Invited paper

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Meta-theory of electric powers and present state of power theory of circuits with periodic voltages and currents

Abstract. *Historically justified goals of the power theories development, general expectations and mathematical tools used for it are discussed in the paper. General features of the power theory of single-phase circuits with sinusoidal voltages and currents, the only commonly accepted power theory, were used as a reference for the presented discussion. Also discussed in this paper are the major issues of power theories development such as these: Should the power theory be formulated in the frequency or in time-domain? Should it be based on quantities averaged over a period or on instantaneous values? It was shown that in spite of the name "power" theory, the load currents rather than powers convey information on power properties of electrical circuits.*

Streszczenie. *Przedmiotem niniejszego artykułu są historycznie uzasadnione cele rowoju teorii mocy, oczekiwania wzgledem niej oraz służące temu rozwojowi matematyczne narzędzia teorii mocy. Odniesieniem dla tej dyskusji są ogólne właściwości teorii mocy obwodów jednofazowych z sinusoidalnymi przebiegami prądu i napięcia, jedynej teorii mocy, co do której nie ma w elektrotechnice zastrzeżeń. W artykule omawia się częstotliwościowe i czasowe podejscia do teorii mocy a także formułowanie tej teorii w oparciu o wielkości otrzymane poprzez uśrednianie lub oparte na wielkościach chwilowych (***Meta-teoria mocy i obecny stan teorii mocy obwodów z okresowymi przebiegami prądów i napięć*)*

Keywords: Power definitions, frequency-domain, time-domain, Currents' Physical Components, CPC
Słowa kluczowe: Definicje mocy, dziedzina częstotliwościowa, dziedzina czasowe, Składowe Fizyczne Prądu, CPC,

"Theories are nets cast to catch what we call 'the world'; to rationalize, to explain, and to master it. We endeavor to make the mesh ever finer and finer".
Karl Popper, The Logic of Scientific Discovery [1].

Introduction

After more than a century of research on power properties of electrical systems and long-lasting attempts aimed at describing them in power terms, electrical engineers have a right to ask scientists for conclusions on what they achieved.

The debate, still not concluded, on power properties of electrical systems was initiated by Steinmetz's observation in 1892 [2] that the apparent power S in a circuit with an electric arc is higher than the arc's active power P.

The apparent power S is associated with the cost of the electric energy and its delivery to customer loads. Therefore, the difference between the active and apparent powers has an economic importance. This in turn generates the question of how this difference can be reduced. This question cannot be fully answered, however, without understanding energy flow and power related physical phenomena in electrical systems. Therefore, power properties of electrical systems are studied for ***cognitive***, ***practical*** and ***economic*** reasons which are mutually interlaced.

Steinmetz asked the question about powers at the very beginning of power system development, almost at the time of the tough controversy between Edison and Tesla: *should these systems be direct or alternating current systems?* Steinmetz's question at that time had cognitive rather than economic importance. The amount of electric energy produced by power system generators per capita was almost zero. Now the total power of generators in power systems per capita could be even more than twenty times higher than human natural power which is approximately 100 W. Following the increase in the generated power, the importance of Steinmetz's question has increased as well.

Unfortunately, the apparently simple issue raised by Steinmetz is actually one of the most challenging problems of electrical engineering. Theoreticians involved in studies on power phenomena have failed to catch up with the progress in power system development. Economic pressure caused electrical engineers to develop methods of compensation before power phenomena in electrical systems were fully comprehended. Also, to somehow handle powers in electrical systems, various national standards, regulations or recommendations on power terms were adopted in a number of countries.

The century long efforts of theoreticians resulted in several different approaches to description and interpretation of power phenomena in electrical systems. These are referred to as various ***power theories*** which are often regarded as sorts of "schools" of the power theory.

Followers of these schools adhere to them sometimes even with a sort of religious zeal that hampers debate on electric powers. The author of this paper was not able to publish an opinion and a proof that instantaneous reactive power q cannot represent flow of energy ***around*** supply lines, as claimed by authors of the Instantaneous Reactive Power p-q Theory. It was a heresy for its followers.

Analysis of various power theories, with respect to their goals, mathematical tools used, or correctness of physical interpretations they suggest is not the subject of a particular power theory. Such analysis and debate on various theories has to be done from a higher level than particular power theories. Therefore, evaluation of objectives, approaches and mathematical tools used by various power theories can be regarded as a subject of ***meta-theory of electric power***.

Debating on power theories, we should be aware of very logical fundamentals of such theories. They belong to an empirical science. According to Karl Popper [1] such theories are never empirically verifiable. In fact there is an asymmetry between verifiability and falsifiability of such theories. They cannot be derived from a singular observation, but a singular observation can falsify them.

Reference power theory

As the history of development of power theories has demonstrated, the expectations as to their form are very debatable. No one could claim that his concept of it is just the right one. It is also difficult to avoid a bias when these expectations are formulated by an author or followers of a specific concept of this theory. The issue is almost unsolvable.

Some help in formulating these expectations could be gained by looking upon the general form and features of the only power theory that is commonly accepted and not challenged. It is the power theory of single-phase circuits with sinusoidal voltages and currents. The level of acceptance of a description and interpretation of power properties of such circuits is so high that even the term “theory” is commonly not used.

Let us briefly summarize this theory. If a sinusoidal voltage

$$u(t) = \sqrt{2}\, U \cos\omega_1 t$$

is applied to a linear, time-invariant (LTI) load, then the load current is equal to

$$i(t) = \sqrt{2}\, I \cos(\omega_1 t - \varphi).$$

The energy is delivered to the load at the rate

$$\frac{dW}{dt} \overset{\text{df}}{=} p(t) = u(t)\, i(t)$$

which is the instantaneous power of the load. This power can be decomposed into unidirectional and bidirectional components

$$p(t) = 2\,UI \cos\omega_1 t \times \cos(\omega_1 t - \varphi) = p_{\text{u}}(t) + p_{\text{b}}(t)$$

where

$$p_{\text{u}}(t) \overset{\text{df}}{=} UI\cos\varphi\,(1 + \cos 2\omega_1 t)$$

$$p_{\text{b}}(t) \overset{\text{df}}{=} UI\sin\varphi \times \sin 2\omega_1 t.$$

The mean value, over the period T, of the instantaneous power

$$\frac{1}{T}\int_0^T p(t)\,dt \overset{\text{df}}{=} P = U\,I\cos\varphi$$

is the load active power. The amplitude of the bidirectional component of the instantaneous power

$$UI\sin\varphi \overset{\text{df}}{=} Q$$

is the reactive power. The load current can be decomposed into an in-phase with the load voltage component and a component shifted by quarter of the period

$$i(t) = \sqrt{2}I\cos(\omega_1 t - \varphi) =$$
$$= \sqrt{2}I\cos\varphi\cos\omega_1 t + \sqrt{2}I\sin\varphi\sin\omega_1 t = i_{\text{a}}(t) + i_{\text{r}}(t)$$

The active and reactive currents

$$i_{\text{a}}(t) \overset{\text{df}}{=} \sqrt{2}I\cos\varphi\cos\omega_1 t = \sqrt{2}I_{\text{a}}\cos\omega_1 t$$

$$i_{\text{r}}(t) \overset{\text{df}}{=} \sqrt{2}I\sin\varphi\sin\omega_1 t = \sqrt{2}I_{\text{r}}\sin\omega_1 t$$

are mutually orthogonal, because their scalar product, defined as

$$(i_{\text{a}}, i_{\text{r}}) \overset{\text{df}}{=} \frac{1}{T}\int_0^T i_{\text{a}}(t)\, i_{\text{r}}(t)\,dt$$

is equal to zero. Consequently, the rms value of the load current, defined as

$$||i|| \overset{\text{df}}{=} \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

satisfies the relationship

$$||i||^2 = ||i_{\text{a}}||^2 + ||i_{\text{r}}||^2, \qquad \text{or} \qquad I^2 = I_{\text{a}}^2 + I_{\text{r}}^2.$$

Multiplying the last equation by the square of the load voltage rms value $||u|| = U$, the power equation is obtained

$$S^2 = P^2 + Q^2.$$

When the load has equivalent parameters

$$\boldsymbol{Z} = Z\,e^{j\varphi} = R + jX, \qquad \text{or} \qquad \boldsymbol{Y} = Y\,e^{-j\varphi} = G + jB$$

then the rms value of the load current and its components can be expressed in terms of these parameters

$$I = YU, \qquad I_{\text{a}} = GU, \qquad I_{\text{r}} = |B|U$$

and powers

$$S = YU^2, \qquad P = GU^2, \qquad Q = -BU^2.$$

The effectiveness of the supply source utilization is specified by the power factor

$$\lambda = \frac{P}{S},$$

and it can be expressed in terms of powers, current components rms values or the load equivalent parameters

$$\lambda = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{I_{\text{a}}}{\sqrt{I_{\text{a}}^2 + I_{\text{r}}^2}} = \frac{G}{\sqrt{G^2 + B^2}}.$$

Inductive loads can be compensated to unity power factor which reduces the supply current to its active component, $i_{\text{a}}(t)$, by a shunt capacitor of susceptance

$$B_{\text{C}} = \omega_1 C = \frac{Q}{U^2} = \frac{I_{\text{r}}}{U} = -B.$$

Now, we can summarize general features of this power theory.

- (i) It provides undisputable physical interpretation of power phenomena in the system.
- (ii) Power quantities in this theory specify technical parameters of the system components and provide fundamentals for measurements, needed for evaluation of the power system operation.
- (iii) Power quantities can be expressed in terms of equivalent parameters of the load.
- (iv) It specifies the power factor and relates it to the load parameters.
- (v) It provides data needed for compensator design.

The first feature (i) has cognitive merits. The remaining have practical ones. The second feature (ii) enables system design, its supervision and energy accounts. The third and fourth features, (iii) and (iv), enable improvement of the system performance by modification of the load parameters. The last one (v) enables this improvement by compensation.

These five features of the power theory of single-phase systems with sinusoidal voltages and currents provide very solid fundamentals for such system technology. Power systems now have some properties not existing in those described above and consequently, they need more sophisticated power theories. It would be beneficial if these theories could provide equally solid fundamentals, as the above drafted power theory of single-phase systems with sinusoidal voltages and currents.

Original objectives of the power theory

After Steinmetz’s observation that the power factor in circuits with an electric arc is lower than one, studies were focused on the question: “***Why can apparent power*** S ***be higher than active power*** P***?***” Because powers in single-phase circuits with sinusoidal voltages and currents satisfy a geometric relation, the above question was formulated in terms of power squares

(1) $$S^2 - P^2 = \ ?$$

Budeanu [3] explained in 1927 this difference in terms of the reactive power which he defined in terms of the voltage and current harmonic rms values U_n I_n and their mutual phase-shift as

$$Q_{\rm B} \overset{\rm df}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$$

and distortion power D, defined in such a way that

$$S^2 - P^2 = Q_{\rm B}^2 + D^2.$$

Fryze [4] explained in 1931 this difference in terms of reactive power $Q_{\rm F}$

$$S^2 - P^2 = Q_{\rm F}^2$$

defined as a product of useless current and the supply voltage rms values

$$Q_{\rm F} \overset{\rm df}{=} ||i_{\rm Fr}||\,||u||$$

where

$$i_{\rm Fr}(t) \overset{\rm df}{=} i(t) - \frac{P}{||u||^2} u(t).$$

Fryze was also probably the first scientist who used the term "***power theory***."

Shepherd and Zakikhani [5] (1972) did not explain this difference, but concluded that apparent power S, in square, is composed of following powers

$$S_{\rm R} \overset{\rm df}{=} ||i_{\rm R}||\,||u||, \qquad Q_{\rm S} \overset{\rm df}{=} ||i_{\rm r}||\,||u||.$$

Assuming that at the supply voltage

$$u = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n)$$

the load current has the waveform

$$i = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \alpha_n - \varphi_n)$$

then currents $i_{\rm R}$ and $i_{\rm r}$ were defined by Shepherd and Zakikhani as follows

$$i_{\rm R} = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos\varphi_{\rm n}\, \cos(n\omega_1 t + \alpha_n)$$

$$i_{\rm r} = \sqrt{2}\sum_{n=1}^{\infty} I_n \sin\varphi_{\rm n}\, \sin(n\omega_1 t + \alpha_n).$$

The power equation according Shepherd and Zakikhani has the form

$$S^2 = S_{\rm R}^2 + Q_{\rm S}^2$$

meaning, without the active power.

Kusters and Moore [6] explained in 1980 the difference between apparent and active powers in terms of capacitive reactive power $Q_{\rm C}$

$$S^2 - P^2 = Q_{\rm C}^2 + Q_{\rm Cr}^2$$

defined as

$$Q_{\rm C} \overset{\rm df}{=} ||u||\,||i_{\rm qC}||\ {\rm sgn}(\dot{u}, i).$$

In this definition

$$i_{\rm qC}(t) \overset{\rm df}{=} \frac{(\dot{u}, i)}{||\dot{u}||^2}\,\dot{u}(t)$$

where $\dot{u} = du/dt$. The power $Q_{\rm Cr}$ was introduced simply to satisfy the power equation.

This review of the most publicized approaches to studies on the power theory, which filled almost a hundred years of research, identifies its historically major issue: the physical cause of the difference between the apparent and the active power. It is of cognitive (i) rather than of a practical importance. Practical issues have occurred in Shepherd & Zakikhani's and Kusters & Moore's studies. They attempted to solve a problem of compensation in the presence of harmonics.

Tools for power theory development

While the question (1) refers to the primary objective of power theory, the question on tools that could provide the answer leads to two questions:

1. Should the power theory be formulated in the frequency-domain, as postulated by Budeanu or in the time-domain, as postulated by Fryze?
2. Should the power theory be formulated based on quantities calculated by averaging over a period of the supply voltage or on instantaneous values, as postulated by Akagi and Nabae [7]?

Some results in studies on electric powers, particularly with respect to compensation, can be obtained using methods of optimization. However, even if such methods can provide in some situations excellent results, especially for compensator design, they do not contribute to power theory development. Power theory is a set of general statements on powers in electrical systems, while optimization provides only specific statements.

Frequency versus time-domain

For any quantity specified in the time-domain the Fourier Transform enables calculation of its unique equivalent in the frequency-domain, and for any quantity specified in a frequency-domain the Inverse Fourier Transform enables calculation of its unique equivalent in the time-domain. Thus these two domains are mathematically equivalent. This is particularly true in electrical systems where voltages and currents have to be continuous and frequency bounded, thus they satisfy necessary conditions for the Fourier series convergence.

The question whether the power theory should be formulated in a frequency or in a time-domain, as suggested in Refs. [7, 9, 10], or not boils down to the question: should the concept of harmonics be used or not?

To answer such a question two aspects of the issue should be taken into account. The first of them is metrologycal availability of harmonics. Such availability can be crucial for technical implementations of a harmonics-based power theory. When the concept of harmonics was introduced to the power theory, it was possible to measure, using tuned filters, only the rms value of harmonics. Their phase was practically beyond measurement possibility. Now, sampling and digital signal processing (DSP) are capable of providing complex rms value of harmonics up to relatively high order in real time. Thus availability is not an issue.

The second aspect of the harmonics issue is more crucial: does the concept of harmonics contribute to our comprehension of power phenomena or hinder this comprehension?

Thinking of voltages and currents as sums of harmonics when applied to the energy flow studies may lead to unacceptable conclusions. This was demonstrated by Fryze who analyzed the energy flow in the circuit shown in Fig. 1, with dc supply voltage and a periodic switch.

According to the frequency-domain approach, the load voltage and current can be expressed in terms of harmonics in the forms

(2) $$u(t) = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n) = \sum_{n=0}^{\infty} u_n$$

(3) $$i(t) = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \beta_n) = \sum_{n=0}^{\infty} i_n.$$

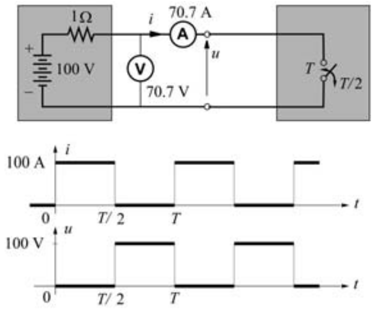

Fig. 1. Circuit with a periodic switch.

The instantaneous power $p(t)$ at the load terminals, i.e., the rate of energy, $W(t)$, that flows from the supply source to the load is equal to

(4) $$p(t) = \frac{dW}{dt} = u(t)\, i(t) = \sum_{r=0}^{\infty} u_r \sum_{s=0}^{\infty} i_s = \sum_{n=0}^{\infty} S_n \cos(n\omega_1 t + \psi_n)\ .$$

Thus, it is equal to an infinite sum of oscillating components. This formula suggests the existence of an infinite number of oscillating components in the instantaneous power. To find the instantaneous power at any instant of time, this series has to be calculated.

In the time-domain, however, it is visible from the voltage and current waveforms that their product

(5) $$p(t) = u(t)\, i(t) \equiv 0$$

apart only from points of discontinuity. There is no flow of energy and no energy oscillation. The time-domain approach is clearly superior to the frequency-domain in such a situation. There are also quite opposite situations, however.

The power theory of single-phase circuits with linear, time-invariant (LTI) loads at nonsinusoidal supply voltage was formulated eventually [8] not in time, but in the frequency-domain. Such loads, shown in Fig. 2, can be specified in terms of admittance for harmonic frequencies

$$\boldsymbol{Y}_n = G_n + jB_n$$

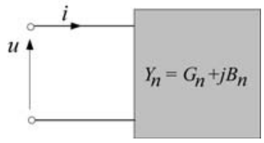

Fig. 2. LTI load and its admittance for harmonic frequencies.

If the supply voltage is expressed in terms of harmonics

(6) $$u(t) = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

the load current, apart from the active current defined by Fryze [4], contains two currents which are defined [8] in the frequency-domain. These are the reactive current

(7) $$i_{\mathrm{r}}(t) \stackrel{\mathrm{df}}{=} \sqrt{2}\ \mathrm{Re}\sum_{n=1}^{\infty} jB_n \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

and the scattered current

(8) $$i_{\mathrm{s}}(t) \stackrel{\mathrm{df}}{=} (G_0 - G_{\mathrm{e}})U_0 + \sqrt{2}\ \mathrm{Re}\sum_{n=1}^{\infty} (G_n - G_{\mathrm{e}})\boldsymbol{U}_n\, e^{jn\omega_1 t}\ .$$

The presence of these two currents in the supply current is the cause for which the apparent power S is higher than the load active power P. Thus the answer to the main question (1) of the power theory was not provided in the time-domain, but in the frequency-domain. Equivalent decomposition in the time-domain that would reveal the cause of the apparent power increase is not known.

Because the time and frequency-domains are mutually equivalent, the scattered current, originally identified in the frequency-domain, can be also somehow recalculated and expressed in the time-domain. The physical interpretation of this current existence, namely, as the effect of a change of the load conductance with harmonic frequency cannot be used in the time-domain, however.

Averaging versus instantaneous approach

The most fundamental power quantity in electrical systems, the instantaneous power $p(t)$, specifies the flow of the electric energy at each instant of time. Its physical interpretation is the most unquestionable power quantity. This can imply a conclusion that the whole power theory should be based on quantities defined as instantaneous ones.

There are situations in power systems where indeed instantaneous values of voltages and currents are crucial, as it is during disturbances or faults. Performance of electrical systems with periodic voltages and currents is specified at normal operation entirely in terms of quantities defined as some integrals over the supply voltage period T, however. These are the active and apparent powers, voltage and current rms value, the power factor, rms value of harmonics, harmonic distortion, or the voltage and current symmetrical components. The instantaneous power $p(t)$ usually is not a matter of interest for system designers and operators. In fact, terms like "*rms*", "*apparent power*", "*harmonic*", "*symmetrical component*", are alien for theories that describe instantaneous properties of electrical systems.

The major difficulty of power theories that claim to be "instantaneous" stems from the fact that power properties of the load cannot be identified instantaneously. This is illustrated in Fig. 3, with an unknown load and a pair of instantaneous values of the load voltage and current.

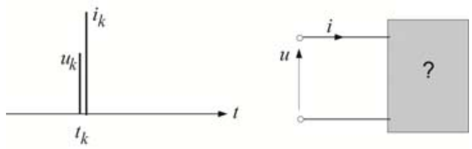

Fig. 3. Unknown load and pair of instantaneous values of the load voltage and current.

As shown in Fig. 4, in the "black box" could be a resistor, an inductor or a capacitor.

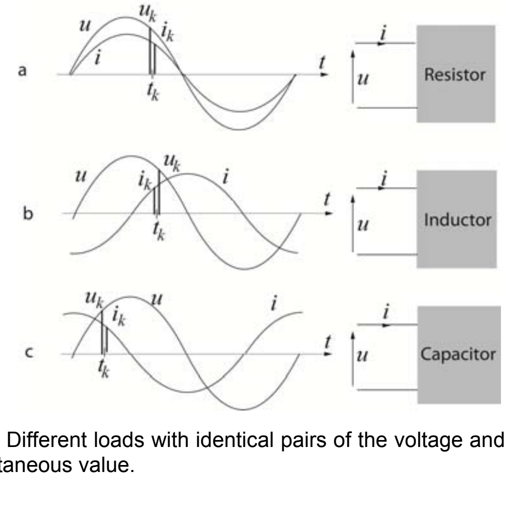

Fig. 4. Different loads with identical pairs of the voltage and current instantaneous value.

In fact, an infinite number of different loads can have iden-tical pairs of the voltage and current samples. Taking into account that the load voltage and current are in general nonsinusoidal, the instantaneous values of the voltage and current over the whole period T have to be measured to draw conclusions on the load power properties.

Quantities obtained by averaging could be added to the instantaneous theory, but this would undermine the claim that the theory is indeed instantaneous. It is just the case of the Instantaneous Reactive Power *p-q* Theory, where the active power P can be found by averaging the instantaneous active power p, but even with this, the co-cepts of the apparent power S and consequently, the power factor do not exist inside of it.

Carriers of information on power properties

It seems to be quite evident that the power properties of electrical systems are explained in terms of powers. This is for sure true in single-phase systems with sinusoidal voltages and currents. However, power theories are to describe power properties of electrical loads when the supply voltage is nonsinusoidal and, in the case of three-phase systems, asymmetrical.

Powers of electrical loads are defined with a use of voltages and currents at the load terminals. Their features depend both on the load properties, such as generated harmonics or imbalance, and on properties of the supply voltage, such as distribution voltage harmonics and/or asymmetry. Observing specific features of a power it could be impossible to conclude whether the load or the supply voltage is responsible for it.

Let us illustrate this by calculating the instantaneous power $p(t)$ of two loads shown in Fig 5.

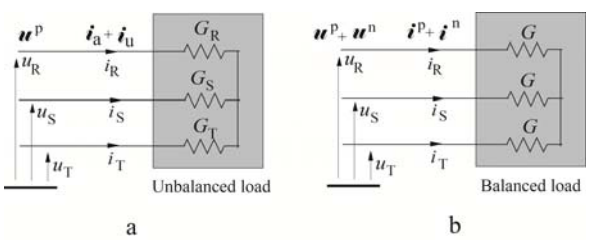

Fig. 5. Two different loads that cannot be distinguished in terms of instantaneous power $p(t)$.

The first, in Fig.5a, is purely resistive, but unbalanced and supplied from a source of sinusoidal and symmetrical voltage of positive sequence $\boldsymbol{u}^{\mathrm{p}}$.

The vector $\boldsymbol{i}$ of line currents of such a load is com-posed of [11] the active and unbalanced currents, $\boldsymbol{i}_{\mathrm{a}}$, $\boldsymbol{i}_{\mathrm{u}}$, and the instantaneous power of the load is equal to

$$(9)\quad p(t) = \frac{dW(t)}{dt} = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{i}_{\mathrm{a}}+\boldsymbol{i}_{\mathrm{u}}) = p_{\mathrm{a}}(t) + p_{\mathrm{u}}(t) = = P + 3U I_{\mathrm{u}}\cos(2\omega t+\Psi)$$

where I_{u} is the rms value of the unbalanced current and Ψ is the phase of the load unbalanced admittance. Thus this power contains a constant and an oscillating component.

The second load, shown in Fig. 5b, is a balanced resistive load of the phase conductance G, supplied with a sinusoidal, but asymmetrical voltage, composed of a vector of positive sequence voltages $\boldsymbol{u}^{\mathrm{p}}$ and negative sequence voltages $\boldsymbol{u}^{\mathrm{n}}$. The instantaneous power is equal to

$$p(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = [\boldsymbol{u}^{\mathrm{p}}+\boldsymbol{u}^{\mathrm{n}}]^{\mathrm{T}}[\boldsymbol{i}^{\mathrm{p}}+\boldsymbol{i}^{\mathrm{n}}] = [\boldsymbol{u}^{\mathrm{p}}+\boldsymbol{u}^{\mathrm{n}}]^{\mathrm{T}}G[\boldsymbol{u}^{\mathrm{p}}+\boldsymbol{u}^{\mathrm{n}}] = = G[\boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{p}}].$$

The first two terms are constant components of the instan-taneous power, since

$$G\boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{p}} = G\|\boldsymbol{u}^{\mathrm{p}}\|^2 = P^{\mathrm{p}}, \qquad G\boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{n}} = G\|\boldsymbol{u}^{\mathrm{n}}\|^2 = P^{\mathrm{n}}$$

where P^{p} and P^{n} are active powers of the positive and negative sequence voltages. The remaining terms can be rearranged as follows

$$G(\boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{p}}) = G(u_{\mathrm{R}}^{\mathrm{p}}u_{\mathrm{R}}^{\mathrm{n}} + u_{\mathrm{S}}^{\mathrm{p}}u_{\mathrm{S}}^{\mathrm{n}} + u_{\mathrm{T}}^{\mathrm{p}}u_{\mathrm{T}}^{\mathrm{n}}) + + G(u_{\mathrm{R}}^{\mathrm{n}}u_{\mathrm{R}}^{\mathrm{p}} + u_{\mathrm{S}}^{\mathrm{n}}u_{\mathrm{S}}^{\mathrm{p}} + u_{\mathrm{T}}^{\mathrm{n}}u_{\mathrm{T}}^{\mathrm{p}}) = = 2G(u_{\mathrm{R}}^{\mathrm{p}}u_{\mathrm{R}}^{\mathrm{n}} + u_{\mathrm{S}}^{\mathrm{p}}u_{\mathrm{S}}^{\mathrm{n}} + u_{\mathrm{T}}^{\mathrm{p}}u_{\mathrm{T}}^{\mathrm{n}}) =$$

$$= 4GU^{\mathrm{p}}U^{\mathrm{n}}[\cos\omega t\times\cos\omega t + + \cos(\omega t-120^0)\times\cos(\omega t+120^0) + + \cos(\omega t+120^0)\times\cos(\omega t-120^0)] = = 6GU^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega t.$$

Consequently,

$$(10)\qquad p(t) = P^{\mathrm{p}} + P^{\mathrm{n}} + 6GU^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega t.$$

Thus, as in the previous case, the instantaneous power contains a constant and an oscillating component. Conse-quently, these two entirely different loads cannot be distin-guished in terms of the instantaneous power $p(t)$. One could say, that the oscillating components could be differentiated by rms value of unbalanced current I_{u} in formula (9) and the rms value of negative sequence voltage U^{n} in formula (10). However, they are explicitly visible only because these for-mulae were obtained by analysis. When the instantaneous power is measured and recorded, it is obtained in the form

$$(11)\qquad p(t) = \mathrm{Const.} + \mathrm{A}\times\cos 2\omega t.$$

Such a measurement is not capable of revealing the cause of the presence of the oscillating component. Is it caused by the load imbalance or by the supply voltage asymmetry?

This difficulty applies not only to loads with sinusoidal voltages and currents, as considered above, but also to loads when these quantities are nonsinusoidal. To show this, let as calculate the instantaneous power of resistive loads shown in Fig. 6.

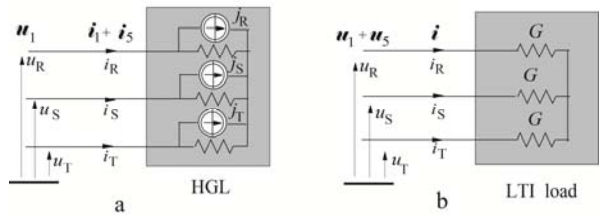

Fig. 6. Two different loads that cannot be distinguished in terms of instantaneous power $p(t)$.

Fig. 6a shows a balanced harmonic generating load (HGL), which supplied with a symmetrical sinusoidal voltage, gene-rates the 5th order symmetrical current. Fig. 6b shows a balanced linear-time invariant (LTI) load supplied with sym-metrical voltage with the 5th order harmonic.

In the case of the load in Fig. 6a, assuming that the current in line R contains the 5th order harmonic of the value

$$i_{\mathrm{R5}}(t) = \sqrt{2}I_5\cos(5\omega_1 t + \alpha_5).$$

the instantaneous power of the load is equal to

$$(12)\qquad p(t) = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}_1^{\mathrm{T}}(\boldsymbol{i}_1+\boldsymbol{i}_5) = P + 3U_1 I_5\cos(6\omega_1 t + \alpha_5).$$

In the case of the load in Fig. 6b, the instantaneous power of the load is

$$p = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}} G\,\boldsymbol{u} = [\boldsymbol{u}_1 + \boldsymbol{u}_5]^{\mathrm{T}} G[\boldsymbol{u}_1 + \boldsymbol{u}_5] = \\ = G\,\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_1 + G\,\boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_5 + G(\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_1). \tag{13}$$

The first two terms are constant components of the instantaneous power

$$G\,\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_1 + G\,\boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_5 = G||\boldsymbol{u}_1||^2 + G||\boldsymbol{u}_5||^2 = P.$$

The last term is equal to

$$\begin{aligned} G(\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_1) &= G[u_{1\mathrm{R}}\,u_{5\mathrm{R}} + u_{1\mathrm{S}}\,u_{5\mathrm{S}} + u_{1\mathrm{T}}\,u_{5\mathrm{T}}] + \\ &+ G[u_{5\mathrm{R}}\,u_{1\mathrm{R}} + u_{5\mathrm{S}}\,u_{1\mathrm{S}} + u_{5\mathrm{T}}\,u_{1\mathrm{T}}] = \\ &= 2G[u_{1\mathrm{R}}\,u_{5\mathrm{R}} + u_{1\mathrm{S}}\,u_{5\mathrm{S}} + u_{1\mathrm{T}}\,u_{5\mathrm{T}}] = \\ = 4GU_1U_5[\cos\omega_1 t \times \cos 5\omega_1 t &+ \\ &+ \cos(\omega_1 t - 120^0) \times \cos(5\omega_1 t + 120^0) + \\ &+ \cos(\omega_1 t + 120^0) \times \cos(5\omega_1 t - 120^0)] = \\ = 6GU_1U_5\cos 6\omega_1 t. \end{aligned}$$

Consequently, the instantaneous power of the load is

$$p(t) = P + 6GU_1U_5\cos 6\omega_1 t. \tag{14}$$

Thus again, when this power is measured and recorded, it is obtained in the form

$$p(t) = \mathrm{Const.} + \mathrm{A} \times \cos 6\omega_1 t\ ,$$

and it is not possible to conclude why it contains the oscillating component.

To summarize this observation we can conclude, that when power properties of a load are described by a power theory exclusively in terms of powers, which always are some forms of products of voltages and currents, these products hide causes of power related phenomena. It is not possible to answer the question: are they caused by the load current or by the supply voltage?

Consequently, although it is rather a strange conclusion for power theories, powers could not carry information on power properties of electrical loads.

Causes of various power properties can be visible much more explicitly when a power theory is founded on the load current decompositions, as it was initiated by Fryze, at clearly specified supply voltage properties, respective harmonic distortion and asymmetry.

Identification of distortion and asymmetry sources

Unfortunately, even a power theory founded on the current decomposition at a specified voltage could fail, meaning such a power theory might not enable for reliable identification of the load power properties.

This is because the supply voltage distortion and/or asymmetry at the load terminals could be caused by a distortion and/or asymmetry of the internal distribution voltage or it could be only a response to the load generated harmonics and the load imbalance. Usually both these two causes of the supply voltage distortion and/or asymmetry coexist together. The right answer to the question, "*where are the sources of harmonics and/or asymmetry located?*" which is illustrated in Fig. 7, is crucial for description of the load power properties and for compensation.

Therefore, formulation of a power theory cannot be separated from the issue of identification of harmonic distortion and asymmetry sources.

This identification could be as simple as just the knowledge of where the crucial equipment that could cause distortion and/or asymmetry is located in the distribution system. This could be the knowledge of the system response to ON/OFF switching of some equipment. However, in some cases sophisticated metrological methods might be required for that purpose.

In general, metrological identification requires usually that the system is observed not in one, but in a number of different states, thus identification in its very nature is a dynamic process. Consequently, a power theory integrated with identification of sources of harmonics and asymmetry would be not static, but a dynamic theory.

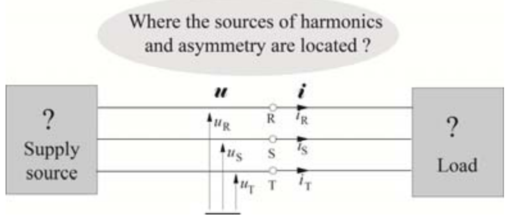

Fig. 7. Illustration to the major question of identification of distortion and asymmetry sources.

Observe that adaptive compensators are also dynamic in their nature. A power theory oriented at control of such compensators could exploit their dynamic behavior to identify location of harmonics and asymmetry sources.

Conclusions

In spite of a long history of power theory development, this process is far from completion. This paper returns to very fundamental questions of power theory. It is an invitation for a debate about how well particular concepts of a power theory provide a response to these questions. Also it is an invitation to a debate on how to face new challenges in power theory development, in particular for a very sophisticated micro-grid environment.

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