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Considerations on the Concept of Poynting Vector Contribution to Power Theory Development

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***Summary:** There are spreading opinions in electrical engineering that power theory of electric circuits should be founded on the concept of the Poynting vector and its properties. The correctness of such an approach is investigated in this paper.*

To eliminate possible ambiguity in this investigation, the notion of "power theory" is first clarified in the paper. It is shown that the subject of power theory is broader than only a description of energy flow phenomena. Indeed, the apparent power, S, and the reactive power, Q, i.e., two major quantities of power theory that are essential for its practical implementations in three-phase systems do not have interpretation in terms of energy flow.

Therefore, the flux of the Poynting vector over a load boundary, which provides only information on the load instantaneous power, does not provide information on the apparent and reactive powers of three-phase loads. Therefore, power theory cannot be founded on the concept of the Poynting vector and its properties. Consequently, the Poynting vector and its flux do not provide any information useful for academic interpretation of power properties of electric circuits and for practical applications of power theory in industrial systems.

I. INTRODUCTION

In recent discussions [10] on power definitions, interpretations, i.e., on power theory development, the Poynting vector

$$\vec{\boldsymbol{P}} = \vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}} \,, \tag{1}$$

where $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{H}}$ denote the electric and magnetic field intensities, is increasingly often referred to as the very fundamental of this theory. This is because the flux of the Poynting vector to a volume, V, through its surface, S, is equal to the rate of energy, *W*, flow to this volume, i.e., to the instantaneous power, $p(t) = dW/dt$, of the load. Indeed, let us assume that electrical energy is delivered to a load exclusively by its three-phase, three-wire supply , denoted in this paper by R, S and T, with line-to-ground voltages u_R, u_S and u_T and line currents i_R, i_S and i_T. In such a case, the flux of the Poynting vector through any surface, S, that crosses the sup-ply lines only once, is equal to

$$\oiint_S \vec{\boldsymbol{P}} \bullet d\vec{\boldsymbol{S}} = u_R i_R + u_S i_S + u_T i_T = p(t) \,. \tag{2}$$

Moreover, a direct relation of the Poynting vector to Maxwell equations and to basic properties of energy flow in electromagnetic fields makes the claim that power theory should be founded on the Poynting vector very appealing.

Unfortunately, power theory has different objectives than only a description of energy flow in electric circuits and consequently, this claim could be deeply misleading.

The main concern of power theory is focused on the fact that even at a constant rate of energy transfer to a load, i.e., at a constant active power, *P*, various phenomena associated with energy delivery can cause an increase in the supply current RMS value. This increases the energy loss on delivery and increases the power rating of the distribution system equipment *without* any contribution to net energy flow.

Therefore, before any relationship between the concept of the Poynting vector and power theory is discussed, the notion of the term *power theory* has to be recollected. In particular, we should recollect the objectives of a power theory, i.e., the reasons for which we develop this theory. Having this settled, we could try to answer the question: *to what degree does the concept of the Poynting vector and its flux facilitate us to fulfill the objectives for which the power theory has been developed?*

II. POWER THEORY OBJECTIVES

The notion *power theory* has a particular meaning in electrical engineering, the meaning allocated to it by scientists who have worked on its development, such as Buchholz, Fryze, Budeanu, Shepherd and Zakikhani, Depenbrock [1-6] The increase of the apparent power, not associated with energy delivered to the load, was the major subject of their investigations. At the same time, it can be observed that not everything that relates to active power is a subject of power theory. There are numerous situations where the active power, *P*, is the subject of a major interest and investigation, but not the subject of power theory. The dependence of power loss in a magnetic core on the winding current frequency; the dependence of power in the air gap of a motor on the rotor speed or the dependence of the power radiated by an antenna on its geometry, are just a few examples of such a situation.

Two objectives of power theory are clearly visible in the publications by Fryze, Budeanu, Shepherd and Zakikhani, Depenbrock. One of them, academic in its very nature, was motivated only by our curiosity, our need to understand the world around us. However, the second objective was very practical,. It was to provide fundamentals needed for electrical systems design. The apparent power, *S*, of a load is one of the data of major importance needed for selecting the power ratings of the distribution system equipment. This is very closely related to the issue of the effectiveness of energy delivery, i.e., to power factor, $\lambda = P/S$, and tariffs for this delivery. This was just the reason that the question of why and how harmonics contribute to the apparent power increase was so long and so hotly debated between followers of the Budeanu and Fryze approaches to the reactive power definition. This practical question was soon extented by another very practical issue, namely, how to improve power factor by compensation of useless components of the apparent power. The first answers to this question were given by Shepherd and Zakikhani [4] as well as by Kusters and Moore [6].

To summarize this, the objectives of power theory seem to be:

(i) Explanation of physical phenomena that cause an increase of the apparent power, *S*, above the active power, *P*, of the load and consequently, the increase of the supply current RMS value.

(ii) Providing data needed for selecting the power ratings of power system equipment.

(iii) Providing data on the load power factor, needed for energy accounts between energy suppliers and customers.

(iv) Providing fundamentals for power factor improvement by compensation of useless components of the apparent power.

This roster of power theory objectives seems to show a somewhat surprising feature of this theory. It does not have the active power and the flow of energy between the supplier and the load, in the center of interest, but rather the difference between this power and the apparent power. We would not have power theory if power equipment could be rated with respect to only the active power, *P*, as it is in DC systems. To draw a conclusion that the difference between active power and the apparent power is the core of interest of power theory, we have to remember that the apparent power, *S*, is not related to energy flow. As it was demonstrated by Fryze [3] the apparent power could have a non-zero value in a circuit with no energy flow. The same is with the reactive power. In spite of a common, deeply rooted belief that the reactive power is a measure of energy oscillations, the physical interpretation of reactive power is not revealed jet [8]. This is visible particularly in three-phase balanced systems with sinusoidal voltages and currents, where the instantaneous power $p(t)$, i.e., the rate of energy flow between the supply and the load, $p(t) = dW(t)/dt$, is independent of the reactive power, *Q*, of the load. Thus, two major power quantities, the apparent and reactive powers, essential for practical applications in electrical engineering, are not related to energy flow between the load and the supply, while the concept of the Poynting vector and the Poynting theorem are focused only on energy flow. It makes their contribution to power theory suspicious.

III. COMPUTATIONAL ASPECT

Even now, in the age of computers, when powerful tools for modeling electric and magnetic fields are available, a simplicity of calculations and analysis is still important. Especially because computers do not supersede us in drawing intellectual conclusions. Having this in mind, one could ask the equstion: *Why should I calculate the flux of the Poynting vector P over the load boundary and calculate first the electric and magnetic fields on this boundary to know the rate of energy flow to the load,*

$$\oiint_{S} (\vec{E} \times \vec{H}) \bullet d\vec{S} = \frac{dW}{dt} = p(t)\,, \tag{3}$$

if I can calculate this rate of energy flow, for my three-phase load, having supply voltages and currents, according to

$$\frac{dW}{dt} = p(t) = u_R i_R + u_S i_S + u_T i_T\ ? \tag{4}$$

Furthemore, the flux of the Poynting vector even does not depend on the selected boundary, S.

Indeed, for a not radiating, common loads, such as, for example, a motor supplied through a three-phase line as shown in Fig. 1, the flux of the Poynting vector has the same value for boundary S_1, S_2, S_3 or for any other boundary that confines the load. Only the voltages and currents at the load terminals affect this flux.

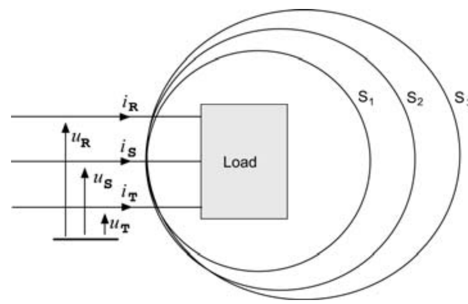

Fig.1. Three-phase loads and a few surfaces for the flux of the Poynting vector calculation

Only for a very simple geometry of the load and isotropic media of the space does the Poynting vector flux provide the instantaneous power of the load with a reasonable amount of calculation. However, even in such a situation calculating this flux could be considered only as an intellectual exercise for students rather than as a scientific approach to engineering problems. Geometry of power equipment is not simple and the medium is not isotropic. When a device or its element such as a screen, cover or a rotor is not supplied directly by conductors with distinctive voltages and currents, the Poynting vector approach provides a very useful, often the only tool for energy flow analysis. The same is with antennas and other energy radiating devices. However, when voltages and currents at the terminals of a device are known, and this device could be considered as a lumped RLC circuit, then the Poynting vector approach seems to be entirely redundant from the computational point of view. When a load is considered from the point of view of power theory, it can usually be assumed that voltages and currents at the load terminals are available, therefore, it seems that there is no justification in such a situation, at least as to computational reasons, for using the Poynting vector approach. Also observe, that voltages and currents are the input quantities for power and energy meters.

IV. COGNITIVE ASPECT

Since calculation of the flux of the Poynting vector seems to be too toilsome for power theory applications to have any practical benefits, let us focus our attention on a cognitive aspect of the Poynting vector for this theory.

The Poynting Theorem, which has the form

$$\int_V \vec{\boldsymbol{E}}\bullet\vec{\boldsymbol{j}}\,dV = -\int_V (\vec{\boldsymbol{H}}\bullet\frac{\partial\vec{\boldsymbol{B}}}{\partial t} + \vec{\boldsymbol{E}}\bullet\frac{\partial\vec{\boldsymbol{D}}}{\partial t})dV - \oiint_S (\vec{\boldsymbol{E}}\times\vec{\boldsymbol{H}})\bullet d\vec{\boldsymbol{S}}\,, \qquad (5)$$

says that energy that enters a volume V confined by a surface S is dissipated over that volume and is stored in electric and magnetic fields. This is true, but this truth is rather trivial now. It is a common knowledge in electrical engineering for a century. Thus, the cognitive aspect of the Poynting vector is not impressive. This theorem also says how to calculate the rate of energy dissipation and storage in the volume V, but again, only when this volume is geometrically simple and the medium isotropic, then these calculations are not very toilsome. For a common power system, which is geometrically complex and anisotropic, calculation of the rate of energy dissipation and storage is much more efficient when such a system is considered as a circuit with lumped RLC parameters.

One could say that the Poynting vector provides us with an important physical interpretation relevant to energy flow, as a local power flux, or a power flux density, i.e., it informs us how energy enters the volume V. Unfortunately, even the existence of the Poynting vector in the presence of electric and magnetic fields, $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{H}}$ could be controversial or at least, a common electrical engineer may need the expertise of an expert in electromagnetic fields to have an opinion in this matter. Consider for example the situation shown in Fig. 2, where an electric field is created by a charged capacitor and magnetic field is created by a permanent magnet. For sure, there is no energy flow in such a situation, although apparently the Poynting vector is not equal to zero. Another such situation, maybe more confusing for engineers who are not experts on electrodynamics, is shown in Fig. 3. Thus, it is not enough that electric and magnetic fields exist in a point of space for the Poynting vector to exist and consequently, energy flow. The Poynting vector is the energetic aspect of the mutual dependence of electric and magnetic fields, specified by the Maxwell equations.

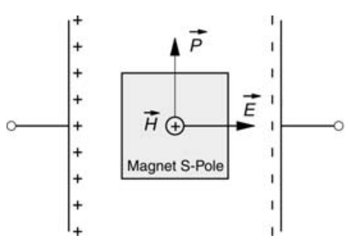

Fig. 2. Situation where apparently the Poynting vector occurs

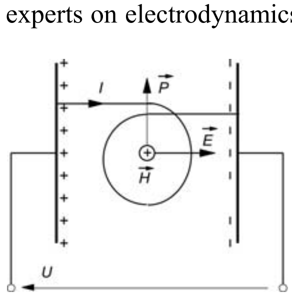

Fig. 3. Situation where apparently the Poynting vector occurs

Unfortunately, there are numerous situations in power systems where sources of magnetic fields are independent of sources of electric fields. Motors with permanent magnets, DC motors, synchronous generators are just a few examples.

When electric and magnetic fields are related by Maxwell equations, then indeed the Poynting vector can be considered as a power flux density. However, even if this information could be interesting from the point of view of energy flow studies and often useful for power loss investigation in power equipment, it is irrelevant for power properties at the load terminals, properties that should be specified in terms of powers.

V. POYNTING VECTOR AND POWER THEORY

To evaluate the relationship of the Poynting vector approach to power theory, let us summarize the up-to-date development of this theory, as it applies to three-phase, three-wire systems, shown in Fig. 4, with sinusoidal voltages and currents. Such systems form a major subset of all three-phase systems. Any conclusion drawn from the Poynting vector approach with respect to power theory of three-phase systems under more complex conditions, such as for example, non-sinusoidal, has to be true when such a conclusion is applied to this particular subset. Also, this is the major type of system for energy delivery.

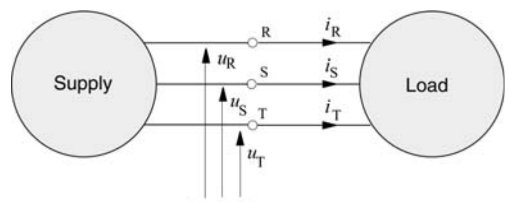

Fig. 4. Three-phase three-wire system

The power equation of such systems, commonly used in electrical engineering, has the form

$$S^2 = P^2 + Q^2\,, \qquad (6)$$

where, *S*, *P* and *Q* are the apparent, active and reactive powers, respectively. It is valid only on the condition that the load is balanced. Otherwise, this equation is erroneous [8] and results in an incorrect value of power factor, λ, and incorrect value of the apparent power *S*.

Assuming that the voltage and current vectors have the form

$$\boldsymbol{u} = \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix}, \qquad \boldsymbol{i} = \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix}, \qquad (7)$$

and $||\boldsymbol{u}||$ and $||\boldsymbol{i}||$ are their RMS values [7], in general equal to

$$||\boldsymbol{x}|| = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{x}^{T}(t)\;\boldsymbol{x}(t)\,dt} = \sqrt{X_R^2 + X_S^2 + X_T^2}\,, \qquad (8)$$

then, the apparent power should be defined as [1, 7, 8]

$$S = ||\boldsymbol{u}||\,||\boldsymbol{i}|| = \sqrt{U_R^2 + U_S^2 + U_T^2}\bullet\sqrt{I_R^2 + I_S^2 + I_T^2}\,, \qquad (9)$$

The load in the situation considered has the equivalent circuit shown in Fig. 5,

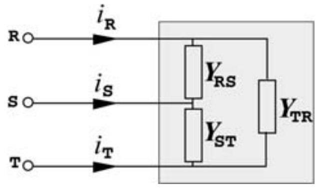

Fig. 5. Equivalent circuit of three-phase, static linear load

and can be characterized by two admittances. Namely, by the equivalent admittance

$$\boldsymbol{Y}_{\mathrm{e}} = G_{\mathrm{e}} + jB_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}}\,, \tag{10}$$

and the unbalanced admittance

$$\boldsymbol{A} = Ae^{j\psi} = -\,(\boldsymbol{Y}_{\mathrm{ST}} + \alpha\boldsymbol{Y}_{\mathrm{TR}} + \alpha^{*}\boldsymbol{Y}_{\mathrm{RS}}), \quad \alpha = 1e^{j2\pi/3}\,. \tag{11}$$

The line-to-line admittances $\boldsymbol{Y}_{\mathrm{RS}}$, $\boldsymbol{Y}_{\mathrm{ST}}$ and $\boldsymbol{Y}_{\mathrm{TR}}$ of the load, needed for the equivalent and unbalanced admittances, $\boldsymbol{Y}_{\mathrm{e}}$ and $\boldsymbol{A}$, calculation, can be obtained by a simultaneous sampling of two line currents and two line-to-line voltages and digital signal processing of these samples to calculate the complex RMS (CRMS) values of the supply voltages and currents.

If the CRMS values of the line-to-artificial zero point voltages of the supply are arranged into vectors defined as follows

$$\begin{bmatrix}\boldsymbol{U}_{\mathrm{R}}\\ \boldsymbol{U}_{\mathrm{S}}\\ \boldsymbol{U}_{\mathrm{T}}\end{bmatrix} = \boldsymbol{U}, \qquad \begin{bmatrix}\boldsymbol{U}_{\mathrm{R}}\\ \boldsymbol{U}_{\mathrm{T}}\\ \boldsymbol{U}_{\mathrm{S}}\end{bmatrix} = \boldsymbol{U}^{\#}\,, \tag{12}$$

then, the supply current of the load

$$\boldsymbol{i} = \begin{bmatrix}i_{\mathrm{R}}\\ i_{\mathrm{S}}\\ i_{\mathrm{T}}\end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix}\boldsymbol{I}_{\mathrm{R}}\\ \boldsymbol{I}_{\mathrm{S}}\\ \boldsymbol{I}_{\mathrm{T}}\end{bmatrix} e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\,\boldsymbol{I}\,e^{j\omega_1 t}\,, \tag{13}$$

can be decomposed into three components, namely

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}\,, \tag{14}$$

where

$$\boldsymbol{i}_{\mathrm{a}} = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}}\,\boldsymbol{U}\,e^{j\omega_1 t}\}\,, \tag{15}$$

is the active current,

$$\boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re}\{\,jB_{\mathrm{e}}\,\boldsymbol{U}e^{j\omega_1 t}\}\,, \tag{16}$$

is the reactive current and

$$\boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\,\boldsymbol{U}^{\#}e^{j\omega_1 t}\}\,, \tag{17}$$

is the unbalanced current. Since these three components are orthogonal [7], thus their RMS values satisfy the relationship

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2\,, \tag{18}$$

where

$$||\boldsymbol{i}_{\mathrm{a}}|| = G_{\mathrm{e}}\,||\boldsymbol{U}||, \quad ||\boldsymbol{i}_{\mathrm{r}}|| = |B_{\mathrm{e}}|\,||\boldsymbol{U}||, \quad ||\boldsymbol{i}_{\mathrm{u}}|| = A\,||\boldsymbol{U}||. \tag{19}$$

Multiplication of the above equation by the voltage RMS value $||\boldsymbol{U}||$, results in the power equation of the form

$$S^2 = P^2 + Q^2 + D^2\,, \tag{20}$$

where D is the unbalanced power [7, 9]

$$D = ||\boldsymbol{U}||\,||\boldsymbol{i}_{\mathrm{u}}|| = A\,||\boldsymbol{U}||^2. \tag{21}$$

This power equation is fundamental for evaluating the system performance in the sinusoidal situation, in particular, for calculating its power factor $\lambda = P/S$ needed for tariff for energy delivery to customers. It is also needed for selecting the power ratings of the supply equipment and for design of a compensator of the reactive power and for the load balancing.

If the Poynting vector approach is fundamental for power theory, as it is suggested by Authors of Refs. [10], it should provide information on powers and their relation, specified by the power equation. In particular, the following questions important from the point of view of objectives of this theory, as compiled in Section II, should be answered:

How can powers in power systems and the power equation be interpreted and specified in terms of the Poynting vector or its flux? Do they solve something previously unsolved or do they deepen their physical interpretation? Do they provide electrical engineers with new and more convenient tools for design and evaluation of the power system performance? Do they provide data for compensator control and fundamentals for tariffs?

To answer these questions, let us consider a balanced load shown in Fig. 6 supplied with a symmetrical sinusoidal voltage, such that $u_{\mathrm{R}} = U\sqrt{2}\cos\omega t$

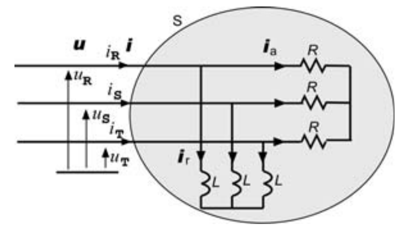

Fig. 6. Balanced RL load

The unbalanced current of such a load, $\boldsymbol{i}_{\mathrm{u}} = 0$, thus, the flux of the Poynting vector through surface S is equal to

$$\oiint_{S} \vec{\boldsymbol{P}}\bullet d\vec{\boldsymbol{S}} = p(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}}) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{a}} = P\,, \tag{22}$$

since

$$\boldsymbol{u}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\,U\begin{bmatrix}\cos\omega t\\ \cos(\omega t - 120^0)\\ \cos(\omega t + 120^0)\end{bmatrix}^{\mathrm{T}} \bullet \sqrt{2}I_{\mathrm{r}}\begin{bmatrix}\sin\omega t\\ \sin(\omega t - 120^0)\\ \sin(\omega t + 120^0)\end{bmatrix} =$$

$$= UI_{\mathrm{r}}[\sin 2\omega t + \sin(2\omega t + 120^0) + \sin(2\omega t - 120^0) \equiv 0. \tag{23}$$

The same value of the Poynting vector flux is obtained, of course, when the load is purely resistive, as shown in Fig. 7

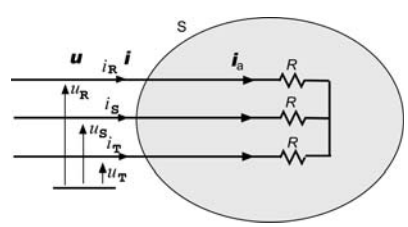

Fig. 7. Balanced resistive load

This means that the loads in Fig. 6 and in Fig. 7, different with respect to reactive and apparent powers, Q and S, and power factor λ, cannot be distinguished in terms of the Poynting vector flux through the surface S. This result is trivial, of course, because it is known that the reactive power Q of three-phase loads does not affect the instantaneous power $p(t)$ of such a load, i.e., the rate of energy flow. Thus, there is no relationship between the flux of the Poynting vector through a surface that enclosures a three-phase load and the reactive power Q of such a load.

In single-phase systems under sinusoidal conditions, when the supply voltage and current are generalized mathematically to complex voltages and currents, the notion of the complex apparent power can be introduced. It is defined as

$$\boldsymbol{S} = S e^{j\varphi} = \boldsymbol{U}\boldsymbol{I}^{*} = P + jQ\,. \tag{24}$$

Using such an approach, the electric and magnetic field intensities, $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{H}}$, and consequently, the Poynting vector, become complex vectors. The flux of the imaginary part of the Poynting vector is equal to the reactive power, thus there is a relation between this vector and the reactive power, Q. Unfortunately, all attempts aimed at application of this kind of approach to nonsinusoidal systems or three-phase unbalanced systems have failed. Such attempts have resulted in erroneous definitions of the reactive and apparent powers and in erroneous power equations. Moreover, such approaches did not revel the presence of the unbalanced power, D, in the power equation. Therefore, there is no ground for extrapolation of the relationship between the Poynting vector and the reactive power, valid in single-phase sinusoidal systems, to a similar relationship in nonsinusoidal and three-phase unbalanced systems.

The presented analysis demonstrates that the Poynting vector approach does not provide an affirmative answer to any of questions compiled in this Section. This approach does not provide basic data that would enable us to calculate the apparent and reactive powers, power factor and to evaluate three-phase system performance, as well as to design its components.

VI. CONCLUSIONS

The Poynting Theorem and the Poynting vector are fundamental mathematical tools for calculating energy flow and its storage in electromagnetic fields. Their applications in various fields of electrical engineering are countless. However, it is the author of this paper's opinion, drawn from the history of the power theory development, that such questions of how energy enters a conductor or a capacitor are not the subject of power theory. The interest of power theory is focused on powers at the junction of the energy supplier and customer. It provides theoretical fundamentals needed for the supply equipment design, for creating justified tariffs for accounts for energy delivery and for design and control of compensators, capable of improving the effectiveness of energy delivery. Unfortunately, the Poynting Theorem and vector have nothing to offer in this respect to power theory. This is because the basic power quantities of this theory, when applied to three-phase systems, namely the apparent and reactive powers, S and Q, have no relationship to energy flow between the supplier and the load.

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