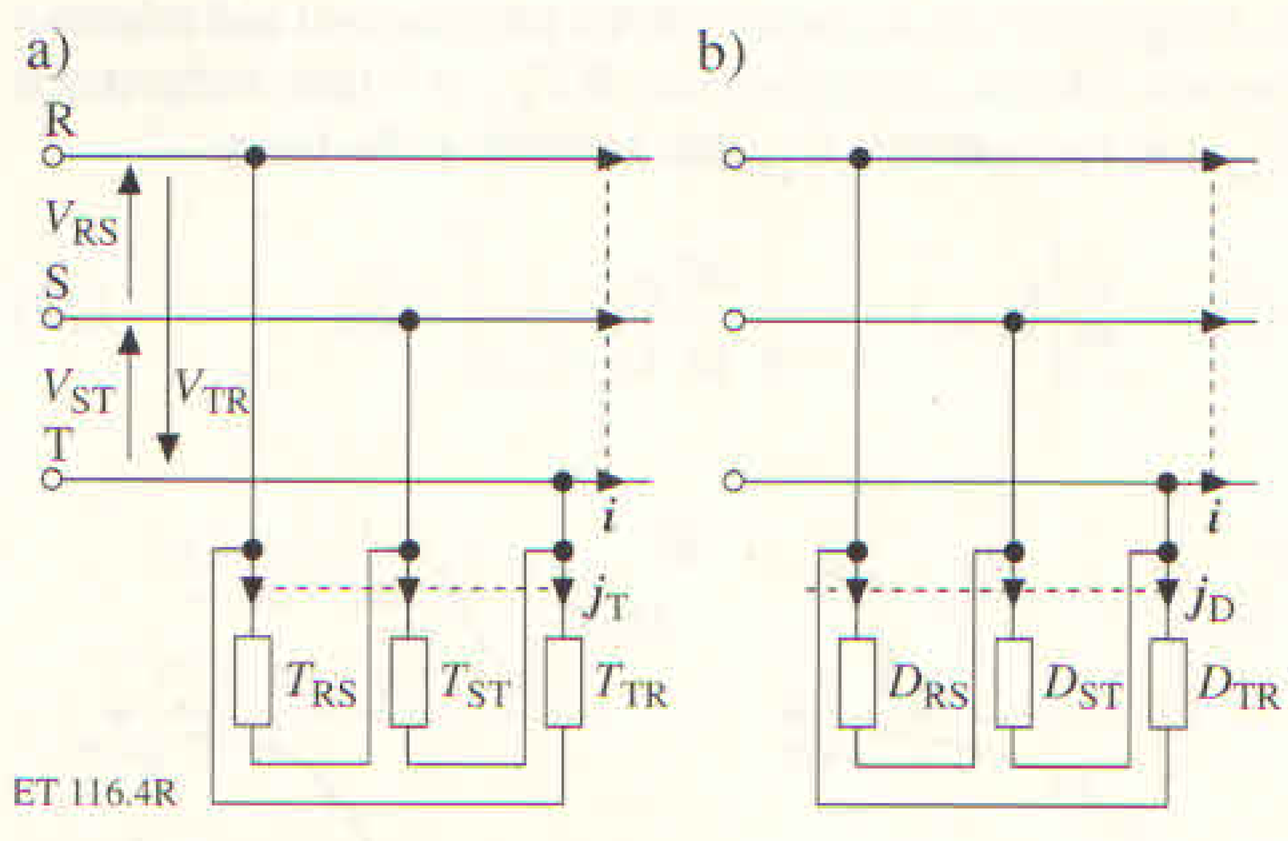


Fig. 4. BaCo circuit structures
a) Phase susceptances T_{XY}
b) Phase susceptances D_{XY}

As it is composed of three groups of terms of the form

$$d_{XY}^2 := \sum_{n\in N} |T_{XYn} - D_{XYn}|^2 V_{XYn}^2 \tag{28}$$

of positive value, therefore, distance d is minimum on the condition that each term d_{XY} is minimum. Observe, that susceptance T_{XY_n} in eq. (28) depends only on the load unbalanced admittance $\underline{A}_n$ and on the load equivalent susceptance B_{en} , while the susceptance D_{XY_n} depends only on the chosen structure and parameters of the BaCo circuit. Therefore, eq. (28) may be considered as a starting point for the BaCo circuit design.

If the difference $T_{XY_n} - D_{XY_n}$ is not equal to zero for each $n \in N$, then the distance d_{XY} depends not only on susceptance D_{XY_n}, but also on the voltage RMS value V_{XY_n} which changes with the source current, i. e., with the BaCo circuit parameters. None the less, if they are chosen in such a way that any resonance cannot occur in the circuit, then the relative changes of the voltage RMS value V_{XY_n} are in practical situations much smaller than relative changes of the source current, since the source impedance stands usually for only a few per cent of the load impedance. Therefore, one may expect that minimization of distance d_{XY} at the assumption that V_{XY} = const, should not involve any substantial error in the minimization result.

To eliminate the treat of a resonance amplification of the source harmonics by the series resonance of the BaCo circuit with usually inductive impedance of the power system, the branches of this circuit should have an inductive impedance for harmonic frequencies.

The simplest branches of this kind are an L-branch (**Fig. 5a**) and a series LC-branch (**Fig. 5b**), with sufficiently large inductance L.

In practical situations, for $n > 1$, $V_{XYn}^2 \ll V_{XY1}^2$. Therefore, the difference $T_{XY1} - D_{XY1}$ contributes mainly to the distance d_{XY}, so that its reduction requires that susceptance D_{XY1} is of the same sign as susceptance T_{XY1}. Hence, if $T_{XY1} < 0$, then an L-type branch should be chosen as the BaCo circuit branch, and if $T_{XY1} > 0$, then an LC-type branch should be chosen. In the first case, if $T_{XY1} < 0$, the inductance L_{XY} that minimizes the form

$$d_{XY}^2 := \sum_{n\in N} \left| T_{XYn} + \frac{1}{n\omega_1 L_{XY}} \right|^2 V_{XYn}^2 \tag{29}$$

is an optimum inductance for the unbalanced and reactive current RMS value reduction. If $T_{XY1} > 0$ then inductance L_{XY} and capacitance C_{XY} that minimize the form

$$d_{XY}^2 := \sum_{n\in N} \left| T_{XYn} + \frac{nC_{XY}}{1 - n^2\omega_1^2 L_{XY}C_{XY}} \right|^2 V_{XYn}^2 \tag{30}$$

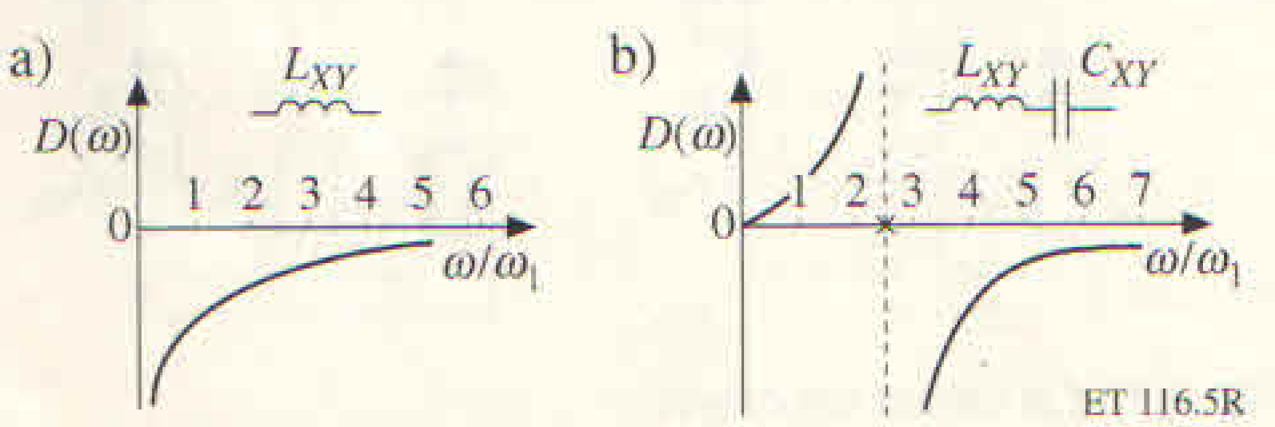


Fig. 5. Branches of the BaCo circuit
a) L-branch b) LC-branch

are the optimum parameters of the BaCo circuit. Comparing this formula with eq. (3) in [13], it occurs that minimization of the distance d_{XY} is equivalent to minimization of the source current RMS value, discussed in [13]. The only difference is that the term T_{XY_n} stands for the phase susceptance of an BaCo circuit that entirely compensates the unbalanced and reactive currents, but not for the load susceptance.

Since the load impedance is usually at least by one order higher then the source impedance, it may be assumed that the resonance frequency of the LC-branch is approximately equal to

$$1/\sqrt{(L_s + L_{XY})C_{XY}} := \omega_r \tag{31}$$

where L_s is the source internal inductance. To eliminate a threat of a resonance for harmonic frequencies, inductance L_{XY} should have at least such a value that

$$\omega_r < n_c\omega_1 \tag{32}$$

where n_c denotes the order of the lowest characteristic harmonic. The efficiency of the distance d_{XY} reduction increases with the the inductance increase, i. e., with the resonance frequency ω_r reduction, similarly as it was discussed in [13], regarding the current RMS value minimization, however, at the price of compensator ratings and its cost increase. Therefore, the choice of inductance is not only a technical matter, but also a matter of a trade off between the BaCo circuit efficiency and its cost. Discussion of this trade off is out of the scope of this paper, however. The method is illustrated on the following example.

– *Example:*

The RMS values of voltage source harmonics at phase R in the circuit, shown in **Fig. 6**, are equal to

$E_1 = 100$ V, $E_5 = 0.1\,E_1$, $E_7 = 0.05\,E_1$.

The source was assumed to be symmetrical and the frequency ω_1 was normalized to $\omega_1 = 1$ rad/s. At such assumptions, a circuit analysis program provides the following results for the not compensated circuit:

$$\underline{I}_1 = \begin{bmatrix} 45.3e^{j44^\circ} \\ 81.0e^{-j147^\circ} \\ 37.5e^{j20^\circ} \end{bmatrix} \text{A},\ \underline{I}_5 = \begin{bmatrix} 5.1e^{-j95^\circ} \\ 3.8e^{j68^\circ} \\ 1.9e^{j123^\circ} \end{bmatrix} \text{A},\ \underline{I}_7 = \begin{bmatrix} 1.7e^{-j75^\circ} \\ 1.3e^{j123^\circ} \\ 0.6e^{j66^\circ} \end{bmatrix} \text{A}.$$

The phase voltages and currents RMS values are shown in Fig. 6. The source current RMS value is $\|\boldsymbol{i}\| = 100.3$ A. Its active, reactive, scattered and unbalanced components have RMS values:

$\|\boldsymbol{i}_a\| = 56.3$ A, $\|\boldsymbol{i}_r\| = 25.2$ A, $\|\boldsymbol{i}_s\| = 5.7$ A, $\|\boldsymbol{i}_u\| = 78.8$ A.

Particular powers are equal to

$P = 9.8$ kW, $Q_r = 4.4$ kVA, $D_s = 1.0$ kVA,
$Q_u = 13.7$ kVA, $S = 17.5$ kVA

and the power factor has the value

$\lambda = P/S = 0.56$.

Having the complex RMS values of the phase voltages and currents harmonics $\underline{U}_{Xn}$ and $\underline{I}_{Xn}$, the equivalent admittances $\underline{Y}_{en}$ and unbalanced admittances $\underline{A}_n$ of the load can be calculated, namely

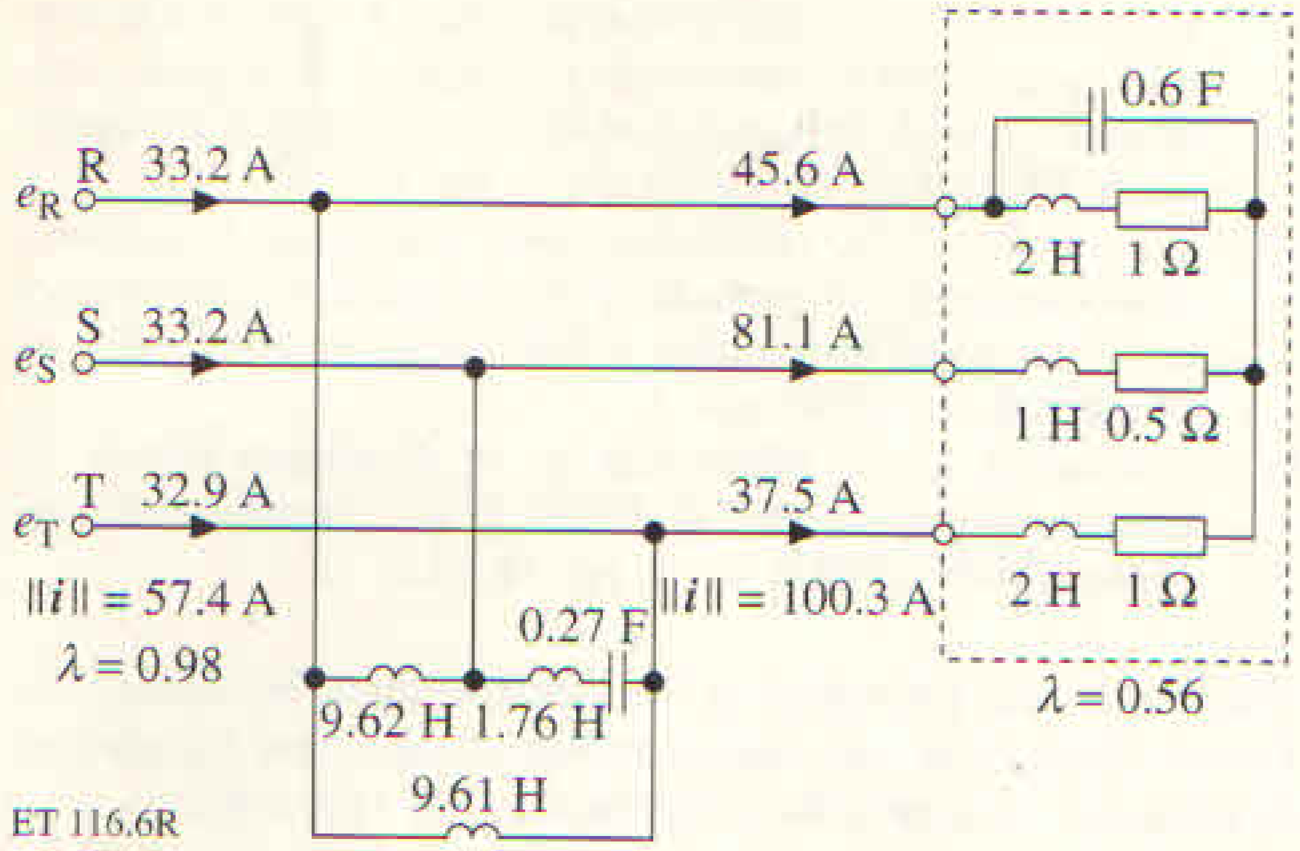


Fig. 6. Unbalanced load with BaCo circuit

$\underline{Y}_{e1} = 0.358\ e^{-j23°}$ S, $\underline{Y}_{e5} = 0.322\ e^{-j83°}$ S, $\underline{Y}_{e7} = 0.221\ e^{-j85°}$ S;

$\underline{A}_1 = 0.455\ e^{j90°}$ S, $\underline{A}_5 = 0.198\ e^{-j112°}$ S, $\underline{A}_7 = 0.132\ e^{-j56°}$ S.

This enables us to calculate the susceptances T_{XYn} of the BaCo circuit for a total compensation, namely

$T_{RS1} = 0.104$ S, $T_{RS5} = 0.221$ S, $T_{TR7} = 0.153$ S;

$T_{ST1} = 0.350$ S, $T_{ST5} = 0.014$ S, $T_{ST7} = 0.001$ S;

$T_{TR1} = 0.104$ S, $T_{TR5} = 0.125$ S, $T_{TR7} = 0.068$ S.

Because T_{RS1} and T_{TR1} are negative, while T_{ST1} is positive, the *L*-type branches are chosen as RS and TR branches of the BaCo circuit and *LC*-type as ST branch. It was assumed that the series resonance of this branch is at 1/3 of the 5th harmonic frequency. Eqs. (29) and (30) result in the following BaCo circuit parameters

$L_{RS} = 9.62$ H, $L_{ST} = 1.76$ H, $C_{ST} = 0.217$ F, $L_{TR} = 9.61$ H.

With these parameters the BaCo circuit modifies the phase currents harmonics to the values

$$\underline{I}_1 = \begin{bmatrix} 32.7e^{j0°} \\ 32.7e^{-j120°} \\ 32.7e^{j120°} \end{bmatrix} \text{A},\ \underline{I}_5 = \begin{bmatrix} 5.7e^{-j94°} \\ 5.4e^{j45°} \\ 3.9e^{j151°} \end{bmatrix} \text{A},\ \underline{I}_7 = \begin{bmatrix} 1.9e^{-j77°} \\ 1.9e^{j142°} \\ 1.3e^{j33°} \end{bmatrix} \text{A}$$

and their RMS values to those shown in Fig. 6. The unbalanced admittances are modified to

$\underline{A}_1 = 0$, $\underline{A}_5 = 0.11 \cdot e^{-j134°}$ S, $\underline{A}_7 = 0.08 \cdot e^{-j28°}$ S.

It reduces the source current RMS value from $\|\boldsymbol{i}\| = 103.3$ A to $\|\boldsymbol{i}\| = 57.4$ A, and modifies the reactive and unbalanced components RMS values to

$\|\boldsymbol{i}_r\| = 9.0$ A, $\|\boldsymbol{i}_u\| = 2.8$ A

without affecting the active and scattered currents. Particular powers are modified to

$Q_r = 1.6$ kvar, $Q_u = 0.3$ kVA, $S = 9.5$ kVA

and the power factor is improved to

$\lambda = P/S = 0.98$.

Taking into account how strongly asymmetrical the load is in the example considered, the results of symmetrization and improvement of the power factor with relatively simple BaCo circuit obtained show that the method suggested may be really effective.

4 Conclusions

Determining the load asymmetry at non-sinusoidal voltage by means of unbalanced admittances $\underline{A}_n$ enables us not only to express quantitatively the source overloading caused by this asymmetry, but also their values provide the input data for balancing circuit design. Such a circuit, even at a low complexity, may minimize effectively the unbalanced current. If equivalent susceptances B_{en} of the load are known, then the unbalanced current minimization may be integrated with minimization of the reactive current $\boldsymbol{i}_r$ as well. This means that the unbalanced admittances and equivalent susceptances can be considered as very useful parameters of the load.

They can be determined based on a harmonic analysis of the phase voltages and currents, that provides the complex RMS values of the voltage and current harmonics.

5 List of symbols

i_R, i_S, i_T	phase currents
$\boldsymbol{i}, \|\boldsymbol{i}\|$	current vector $[i_R, i_S, i_T]^T$ and its RMS value
n	harmonic order
N	set of the voltage and current harmonic orders n
n_c	order of the lowest significant harmonic
I_{Rn}, I_{Sn}, I_{Tn}	RMS values of phase currents harmonics
$\underline{I}_{Rn}, \underline{I}_{Sn}, \underline{I}_{Tn}$	complex RMS values of phase currents harmonics
$\boldsymbol{i}_a, \boldsymbol{i}_s, \boldsymbol{i}_r, \boldsymbol{i}_u$	vectors of active, scattered, reactive, and unbalanced currents
K	number of harmonics
M	number of BaCo circuit components
$\underline{U}_{Rn}, \underline{U}_{Sn}, \underline{U}_{Tn}$	complex RMS values of phase voltages harmonics
$\boldsymbol{U}_n$	vector of complex RMS values $[\underline{U}_{Rn}, \underline{U}_{Sn}, \underline{U}_{Tn}]^T$
$\underline{A}_{Rn}, \underline{A}_{Sn}, \underline{A}_{Tn}$	unbalanced admittances of phases R, S, and T
A_n	magnitude of unbalanced admittance $\underline{A}_n$
$\underline{\boldsymbol{A}}_n$	diagonal matrix built of admittances $\underline{A}_{Rn}, \underline{A}_{Sn}$, and $\underline{A}_{Tn}$
$\|\boldsymbol{u}_n\|$	RMS value of the vector of line voltage harmonics
$\underline{Y}_{XYn}$	phase X to phase Y load admittance for n-order harmonic
$\underline{A}^+, \underline{A}^-$	unbalanced admittances for a positive and negative sequence harmonic
$\underline{\alpha} = 1 \cdot e^{j2\pi/3}$	turn coefficient
s	sequence index
$\underline{\beta} = 1 \cdot e^{-js2\pi/3}$	sequence turn coefficient
$\underline{I}_{Xnu}$	complex RMS value of unbalanced current harmonic in phase X = R, S, or T

$\underline{S}_n$	complex apparent power of n order harmonic
$\underline{Y}_{en}$, G_{en}, B_{en}	equivalent admittance, conductance and susceptance of the load for n-order harmonic
T_{XYn}	phase X to phase Y susceptances of BaCo circuits which compensates unbalance current entirely
$\boldsymbol{T}_n$	diagonal matrix of susceptances T_{XYn}
$\boldsymbol{j}_T$	vector of phase to phase currents of BaCo circuits built of susceptances T_{XYn}
V_{XYn}, $\underline{V}_{XYn}$	phase X to phase Y voltage harmonic RMS and complex RMS value
$\boldsymbol{V}_n$	vector of harmonic complex RMS values $\underline{V}_{XYn}$
D_{XYn}	phase X to phase Y susceptances of BaCo circuits which minimises the unbalance current
$\boldsymbol{D}_n$	diagonal matrix of susceptances D_{XYn}
$\boldsymbol{j}_D$	vector of phase to phase currents of BaCo circuits built of susceptances D_{XYn}
d	distance of vectors $\boldsymbol{j}_T$, $\boldsymbol{j}_D$
L_{XY}, C_{XY}, L_s	inductance and capacitance of XY branch of the BaCo circuit and the supply source inductance
ω_r, ω_1	angular resonance frequency of an LC-branch and fundamental frequency
E_n	RMS value of the source internal voltage harmonic
P, S	active and apparent powers
Q_r, D_s, Q_u	reactive, scattered, and unbalanced powers

References

[1] *Lyon, W. V.:* Reactive power and and unbalanced circuits. Electr. World (1920) pp. 1419–1425

[2] *Bucholz, F.:* Die Drehstrom-Scheinleistung bei ungleichmäßiger Belastung der drei Zweige. Licht und Kraft, Org. Elektrotech. Ver. München (1922) no. 2, pp. 9–113

[3] *Quade, W.:* Zusammensetzung der Wirk-, Blind- und Scheinleistung bei Wechselströmen beliebiger Kurvenform und neue Leistungsdefinitionen für unsymmetrische Mehrphasensysteme beliebiger Kurvenform. ETZ Elektrotech. Z. 58 (1937) pp. 1312–1332

[4] *Nedelcu, V. N.:* Die einheitliche Leistungstheorie der unsymmetrischen und mehrwelligen Mehrphasensysteme. ETZ-A Elektrotech. Z. 84 (1963) no. 5, pp. 153-157

[5] *Nowomiejski, Z.; Cichowska, Z.:* Unbalanced three-phase systems. Sci. Letters "Elektryka" (1964) no. 17, pp. 25–76 (in Polish)

[6] *Harms, G.:* Blindleistung im Energieeinsatz. Elektrotech. 62 (1980) no. 6, pp. 20–22

[7] *Klinger, G.:* L-C Kompensation und Symmetrierung für Mehrphasensysteme mit beliebigem Spannungsverlauf. etzArchiv 1 (1979) no. 2, pp. 57–61

[8] *Mayer, D.; Pav, M.:* A comment on nonlinear 3-phase system compensation. Electr. Obzor 77 (1988) no. 2, pp. 75–77

[9] IEEE Standard Dictionary of Electrical and Electronics Terms (Editor: F. Jay). New York/USA: The Inst. of Electr. and Electron. Eng. Inc., 1988

[10] *Czarnecki, L. S.:* What is wrong with the Budeanu concept of reactive and distortion power and why it should be abandoned. IEEE Trans. Instrumentat. a. Meas. IM-36 (1987) no. 3, pp. 834–837

[11] *Czarnecki, L. S.:* Orthogonal decomposition of the current in a 3-phase nonlinear asymmetrical circuit with non-sinusoidal voltage source. IEEE Trans. Instrumentat. a. Meas. IM-37 (1988) no. 1, pp. 30–34

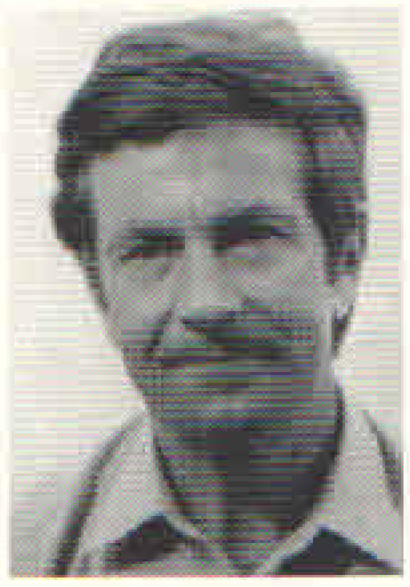
[12] *Czarnecki, L. S.:* Reactive and unbalanced currents compensation in 3-phase asymmetrical circuit under non-sinusoidal conditions. IEEE Trans. Instrumentat. a. Meas. IM-38 (1989) no. 3

[13] *Czarnecki, L. S.:* Minimization of distortion power of nonsinusoidal situations. IEEE Trans. Instrumentat. a. Meas. IM-36 (1987) no. 1, pp. 18–22

This paper was presented at the "International Workshop on Power Definitions and Measurements under Non-Sinusoidal Conditions" in Como/Italy (September 10–12, 1991)

Manuscript received on February 27, 1992

The Author



Leszek S. Czarnecki (1939) received the M. Sc. and Ph. D. degrees in electrical engineering, and the D. Sc. degree from the Silesian Technical University, Gliwice/Poland in 1963, 1969 and 1984, respectively, where he was employed as an Assistant Professor. Since 1984 he worked for two years at the Power Engineering Section, Division of Electrical Engineering, National Research Council of Canada as a Visiting Research Officer. In 1987 he joined the Electrical Engineering Dept. at Zielona Gora Technical University. He is currently working as an Associate Professor in the Electrical and Computer Engineering at the Louisiana State University (LSU), Baton Rouge/USA. His research interests include network analysis, synthesis, power flow in non-sinusoidal systems and power electronics. (Electrical and Computer Eng., Louisiana State University, Baton Rouge/USA, Louisiana 70803, T + 15 04/3 88-52 39, Fax + 15 04/3 88-52 00)

Discussion

J. L. Willems (Univ. of Gent/Belgium):

I do have a remark with respect to the criterion, given by eq. (25), expressing the quality of the compensating circuit. The ideal compensator is indeed such that $\boldsymbol{j}_D$ equals $\boldsymbol{j}_T$. However, since the objective of the compensating circuit is to realize a line current $\boldsymbol{i}$ which is as small as possible, the optimization criterion for the circuit (given that the ideal one is not realizable because of restrictions on the number of elements) should be: minimize $\|\boldsymbol{i}\|$, or minimize $\|\boldsymbol{i}_r\|^2 + \|\boldsymbol{i}_u\|^2$ (since $\boldsymbol{i}_a$ and $\boldsymbol{i}_s$ are not affected by the compensation). The primary purpose of the compensation is indeed *not* to make $\boldsymbol{j}_D$ as close to $\boldsymbol{j}_T$ as possible. However, I think that because of the orthogonality both objectives lead to the same result. Nevertheless it is my opinion that the other problem statement is to be preferred for conceptual reasons.

Reply of L. S. Czarnecki:

Prof. *J. L. Willems* is right that there is a reasoning gap between the primary purpose of the source current RMS value i minimization and making the current j_D as close to j_T as possible. This was an intuitive assumption rather than a proven equivalence. Numerical analysis seems to confirm the validity of such an assumption, but it would be better to prove it formally.

A. Ferrero, G. Superti Furga (Politech. of Milano/Italy):

We are of the opinion that Prof. *L. S. Czarnecki* should be commended for his work concerned with the definitions of power components under non-sinusoidal conditions. In his papers [D1] and [D2] he achieves the important result of transferring the time-domain based approach proposed by *Fryze* to the frequency domain and finds a rigorous physical meaning to all defined quantities.

Although the extension of his theory to three-phase systems is not yet completed, we do think that the frequency-domain decomposition of currents proposed by *L. S. Czarnecki* is determining in understanding the behaviour of the different loads under non-sinusoidal conditions.

Indeed, we are of the opinion that this theoretical aspect represents the most attractive part of the whole *Czarnecki*'s work, while the parts concerned with the practical problems of power compensation give rise to some doubts about their practical utility.

In this paper, as well as in other recent works [D3] and [D4], the Author claims that some of the current components (namely the reactive current, the unbalanced current and the scattered current) in which he decomposes the line current can be completely compensated (or at least minimized) by means of a proper network of passive reactive elements.

However, the following conditions must be satisfied to achieve this goal:

- The defined current components must be known (and hence accurately measured).
- The compensating network must be realized.

Putting into practice these statements is not so easy as the Author claims because of some drawbacks that we are going to summarize in the following part of our discussion, in order to submit them to the Author's consideration.

I. As far as the measurement of the current components is concerned, the frequency-domain analysis is the only way they can be determined. The modern digital instrumentation allows real-time measurements of the frequency-domain components [D5].
However, our experience in using such techniques suggests that the result of such measurements can be misleading when they are applied to the determination of the current components defined by the Author in fact, the signal frequency components obtained employing Fast-Fourier-Transformation (FFT) algorithms are affected by both the short and long range leakage errors.
These errors can be reduced employing suitable tapered windows, but in most cases it still remains impossible to ascertain whether a small amplitude component is actually present in the signal spectrum or it is due to a measurement error. On the other hand, small amplitude harmonic components can be masked by the side lobes of the employed windows. In order to focus the consequences of this ambiguity, suppose that the voltage components $\boldsymbol{u}_n$ has small amplitude and suppose that the current component $\boldsymbol{i}_n$ is present too with non negligible amplitude. In this case there is no way to decide whether the current component $\boldsymbol{i}_n$ is due to load non-linearity (that means that the measured voltage component $\boldsymbol{u}_n$ was supposed to be a measurement error) or to a resonance at harmonic n (that means that the measured voltage component $\boldsymbol{u}_n$ was supposed to be an actual harmonic component of voltage $\boldsymbol{u}$).
Quite similar problems arise in the measurement of the sign of the harmonic active powers P_n, which is necessary to distinguish harmonics generated in the load non-linearity and/or time-variance in the case of non-zero source impedance (that represents the practical case).
The above mentioned problems can be overcome if a coherent sampling technique is adopted [D6]. The drawback of this approach is that a good synchronization of the sampling frequency with the signal fundamental frequency can be attained only in steady state conditions. However, this is not the case of some practical applications (i. e. arc furnaces) whose normal working condition is the transient condition.

II. The practical realization of a passive, reactive element 1-port network represents, in our opinion, a further critical point of *Czarnecki*'s approach.
In his papers he proves that some of the defined current components can be compensated by means of such a network. He also gives some examples, but he refers always to ideal reactive elements.
However, the actual equivalent circuit of the reactive elements employed for the practical realization of the compensating network is rather different from the ideal one. For example, the equivalent circuit of the ideal inductor in **Fig. D1a** is shown in **Fig. D1b**. We think that in practical applications employing power components, the effect of the parasitic elements can no longer be considered negligible. Did the Author never consider such effects on the behaviour of the compensating network ?

In conclusion, we think that the applications of *Czarnecki*'s theory to the minimization of the apparent

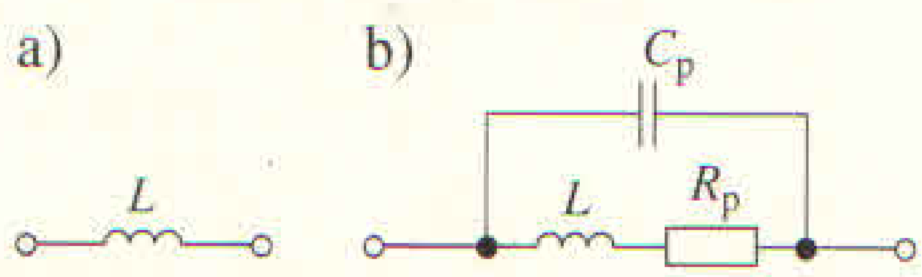


Fig. D1. Equivalent circuits
a) Ideal inductor b) Real inductor

power under practical circumstances is critical and probably less effective (both from a technical and an economical point of view) than other methods, like those employing active filters.

On the other hand, we still remain firmly persuaded that these drawbacks does not lessen the real power of *Czarnecki*'s theory which actually lays in the theoretical approach to the non-sinusoidal situation. The knowledge that a particular network is able to generate a single current component is extremely important from the theoretical point of view, since it allows to assess the behaviour of the different loads.

References

[D1] *Czarnecki, L. S.:* Consideration on the Reactive Power in Nonsinusoidal Situations. IEEE Trans. Instrumentat. a. Meas. IM-34 (1985) pp. 399–404

[D2] *Czarnecki, L. S.:* Orthogonal Decomposition of the Currents in a Three-phase Nonlinear Asymmetrical Circuit with a Nonsinusoidal Voltage Source. IEEE Trans. Instrumentat. a. Meas. IM-37 (1988) pp. 30–34

[D3] *Czarnecki, L. S.:* Reactive and Unbalanced Currents Compensation in Three-Phase Asymmetrical Circuits under Nonsinusoidal Conditions. IEEE Trans. Instrumentat. a. Meas. IM-38 (1989) pp. 754–759

[D4] *Czarnecki, L. S.:* Scattered and Reactive Current, Voltage and Power in Circuits with Nonsinusoidal Waveforms and their Compensation. IEEE Trans. Instrumentat. a. Meas. IM-40 (1991) pp. 563–567

[D5] *Bertocco, M.; Offelli, C.; Petri, D.:* Numerical Algorithm for Power Measurements. Int. Workshop on Power Definitions a. Measurements under Non-Sinusoidal Conditions, Como/Italy 1991, Proc. pp. 171–199

[D6] *Ferrero, A.; Ottoboni, R.:* A Low-Cost Frequency Multiplier for Synchronous Sampling of Periodic Signals. IEEE Instrumentat. a. Meas. Technol. Conf. (IMTC) Atlanta, Georgia/USA 1991, Proc. pp. 247–250

Reply of L. S. Czarnecki:

I appreciate very much a high opinion on my work concerning power definitions, expressed by *A. Ferrero* and *G. Superti Furga* in their comment. However, I do not share their deep concern with regard to the effects of the measurement and compensator parameters accuracy on the compensation results.

They are absolutely right that the compensation or minimization of some components of the load current requires that these components are accessible for measurement and a compensating circuit is realized. This is true, however, for any method of the compensation which was developed in the past or will be developed in the future. Measurement accuracy, the errors involved in signal processing and parasitic parameters of the compensator components will always affect the efficiency of the compensation, independently of theoretical fundamentals employed for the compensator design. They will affect the performance of the compensator developed with the time-, or with frequency-domain approach.

For example, if the voltage and current samples are used for generating the control signal of the compensator or for this compensator design and the sampling rate is lower than the Nyquist frequency, then some information on the voltage and current waveform is lost, independently how this signal is generated, in the time-, or in frequency-domain. Also, an error occurs at the RMS value or the active power calculation both in time- and in frequency-domain at the lack of sampling synchronization.

Hopefully, the total compensation or exactly optimum minimization are only theoretical goals. The practical goals are rather limited to reduction of the current RMS value and its distortion to an economically justified level. Compensation of the minute components of the current, even of possible, may not be economically justified.

It should be noted that contrary to *Budeanu*'s and *Fryze*'s power theories which waited for more than 30 years until the first meters of particular powers or current components were built, all quantities suggested in my theory are measurable now, only the measurement accuracy is affected by the development in signal acquisition and processing.