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Energy flow and power phenomena in electrical circuits: illusions and reality

L. S. Czarnecki

Contents Common opinions respective to the nature of the reactive power, energy flow and oscillations, as well as the notion of the apparent power in single- and in three-phase systems are discussed in this paper. It is shown that some interpretations of powers and energy flow in linear, single-phase circuits are often generalized for more complex situations where these interpretations are not longer valid. Consequently, power phenomena in electrical systems are often misinterpreted. This relates to the reactive power which occurs in three-phase systems without energy oscillation between the supply source and the load, as well as it occurs in time-variant systems without energy storage capability. Also, it was demonstrated in the paper that the arithmetic and geometric apparent powers, commonly used in three-phase systems, do not characterize the supply loading correctly when the load is unbalanced.

Energiefluß- und Leistungs-Phänomene in elektrischen Schaltungen – Illusionen und Wirklichkeit

Übersicht In dem Beitrag werden allgemeine Ansichten über die Natur von Blindleistung, Energiefluß und Schwingungen sowie der Bergriff der Scheinleistung in einphasigen und dreiphasigen Systemen diskutiert. Es wird gezeigt, daß einige Interpretationen der Leistung und des Energieflusses in linearen einphasigen Schaltungen oft für komplexere Fälle verallgemeinert werden, wo diese nicht mehr gelten. Folglich werden Leistungs-Phänomene in elektrischen Systemen häufig falsch interpretiert. Dies bezieht sich auf die Blindleistung in dreiphasigen Systemen ohne Energieschwingungen zwischen der speisenden Quelle und der Last sowie auf auf zeitvariante Systemen ohne Energiespeicher. Es wird auch gezeight, daß die gewöhnlich in dreiphasigen Systemen verwendete arithmetische und geometrische Scheinleistung bei Schieflast die Belastung der Quelle nicht richtig wiedergibt.

1 Introduction

After more than a century of dealing with energy flow and electric powers, one might take it for granted that electrical engineers cannot be confused regarding power phenomena in electrical circuits. Unfortunately, though electric energy has become 'blood and muscles' of our technical civilization, numerous publications and discussions show that the relation between energy flow and powers is often misinterpreted and power phenomena in electrical circuits are not well comprehended. Some opinions, for example, that reactive power occurs because of energy oscillation between the source and the load, are consolidated by decades of university teaching to such a degree that even the question whether this is really true, may seem like a heresy. Also, it is the authors experience that electrical engineers when asked "what is the apparent power S in three phase circuits?" usually answer, that this is a quantity calculated as

$$S = U_R I_R + U_S I_S + U_T I_T \ , \qquad (1)$$

where U and I with indices R, S and T denote the RMS values of the voltage and current at the load terminals. Some of them will answer that this is the value calculated from the formula

$$S = \sqrt{P^2 + Q^2} \ , \qquad (2)$$

where P and Q stand for the active and reactive powers of the load. It does not mean that the right answer for some engineers is not known. As far as in 1922 Buchholz defined [1] the apparent power S in three-phase systems, as

$$S = \sqrt{I_R^2 + I_S^2 + I_T^2}\sqrt{U_R^2 + U_S^2 + U_T^2} \ , \qquad (3)$$

but it seems that it is still unclear for electrical engineers why this definition is better than the former ones, especially that definitions (1) and (2), introduced in Ref. [3] are supported by the IEEE Standard [4], while in that Standard the Buchholz's definition is not even mentioned. It is rather unknown in the electrical engineers community.

Also, as long ago as in 1931, Fryze warned [2] that oscillation of energy between a load and a supply source could be only apparent. A conviction that there is such an oscillation could be only an effect of a misuse of the Fourier series. Fryze supported this conclusion with considerations on power phenomena in the circuit shown in Fig. 1. The product of the Fourier series of the voltage and current at the supply terminals contains an infinite number of sinusoidal terms, and each of them can be interpreted as an oscillating component of energy flow. But there is no flow of energy between the load and the supply in this circuit, since the voltage and current product is equal to zero. There is an exchange of the energy stored in

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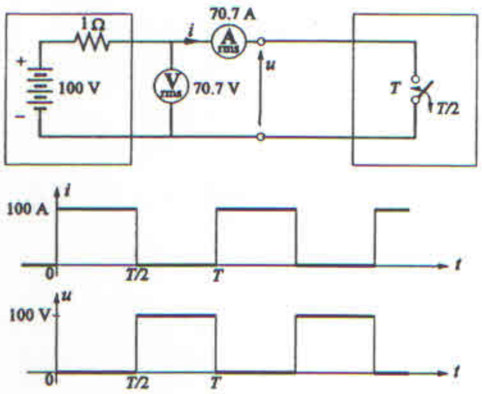

Fig. 1. Example of a circuit without energy flow to the load but with a non zero apparent power

the magnetic field of the closed circuit since it has a stray inductance and energy stored in the electric field of the open circuit since it has a stray capacitance, thus there is an oscillation of energy at switching instants. These marginal oscillations do not account, of course, for the presence of the apparent power, actually equal to $S = 5$ kVA, in such a circuit. The voltage u in the absence of current and the current i in the absence of the voltage, that means in intervals when there is no energy flow, contribute to this apparent power.

The author of this paper is aware that some Readers may find the discussed questions and conclusions quite trivial, while other may reject them as an unacceptable heresy, and therefore, some very fundamental concepts related to the power phenomena in electrical circuits should and will be investigated in this paper. After more than a century of investigations and discussions on power phenomena and powers in electrical circuits, some solid conclusions should be drawn and some long lasting misconceptions should be abandoned. Our misinterpretations of power phenomena do not cause any malfunctions of electrical equipment. May be, only some systems are not properly optimized, since formulae (1), (2) and (3) may provide different values of the power factor, $\lambda = P/S$. Most important is that our misinterpretations do not propagate to the next generations of electrical engineers.

2 Does reactive power occur because of energy oscillation?

It seems that it would be difficult to find an electrical engineer who would not provide an affirmative answer to such a question. It is even easy to guess why there is a common opinion that the reactive power is related to the energy oscillation between the load and the source. This is because the very concept of the reactive power Q is explained very often using a linear, single-phase load supplied with a sinusoidal voltage. In such a circuit, at the load voltage equal to $u(t) = \sqrt{2}U\cos\omega_1 t$, and the load current $i(t) = \sqrt{2}I\cos(\omega_1 t - \varphi)$, the rate of energy $W(t)$ flow from the supply source to the load, that means, the instantaneous power $p(t)$, can be expressed in the form:

$$p(t) = \frac{\mathrm{d}}{\mathrm{d}t}W(t) = u(t)i(t) = P(1 + \cos 2\omega_1 t) + Q\sin 2\omega_1 t \ , \tag{4}$$

where P denotes the active power and

$$Q = UI\sin\varphi \ , \tag{5}$$

denotes the reactive power of the load.

The first term on the right side of formula (4), $P(1 + \cos 2\omega_1 t)$, represents an unidirectional component of the instantaneous power. It occurs even if the load is purely resistive which means that there is no energy oscillation between the source and the load. The second term, $Q\sin 2\omega_1 t$, represents a bidirectional, that means an oscillating component of the instantaneous power. The reactive power Q is the amplitude of this oscillating component. When the reactive power Q is not equal to zero, then there are intervals of time when energy flows from the load back to the supply source. This common interpretation is so convincing, that the reactive power is associated usually with the energy oscillation, irrespective of situation.

Difficulties begin when we attempt to apply this interpretation to three-phase loads. This is particularly visible when a balanced linear load is supplied with a symmetrical sinusoidal voltage in a three-wire system, where there is no energy oscillation between the load and the supply source. The load reactive power Q in such a situation, at voltages and currents denoted as it is shown in Fig. 2, is equal to

$$Q = 3U_R I_R \sin\varphi \tag{6}$$

Let us assume that the supply voltage is of a positive sequence, and the voltage at terminal R is equal to $u_R(t) = \sqrt{2}U_R\cos\omega_1 t$. The instantaneous power in a cross-section between the load and the supply source is constant in such a situation, since

$$\begin{aligned} p(t) &= \frac{\mathrm{d}}{\mathrm{d}t}W(t) = u_R(t)i_R(t) + u_S(t)i_S(t) + u_T(t)i_T(t) \\ &= 2U_R I_R\Big[\cos\omega_1 t\cos(\omega_1 t - \varphi) + \cos\left(\omega_1 t - \frac{2\pi}{3}\right) \\ &\quad \times\cos\left(\omega_1 t - \frac{2\pi}{3} - \varphi\right) + \cos\left(\omega_1 t + \frac{2\pi}{3}\right) \\ &\quad \times\cos\left(\omega_1 t + \frac{2\pi}{3} - \varphi\right)\Big] \\ &= 3U_R I_R\cos\varphi = \text{Const.} \end{aligned} \tag{7}$$

Thus, there is no energy oscillation between the load and the source, irrelevant of the reactive power Q of the load.

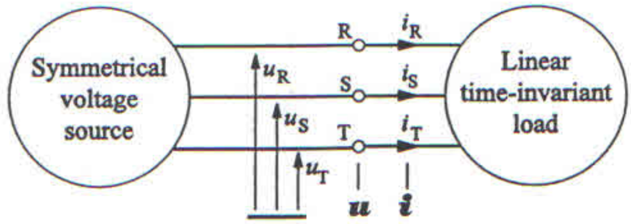

Fig. 2. Three-phase, three-wire circuit

This is true, of course, for any single harmonic of the n-th order. Namely, for the voltage and current harmonic of the n-th order, expressed in a form of three-phase vectors,

$$\mathbf{u}_n(t) = \begin{bmatrix} u_{Rn}(t) \\ u_{Sn}(t) \\ u_{Tn}(t) \end{bmatrix}, \quad \mathbf{i}_n(t) = \begin{bmatrix} i_{Rn}(t) \\ i_{Sn}(t) \\ i_{Tn}(t) \end{bmatrix} , \tag{8}$$

the instantaneous power of a single harmonic

$$p_n(t) = \mathbf{u}_n^{\mathrm{T}}(t)\mathbf{i}_n(t) = \text{Const.} , \tag{9}$$

where the upper index T denotes a transposed vector. The instantaneous power of the load $p(t)$ is not constant, however, when the voltage is distorted. It is enough that for a resistive balanced load, as this shown in Fig. 3, the supply voltage contains the fundamental and the second order harmonics, i.e., $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$, then the current vector can be expressed as

$$\mathbf{i} = \mathbf{i}_1 + \mathbf{i}_2 = G\mathbf{u}_1 + G\mathbf{u}_2 . \tag{10}$$

The instantaneous power is equal to

$$p(t) = (\mathbf{u}_1^{\mathrm{T}} + \mathbf{u}_2^{\mathrm{T}})(\mathbf{i}_1 + \mathbf{i}_2) = \mathbf{u}_1^{\mathrm{T}}\mathbf{i}_1 + \mathbf{u}_2^{\mathrm{T}}\mathbf{i}_2 + \mathbf{u}_1^{\mathrm{T}}\mathbf{i}_2 + \mathbf{u}_2^{\mathrm{T}}\mathbf{i}_1 = \text{Const} + 2G\mathbf{u}_1^{\mathrm{T}}\mathbf{u}_2 \tag{11}$$

Assuming, for simplicity sake, that $u_{R1}(t) = \sqrt{2}U_1 \cos\omega_1 t$ and $u_{R2}(t) = \sqrt{2}U_2 \cos 2\omega_1 t$, we obtain

$$\begin{aligned} \mathbf{u}_1^{\mathrm{T}}\mathbf{u}_2 &= u_{R1}(t)u_{R2}(t) + u_{S1}(t)u_{S2}(t) + u_{T1}(t)u_{T2}(t) \\ &= 2U_1U_2\Big[\cos\omega_1 t \cos 2\omega_1 t + \cos\Big(\omega_1 t - \frac{2\pi}{3}\Big) \\ &\quad \times \cos\Big(2\omega_1 t + \frac{2\pi}{3}\Big) + \cos\Big(\omega_1 t + \frac{2\pi}{3}\Big) \\ &\quad \times \cos\Big(2\omega_1 t - \frac{2\pi}{3}\Big)\Big] \\ &= 3U_1U_2 \cos 3\omega_1 t . \end{aligned} \tag{12}$$

Thus, the instantaneous power in the cross-section between the load and the source changes as

$$p(t) = \text{Const.} + 6GU_1U_2 \cos 3\omega_1 t . \tag{13}$$

One could interpret this result that an oscillating component occurs in the instantaneous power at the supply voltage distortion. Since the load is purely resistive, the power factor λ remains equal to unity, however, and the reactive power does not occur, in spite of the presence of such an oscillating component in the instantaneous power. As a matter of fact, in spite of the presence of the term $6GU_1U_2 \cos 3\omega_1 t$ in the instantaneous power $p(t)$, energy does not oscillate between the supply source and the load in the circuit considered, since the term

$$\text{Const.} = P = 3G(U_1^2 + U_2^2) , \tag{14}$$

and consequently, the instantaneous power $p(t)$ cannot be negative.

One could argue that even if the instantaneous power $p(t)$ is constant, there is an oscillation of energy in individual phases R, S and T which cancel mutually, since the products $u_R(t)i_R(t), u_S(t)i_S(t)$ and $u_T(t)i_T(t)$ do may have the oscillating components. Such an explanation raises a question, however, whether or not are these products an instantaneous power? Indeed, it is possible to find publications where these products are considered to be the instantaneous power of individual phases of three-phase systems.

To be the instantaneous power, the products $u_R(t)i_R(t)$, $u_S(t)i_S(t)$ and $u_T(t)i_T(t)$ have to be a derivative of energy of individual phases. Unfortunately, it may not be possible to separate a single phase from the three-phase system as a one-port and to specify its instantaneous power in terms of the voltage and current at its terminals. It can be done for some equivalent circuits but not for circuits with phases mutually coupled with magnetic or electric fields. Moreover, the voltages $u_R(t)$, $u_S(t)$ and $u_T(t)$, and consequently, the products $u_R(t)i_R(t)$, $u_S(t)i_S(t)$ and $u_T(t)i_T(t)$, depend on the reference point. Any point, even one of terminals R, S and T can be chosen as such a point, making the respective product $u_R(t)i_R(t)$, $u_S(t)i_S(t)$ or $u_T(t)i_T(t)$ equal to zero, without affecting the power phenomena in the circuit and the instantaneous power $p(t)$. Therefore, such voltage and current products for individual phases cannot be considered as the instantaneous power. Thus, the question of how to interpret the reactive power Q in terms of energy oscillation in three-phase systems remains without answer. It seems that the association of the reactive power with the energy oscillation is only a misinterpretation of power phenomena in three-phase systems.

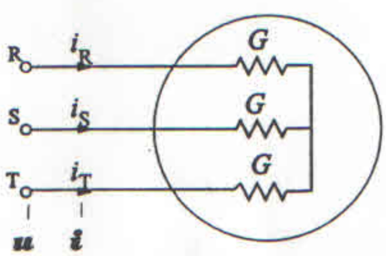

Fig. 3. Balanced, resistive, three-phase load

3 Does the reactive power occur because of energy storage?

Asking such a question, we also can expect an affirmative response. This is because the reactive power in single-phase circuits with sinusoidal voltages and currents is often interpreted in terms of the energy stored in the magnetic fields of inductors and in the electric fields of capacitors. Indeed, if the current of an inductor of inductance L changes as $i(t) = I_{\max} \sin\omega_1 t$, then the energy stored in the magnetic field, T, of the inductor changes as

$$T = \frac{1}{2}Li^2(t) = \frac{1}{2}LI_{\max}^2 \sin^2\omega_1 t = T_{\max}\sin^2\omega_1 t , \tag{15}$$

while the reactive power of such an inductor is equal to

$$Q = \omega_1 \frac{1}{2}LI_{\max}^2 = \omega_1 T_{\max} . \tag{16}$$

Similarly, if the voltage on a capacitor of capacitance C changes as $u(t) = U_{\max} \sin\omega_1 t$, then the energy stored in the electric field, V, of the capacitor changes as

$$V = \frac{1}{2}Cu^2(t) = \frac{1}{2}CU_{\max}^2 \sin^2 \omega_1 t = V_{\max} \sin^2 \omega_1 t \ , \tag{17}$$

while the reactive power of such a capacitor is equal to

$$Q = -\omega_1 \frac{1}{2} CU_{\max}^2 = -\omega_1 V_{\max} \ . \tag{18}$$

Thus, there is a direct relation between the reactive power and maximum value of the energy stored in the electric and in the magnetic fields, and consequently, a common conviction among electrical engineers that the reactive power is associated with the phenomenon of energy storage in magnetic and/or electric fields.

Unfortunately, it is enough to have a periodic switch in a circuit and the association of the reactive power with the energy storage in the load becomes questionable. The reactive power may occur in such a situation even in circuits that do not contain devices with energy storage capability, namely, even in a purely resistive circuit. A resistive load with a TRIAC controlled power, shown in Fig. 4, is a very common example of such a circuit.

Example 1. At the supply voltage $u(t) = 220\sqrt{2}\sin\omega_1 t$ [V], the load resistance $R = 2$ [Ω] and the TRIAC firing angle $\alpha = 135°$, the current RMS value in the circuit shown in Fig. 4 is equal to $\|i\| = 21.31$ [A]. The load active power has the value $P = R\|i\|^2 = 908$ [W] while the source apparent power is equal to $S = U\|i\| = 220\ 21.31 =$ 4.69 kVA, thus, the power factor of the load has the value of $\lambda = P/S = 0.19$. The fundamental harmonic of the supply current shown in Fig. 5 is equal to

$$i_1(t) = 13.0\sqrt{2}\sin(\omega_1 t - 60°) \text{ [A]} \ ,$$

thus, the supply source is loaded with the reactive power of

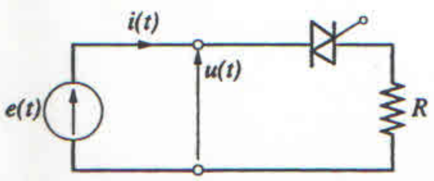

Fig. 4. Circuit with a TRIAC controlled load power

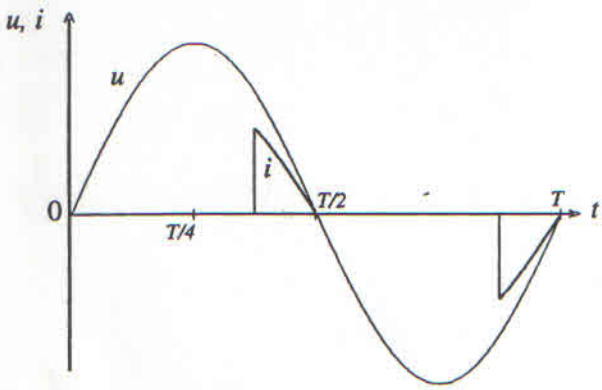

Fig. 5. Voltage and current waveforms in the circuit with the TRIAC controlled load power

$$Q_1 = UI_1 \sin\varphi_1 = 220\ 13.0 \sin 60° = 2.48 \text{ kVAr} \ ,$$

while there is no device that could be able to store energy in the load and send it back to the supply source. Moreover, since it was assumed that the supply voltage is sinusoidal, thus there is no reactive power in the circuit other than only the reactive power of the fundamental harmonic.

The reactive power Q_1 occurred at the lack of energy storage capability of the load, as well as at unidirectional flow of energy, since the instantaneous power $p(t)$ is non-negative. This means, that the common physical interpretations of the reactive power, valid in time-invariant, single-phase circuits, cannot be applied to three-phase circuits, as well as to time-variant circuits.

4 Does the power factor decline because of energy oscillations?

A probable affirmative response to such a question is a conclusion from the common believe that the reactive power, which reduces the power factor, is caused by energy oscillations, which as shown in previous sections, remains questionable. In the time-variant circuit discussed in Example 1, the power factor, defined as $\lambda = P/S$ is equal to $\lambda = 0.19$, in spite of the lack of energy oscillation. Also, the power factor of three-phase balanced systems is usually lower than unity without energy oscillations even at sinusoidal supply voltage. The power factor in three-phase systems will be discussed in a separate section, since the power factor is not univocally defined in such systems, so in this Section the discussion is confined to relation between energy oscillations and power factor in single-phase circuits.

The relationship between energy oscillations and power factor seems to be obvious in linear, time-invariant circuits. The increase in the amplitude of the energy oscillation between the supply source and the load in such circuits, at the constant active power, P, always contributes to the power factor, λ, decline. However, quite opposite effect can occur in circuits with time-variant parameters. Namely, an increase in energy oscillation between the source and the load can reduce the supply current RMS value, and consequently, improve the power factor, λ.

Example 1 (continuation). The reactive power in the circuit shown in Fig. 4 can be compensated by a capacitor connected as shown in Fig. 6. For the conditions assumed in Example 1, at the total compensation of the reactive power of the value $Q_1 = 2.48$ kVAr, the supply current fundamental harmonic is reduced to

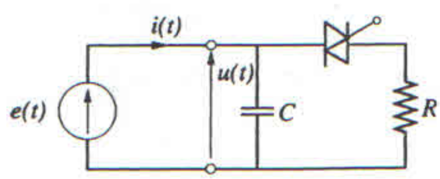

Fig. 6. Circuit with a TRIAC controlled load power and with a capacitor for the reactive power compensation

$$i_1(t) = 6.5\sqrt{2}\sin\omega_1 t \text{ [A]}$$

while other current harmonics remain unchanged. This reduces the supply current RMS value to $\|i\| = 18.1$ [A] and improves the power factor from the value of $\lambda = 0.19$ to $\lambda = 0.23$. It is important to note, that the power factor improves in a situation where there are intervals of time, as shown in Fig. 7, where the instantaneous power $p(t) = u(t)i(t)$ is negative, thus the energy oscillates between the compensated load and the source. Hence, energy oscillations improve the power factor in the situation considered.

Thus, there is no universally valid relation between energy oscillation and the power factor. This is because there is no universally valid relation between the instantaneous power $p(t)$ and the apparent power S. Only for linear, time-invariant circuits there is a pair of relations:

$$p(t) = P(1 - \cos 2\omega_1 t) + Q\sin 2\omega_1 t \tag{19}$$

$$S^2 = P^2 + Q^2 \tag{20}$$

that might suggest such a relationship. Unfortunately, this kind of relationship between the instantaneous power $p(t)$ and the apparent power S cannot be specified for time-variant or three-phase loads. Linear, time-invariant single-phase loads are only a particular kind of loads where such a relationship exists. The lack of any physical interpretation of the apparent power S, that means, the lack of any relationship between this power and physical phenomena in the circuit is one of reasons for that. This is, however, an issue where also there is no common consensus between electrical engineers. Some engineers claim that the apparent power has a clear physical interpretation.

5 Does the apparent power S have any physical interpretation?

The apparent power S is one of the main power quantities in electrical engineering. Various power equipment, for example, transformers or breakers are rated in terms of their apparent power. The supply capability of distribution systems are specified in terms of their short-circuit power, which also is an apparent power. Therefore, the question regarding the physical interpretation of the apparent power is a very natural question for electrical engineers. If the apparent power S is such an important quantity, then what physical phenomenon in electric circuits is characterized by this power?

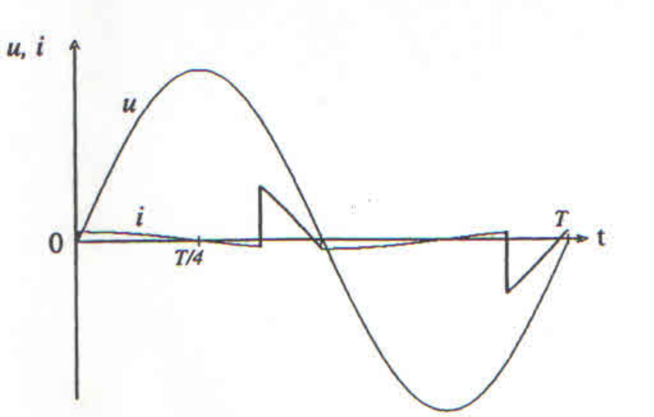

Fig. 7. Voltage and current waveforms in the circuit with the TRIAC controlled load power and with a capacitor for reactive power compensation

The most established and a time-honored Lienard's interpretation of the apparent power in single phase circuits with sinusoidal voltages and currents was provided by Curtis and Silsbee in Ref [5]. According to this interpretation, the active power P of the load in the circuit shown in Fig. 8, at a constant RMS value of the supply voltage U and the constant RMS value of the supply current I, changes with the change of the phase angle φ of the load impedance Z as shown in Fig. 9. The apparent power S stands for the maximum value of the active power P of the load. Since the active power P is a quantity with a clear physical interpretation, the authors have concluded, that the apparent power S, as the maximum value of the active power P, also has the physical interpretation.

Unfortunately, the condition that the supply voltage RMS value U and the supply current RMS value I, are kept constant at a variable load phase angle φ cannot be satisfied at the same time in real systems. Let us assume that the supply source impedance be inductive, as shown in Fig. 10. To keep the voltage RMS value U and current RMS value I constant at a variable load phase requires that the RMS value of the internal voltage E of the supply source is

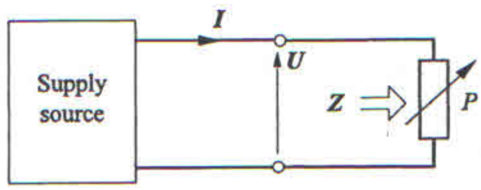

Fig. 8. A single-phase circuit with a load of variable impedance Z and active power P

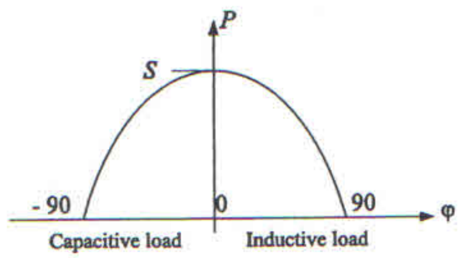

Fig. 9. Dependence of the load active power P on the load impedance phase φ at a constant load voltage and current RMS value, U and I

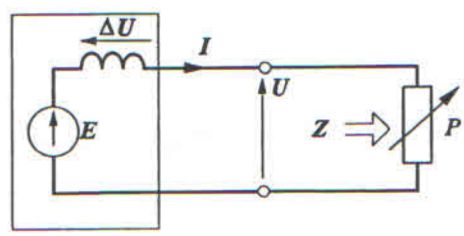

Fig. 10. A single-phase circuit with a load of variable impedance Z and active power P supplied from an inductive voltage source

continuously modified as shown in Fig. 11. Thus, the apparent power S does not characterize any phenomenon in the actual circuit, but a phenomenon in a circuit with a different load impedance Z and a different internal voltage E. Even such an experiment that would provide the characteristics shown in Fig. 9 cannot be performed in real conditions, since customers have no access to the control of the internal voltage E of the distribution system. Thus, the reasoning presented in Ref. [5] does not provide convincing physical interpretation of the apparent power S. Consequently, the apparent power in single-phase circuits remains not a physical but a conventional quantity; a conventional measure of the current loading at a voltage provided by the supply source, that means, a product of the voltage and current RMS values, namely

$$S = UI \ , \tag{21}$$

or, using symbols adopted in Ref [6] for RMS values of nonsinusoidal quantities,

$$S = \|u\|\|i\| \ . \tag{22}$$

Nonetheless, in spite of the lack of the physical interpretation, the conventional meaning of the apparent power S seems to be clear, which means, that the existence of a physical interpretation for some quantities is not so crucial as someone seems to believe.

The matter of the apparent power definition in three-phase systems is, however, more confusing. This is mainly because different quantities are considered to be an apparent power in three-phase systems.

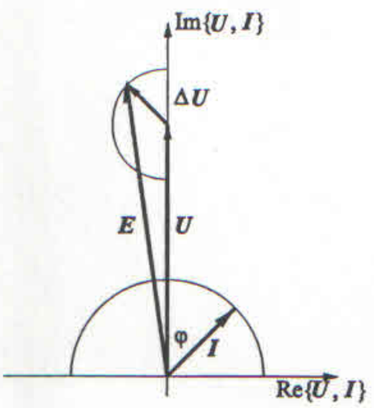

Fig. 11. Diagram of the complex RMS values of voltages and current in the circuit shown in Fig. 10 at a variable phase φ and constant load voltage and current RMS values, U and I

6 What is the apparent power in three-phase three-wire systems?

Asking such a question we may receive three different answers, namely, that this is a quantity defined according to formulae (1), (2) or (3). In order to distinguish these definitions, indices A, G and B are allocated to the symbol S of the apparent power, as follows

$$S_A = U_R I_R + U_S I_S + U_T I_T \tag{23}$$

$$S_G = \sqrt{P^2 + Q^2} \tag{24}$$

$$S_B = \sqrt{I_R^2 + I_S^2 + I_T^2}\sqrt{U_R^2 + U_S^2 + U_T^2} \tag{25}$$

Essentially, these are three different power quantities: the arithmetic apparent power, the geometric apparent power, as defined in the IEEE Standard Dictionary of Electrical and Electronics Terms [4] and the apparent power as defined by Buchholtz in Ref. [1].

As long as the supply voltage is sinusoidal, symmetrical and the load is balanced, these answers are not erroneous, since these three definitions provide the same numerical value of the apparent power. Unfortunately, it is enough that one of these conditions is not satisfied and serious ambiguities may occur, because the apparent power calculated with these definitions may have different values. Consequently, the power factor, defined as the ratio of the active and apparent powers

$$\lambda = \frac{P}{S} \tag{26}$$

depends on the choice of the apparent power definition. This is illustrated below on the example.

Example 2. A balanced resistive load shown in Fig. 12, is supplied with a sinusoidal symmetrical voltage. The internal voltage RMS value of the reference phase R was assumed to be equal to $E_R = 220$ V. The load and the supply source parameters where chosen such that at the load active power $P = 100$ kW, the active power loss in the supply source is equal to $\Delta P = 5$ kW.

An unbalanced load, as shown in Fig. 13, supplied from the same source has the same active power $P = 100$ kW as the power of the balanced load when its phase-to-phase resistance is equal to 1.173 Ω. Depending on the definition of the apparent power, its value and power factor is equal to $S_A = 119$ kVA, $\lambda_A = 0.84$, $S_G = 100$ kVA, $\lambda_G = 1$, $S_B = 149.4$ kVA, $\lambda_B = 0.67$. Thus, the question arises,

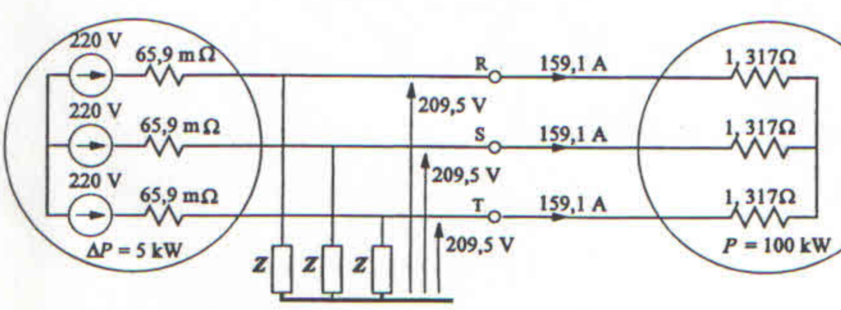

Fig. 12. A three-phase circuit with a balanced resistive load

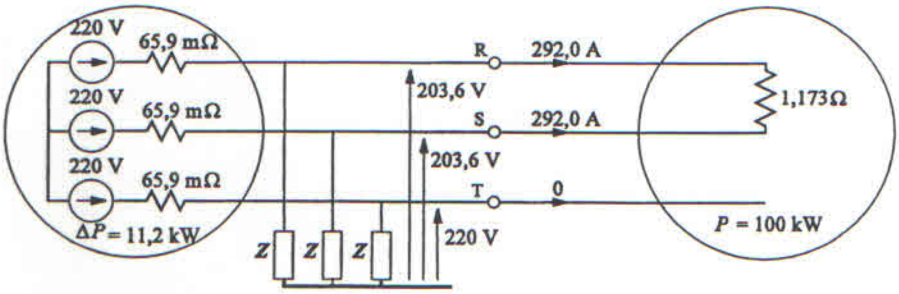

Fig. 13. A three-phase circuit with an unbalanced resistive load but the same active power P as the load shown in Fig. 12

which value and why should be chosen as the value of the apparent power, and consequently, the power factor?

Similarly as in the case of single-phase circuits, there is no physical phenomenon that could be characterized by any of these three apparent powers. They represent nothing else than only three different conventions. Thus, on what kind of criteria the choice of the best convention should be based upon?

When a convention is a matter of concern, we can discuss only whether such a convention is useful or not. The convention of the apparent power S in single-phase circuits is well established. It is a kind of a measure of the loading of the supply source. If i_a denotes the active component of the load current i and $\|i_a\|$ is its RMS value, then, the load current related loss of the active power in the supply source is equal to

$$\Delta P = R_s\|i\|^2 = R_s\frac{\|i_a\|^2}{\lambda^2} = \frac{\Delta P_{min}}{\lambda^2} \tag{27}$$

where R_s is the equivalent resistance of the supply source. Therefore, the power factor λ that has a similar property in three-phase systems and the apparent power that provides such a power factor could be chosen as the best convention as to the apparent power definition.

Example 2 (continuation). In the case of the circuit considered in the Example 2, the load imbalance caused an increase of the active power loss in the supply from 5 kW to 11.2 kW. Since there is no differences between value of various apparent powers when the load is balanced, let us find what kind of a balanced load of the same active power, $P = 100\,\mathrm{kW}$, causes such increase of the active power loss in the supply source. The circuit analysis shows, that such a property has a balanced resistive-inductive load, shown in Fig. 14, with the phase impedance equal to $Z = 0.879\,e^{j48.0^\circ}\,\Omega$. Such a load has the power factor $\lambda = \cos(48.0^\circ) = 0.67$ and the apparent power $S = 149.4\,\mathrm{kVA}$.

Comparing these values with those for the unbalanced load, it occurs, that the unbalanced load is equivalent with respect the active power loss in the supply, to a balanced load, if the apparent power S is calculated according to Buchholz's definition. Thus, the arithmetic and geometric apparent powers, S_A and S_G, and the power factor calculated with their use, do not characterize properly the power loss increase in the supply source due to imbalance of three-phase loads. It can be proven, moreover, that Buchholz's definition can be extended to systems with nonsinusoidal voltages and currents, with the RMS values $\|\mathbf{x}\|$ of three-phase vectors $\mathbf{x}(t)$ defined as

$$\|\mathbf{x}\| = \sqrt{\frac{1}{T}\int_0^T \mathbf{x}^T\mathbf{x}\,dt} \tag{28}$$

where $\mathbf{x} = \mathbf{x}(t)$ is a common symbol for the three-phase voltages $\mathbf{u}(t)$ and currents $\mathbf{i}(t)$. These vectors can be composed of voltage and current harmonics, as specified with formula (8), of the order n from a set N, namely

$$\mathbf{u}(t) = \sum_{n\in N}\mathbf{u}_n(t), \quad \mathbf{i}(t) = \sum_{n\in N}\mathbf{i}_n(t) \tag{29}$$

At such symbols the apparent power S in three-phase systems can be defined [7] as

$$S = \|\mathbf{u}\|\|\mathbf{i}\| \tag{30}$$

formally analogous to the apparent power definition in single-phase systems.

7 Conclusions

Reactive power may occur in circuits without energy storage capability and without energy oscillations between the supply source and the load. Therefore, common

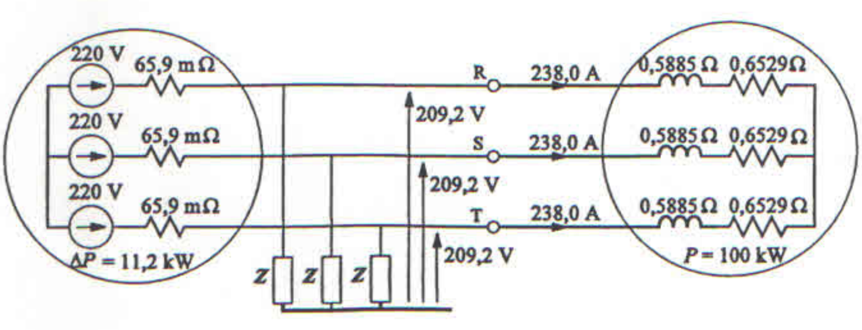

Fig. 14. A three-phase circuit with a balanced resistive-inductive load and the same active power P as the load shown in Fig. 12

interpretations of reactive power, and in particular, its relation to energy oscillation and energy storage, convincing in single-phase, linear and time-invariant circuits, should not be extrapolated to more complex situations, where these interpretations are not longer valid. Such interpretations should be abandoned, similarly as the Lienard's interpretation of the apparent power S. Also, commonly used definitions of the apparent power, sometimes referred to as the arithmetic and geometric apparent powers, do not provide information on the real loading of the supply source and its power factor in circuits with unbalanced loads. Definitions of the arithmetic and geometric apparent powers should not be used in situations where the load is unbalanced, since they result in an erroneous value of the load power factor. Unfortunately, it seems that these interpretations and definitions are so common that their elimination from the field of electrical engineering will not be easy.

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