Critical Comments on the Conservative Power Theory (CPT)

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***Abstract*—Main features of the Conservative Power Theory (CPT) are examined in the paper. It was concluded that the "reactive energy" as defined in CPT cannot be regarded as a physical quantity. It was demonstrated that separation of the void reactive current can lead to wrong conclusions at a reactive compensator design. It was shown that distortion power defined in the CPT has no relation to mutual distortion of the load voltage and current. The meaning of the term "conservative" as used in the CPT, is discussed as well. It is emphasized that the conservative property of the "reactive energy" is only mathematical, but not physical property of that "energy".**

Keywords-power definitons; power phenomena; reactive current; reactive power; distortion power; Currents' Physical Components; CPC; reactive compensation.

I. INTRODUCTION

The Conservative Power Theory (CPT) becomes one of the major power theories of electrical systems with non-sinusoidal voltages and currents. The number of papers that use the CPT concept, published recently [1-5], support this opinion. As such a major power theory, it serves for interpretation of energy flow phenomena in electrical systems as well as a theoretical fundamental for compensator design.

An examination of the CPT reveals that some terms and concepts suggested by this theory are similar to those, suggested by the abandoned now theories, as developed by Budeanu [7] and by Kusters and Moore [12], however.

Taking into account that the CPT becomes one of the main power theory, its fundamental have to be carefully verified and revised. Fundamentals to be reliable have to be challenged with much more stringent verifications than a verification of a common construction built on such fundamentals.

According to authors of the CPT, it can be applied to nonlinear and three-phase systems, but this study is confined to a very small sub-set of such systems, namely to systems with purely reactive single-phase loads. It is also assumed that the supply voltage has a form of only cosine function, without any phase-shift, or if distorted, all harmonics are only cosine functions, also without any phase-shift. This assumption is a result of a conclusion that if the CPT does not have some specified property in such a simple sub-set, it cannot have this property in the whole set of non-linear and three-phase systems. Also, the system complexity could hide some analyzed properties or such properties could not be clearly visible. Moreover, this will reduce complexity of the presented analysis, making the results obtained much easier to verify.

II. UNBIASED VOLTAGE INTEGRAL

One of the main quantities in the CPT is the "unbiased voltage integral". It is defined as

$$\hat{u}(t) = \int_0^t u(\tau)\,d\tau - \frac{1}{T}\int_0^T [\int_0^t u(\tau)\,d\tau]\,dt\ . \tag{1}$$

Assuming that the supply voltage, as assumed is

$$u(t) = \sqrt{2}\,U\cos\omega t$$

the unbiased voltage integral is equal to

$$\hat{u}(t) = \int_0^t u(\tau)\,d\tau - \frac{1}{T}\int_0^T [\int_0^t u(\tau)\,d\tau]\,dt = \sqrt{2}\,\frac{U}{\omega}\sin\omega t\ . \tag{2}$$

Let the supply voltage be periodic and composed of harmonics of order n from a set N

$$u(t) = \sum_{n\in N} u_n(t) = \sqrt{2}\sum_{n\in N} U_n \cos n\omega_1 t\ . \tag{3}$$

The unbiased integral of such a voltage is

$$\hat{u}(t) = \sum_{n\in N} \hat{u}_n(t) = \sqrt{2}\sum_{n\in N} \frac{U_n}{n\omega_1}\sin n\omega_1 t\ . \tag{4}$$

III. "REACTIVE ENERGY" W

The second main quantity defined in the CPT is the "reactive energy". It is defined as

$$W = \langle \hat{u}, i\rangle = \frac{1}{T}\int_0^T \hat{u}(t)\,i(t)\,dt\ . \tag{5}$$

Its name is written in this paper in quotation marks, since, as it will be shown in this paper, this quantity should not be regarded as an energy.

This quantity has occurred, although without the name of "reactive energy", in the Power Theory developed by Kusters and Moore (K&M) [12], recommended next by the International Electrotechnical Commission in 1979 for use in electrical engineering. This Theory was challenged by the author of this paper in Refs. [13, 14], where it was demonstrated that it does not have properties as claimed.

The K&M concept was based on the load current decomposition in the time-domain and was confined to single-phase loads. They separated the active current, exactly as it was done by Fryze [8] and a reactive current. This reactive current for RL load, was a current of an equivalent inductor, connected in parallel to a resistive branch with the active current, as shown in Fig. 1.

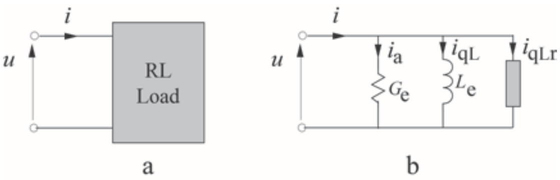

Fig. 1. RL load and its equivalent load

If an inductor of inductance L is supplied with nonsinusoidal voltage

$$u(t) = \sum_{n\in N} u_n(t) = \sqrt{2}\sum_{n\in N} U_n \cos n\omega_1 t \tag{6}$$

then its current in a steady state is

$$i = \frac{1}{L}\int u\,dt = \frac{1}{L}\hat{u}\ . \tag{7}$$

The "reactive energy" W of such an inductor is

$$W = \frac{1}{T}\int_0^T \hat{u}(t)\,i(t)\,dt = \frac{1}{T}\int_0^T \sum_{r\in N}\hat{u}_r(t)\sum_{s\in N} i_s(t)\,dt = $$
$$= \sum_{n\in N}\frac{1}{T}\int_0^T \hat{u}_n(t)\,i_n(t)\,dt = L\sum_{n\in N}\frac{1}{T}\int_0^T i_n^2(t)\,dt = L||i||^2 \tag{8}$$

hence, the inductance L is

$$L = \frac{W}{||i||^2} = \frac{\frac{1}{T}\int_0^T \hat{u}(t)\,i(t)\,dt}{||i||^2}\ . \tag{9}$$

Due to (7), the inductor current rms value is

$$||i|| = \frac{1}{L}||\hat{u}|| \tag{10}$$

therefore

$$\frac{1}{L} = \frac{\frac{1}{T}\int_0^T \hat{u}(t)\,i(t)\,dt}{||\hat{u}||^2} \tag{11}$$

and consequently, the inductor current is,

$$i = \frac{1}{L}\hat{u} = \frac{\frac{1}{T}\int_0^T \hat{u}(t)\,i(t)\,dt}{||\hat{u}||^2}\hat{u}\ . \tag{12}$$

The term "reactive current" in systems with nonsinusoidal voltages and currents can denote, depending on a particular power theory, quite different quantities. To avoid readers' confusion, the meaning of this term has to be specified and strictly differentiated by symbols.

"Reactive current" refers in this paper to a quadrature component of the current, as defined by Shepherd and Zakikhani (S&Z) [11]. In the same sense this term is used in the Currents' Physical Components (CPC) power theory [16]. Using CPC symbols, if the load voltage is

$$u(t) = \sqrt{2}\ \mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{13}$$

then the load reactive current is

$$i_{\mathrm{r}}(t) = \sqrt{2}\ \mathrm{Re}\sum_{n\in N} jB_n \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{14}$$

Only a current defined by (14) will be called the reactive current in this paper and only such a current shall be denoted by the symbol $i_{\mathrm{r}}(t)$. The "reactive current" in the CPT will be denoted by $i_{\mathrm{rT}}(t)$.

The inductive reactive current $i_{\mathrm{qL}}(t)$ in the K&M Power Theory [12], (5), and the "reactive current" $i_{\mathrm{rT}}(t)$ in the CPT [1], (7b), are defined in the same way, namely

$$i_{\mathrm{qL}}(t) = i_{\mathrm{rT}}(t) = \frac{\frac{1}{T}\int_0^T \hat{u}(t)\,i(t)\,dt}{||\hat{u}||^2}\hat{u} \tag{15}$$

The current in the nominator $i(t)$ in this definition is not the inductor current, as it is in (12), but the load current.

Because the concept of the active current $i_{\mathrm{a}}(t)$ and the "reactive currents" $i_{\mathrm{qL}}(t) = i_{\mathrm{rT}}(t)$ for K&M Theory and for CPT are identical, the load current decomposition in these two Theories are almost identical. The only difference is in the definition of the *residual reactive current* $i_{\mathrm{qLr}}(t)$. It is composed of not only the *void current* $i_{\mathrm{v}}(t)$, defined in the CPT, but also a *scattered current* $i_{\mathrm{s}}(t)$. When K&M suggested [12] their Theory in 1980, the scattered current was not yet known. It was revealed later, in a frame of the CPC concept, in 1984 [16].

The physical meaning of the "reactive energy" as defined by (5) is not clear. It is not the energy E_{m} stored in the magnetic field of the inductor, since this energy is a time-varying quantity. Its value for an individual inductor is given by (8). It can be rearrange as follow

$$W = L||i||^2 = L\sum_{n\in N}||i_n||^2 = \sum_{n\in N} L||i_n||^2 = \sum_{n\in N} W_n \tag{16}$$

where

$$W_n = L||i_n||^2 = L\,I_n^2 = \frac{1}{2}L\,I_{n,\max}^2 = W_{n,\max} \tag{17}$$

is the maximum value of the energy stored in the inductor by the n^{th} order harmonic. Thus

$$W = \sum_{n\in N} W_n = \sum_{n\in N} W_{n,\max} \tag{18}$$

The "reactive energy" is the sum of maximum values of the energy stored by individual harmonics. It is not the maximum value of this energy $W_{\max}$, however, because maxima for individual harmonics occur at different instants of time. It could be regarded only as double value of the mean value of this energy.

Let us find relation between the "reactive energy" W and the energy E_{m} of the magnetic field of a lossless inductor of inductance L supplied with the a single voltage harmonic of the n^{th} order

$$u_n(t) = \sqrt{2}\,U_n \cos n\omega_1 t \tag{19}$$

so that the unbiased integral of this voltage is

$$\hat{u}_n(t) = \sqrt{2}\frac{U_n}{n\omega_1}\sin n\omega_1 t\ . \tag{20}$$

The inductor current is

$$i_n(t) = \sqrt{2}\,\frac{U_n}{n\omega_1 L}\sin n\omega_1 t\,. \tag{21}$$

Thus the "reactive energy" of the inductor is

$$W_{\rm L} = \langle \hat{u}, i\rangle = \langle \hat{u}_n, i_n\rangle = W_{{\rm L}n} = \frac{1}{T}\int_0^T \hat{u}_n(t)\,i_n(t)\,dt = \frac{U_n^2}{n^2\omega_1^2 L}. \tag{22}$$

The energy stored in the magnetic field of the inductor is

$$E_{\rm m}(t) = \frac{1}{2}L\,i_n^2(t) = \frac{U_n^2}{n^2\omega_1^2 L}\sin^2 n\omega_1 t = W_{{\rm L}n}\sin^2 n\omega_1 t. \tag{23}$$

Let us repeat this reasoning replacing the inductor by a capacitor of capacitance C. Its current is

$$i_n(t) = -\sqrt{2}\;n\omega_1 C U_n \sin n\omega_1 t\,. \tag{24}$$

Thus the "reactive energy" of the capacitor is

$$W_{{\rm C}n} = \langle \hat{u}_n, i_n\rangle = \frac{1}{T}\int_0^T \hat{u}_n(t)\,i_n(t)\,dt = -U_n^2\,C\,. \tag{25}$$

Since this value is negative, quantity W cannot be regarded as the energy. It explains why the term "reactive energy" is written in quotation marks in this paper. The energy stored in the electric field of the capacitor is

$$E_{\rm e}(t) = \frac{1}{2}C\,u_n^2(t) = U_n^2 C\cos^2 n\omega_1 t = -W_{{\rm C}n}\cos^2 n\omega_1 t\,. \tag{26}$$

When an inductor and a capacitor are connected in parallel and they are supplied as previously, then the total energy stored in such an LC load is

$$E(t) = E_{\rm m}(t) + E_{\rm e}(t) = \frac{U_n^2}{n^2\omega_1^2 L}\sin^2 n\omega_1 t + U_n^2 C\cos^2 n\omega_1 t \tag{27}$$

but not

$$W_n = W_{{\rm L}n} + W_{{\rm C}n} = \frac{U_n^2}{n^2\omega_1^2 L} - U_n^2 C\,. \tag{28}$$

This is particularly visible at a resonance, when

$$\frac{1}{n^2\omega_1^2 L} = C\,. \tag{29}$$

In such a case, the energy stored in such LC load is

$$E(t) = \frac{U_n^2}{n^2\omega_1^2 L} = {\rm Const.} \tag{30}$$

while the "reactive energy" $W_n = 0$. This raises a question on the physical meaning of the "reactive energy" W. This observation does not confirm the opinion expressed in [5] that "*…the reactive energy accounts for inductive and capacitive energy stored in the load circuit*."

These conclusions resulted from calculation of the energy inside of a reactive load. Power Theories are formulated having the voltage and current outside of the load as the input data, however, i.e., the voltage and current at the load terminals. Let us calculate the "reactive energy" W of a purely reactive load of any complexity at nonsinusoidal supply voltage composed of harmonics of order n from a set N as specified by (3). The unbiased integral of such a voltage is specified by (4).

The load has admittance for harmonic frequency of the $n^{\rm th}$ order harmonic equal to

$$\boldsymbol{Y}_n = G_n + jB_n = jB_n \tag{31}$$

and the current

$$i(t) = \sum_{n\in N} i_n(t) = -\sqrt{2}\sum_{n\in N}{\rm sgn}\{B_n\}|B_n|U_n\sin n\omega_1 t \tag{32}$$

which means that if for the $n^{\rm th}$ order harmonic the load is inductive, i.e., $B_n < 0$, then

$$i_n(t) = \sqrt{2}\,|B_n|U_n\sin n\omega_1 t\,. \tag{33}$$

If for such a harmonic the load is capacitive, i.e., $B_n > 0$, then

$$i_n(t) = -\sqrt{2}\,|B_n|U_n\sin n\omega_1 t\,. \tag{34}$$

The "reactive energy" of such a reactive load is

$$W = \langle \hat{u}, i\rangle = \sum_{n\in N}\langle \hat{u}_n, i_n\rangle = \sum_{n\in N} W_n = -\sum_{n\in N}{\rm sgn}\{B_n\}|B_n|\frac{U_n^2}{n\omega_1}. \tag{35}$$

Individual terms of this sum can be, depending on the sign of the load susceptance B_n, positive or negative, thus they can cancel mutually. Therefore, this sum does not specify the maximum value of the stored energy in the load.

Formula (36) has strong analogy with definition of the reactive power $Q_{\rm B}$ in Budeanu's power theory [7]. Namely, for purely reactive loads this definition can be rearranged as follows

$$Q_{\rm B} = \sum_{n\in N} U_n\,I_n\sin\varphi_n = -\sum_{n\in N}{\rm sgn}\{B_n\}|B_n|U_{\rm n}^2\,. \tag{36}$$

Individual terms in Budeanu's definition of the reactive power stand for the amplitude of the energy oscillation, since the bidirectional component of the instantaneous power $p(t)$ is equal to

$$\tilde{p}_n = U_n\,I_n\sin\varphi_n\,\sin 2n\omega_1 t = Q_n\sin 2n\omega_1 t\,. \tag{37}$$

The sum of these amplitudes, i.e., the Budeanu's reactive power, does not specify, as shown in [15], any physical phenomenon in the circuit, however.

The "reactive energy" W occurs to be almost identical with the reactive power $Q_{\rm I}$ defined in 1925 [6] by Illovici. These two quantities differ only by a dimensional coefficient. Namely, according to Illovici, the reactive power should be defined as

$$Q_{\rm I} = \sum_{n\in N}\frac{1}{n}U_n I_n\sin\varphi_n\,. \tag{38}$$

This is a quantity measured by a wattmeter with the resistor in the voltage branch is replaced by an inductor L, as shown in Fig. 2.

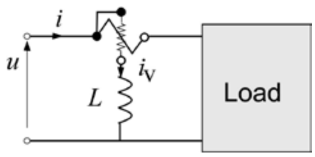

Fig. 2. Meter of the reactive power $Q_{\rm I}$.

At terminals of a purely reactive load such an instrument measures the quantity

$$Q_{\rm I} = \sum_{n\in N}\frac{1}{n}U_n I_n\sin\varphi_n = -\sum_{n\in N}{\rm sgn}\{B_n\}|B_n|\frac{U_n^2}{n} = \frac{1}{\omega_1}W\,. \tag{39}$$

It means that the “reactive energy” W is a measurable quantity. Measurability is not crucial for a conclusion on physical nature of a quantity, however. At the present state of development in the digital technology, whatever is mathematically defined, it could be measured.

IV. CONSERVABILITY OF “REACTIVE ENERGY” W

The “reactive energy” W satisfies the Conservative Property. It means that in any circuit confined by a sphere with zero energy transfer and composed of K branches, as shown in Fig. 3,

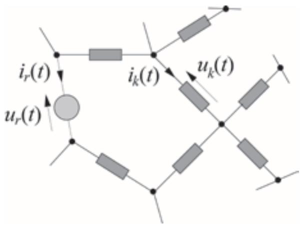

Fig. 3. Circuit with K branches

the sum of “reactive energies” of individual branches W_k is equal to zero, i.e.,

$$\frac{1}{T}\int_0^T \sum_{k=1}^{K} \hat{u}_k(t)\, i_k(t)\, dt = \sum_{k=1}^{K} W_k \;=\; 0\,. \tag{40}$$

This is a very important property. It enables balancing the “reactive energies” and verification of its calculation. Also, if a quantity satisfies the conservative property, this might indicate that this quantity has a physical nature. Such argument was sometimes used in discussions on the physical nature of Budeanu’s reactive power. It also satisfies the conservative property, i.e.,

$$\sum_{k=1}^{K} \sum_{n\in N} U_{kn}\, I_{kn} \sin\varphi_{kn}\, dt \;=\; \sum_{k=1}^{K} Q_{\mathrm{B}k} \;=\; 0\,. \tag{41}$$

The conservative property is an outcome of one of two more fundamental principles. One of them is the Energy Balance Principle (EBP). The second principle is the Tellegen Theorem [9]. According to EBP, if in any circuit confined by a sphere with zero energy transfer and composed of K branches, and if energy E_k is transferred to the k-branch, then

$$\sum_{k=1}^{K} E_k \;=\; \text{Const.} \tag{42}$$

Since the instantaneous power of the k-branch is

$$\frac{dE_k}{dt} = p_k(t) \tag{43}$$

thus the conservative property of the instantaneous power

$$\sum_{k=1}^{K} \frac{dE_k}{dt} = \sum_{k=1}^{K} p_k(t) = 0 \tag{44}$$

is a direct conclusion from the Energy Balance Principle.

As emphasized in [1], the conservative property of the “reactive energy” W, with the importance of this property reflected in the name of the Conservative Power Theory, is a conclusion from the Tellegen Theorem.

This Theorem, concluded by Tellegen from Kirchoff Laws in [9], seems to be not commonly known because it was developed not long ago. Since it is crucial for this discussion on the conservative property of the “reactive energy” W, its meaning is explained below.

According to this Theorem, if we have two circuits of the identical topology, as shown in Fig. 4, then the sum of voltage-currents products over all K branches with voltages taken from the circuit in Fig. 4(a) and the currents taken from the circuit in Fig. 4(b) is equal to zero, i.e.,

$$\sum_{k=1}^{K} u_k^{\mathrm{a}}(t)\, i_k^{\mathrm{b}}(t) \;\equiv\; 0\,. \tag{45}$$

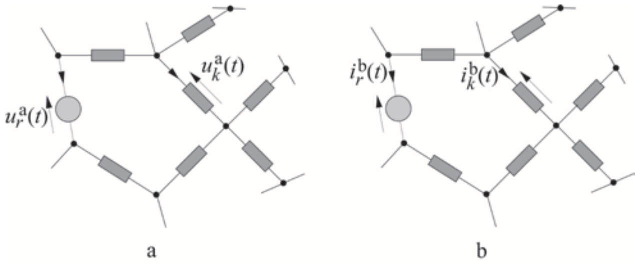

Fig. 4. Two circuits with identical topology

The voltage-current products in (45) do no not stand for any physical quantity, however, because voltages are taken from one circuit while the currents are taken from the other one. Nonetheless, such non-physical products have the conservative property. This property is also valid for any integral operations performed on voltages and currents in these two circuits. Therefore, assuming that

$$u_k^{\mathrm{a}}(t) \equiv \hat{u}_k(t), \qquad i_k^{\mathrm{b}}(t) \equiv i_k(t) \tag{46}$$

from the Tellegen Theorem (45) we obtain

$$\sum_{k=1}^{K} u_k^{\mathrm{a}}(t)\, i_k^{\mathrm{b}}(t) = \sum_{k=1}^{K} \hat{u}_k(t)\, i_k(t) \;\equiv\; 0, \tag{47}$$

and hence

$$\frac{1}{T}\int_0^T \sum_{k=1}^{K} \hat{u}_k(t)\, i_k(t)\, dt = \sum_{k=1}^{K} \langle \hat{u}_k, i_k \rangle = \sum_{k=1}^{K} W_k \equiv 0. \tag{48}$$

It means that the conservative property of the reactive energy W does not strengthen arguments for its physical nature. This has a strong analogy with the conservative property of the reactive power as defined by Budeanu’s.

The Budeanu’s reactive power Q_{B} can be expressed as demonstrated in [10]

$$Q_{\mathrm{B}} = \sum_{n\in N} U_n\, I_n \sin\varphi_n = \frac{1}{T}\int_0^T u(t)\, \mathrm{H}\{i(t)\}\, dt \tag{49}$$

where

$$\mathrm{H}\{i(t)\} = \mathrm{PV}\,\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{i(\tau)}{\tau - t}\, dt \tag{50}$$

is the Hilbert Transform of the load current $i(t)$. Symbol PV denotes the principal value of the integral.

Assuming that in circuits in Fig. 4(a) and (b)

$$u_k^{\mathrm{a}}(t) \equiv u_k(t), \qquad i_k^{\mathrm{b}}(t) \equiv \mathrm{H}\{i_k(t)\} \tag{51}$$

then from Tellegen Theorem

$$\sum_{k=1}^{K} u_k^{\mathrm{a}}(t)\, i_k^{\mathrm{b}}(t) = \sum_{k=1}^{K} u_k(t)\, \mathrm{H}\{i_k(t)\} \;\equiv\; 0\,. \tag{52}$$

Hence

$$\frac{1}{T}\int_0^T \sum_{k=1}^{K} u_k(t)\,\mathrm{H}\{i_k(t)\}\,dt = \sum_{k=1}^{K}\frac{1}{T}\int_0^T u_k(t)\,\mathrm{H}\{i_k(t)\}\,dt = \sum_{k=1}^{K} Q_{\mathrm{B}k} = 0. \qquad (53)$$

The conservative property of the Budeanu reactive power Q_{B} is not a consequence of the EBP, i.e., a physical principle, but Tellegen Theorem, which is a sort of mathematiccal, but not a physical property of electrical systems. Consequently, the CPT is no more conservative than the Budeanu Power Theory. From the fact, that the "reactive energy" W has a conservative property, we should not draw the conclusion that it is a physical quantity. The same was with the Budeanu's reactive power Q_{B}.

V. "REACTIVE ENERGY" W AND ENERGY STORAGE

In a quest for the physical meaning of the "reactive energy", let us check if it has a relation to energy storage.

A partial answer to this question was given in Section III. In the same section it was shown, however, that at sinusoidal supply voltage and resonance condition in an LC load the energy stored in the load, specified by (31), has a non-zero value, while the "reactive energy" W, specified by (29), is equal to zero. It means that there is no relation between amount of energy stored in purely reactive load and "reactive energy" W of such a load.

The lack of any relationship between the "reactive energy" W and the energy storage in the load is even more visible if we leave linear loads for non-linear ones. To demonstrate this, let us consider a purely resistive load with a periodic switch, made of a TRIAC, shown in Fig. 5, supplied from an ideal source of a sinusoidal voltage.

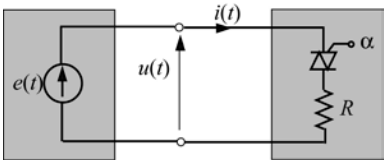

Fig. 5. Resistive load with periodic switch

At sinusoidal supply voltage

$$u(t) = \sqrt{2}\,U \sin \omega_1 t$$

the load current at the TRIAC firing angle α has the waveform as shown in Fig. 6.

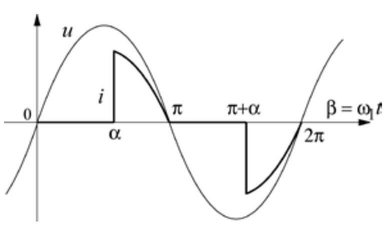

Fig. 6. Voltage and current waveforms.

The load current is composed of harmonics

$$i(t) = \sum_{k=1}^{\infty} i_n(t) = i_1(t) + \sum_{k=2}^{\infty} i_n(t) \qquad (54)$$

with the current fundamental harmonic

$$i_1(t) = \sqrt{2}\,I_1 \sin(\omega_1 t - \varphi_1)\,. \qquad (55)$$

i.e., shifted with respect to the voltage as shown in Fig. 7. The supply voltage unbiased integral is

$$\hat{u}(t) = -\sqrt{2}\,\frac{U}{\omega}\cos\omega_1 t\,, \qquad (56)$$

and consequently, the reactive energy W is equal to

$$W = \langle \hat{u}, i\rangle = \sum_{n=1}^{\infty}\langle \hat{u}_n, i_n\rangle = \langle \hat{u}, i_1\rangle = \frac{1}{T}\int_0^T \hat{u}(t)\, i_1(t)\, dt = \frac{U I_1}{\omega_1}\sin\varphi_1. \qquad (57)$$

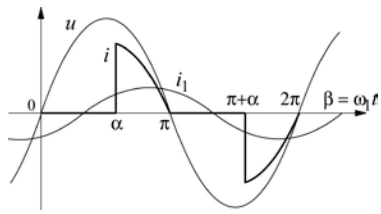

Fig. 7. Voltage, current and the current fundamental harmonic i_1 waveforms in resistive circuit with TRIAC

Thus, loads without any capability of energy storage could have a "reactive energy" W. This confirms the previous conclusion that the reactive energy W is not associated with the phenomenon of energy storage.

VI. VOID CURRENT AND DISTORTION POWER

The supply current of a purely reactive single-phase load supplied with a nonsinusoidal supply voltage is composed of a reactive current defined by S&Z in [11].

According to CPT the load current of a purely reactive load is composed of the reactive current and the void current. To differentiate symbols of reactive currents in the CPC power theory and in CPT, its symbol in the CPT will have index rT in this paper. The reactive current in the CPT is defined as

$$i_{\mathrm{rT}}(t) = \frac{W}{\|\hat{u}\|^2}\,\hat{u}(t) \qquad (58)$$

meaning this current is proportional to "reactive energy" W. Since the physical meaning of the "reactive energy" remains unclear, the same applies to the "reactive current" $i_{\mathrm{rT}}(t)$. The remaining current is referred to as a void current

$$i_{\mathrm{v}}(t) = i(t) - [\,i_{\mathrm{a}}(t) + i_{\mathrm{rT}}(t)] \qquad (59)$$

If the load is purely reactive, and consequently, its current does not contain the active components, then the void current is

$$i_{\mathrm{v}}(t) = i(t) - i_{\mathrm{rT}}(t)\,. \qquad (60)$$

Thus the supply current of reactive single-phase loads can be decomposed as follows

$$i(t) = i_{\mathrm{rT}}(t) + i_{\mathrm{v}}(t)\,. \qquad (61)$$

Since the physical meaning of the "reactive current" $i_{\mathrm{rT}}(t)$ is unclear, the physical meaning of the void current is also unclear.

The reactive and void currents are mutually orthogonal, so that their rms values satisfy the relationship

$$\|i\|^2 = \|i_{\mathrm{rT}}\|^2 + \|i_{\mathrm{v}}\|^2\,. \qquad (62)$$

Multiplying this formula by the square of the supply voltage rms value $\|u\|$, the power equation of reactive loads is obtained. It has the form

$$S^2 = Q_{\mathrm{T}}^2 + D_{\mathrm{T}}^2 . \qquad (63)$$

According to [1], the quantity

$$D_{\mathrm{T}} = ||i_{\mathrm{v}}|| \times ||u|| \qquad (64)$$

is a distortion power of the load.

The concept of a distortion power occurred for the first time in Budeanu's power theory. It was defined as

$$D_{\mathrm{B}} = \sqrt{S^2 - P^2 - Q_{\mathrm{B}}^2} . \qquad (65)$$

Indices T and B were used in (65) and (66) to distinguish these two distortion powers. Despite having the same name, these are two different quantities.

Distortion power D_{B} was interpreted as a measure of the effect of the mutual voltage and current distortion on the apparent power S of the load. This interpretation was challenged in [15], where it was demonstrated that such interpretation was not right. Let us check whether distortion power D_{T} defined in the CPT is related to the load voltage and current mutual distortion. This is done below with a numerical analysis of a purely reactive load shown in Fig. 8 supplied with the voltage:

$$u(t) = \sqrt{2}\,(100 \sin\omega_1 t + 30 \sin 3\omega_1 t)\,\mathrm{V}, \qquad \omega_1 = 1 \text{ rad/s}.$$

The admittances of such a load for the voltage harmonics are, respectively, $\boldsymbol{Y}_1 = -j1/2$ S and $\boldsymbol{Y}_3 = j1/2$ S. The "reactive energy" W of such load is equal to

$$W = -\sum_{n\in\{1,\,3\}} \mathrm{sgn}\{B_n\}\,|B_n|\,\frac{U_n^2}{n\,\omega_1} = 4.85 \text{ kJ}.$$

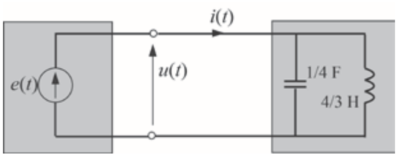

Fig. 8. Circuit with reactive load

Since

$$||\hat{u}|| = \sqrt{\sum_{n\in\{1,\,3\}} \left(\frac{U_n}{n\,\omega_1}\right)^2} = \sqrt{\left(\frac{U_1}{\omega_1}\right)^2 + \left(\frac{U_3}{3\omega_1}\right)^2} = 100.50 \ \ \mathrm{Vt}$$

the "reactive current" $i_{\mathrm{rT}}(t)$ rms value is

$$||i_{\mathrm{rT}}|| = ||\frac{W}{||\hat{u}||^2}\,\hat{u}(t)|| = \frac{|W|}{||\hat{u}||} = 48.26 \text{ A}.$$

The load current rms value is

$$||i|| = \sqrt{\sum_{n\in\{1,\,3\}} (Y_n U_n)^2} = \sqrt{(0.5\times 100)^2 + (0.5\times 30)^2} = 52.20 \text{A}.$$

Since the active current does not exist, the rms value of the void current is equal to

$$||i_{\mathrm{v}}|| = \sqrt{||i||^2 - ||i_{\mathrm{rT}}||^2} = \sqrt{52.2^2 - 48.26^2} = 19.90 \text{A},$$

so that the distortion power

$$D_{\mathrm{T}} = ||i_{\mathrm{v}}||\,||u|| = 19.90\times 104.40 = 2.08 \text{ kVA}.$$

The load current is equal to

$$i(t) = \sqrt{2}\,[50\sin(\omega_1 t - 90^0) + 15\sin(3\omega_1 t + 90^0)] =$$
$$= \sqrt{2}\,[50\sin\omega_1(t - \frac{T}{4}) + 15\sin 3\omega_1(t - \frac{T}{4})]\,\mathrm{A} = \frac{1}{2}\,u(t - \frac{T}{4}).$$

The load current is shifted with respect to the voltage by $T/4$, as shown in Fig. 9, but it is not distorted, in spite of non-zero distortion power D_{T}. It demonstrates that there is no relation between distortion power D_{T} and distortion of the load current with respect to the supply voltage.

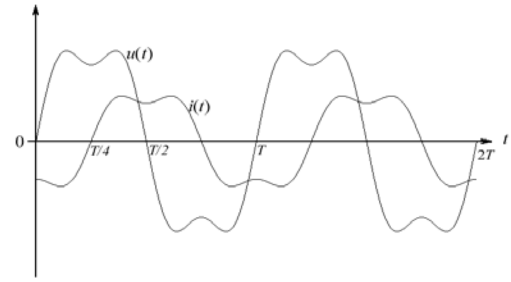

Fig. 9. Voltage and current waveform in the circuit shown in Fig. 7.

VII. REACTIVE COMPENSATION

The CPT was developed mainly from a perspective of needs of compensation using tools of the time-domain. Compensation in time-domain is one of two approaches to compensation, however. The second one is reactive compensation, which uses tools of the frequency-domain for the compensator design.

The K&M theory was developed having in mind compensation by a reactive compensator, with parameters calculated in the time-domain. In the case of RL load, K&M attempted to find a shunt capacitive compensator. They concluded, that the "reactive current" $i_{\mathrm{qL}}(t)$ cannot by compensated entirely by a capacitor. It can be only minimized and they proposed of a method of calculation of the optimum capacitance C_{opt}, which minimizes the rms value of the $i_{\mathrm{qL}}(t)$ current. They claimed that the residual reactive current $i_{\mathrm{qLr}}(t)$ cannot be compensated by any reactive compensator. In the paper [13] it was demonstrated that the method of the optimum capacitance C_{opt} calculation was not right. In the paper [14] it was proven that the residual reactive current $i_{\mathrm{qLr}}(t)$ can be reduced by a reactive compensator to the scattered current.

Since the load current components in the CPT are defined in the same way as in the K&M power theory, all conclusions with respect to reactive compensation are the same. The "reactive current" $i_{\mathrm{rT}}(t)$ cannot be entirely compensated by a capacitor. The void current $i_{\mathrm{v}}(t)$ does not need switching compensator for compensation. It can be compensated by a reactive compensator

Let us observe that compensation of the reactive current $i_{\mathrm{rT}}(t)$ as defined in the CPT is not equivalent to compensation of the reactive current $i_{\mathrm{r}}(t)$ as defined in the CPC power theory. In particular, when the reactive energy W is equal to zero, the "reactive current" $i_{\mathrm{rT}}(t)$ does not even exist. This is illustrated in the following example.

Let us assume that the supply voltage of the load shown in Fig. 10 contains only fundamental harmonic and a harmonic of the third order.

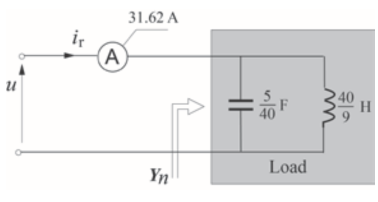

Fig. 10. Example of a load with zero reactive energy W, but non-zero reactive current $i_{\mathrm{r}}(t)$

The admittances for the fundamental and for the third order harmonics of this load, assuming that ω_1 =1 rad/s, are

$$\boldsymbol{Y}_1 = -\,j\,0.1\,\mathrm{S}, \quad \boldsymbol{Y}_3 = j\,0.3\,\mathrm{S}\,.$$

The "reactive energy" W of such a load is

$$W = -\sum_{n\in\{1,\,3\}} \mathrm{sgn}\{B_n\}|B_n|\frac{U_n^2}{n\,\omega_1} = \frac{1}{\omega_1}(U_1^2|B_1| - \frac{U_3^2}{3}|B_3|). \quad (66)$$

It is easy to observe, that at some voltage this energy is equal to zero, namely, it is enough that

$$\frac{U_3}{U_1} = \sqrt{3\frac{|B_1|}{|B_3|}}\,. \quad (67)$$

Such condition is satisfied, for example, with the voltage

$$u(t) = u_1(t) + u_3(t) = 100\sqrt{2}\cos\omega_1 t + 100\sqrt{2}\cos 3\omega_1 t \ \mathrm{V}\,.$$

The load current at such a voltage is

$$i(t) = i_{\mathrm{r}}(t) = 10\sqrt{2}\sin\omega_1 t - 30\sqrt{2}\sin 3\omega_1 t \ \mathrm{A}$$

and its rms value

$$||i_{\mathrm{r}}|| = \sqrt{10^2 + 30^2} = 31.62\mathrm{A}\,.$$

At such a voltage the load current contains only the void current, $i_{\mathrm{V}}(t) = i(t)$. It can be entirely compensated by a reactive compensator which for the fundamental and the third order harmonic has admittances

$$\boldsymbol{Y}_{\mathrm{C}1} = j\,0.1\,\mathrm{S}, \quad \boldsymbol{Y}_{\mathrm{C}3} = -\,j\,0.3\,\mathrm{S}\,.$$

The load with the reactive compensator of the void current and compensation results are shown in Fig. 11.

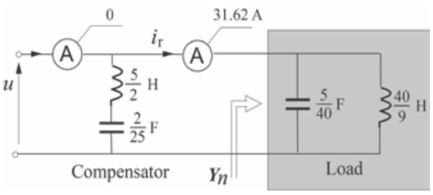

Fig. 11. Load with zero "reactive energy" W and a compensator of the void current $i_{\mathrm{V}}(t)$.

When the supply voltage is changed in such a way that condition (68) is not satisfied, the "reactive energy" W is no longer zero, and consequently the "reactive current" $i_{\mathrm{rT}}(t)$ occurs. A second compensator seems to be needed to eliminate it. It will affect the void current $i_{\mathrm{V}}(t)$ and the parameters of the compensator for its compensation. It means, that separate compensation of the currents $i_{\mathrm{rT}}(t)$ and $i_{\mathrm{V}}(t)$ requires compensators with the supply voltage depended compensators. Therefore, from the reactive compensation perspective there seems to be no rationale for decomposition of the reactive current $i_{\mathrm{r}}(t)$ as defined in the CPC power theory into currents $i_{\mathrm{rT}}(t)$ and $i_{\mathrm{V}}(t)$. These two currents can be compensated together, by a single reactive compensator. If N is the set of harmonic orders n of the supply voltage harmonics, then susceptance $B_{\mathrm{C}n}$ of the compensator, as it was shown in [16] should satisfy the condition

$$\text{For } n \in N, \quad B_{\mathrm{C}n} = -B_n\,. \quad (68)$$

Susceptances of such a compensator do not depend on the supply voltage. Parameters of the compensator shown in Fig. 11 were found just from this condition.

VIII. CONCLUSIONS

The paper shows that the CPT, as it applies to single-phase circuits is almost identical to the Power Theory developed by Kusters and Moore in [12]. Similar are also conclusions respective reactive compensation.

The paper was focused on the question whether the reactive energy W could be regarded as a physical quantity or not. In other words, is there any physical phenomenon in electrical circuits which is characterized by this energy? The paper provides a negative answer to this question. It was also concluded that the CPT is no more conservative Power Theory than the Budeanu's Theory.

It was also concluded that separation of the void current from the reactive current is not beneficial from the reactive compensation perspective.

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