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Power theories and meta-theory of powers in electrical circuits

Abstract. *Some ideas and rules used in the past for the development of the currently existing power theories of electrical circuits are compiled in the paper. Such rules can be regarded as a meta-theory of electric power, meaning a theory that has other theories as its subject matter, in this case, power theories. Distinction of* ***power theories*** *from a* ***meta-theory*** *of electric power provides a clarification in debates on electric powers. Moreover, motivations, validation and comparison of power theories cannot be a subject of these theories, but a meta-theory of electric power. Also history of various approaches to power theories development, such as averaging and instantaneous approaches, time-domain and frequency-domain approaches, are compiled and discussed in the paper.*

Streszczenie. *W artykule zestawiono użyte w przeszłości idee i zasady odnośnie formułowania istniejących obecnie teorii mocy obwodów elektrycznych. Idee takie i zasady mogą być traktowane jako meta-teoria mocy obwodów elektrycznych, w tym sensie, że jej przedmiotem są inne teorie, w tym przypadku, teorie mocy. Rozróżnienie pojęć* ***teoria*** *i* ***meta-teoria,*** *w odniesieniu do mocy, umożliwia uporządkowanie dyskusji nad mocami w obwodach elektrycznych. Ponadto, motywacje rozwoju teorii mocy, ocena ich wiarogodności oraz ich porównywanie nie mogą być przedmiotem teorii, lecz meta-terorii, w tym przypadku, meta-teorii mocy obwodów elektrycznych. W artykule zestawiono także historię różnych podejść do rozwoju teorii mocy, takich jak opartych na uśrednianiu i podejściu chwilowym, w dziedzinie czestotliwości i w dziedzinie czasu.(****Teorie mocy i meta-teoria mocy obwodów elektrycznych****)*

Keywords: Power definitions, nonsinusoidal waveforms, time-domain, frequency-domain, Current's Physical Components, CPC.

Słowa kluczowe: Definicje mocy, przebiegi niesinusoidalne, dziedzina czasowa, dziedzina częstotliwościowa, Składowe Fizyczne Prądu, CPC.

There is nothing more necessary for the man of science than its history and the logic of discovery…the way error is detected…

LORD ACTON

Power theory and meta-theory of electric power

As to my best knowledge, the term ***power theory*** has occurred in literature for the first time, in Polish, in Fryze's paper in 1931 [1]. It is used now as a classifier of various power concepts developed by scientists who studied power properties of electrical circuits. In such a meaning it is used in phrases such as, *Budeanu's power theory, Fryze's power theory*, *Shepherd and Zakikhani's (S&Z) power theory, Kusters and Moore's (K&M) power theory, Depenbrock's power theory* also known as FDB method*, Nabae and Akagi's Instantaneous Reactive Power (IRP) p-q Theory, Czarnecki's CPC power theory, Tenti and Tedeschi (T&T) conservative power theory* and so on. When a phrase like *Budeanu's power theory* is used, we can have an idea, of what the phrase refers to, and can find its details in literature. Used in such a way, the term *theory* can be regarded [2] as a system of terms defined in this theory, relations between these terms, and their interpretations.

The term ***power theory*** is also used sometimes in a different meaning, namely, as a kind of a database of what is known on power properties of electrical circuits, thus as a collection of true statements on power related phenomena, and/or mathematical expressions related to these properties and interpretations. Individual statements belong to this database, meaning, to power theory, as long as it is not proven that these statements do not hold truth. Regarded as such a database, the *power theory* is not a system of terms and relationships between them, i.e., it is not a *theory* in the previous meaning. At the same time, however, anyone who reveals even a single, but a new power related property, contributes to the development of the power theory regarded as a database.

Statements or opinions on the meaning of the term *theory*, as these presented above, are not a subject of a theory, but a meta-theory, in our case of a ***meta-theory of electric power***. Similarly, statements and opinions on the reasons of the power theory development, expectations and opinions on methods used in a specific power theory, belong to meta-theory of electric power. Also, to the meta-theory belong proofs of inconsistency in specific power theories.

Distinction of the theory from meta-theory of electric power enables us for a clarification in debates on electric powers. Mathematically expressed statements in power theories are strict, while those in meta-theory may have only a form of descriptive opinions. Statements in power theories should be general and valid for all situations, confined only by a set of assumptions imposed by a particular theory. Statements in the meta-theory may even have a form of an individual observation, but capable of undermining the correctness of a whole power theory.

This paper is written from the perspective of a meta-theory of electric power, therefore, it is composed of only the author's opinions on power theories and consequently, these opinions can be challenged.

Power theory as a physical theory

Power theories have powers in electrical circuits as their subject matter, i.e., a part of the physical realm. They are physical theories in the sense that they describe physical phenomena. Electrical circuits and systems are the subject of electrical engineering, which spans physics, technology and economy. Power theory, like other theories in electrical engineering, such as circuit theory, electrical machinery theory or antenna theory, serves specific needs of electrical engineering. It can be judged by how well it satisfies expectations of electrical engineering. These expectations are not well defined, but do exist.

Mathematics is the main tool of power theories. It is used on various levels of abstraction, from quite elementary to highly sophisticated. Nonetheless, there does not exist any power theory with mathematical errors. Such errors would refute a potential theory instantaneously and consequently, all existing power theories are mathematically correct. Individual power theories differ mainly in their physical interpretation of power related phenomena and power terms used for describing them.

What does the meta-theory of electric power consist of?

The development of each specific power theory was accompanied by statements on intensions, expectations and opinions on electric powers, expressed by the author of the theory in related papers. Such statements do not belong to theory, however, but rather to a meta-theory of electric power. In the process of review, dissemination and debates, these statements have often gained at least some level of acceptance in the electrical engineers community. Some ideas which are common for all power theories developed, that have gained some level of acceptance, even if they lack precision, constitute the meta-theory of electric power.

It is common for all power theories developed to attempt to describe energy flow in power terms and provide them some physical interpretations, even if these interpretations are debatable. Thus, they are developed for cognitive purposes because of our human curiosity. Also, it is common for all power theories to be developed in a response to various practical needs, such as the power system descripttion in power terms, equipment design or energy accounts. Thus, they are developed for practical purposes. Moreover, it is common for all power theories that voltages and currents at the load terminals provide primary data for the load description in power terms. Topology of the load and parameters of its elements may be unknown. Description of an individual element or a device of the circuit in power terms does not constitute power theory. Such a description can serve only as an illustration.

Averaging and instantaneous approach

Main quantities in the majority of power theories are defined with averaging over the period T of the voltage and current variability. This paradigm is basic for the Budeanu [3] Fryze [1], S&Z [4], K&M [5], FDB [6], CPC–based [8, 9] and T&T conservative [10] power theories. They share the concept of the voltage and current rms value as a major power related term and consequently, the concept of the apparent power S, a key power for rating transmission equipment, power factor calculation, energy accounts and compensator design.

The instantaneous active and reactive powers p and q, in the Instantaneous Reactive Power *p-q* theory are defined [7] as instantaneous quantities. This is motivated according to Ref. [11] by the need of instantaneous compensation and an opinion that power properties of electrical systems can be instantaneously identified. According to Ref. [12], instantaneous identification of power properties is not possible, however. In the case of systems with periodic voltages and currents, observation of these quantities over at least one period T is necessary for identification of the system power properties and for conclusions on the possibility of its com-pensation. Even for such a fundamental question for power properties of electrical systems as: "*does a permanent flow of energy from the supply source to the load exist in the circuit?*" no answer can be provided without observing this flow over one cycle of its variability. At a single instant of time, infinite number of loads with different properties can have [12] an identical pair of p and q powers. Power quan-tities such as the active, reactive, unbalanced and apparent powers, and consequently, also the power factor, which are of the major importance for power system design, operation and its performance evaluation, do not belong to the set of basic power quantities in the IRP p-q theory.

Time- and frequency-domains approach

The averaging approach to power theory formulation contains a frequency-domain approach, used by Budeanu [3], Kusters & Moore [5] and continued in the CPC-based power theory [8, 9] and time-domain approach, originated by Fryze [1]. According to Fryze, explanation of power properties of electrical circuits directly, in time-domain, without any use of harmonics, regarded as only mathematical entities, was much more fundamental. Moreover, due to difficulties with measurement of harmonics at the time when Fryze formulated his power theory, technical implementations of the power theory formulated in the frequency-domain were limited.

Fourier Transforms

(1) $$\boldsymbol{F}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt\,,$$

(2) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \boldsymbol{F}(\omega)\, e^{j\omega t}\, d\omega\,,$$

make the time-domain and the frequency-domain formally equivalent in the sense, that any power related quantity defined in the time-domain has its equivalent in the frequency domain. Similarly, any power related quantity defined in the frequency-domain has its equivalent in the time-domain. One can observe, moreover, that voltages and currents in real power systems are confined as to the value, time duration and the frequency. Consequently, Fourier integrals of such voltages and currents or their spectra are convergent at any instant of time and at any frequency.

At formal ideal symmetry of both domains, there is a striking asymmetry as to conclusions which can be drawn using them on power properties of electrical circuits. It was demonstrated by Fryze [1] with the following reasoning.

Let us consider a circuit shown in Fig. 1, with dc supply voltage and periodic switch

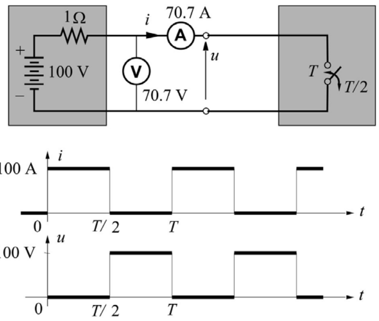

Fig. 1. Circuit with periodic switch

According to frequency-domain approach, the voltage and current can be expressed in terms of harmonics in the forms

(3) $$u(t) = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n) = \sum_{n=0}^{\infty} u_n\,,$$

(4) $$i(t) = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \beta_n) = \sum_{n=0}^{\infty} i_n\,,$$

The instantaneous power $p(t)$ at the load terminals, i.e., the rate of energy, $W(t)$, flow from the supply source to the load, is equal to

(5) $$p(t) = u(t)\, i(t) = \sum_{r=0}^{\infty} u_r \sum_{s=0}^{\infty} i_s = \sum_{n=0}^{\infty} S_n \cos(n\omega_1 t + \psi_n)\,,$$

thus, it is equal to an infinite sum of oscillating components. To find this power at any instant of time, this series has to be calculated. Moreover, this formula suggests the existence of an infinite number of oscillating components in the instantaneous power.

In the time-domain, however, by hand is visible from the voltage and current waveforms that their product

(6) $$p(t) = u(t)\, i(t) \equiv 0\,,$$

apart only from points of the voltage and current discontinuities. There is no flow of energy and no energy oscillations between the supply source and the load in such a circuit.

The fact that the time-domain makes this kind of direct conclusions possible might imply a superiority of the time-domain approach over the frequency-domain. Observation of the history of progress in our comprehension of power related phenomena, contradicts this suggestion, however. This progress was achieved mainly in the frequency-domain. To show this, let us observe evolution of the concept of the reactive current at nonsinusoidal supply voltage.

The concept of a *reactive current* was introduced originally by Fryze [1] in the time-domain. After he introduced the concept of active current $i_{\mathrm{aF}}(t)$, as the only useful component of the load current, the reactive current was defined as the remaining part of this current, namely

(7) $$i_{\mathrm{rF}}(t) \stackrel{\mathrm{df}}{=} i(t) - i_{\mathrm{aF}}(t)\,.$$

The current defined in such a way lacks any physical interpretation; it is not known why it occurs, and it is not known how the load properties affect its value. The only conclusion on this current is that it is a useless current, but this a trivial conclusion, if the active current is defined as the only useful component of the load current.

The concept of the reactive current was equipped with the physical meaning by Shepherd and Zakikhani [4] not in the time-domain, but in the frequency-domain, as follows.

At nonsinusoidal supply voltage

(8) $$u(t) = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n)\,,$$

the load current

(9) $$i(t) = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \alpha_n - \varphi_n)\,,$$

contains a component, composed of current harmonics delayed by $\pi/2$ (in-quadrature) with respect to the voltage harmonics. Each of these current harmonics “in-quadrature” to the voltage harmonics has the same meaning as the reactive current at sinusoidal supply. Shepherd and Zakikhani concluded that their sum should be regarded as the reactive current at nonsinusoidal supply voltage, namely.

(10) $$\sqrt{2}\sum_{n=1}^{\infty} I_n \sin\varphi_n \sin(n\omega_1 t + \alpha_n) \stackrel{\mathrm{df}}{=} i_{\mathrm{r}}(t)\,.$$

This reactive current has nothing in common with the current defined by Fryze in the time-domain, however. It has clear physical interpretation, but this interpretation requires that the concept of harmonics is used, i.e., the frequency-domain. Unlike the Fryze reactive current $i_{\mathrm{rF}}(t)$, defined in the time-domain, the reactive current $i_{\mathrm{r}}(t)$ defined in the frequency-domain, can be expressed in terms of the load equivalent parameters for harmonic frequencies, and again, specified in the frequency-, but not in time-domain.

Let us consider a linear, time-invariant (LTI) load, shown in Fig. 2

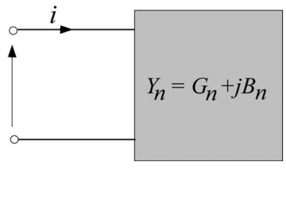

Fig. 2. LTI load and its admittance for harmonic frequencies

Its equivalent parameters can be specified in the frequency-domain by admittance for harmonic frequencies

(11) $$\boldsymbol{Y}_n = G_n + jB_n\,.$$

If the Fourier series of the supply voltage is expressed in the complex form

(12) $$u(t) = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{U}_n\, e^{jn\omega_1 t}\,,$$

then the reactive current has the form

(13) $$i_{\mathrm{r}}(t) \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n \boldsymbol{U}_n\, e^{jn\omega_1 t}\,.$$

The reactive current is expressed in terms of harmonics, meaning in the frequency-domain. It can be recalculated, of course, to the time-domain but in that domain cannot be interpreted as a sum of the current harmonics in-quadrature to the voltage harmonics, since the term *harmonic* is proper only for the frequency-, but not for the time-domain. Independently of how it might be interpreted in the time-domain, we should observe, however, that it was not identified in the time-, but in the frequency-domain.

The same is with the *scattered current*. Its presence in the load current was revealed [8] in the frequency-domain.

If G_{e} is equivalent conductance [8] of the load, then the scattered current is defined as

(14) $$i_{\mathrm{s}}(t) \stackrel{\mathrm{df}}{=} (G_0 - G_{\mathrm{e}})U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} (G_n - G_{\mathrm{e}})\boldsymbol{U}_n\, e^{jn\omega_1 t}\,.$$

This component occurs in the load current when the load conductance G_n changes with harmonic frequency.

Because the time- and the frequency-domains are mutually equivalent, the scattered current, originally identified in the frequency-domain, can be also somehow recalculated and expressed in the time domain. The physical interpretation of this current existence as the effect of a change of the load conductance with harmonic frequency cannot be used in the time-domain, however.

Looking back into history of debates between followers of the time-domain approach and the frequency-domain approach, and observing how the former were forceless in these debates, one can draw a conclusion that the time-domain lacks effective tools in debates on powers. Just this explains the long presence of the Budeanu’s concepts of the reactive power Q_{B} and distortion power D in electrical engineering, in spite of a strong opposition by Fryze, i.e., from the time-domain perspective.

Budeanu defined [3] the reactive power with formula

(15) $$Q_{\mathrm{B}} \stackrel{\mathrm{df}}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n\,.$$

Fryze’s opinion that powers should not be defined in terms of harmonics resulted in a search for an equivalent definition of the Budeanu’s reactive power in the time-domain, which concluded with the formula

(16) $$Q_{\mathrm{B}} = -\frac{1}{T}\int_0^T i(t)\, \mathcal{H}\{u(t)\}\, dt\,,$$

where

(17) $$\mathcal{H}\{u(t)\} = \frac{1}{\pi}\,\mathrm{PV}\int_{-\infty}^{+\infty} \frac{u(\tau)}{\tau - t}\, dt\,,$$

is the Hilbert transform of the load voltage and PV denotes the principle value of the integral.

It is relatively easy to conclude [13] in the frequency-domain that the reactive power as defined by Budeanu for circuits with nonsinusoidal voltages and currents does not refer to energy oscillations, as it is when these quantities are sinusoidal. Indeed, let us assume that the load voltage and current contain the n^{th} order harmonic

$$u_n(t) = \sqrt{2}U_n \cos n\omega_1 t \,, \tag{18}$$

$$i_n(t) = \sqrt{2}I_n \cos(n\omega_1 t - \varphi_n) \,, \tag{19}$$

then the instantaneous power $p(t)$ of the load contains a bi-directional component

$$p_{\mathrm{b}n}(t) = U_n I_n \sin\varphi_n \sin(2n\omega_1 t) = Q_n \sin(2n\omega_1 t) \,, \tag{20}$$

where

$$Q_n = U_n I_n \sin\varphi_n \,, \tag{21}$$

is a generalized amplitude of this oscillation, meaning it can have not only a positive, but also a negative value. The sum of these amplitudes, meaning reactive power Q_B, does not carry any information on energy oscillation since each of its harmonic component $p_{bn}(t)$ has different frequency, $2n\omega_1$. The energy oscillation can exist even if amplitudes Q_n cancel mutually and consequently, $Q_B = 0$.

This reasoning on the physical meaning of the reactive power Q_B applied the frequency-domain approach. As to the author's best knowledge, equivalent reasoning in the time-domain, i.e., without use of the concept of harmonics, but based on time-domain definition (17), does not exist.

Similar is with the relation between the voltage and current distortion and distortion power D, implied by Budeanu. The condition for zero distortion power D, which requires [14] that the load admittance for the supply voltage harmonics does not change with harmonic order, i.e.,

$$\boldsymbol{Y}_n = Y_n e^{-j\varphi_n} = \text{const.} \,, \tag{22}$$

and the condition [14] for the lack of voltage and current mutual distortion

$$\boldsymbol{Y}_n = Y e^{-jn\omega_1} \,, \tag{23}$$

which requires that its phase changes with harmonic order, are formulated [13] in the frequency-domain. Apart from purely resistive loads, these two conditions for the same load cannot be fulfilled. Thus, there is no relation between distortion power D and the voltage and current mutual distortion. Equivalent reasoning based on the time-domain approach, on the relation between the voltage and current distortion and distortion power D is not known for the author of this paper best knowledge, however.

Conclusions

Due to new challenges and new developments in electrical technology, or simply due to the lack of satisfaction from the previous state the development, several papers on power properties of electrical systems are published each year. There is no ground for a conclusion that this might change. Power theories of electrical systems are in the process of continuous development. The meta-theory of electrical power provides some external perspective for this development evaluation. A historical overview of reasons and goals of various concepts of power theories development, as well as presented overview of used mathematical tools, compiled in this paper, might help evaluate this progress and new concepts.

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