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Physical Phenomena that Affect the Effectiveness Of the Energy Transfer in Electrical Systems

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Abstract **– The effectiveness of the energy delivery from its producers to customers is affected by a number of physical phenomena, but the nature of these phenomena is unclear and debated over a century. This debate has resulted in a number of power theories and a great variety of definitions of different power quantities. All of them are mathematically correct but differ as to the interpretation of the physical phenomena which accompany the energy transfer and affect the effectiveness of this transfer. These definitions and physical interpretations affect the method of the power factor improvement and unfortunately, they can cause a compensator malfunction.**

Presently, the most advanced power theory is based on the Currents' Physical Components (CPC) concept. Results of this theory are used in this paper for a discussion on physical phenomena which affect the effectiveness of the energy transfer in electrical systems.

Index Terms **—Definition of powers, instantaneous power, harmonics, asymmetry, Currents' Physical Components, CPC,**

I. INTRODUCTION

Two major powers used for describing power properties of electrical systems, namely, the apparent power S and the active power P, satisfy the inequality

$$S \geq P \tag{1}$$

which affect the power system economy. Explanation of this inequality and the development of methods of its reduction by means of a compensator is the subject of the power theories of electrical systems. Hundreds of papers were devoted to this inequality and several power theories were developed.

The most known are the power theories suggested by Budeanu in 1927 [1], by Fryze in 1931 [2], by Shepherd and Zakikhani in 1972 [3], by Kusters and Moore in 1980 [4], by Nabae and Akagi in 1984 [6], by Depenbrock in 1993 [10], and by Tenti in 2003 [12]. Some of them were regarded even as standards and are currently used for describing power properties of electrical systems. They also provide theoretical fundamentals for methods of the development of compensator for the power factor $\lambda = P/S$ improvement.

Power theories compiled above use different mathematical tools. Although all of them are mathematically correct, these theories differ substantially as to the interpretation of the phenomena that affect the effectiveness of the energy transfer from the supply source to the load.

As long as we think about or discuss power related physical phenomena in electrical systems, the derived conclusions are on a cognitive level and could be subjective. We can have different opinions on the meaning of "a physical entity" or "a physical phenomenon". Such conclusions may not have practical merits. Their application for reactive compensators design or switching compensators control could be a major test for correctness of these conclusions and have practical merits.

Presently the most advanced is the Currents' Physical Components (CPC) – based power theory [5, 7, 9, 17]. It explains and describes power properties of linear and nonlinear loads in single- and in three-phase systems with nonsinusoidal supply voltage. It provides both interpretations of the power-related physical phenomena and fundamentals for a design of reactive compensators and algorithms for control of switching compensators. A cognitive strength of the CPC-based power theory justifies its use as a reference for a discussion on the physical phenomena in electrical loads that can cause a degradation of the effectiveness of the energy transfer in electrical systems.

II. DRAFT OF THE CPC-BASED POWER THEORY

Let us consider a three-phase load, which can be linear or nonlinear, meaning a Harmonics Generating Load (HGL), supplied, as shown in Fig. 1, from a source of a distorted voltage, by a four-wire line.

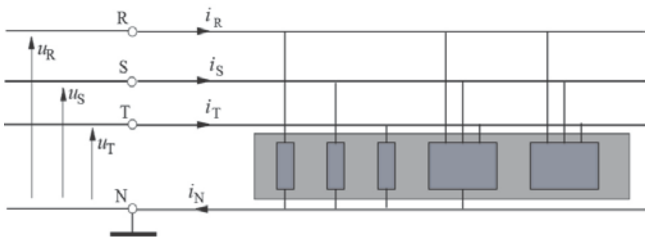

Fig. 1. A load supplied by a three-phase, four-wire line.

The load current, regarded as a three-phase vector of the line currents

$$\boldsymbol{i} = \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} = \sum_{n\in N} \boldsymbol{i}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \begin{bmatrix} \boldsymbol{I}_{Rn} \\ \boldsymbol{I}_{Sn} \\ \boldsymbol{I}_{Tn} \end{bmatrix} e^{jn\omega_1 t} \tag{2}$$

where N denotes the set of the current harmonic orders n, can be decomposed into five Physical Components, namely

$$\boldsymbol{i} = \boldsymbol{i}_{\rm Ca} + \boldsymbol{i}_{\rm Cs} + \boldsymbol{i}_{\rm Cr} + \boldsymbol{i}_{\rm Cu} + \boldsymbol{i}_{\rm G} \tag{3}$$

distinctively associated with five different physical phenomena in the load. These are:

1. Permanent transfer of the energy from the supply source to the load, associated with the ***active current*** $\boldsymbol{i}_{\rm Ca}$.
2. A change of the load conductance with the harmonic order n, associated with the ***scattered current*** $\boldsymbol{i}_{\rm Cs}$.
3. A phase shift between the load current and the voltage harmonics, associated with the ***reactive current*** $\boldsymbol{i}_{\rm Cr}$.
4. The load current asymmetry caused by the load imbalance, associated with the ***unbalanced current*** $\boldsymbol{i}_{\rm Cu}$.
5. Generation of the current harmonics by nonlinear or periodically switched loads, which results in transfer the energy from the load back to the supply source, associated with the ***load generated current*** $\boldsymbol{i}_{\rm G}$.

All these currents are mutually orthogonal, meaning the scalar product $(\boldsymbol{x}, \boldsymbol{y})$ defined as

$$(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^{\rm T}(t)\,\boldsymbol{y}(t)\,dt \tag{4}$$

of each of these currents with the remaining ones, is equal to zero. Consequently, all these currents affect the supply current three-phase rms value $||.||$, defined in [7] as

$$||\boldsymbol{x}|| = \sqrt{(\boldsymbol{x}, \boldsymbol{x})} \tag{5}$$

independently of each other, because, due to their mutual orthogonality, they satisfy the relationship

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\rm Ca}||^2 + ||\boldsymbol{i}_{\rm Cs}||^2 + ||\boldsymbol{i}_{\rm Cr}||^2 + ||\boldsymbol{i}_{\rm Cu}||^2 + ||\boldsymbol{i}_{\rm G}||^2\,. \tag{6}$$

Of all these five currents only the active current $\boldsymbol{i}_{\rm Ca}$ contributes to useful transfer of the energy from the supply source to the customer load. The remaining ones contribute only to an increase of the supply current rms value and the energy loss in the supply source. It means they degrade the effectiveness of the energy transfer in electrical systems.

The presented above results are conclusions from relatively recently developed Currents' Physical Components (CPC) – based power theory of electrical systems. However, according to very common in the electrical engineering community opinions [8], energy oscillations are the cause of the degradation of the effectiveness of the energy transfer. These opinions can have an adverse effect on the development of compensators for power factor improvement. Such opinions should be therefore carefully scrutinized. In particular, properties of the instantaneous power, which can contain an oscillating component, should be analyzed for that.

III. Instantaneous Power

Power properties of electrical systems are described in terms of different powers. The most fundamental of them is the instantaneous power $p(t)$. This power is defined as a rate of the energy $W(t)$ flow to the load. For single-phase loads supplied with a voltage $u(t)$ and a current $i(t)$, this power is equal to

$$p(t) = \frac{d}{dt}W(t) = u(t)\,i(t) \tag{7}$$

For three-phase systems, shown in Fig. 1, the instantaneous power is equal to

$$p(t) = u_{\rm R}i_{\rm R} + u_{\rm S}i_{\rm S} + u_{\rm T}i_{\rm T} = \boldsymbol{u}^{\rm T}\boldsymbol{i}\,. \tag{8}$$

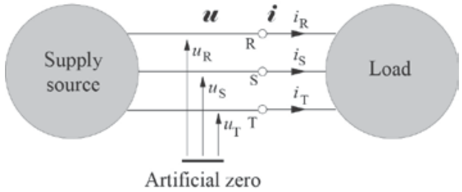

Fig. 2. A structure of three-phase, three-wire system.

The instantaneous power is, from the physical perspective, the only power quantity that has a clear physical interpretation.

The active power, defined as

$$P = \frac{1}{T}\int_0^T p(t)\,dt \tag{9}$$

is commonly regarded in the electrical engineering community like the instantaneous power, as a quantity with a clear physical interpretation. Observe however that definition (9) requires the voltage and current of a load be periodic with a period T, or they are approximated by periodic quantities. Therefore, it could be debatable whether the active power P, as an averaged quantity, is a physical quantity or not.

The fact that the instantaneous power $p(t)$ has such a convincing physical interpretation, motivated opinions of some researchers that just this power should be a central one in a power theory of electrical systems. Observe, however, that the major inequality (1), the power theories are developed around, cannot be expressed in terms of the instantaneous power.

The instantaneous power $p(t)$ is a central quantity in the Instantaneous Reactive Power (IRP) p-q Theory [6]. It is calculated in the frame of that theory, along with the instantaneous reactive power, using three-phase voltages and current recalculated to the α and β coordinates with a Clarke's Transform, which for three-wire systems has the form

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix}\begin{bmatrix} x_{\rm R} \\ x_{\rm S} \end{bmatrix} = \mathsf{C}\begin{bmatrix} x_{\rm R} \\ x_{\rm S} \end{bmatrix}. \tag{10}$$

Having them, the active and the reactive instantaneous powers, p and q, are defined as follows

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} u_\alpha, & u_\beta \\ -u_\beta, & u_\alpha \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathsf{U}_{\rm C}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}. \tag{11}$$

These two powers are essential for control of switching compensators, known mainly as "active power filters". This name is written above in quotation marks, "..", because these devices are not active, do not filter power, and are not filters but compensators. According to the Authors of the IRP p-q Theory [6], the compensator should be controlled in such a way that after compensation the supply source be loaded only by the constant component $\bar{p}$ of the instantaneous power p. It means that the alternating component $\tilde{p}$ of that power, along with the instantaneous reactive power q should be compensated, meaning the compensator should inject the following current

$$\begin{bmatrix} j_{\mathrm{R}} \\ j_{\mathrm{S}} \end{bmatrix} = \mathsf{C}^{-1}\begin{bmatrix} j_{\alpha} \\ j_{\beta} \end{bmatrix} = \mathsf{U}_{\mathrm{C}}^{-1}\begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix} \tag{12}$$

into the system. This conclusion is not right, however. To show this, let us consider an ideal balanced resistive load, with unity power factor, shown in Fig. 3. It does not need, of course, any compensation, but let us suppose that a compensator controlled according to the (IRP) p-q Theory, is connected at its terminals.

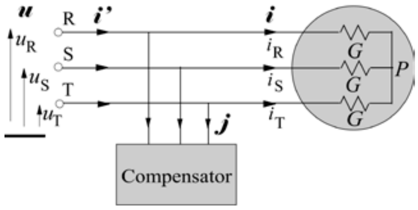

Fig. 3. Balanced resistive load with compensator.

Let the supply voltage is distorted by the 5th order harmonic, so that

$$\boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_5 \tag{13}$$

assuming for the sake of simplicity that

$$u_{\mathrm{R1}} \triangleq \sqrt{2}\,U_1 \cos\omega_1 t, \qquad u_{\mathrm{R5}} \triangleq \sqrt{2}\,U_5 \cos 5\omega_1 t\,. \tag{14}$$

The instantaneous power of the load is

$$p(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}}\, G\, \boldsymbol{u} = G[\boldsymbol{u}_1 + \boldsymbol{u}_5]^{\mathrm{T}}[\boldsymbol{u}_1 + \boldsymbol{u}_5] =$$
$$= G(\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_1 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_1). \tag{15}$$

The first two terms are constant components of the instantaneous power, equal to

$$G\,\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_1 = G\,||\boldsymbol{u}_1||^2 \triangleq P_1, \quad G\,\boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_5 = G\,||\boldsymbol{u}_5||^2 \triangleq P_5 \tag{16}$$

where P_1 and P_5 are harmonic active powers of the fundamental and the 5th order harmonics. The last term

$$G(\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_1) = 6\,G U_1 U_5 \cos 6\omega_1 t\,. \tag{17}$$

Eventually, the instantaneous power of the load is

$$p = P_1 + P_5 + 6\,G U_1 U_5 \cos 6\omega_1 t = \bar{p} + \tilde{p}\,. \tag{18}$$

It has a non-zero oscillating component which should be, according to the (IRP) p-q Theory, reduced by a compensator. The Clarke's components of the compensator current are

$$\begin{bmatrix} j_{\alpha} \\ j_{\beta} \end{bmatrix} = \mathsf{U}_{\mathrm{C}}^{-1}\begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix} = \mathsf{U}_{\mathrm{C}}^{-1}\begin{bmatrix} -6\,G U_1 U_5 \cos 6\omega_1 t \\ 0 \end{bmatrix} \tag{19}$$

or in more details, as shown in [14],

$$\begin{bmatrix} j_{\alpha} \\ j_{\beta} \end{bmatrix} = \frac{1}{u_{\alpha}^2 + u_{\beta}^2}\begin{bmatrix} u_{\alpha}, & -u_{\beta} \\ u_{\beta}, & u_{\alpha} \end{bmatrix}\begin{bmatrix} -6\,G U_1 U_5 \cos 6\omega_1 t \\ 0 \end{bmatrix} =$$

$$= \sqrt{3}\,\frac{-6\,G U_1 U_5 \cos 6\omega_1 t}{u_{\alpha}^2 + u_{\beta}^2}\begin{bmatrix} U_1 \cos\omega_1 t + U_5 \cos 5\omega_1 t \\ U_1 \sin\omega_1 t - U_5 \sin 5\omega_1 t \end{bmatrix}. \tag{20}$$

Thus, the compensator injects some distorted current into the system, reducing the power factor, instead of improving it.

Anyway, there is nothing to improve, because the considered load has unity power factor.

A similar situation occurs [15, 16] when the supply voltage of the load shown in Fig. 3 is sinusoidal but asymmetrical. This asymmetry does not affect the load power factor, which is equal to one, but the instantaneous active power has an oscillating component. If U^{p} and U^{n} are the rms values of the voltage symmetrical components of the positive and the negative sequence and P^{p} and P^{n} are active powers of these components, then the instantaneous power of a load supplied by asymmetrical voltage is

$$p(t) = \frac{dW}{dt} = P^{\mathrm{p}} + P^{\mathrm{n}} + 6\,G U^{\mathrm{p}} U^{\mathrm{n}} \cos 2\omega_1 t\,. \tag{21}$$

According to the (IRP) p-q Theory, the oscillating component of this power should be compensated. Unfortunately, its compensation degrades the power factor and causes the supply current distortion [15].

The oscillating component in the instantaneous power occurs, as discussed above, because of the supply voltage distortion or asymmetry. However, even if this voltage is sinusoidal and symmetrical, the oscillating component in this power is blamed for the presence of the reactive power Q. Thus, we have a cognitive question: "***does the reactive power occur in a system because of the energy oscillation?***"

To answer this question, let us decompose the current of a linear time-invariant (LTI) load, supplied with a sinusoidal, symmetrical voltage, into Physical Components [13]. If the load has an equivalent conductance G_{e}, an equivalent susceptance B_{e} and un unbalanced admittance $\boldsymbol{Y}_{\mathrm{u}}$, then the load current can be decomposed as follows

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}$$

where

$$\boldsymbol{i}_{\mathrm{a}} = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}}\,\mathbf{1}^{\mathrm{p}}\boldsymbol{U}\,e^{j\omega t}\} \tag{22}$$

$$\boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re}\{\,jB_{\mathrm{e}}\,\mathbf{1}^{\mathrm{p}}\boldsymbol{U}\,e^{j\omega t}\} \tag{23}$$

$$\boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}_{\mathrm{u}}\,\mathbf{1}^{\mathrm{n}}\boldsymbol{U}\,e^{j\omega t}\} \tag{24}$$

are the active, reactive and unbalanced currents, respectively while symbols $\mathbf{1}^{\mathrm{p}}$ and $\mathbf{1}^{\mathrm{n}}$ denote three-phase unite vectors, shown in Fig. 4.

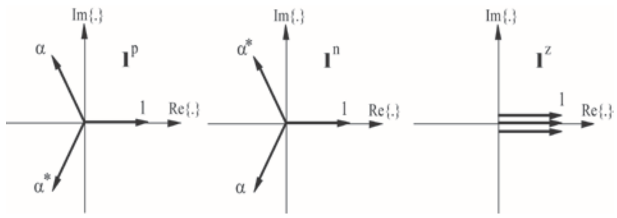

Fig. 4. Three-phase unite vectors.

The instantaneous power $p(t)$ can be expressed as follows

$$p(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}) = p_{\mathrm{a}}(t) + p_{\mathrm{r}}(t) + p_{\mathrm{u}}(t) \tag{25}$$

thus, it has three additive components associated separately with the Currents' Physical Components, i.e., with the active, reactive and unbalanced currents.

The component of the instantaneous power associated with the active current is equal to

$$p_{\rm a}(t) \;=\; \boldsymbol{u}^{\rm T}\boldsymbol{i}_{\rm a} \;=\; \boldsymbol{u}^{\rm T}G_{\rm e}\,\boldsymbol{u} \;=\; G_{\rm e}\,||\boldsymbol{u}||^2 = P \qquad (26)$$

meaning, it does not change in time. Assuming that

$$\boldsymbol{u} \overset{\rm df}{=} \begin{bmatrix} u_{\rm R} \\ u_{\rm S} \\ u_{\rm T} \end{bmatrix} = \sqrt{2}\,U_{\rm R}\begin{bmatrix} \cos\omega t \\ \cos(\omega t - 120^0) \\ \cos(\omega t + 120^0) \end{bmatrix} \qquad (27)$$

hence, the reactive current changes as

$$\boldsymbol{i}_{\rm r} = \sqrt{2}\,{\rm Re}\{jB_{\rm e}\mathbf{1}^{\rm p}\boldsymbol{U}\,e^{j\omega t}\} = -\sqrt{2}B_{\rm e}U_{\rm R}\begin{bmatrix} \sin\omega t \\ \sin(\omega t - 120^0) \\ \sin(\omega t + 120^0) \end{bmatrix}. \qquad (28)$$

Calculating the dot product of the supply voltage and the reactive current vectors, we obtain

$$p_{\rm r}(t) \;=\; \boldsymbol{u}^{\rm T}\boldsymbol{i}_{\rm r} \;=\; 0 \qquad (29)$$

thus, the reactive current does not contribute to the energy flow between the supply source and the load. Thus, the reactive power Q does not occur in the system because of the energy oscillation between the supply source and the load. In fact, oscillations of the energy between the supply source and the load are caused by the load current asymmetry caused by the load imbalance. At such an imbalance, an unbalanced current occurs in the load current

$$\boldsymbol{i}_{\rm u} = \sqrt{2}\,{\rm Re}\{\boldsymbol{Y}_{\rm u}\mathbf{1}^{\rm n}\boldsymbol{U}\,e^{j\omega t}\} = \sqrt{2}Y_{\rm u}U_{\rm R}\begin{bmatrix} \cos(\omega t + \Psi) \\ \cos(\omega t + \Psi + 120^0) \\ \cos(\omega t + \Psi - 120^0) \end{bmatrix} \qquad (30)$$

where Ψ denotes a phase angle of the unbalanced admittance $\boldsymbol{Y}_{\rm u}$. The instantaneous power associated with the unbalanced current is

$$p_{\rm u}(t) \;=\; \boldsymbol{u}^{\rm T}\boldsymbol{i}_{\rm u} = 3Y_{\rm u}\,U_{\rm R}^2\,\cos(2\omega t + \Psi)\,. \qquad (31)$$

Thus, only the unbalanced current but not the reactive current and consequently, the reactive power, causes the energy oscillations.

Sometimes, the energy oscillations between the supply source and the load are blamed for the power factor decline in the presence of the voltage and current harmonics.

This opinion can be supported [11] with results of analysis of the circuit shown in Fig. 5. It is composed of a dc battery and a periodic switch.

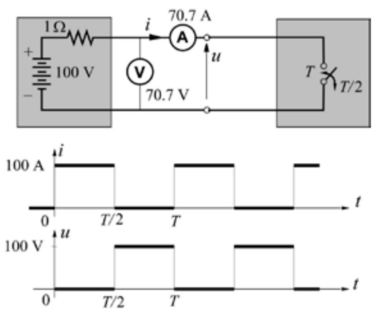

Fig. 5. A circuit with a periodic switch.

The apparent power at the load terminals $S = ||u||\,||i|| = 5$kW, while the active power $P = 0$, thus the power factor λ has zero value. This can be explained in the frequency-domain, using the concept of harmonics as follows.

The load voltage and the current can be expressed in a form of the Fourier Series

$$u(t) \;=\; U_0 + \sqrt{2}\,\sum_{n=1}^{\infty} U_n\cos(n\omega_1 t + \alpha_n) = \sum_{n=0}^{\infty} u_n \qquad (32)$$

$$i(t) \;=\; I_0 + \sqrt{2}\,\sum_{n=1}^{\infty} I_n\cos(n\omega_1 t + \beta_n) = \sum_{n=0}^{\infty} i_n\,. \qquad (33)$$

The instantaneous power at the load terminals

$$p(t) \;=\; \frac{d}{dt}W(t) \;=\; u(t)\,i(t) = \sum_{n=0}^{\infty} S_n\cos(n\omega_1 t + \varphi_n) \qquad (34)$$

is a sum of oscillating components and these oscillations degrade the power factor λ to zero value.

This explanation is erroneous, however. The energy cannot flow across an open circuit so that the instantaneous power harmonic component of the $n^{\rm th}$ order

$$p_n(t) \;=\; S_n\cos(n\omega_1 t + \varphi_n) \qquad (35)$$

cannot exist as a physical quantity. This is only a mathematical object. There are no physical oscillations of the energy in the circuit shown in Fig. 5.

Being aware of that, the concept of harmonics is fundamental for the power theory, however. All attempts aimed at a description of the power phenomena without harmonics have failed. In fact, the Currents' Physical Components in the frame of the CPC-based power theory are defined in the frequency-domain, meaning in terms of harmonics. Such a power theory, of comparable strength, in the time-domain was not yet developed.

IV. CAN THE POWER PROPERTIES BE IDENTIFIED INSTANTANEOUSLY?

A clear physical interpretation of the instantaneous power $p(t)$ has created a strong incentive for specifying and describing power properties of electrical systems in terms of powers defined just as instantaneous quantities. Indeed, the instantaneous power $p(t)$ is an unquestionable power quantity. The Instantaneous Reactive Power p-q Theory is the most visible demonstration of such an approach. This approach is in a clear contrast to a common custom of defining powers through averaging of some quantities over the period T of voltages and currents.

There are situations in power systems where indeed instantaneous values of voltages and currents are crucial, as it is during disturbances or faults. Performance of electrical systems with periodic voltages and currents is specified at normal operation not in terms of instantaneous values of power, but in terms of quantities defined as some integrals over the supply voltage period T, however. These are the active, reactive and apparent powers, the voltage and current rms value, the power factor, rms value of harmonics, harmonic distortion, or the voltage and current symmetrical components. The instantaneous power $p(t)$ usually is not a matter of interest for system designers and operators. However, terms like “*rms*”, “*apparent power*”, and “*harmonic*” are alien for theories that describe instantaneous properties of electrical systems.

The major difficulty of the power theories that claim to be “instantaneous“, stems from the fact that power properties of the load cannot be identified [13] instantaneously. This is illustrated in Fig. 6, with an unknown load and a pair of instantaneous values of the load voltage and current.

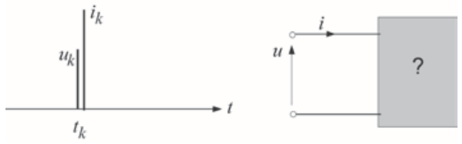

Fig. 6. Unknown load and pair of instantaneous values of the load voltage and current.

As shown in Fig. 7, at the same samples u_k, i_k, a resistor, an inductor or a capacitor or any combination of them could be in the in “black box”.

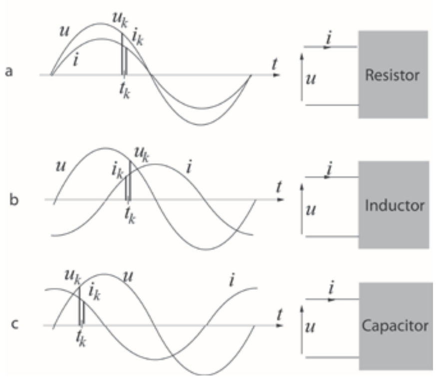

Fig. 7. Different loads with identical pairs of the voltage and current instantaneous value.

In fact, an infinite number of different loads can have identical pairs of voltage and current samples. Taking into account that the load voltage and current are in general nonsinusoidal, the instantaneous values of the voltage and current over the whole period T have to be measured to draw conclusions on the load power properties.

Quantities obtained by averaging could be added to the instantaneous power theory, but this would undermine the claim that the theory is indeed instantaneous. It is just the case of the Instantaneous Reactive Power *p-q* Theory. As it was demonstrated in [13] two entirely different loads, one purely resistive and another one, purely inductive, can have at some instant of time, identical pairs of the *p*, *q* powers. The circuits are shown in Fig. 8 and in Fig. 9.

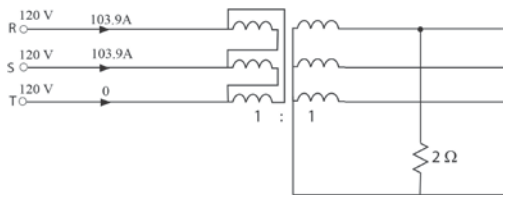

Fig. 8. A three-phase circuit with a purely resistive load.

The instantaneous active and reactive powers in this resistive circuit are equal to

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}UI\cos(2\omega_1 t - 30^{\rm o}) \tag{36}$$

$$q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}UI\sin 2(\omega_1 t + 30^0)\,. \tag{37}$$

At the instant of time such that: $\omega_1 t + 30^{\rm o} = 45^{\rm o}$, these powers are equal to

$$p = -q = \sqrt{3}UI\,. \tag{38}$$

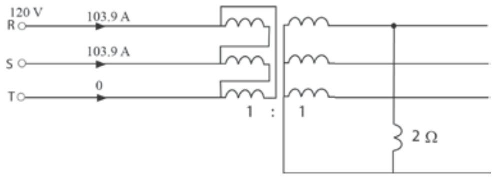

Fig. 9. A three-phase circuit with a purely reactive load.

The instantaneous active and reactive powers in this reactive circuit are equal to

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}UI\cos(2\omega_1 t - 30^{\rm o}) \tag{39}$$

$$q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}UI\,[1 + \sin(2\omega_1 t - 30^{\rm o})]\,. \tag{40}$$

At the instant of time such that: $2\omega_1 t = 30^{\rm o}$, these powers are equal to

$$p = -q = \sqrt{3}UI\,. \tag{41}$$

It means that having such a pair (*p*, *q*), we cannot conclude what are the power properties of the load. The active power *P* can be found only by averaging the instantaneous active power *p*, but even with this averaging, the concepts of the apparent power *S* and consequently, the concept of the power factor does not exist inside of this theory. Thus, the instantaneous approach to the power theory is not capable [16] of providing any physical interpretation of the power properties of electrical systems. It can be done only by the averaging approach. Just this approach provides fundamentals for the Currents’ Physical Components (CPC) – based power theory. This theory provides presently the most advanced description of the power properties of electrical systems, as well as, theoretical fundamentals for their compensation. Explanation of these phenomena is drafted in the next Section.

V. Power Related Physical Phenomena

Loads in commercial buildings or in industrial plants can be reduced to a load of the structure shown in Fig. 1. It can have aggregates of single-phase as well three-phase loads, supplied from three-wire or from four-wire lines. The loads can be linear or nonlinear, i.e., be harmonics generating loads (HGL). The supply voltage can be distorted, but for simplicity sake, it is assumed here that the supply voltage is symmetrical.

Having the voltages and currents at the load terminals RST available, the complex rms values $\boldsymbol{U}_{xn}$ and $\boldsymbol{I}_{xn}$ (x = R, S or T) of the voltage and current harmonics can be measured.

Let the set of all orders *n* of the load voltage and current harmonics for which their crms value can be measured with an acceptable accuracy be denoted by *N*.

For each voltage harmonic of the $n^{\rm th}$ order the equivalent load, shown in Fig. 10, can be found

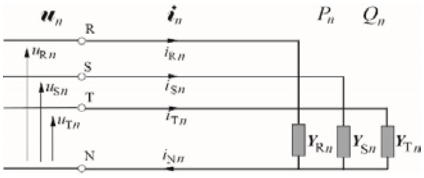

Fig. 10. An equivalent load for the n^{th} order harmonic.

with line-to-neutral admittances equal to

$$\boldsymbol{Y}_{\mathrm{R}n} = \frac{\boldsymbol{I}_{\mathrm{R}n}}{\boldsymbol{U}_{\mathrm{R}n}}, \quad \boldsymbol{Y}_{\mathrm{S}n} = \frac{\boldsymbol{I}_{\mathrm{S}n}}{\boldsymbol{U}_{\mathrm{S}n}}, \quad \boldsymbol{Y}_{\mathrm{T}n} = \frac{\boldsymbol{I}_{\mathrm{T}n}}{\boldsymbol{U}_{\mathrm{T}n}}. \tag{42}$$

The active power of a harmonic of the n^{th} order

$$P_n = \mathrm{Re}\{\boldsymbol{Y}_{\mathrm{R}n} + \boldsymbol{Y}_{\mathrm{S}n} + \boldsymbol{Y}_{\mathrm{T}n}\}\,||\boldsymbol{u}_n||^2 \tag{43}$$

could be positive or negative. If it is positive, then the energy at this harmonic frequency is transported from the supply source to the load. The set of such harmonic orders is denoted by N_{C}. If this harmonic active power P_n is negative, then the energy at this harmonic frequency is transported from the load back to the supply source. The set of such harmonic orders is denoted by N_{G}.

Having sub-sets N_{C} and N_{G}, all harmonics of the load voltage and current can be decomposed [9] into the supply system originated harmonics and the load originated harmonics,

$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \sum_{n\in N_{\mathrm{C}}} \boldsymbol{i}_n + \sum_{n\in N_{\mathrm{G}}} \boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{C}} + \boldsymbol{i}_{\mathrm{G}} \tag{44}$$

$$\boldsymbol{u} = \sum_{n\in N} \boldsymbol{u}_n = \sum_{n\in N_{\mathrm{C}}} \boldsymbol{u}_n + \sum_{n\in N_{\mathrm{G}}} \boldsymbol{u}_n = \boldsymbol{u}_{\mathrm{C}} - \boldsymbol{u}_{\mathrm{G}}\,. \tag{45}$$

The same can be done with the harmonic active powers

$$P = \sum_{n\in N} P_n = \sum_{n\in N_{\mathrm{C}}} P_n + \sum_{n\in N_{\mathrm{G}}} P_n = P_{\mathrm{C}} - P_{\mathrm{G}}\,. \tag{46}$$

The current $\boldsymbol{i}_{\mathrm{G}}$ in (39)

$$\boldsymbol{i}_{\mathrm{G}} = \sum_{n\in N_{\mathrm{G}}} \boldsymbol{i}_n \tag{47}$$

referred to as ***a load generated current***, is associated with a phenomenon of the energy transfer from the load to the supply source due to the current distortion by the load nonlinearity or periodic variance, with the period T, of the load parameters. This distortion, described usually in terms of harmonics, causes that at some harmonic frequencies the energy flows from the load back to the supply source, where it is dissipated on the supply source internal resistance.

Voltage $\boldsymbol{u}_{\mathrm{G}}$ in (45) is a negative sum of harmonics that occur at the load terminals as a response to the load generated current harmonics. As the voltage response of the source, these harmonics have an opposite sign as compared to the supply source originated harmonics.

The current

$$\boldsymbol{i}_{\mathrm{Ca}} = \frac{P_{\mathrm{C}}}{||\boldsymbol{u}_{\mathrm{C}}||^2}\,\boldsymbol{u}_{\mathrm{C}} = G_{\mathrm{Ce}}\,\boldsymbol{u}_{\mathrm{C}} \tag{48}$$

is associated with the phenomenon of the energy transfer from the supply source to the load. This current is referred to as ***an active current***.

For each harmonic of the order n from the sub-set N_{C} the load has an equivalent admittance [17]

$$\boldsymbol{Y}_{\mathrm{e}n} = G_{\mathrm{e}n} + jB_{\mathrm{e}n} = \frac{P_n - jQ_n}{||\boldsymbol{u}_n||^2} = \frac{1}{3}(\boldsymbol{Y}_{\mathrm{R}n} + \boldsymbol{Y}_{\mathrm{S}n} + \boldsymbol{Y}_{\mathrm{T}n}) \tag{49}$$

which can be calculated having measured the harmonic active and reactive powers P_n and Q_n and the three-phase rms value of the supply voltage harmonic $||\boldsymbol{u}_n||$. Having this admittance, two currents can be defined

$$\boldsymbol{i}_{\mathrm{Cs}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N_{\mathrm{C}}} (G_{\mathrm{e}n} - G_{\mathrm{Ce}})\,\mathbf{1}_n \boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{50}$$

referred to as ***a scattered current*** and

$$\boldsymbol{i}_{\mathrm{Cr}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N_{\mathrm{C}}} jB_{\mathrm{e}n}\,\mathbf{1}_n \boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{51}$$

referred to as ***a reactive current.*** The symbol $\mathbf{1}_n$ denotes unite three-phase vector for the n^{th} order harmonic, defined as

$$\mathbf{1}_n = \begin{bmatrix} 1 \\ 1e^{-jn\frac{2\pi}{3}} \\ 1e^{jn\frac{2\pi}{3}} \end{bmatrix} = \begin{bmatrix} 1 \\ \beta_n \\ \beta_n^* \end{bmatrix} = \begin{cases} \mathbf{1}^{\mathrm{p}}, \text{ for } n = 3k-2 \\ \mathbf{1}^{\mathrm{n}}, \text{ for } n = 3k-1, \quad k = 1, 2... \\ \mathbf{1}^{\mathrm{z}}, \text{ for } n = 3k \end{cases} \tag{52}$$

where

$$\beta_n = 1e^{-jn\frac{2\pi}{3}} = (\alpha^*)^n, \quad \alpha = 1e^{j\frac{2\pi}{3}}. \tag{53}$$

The scattered current $\boldsymbol{i}_{\mathrm{Cs}}$ is associated with the phenomenon of the equivalent conductance $G_{\mathrm{e}n}$ change around the G_{Ce} value, while the current $\boldsymbol{i}_{\mathrm{Cr}}$ is associated with the phenomenon of the phase-shift between the voltage and current harmonics. The current

$$\boldsymbol{i}_{\mathrm{Cu}} = \sqrt{2}\mathrm{Re}\sum_{n\in N_{\mathrm{C}}} (\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}\mathbf{1}^{\mathrm{p}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}\mathbf{1}^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}}\mathbf{1}^{\mathrm{z}})\boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{54}$$

is associated distinctively with the phenomenon of the load current asymmetry caused by the load imbalance. It is referred to as ***an unbalanced current***, where [15]

$$\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}} = \frac{1}{3}[(\boldsymbol{Y}_{\mathrm{R}n} + \alpha\beta\boldsymbol{Y}_{\mathrm{S}n} + \alpha^*\beta^*\boldsymbol{Y}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{e}n}(1 + \alpha\beta + \alpha^*\beta^*)] \tag{55}$$

$$\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}} = \frac{1}{3}[(\boldsymbol{Y}_{\mathrm{R}n} + \alpha^*\beta\boldsymbol{Y}_{\mathrm{S}n} + \alpha\beta^*\boldsymbol{Y}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{e}n}(1 + \alpha^*\beta + \alpha\beta^*)] \tag{56}$$

$$\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} = \frac{1}{3}[(\boldsymbol{Y}_{\mathrm{R}n} + \beta\boldsymbol{Y}_{\mathrm{S}n} + \beta^*\boldsymbol{Y}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{e}n}(1 + \beta + \beta^*)] \tag{57}$$

are unbalanced admittances of the positive, negative and zero sequence. Consequently, a three-phase vector $\boldsymbol{i}$ of the load current in the circuit shown in Fig. 1 can be decomposed according to the CPC-based power theory, into five Physical Components, as specified by (3).

VI. An Example of CPC Application for A Reactive Compensator Design

This paper was devoted to explanation, in terms of the CPC concept, the physical phenomena that affect the energy transfer in electrical systems. Nonetheless, the CPC was developed for providing fundamentals for the design of compensators that could improve the effectiveness of this transfer. In fact, the

CPC-based power theory is currently the only one theory that creates fundamentals for compensators design in the presence of the supply voltage distortion. This is illustrated in the following example. Details are in the paper [18], currently in printing.

Example. Let us assume that the load shown in Fig. 11, with $\omega_1 L = R = 0.5\ \Omega$, is supplied with a symmetrical voltage of the fundamental harmonic rms value $U_1 = 240$ V, distorted by the 3rd, 5th, and 7th order harmonics of the relative rms value $U_3 = 2\%U_1$, $U_5 = 3\%U_1$ and $U_7 = 1.5\%U_1$.

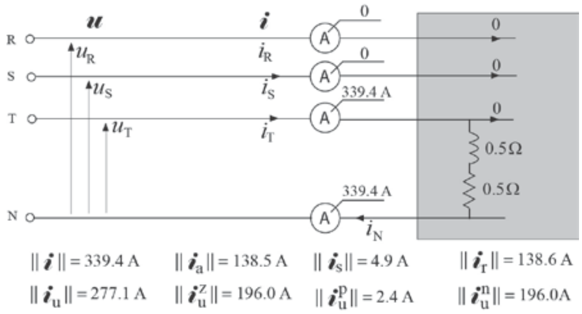

Fig. 11. An example of an unbalanced load.

The load power factor is $\lambda = P/S = 0.408$. The three-phase rms values of the particular physical components of the load current are shown in Fig. 11. Index "C" is omitted in symbols of these current because it was assumed that the load is linear so that the load generated current $\boldsymbol{i}_G$ does not exist. Indices "z", "p" and "n" denote symmetrical components of the zero, positive and the negative sequence of the load unbalanced current.

The CPC – based power theory provides fundamentals for calculating LC parameters of the reactive compensator, compiled in Table 1.

Table 1. LC parameters of a reduced complexity compensator

	Line:	R	S	T	RS	ST	TR
L	mH	1730	770	444	0	2600	1155
C	mF	0	399	691	0	0	266

The results of compensation are shown in Fig. 12. The power factor is improved by the compensator to $\lambda = 0.994$.

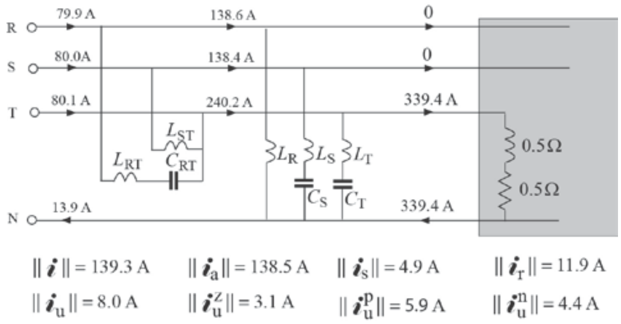

Fig. 12. Results of compensation of an unbalanced load.

VIII. CONCLUSIONS

Explanation of the physical phenomena that affect the effectiveness of the energy transfer, apparently having only cognitive merits, occurs to be crucially important for the development of methods of this effectiveness improvement. Just because of misinterpretations of these phenomena the Instantaneous Reactive Power p-q theory is not capable of providing right control for switching compensators in the presence of the supply voltage distortion or asymmetry. The CPC - based power theory does not have such a deficiency because it is focused on a right interpretation of physical phenomena associated with the energy transfer.

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