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# Currents' Physical Components (CPC) in systems with semi-periodic voltages and currents

***Abstract.*** *There are situations in electrical distribution systems where voltages and currents cannot be regarded as periodic quantities. They could be non-periodic. Nonetheless, properties of electrical systems confine this non-periodicity of voltages and currents to a particular sub-set of non-periodic quantities, referred to as* ***semi-periodic*** *quantities in this paper. The paper presents the concept of semi-periodic quantities and defines major functionals, such as the running active power, the running rms value, the scalar product and the running complex rms (crms) value of quasi-harmonics, needed for describing electrical systems with such voltages and currents in power terms. A recursive approach to calculation of these functionals was presented as well. The paper presents fundamentals of the Currents' Physical Components (CPC) – based power theory of systems with semi-periodic voltages and currents. An application of the semi-periodic concept to a load current decomposition in single-phase circuits with linear loads and with harmonics generating loads is presented as well. The paper presents also a concept of extrapolation of CPC into the closest future, which enables quasi-instantaneous generation of control signals for switching compensators.*

***Streszczenie.*** *Są takie sytuacje w systemach rozdzielczych, w których napięcia i prądy nie mogą być traktowane jako przebiegi okresowe. Mogą one być nieokresowe. Właściwości systemów rozdzielczych ograniczają jednak nieokresowość tych przebiegów do pewnego szczególnego podzbioru przebiegów nieokresowych, określago w tym artykule podzbiorem przebiegów półokresowych (semi-periodic). Artykuł definiuje podstawowe funkcjonały, takie jak biegnąca moc czynna, biegnąca wartość skuteczna, biegnący iloczyn skalarny oraz zespolona wartość skuteczna (crms) quasi-harmonicznych, potrzebnych do opisu właściwości energetycznych w obwodach. Przedstawiona jest także metoda rekursywnego obliczania tych funkcjonałów. Artykuł przedstawia podstawy rozkładu prądu na składowe fizyczne w obwodach z odbiornikami liniowymi oraz z odbiornikami generującymi harmoniczne oraz z półokresowymi przebiegami prądu i napięcia. Przedstawiona jest też metoda ekstrapolacji składowych fizycznych w najbliższą przyszłość, umożliwiająca niemal momentalną generację sygnałów kontrolnych kompensatora. (****Składowe Fizyczne Prądu w systemach z półokresowymi przebiegami prądu i napięcia****).*

**Keywords:** Power definitions, power theory, non-periodic quantities.
**Słowa kluczowe:** Definicje mocy, teoria mocy, przebiegi nieokresowe.

*"Science develops due to two mechanisms: due to problems it generates itself and due to problems that were enforced on it".*

Ian Hacking [4]

## Introduction

It looks like the power theory can eventually provide the answer to Steinmetz's old question, formulated in 1892, [1]: ***why can the apparent power $S$ in a resistive circuit with an electric arc be higher than the active power $P$?*** The answer, in the frame of the Currents' Physical Components (CPC) – based power theory [5], applies to single - and three-phase systems on the condition that voltages and currents are periodic.

There are situations in distribution systems where this condition is not satisfied, however. Voltages and currents of some loads are non-periodic. An example of such a non-periodic voltage and current at a terminal of a three-phase energy pump is shown in Fig. 1.

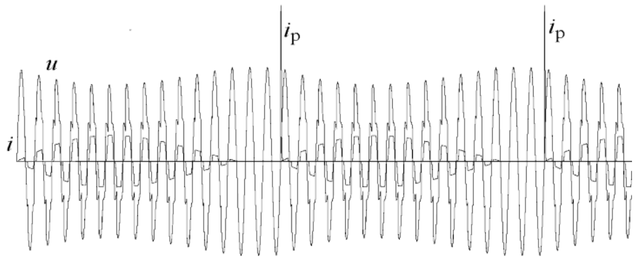


Fig. 1. Waveforms of the load current and the supply voltage of energy pump.

Waveform non-periodicity is mainly due to power electronics devices, which provide versatile technical means for fast control of energy flow, disturbing waveform periodicity. Adjustable speed drives driven by power electronics converters, are common sources of non-periodic phenomena. Also some loads are non-periodic by nature.

Pulsing loads are examples of loads that draw non-periodic currents from a supply source, such as spot welders, generators of beams of X-rays, or electromagnetic guns. These loads release energy in the form of pulses of current of the order of hundreds of kA and only a few milliseconds duration. The energy of the pulse is stored, by a sort of energy pump, over several periods of the supply voltage in a capacitor, to be released next in a short interval of time, and the process is repeated. An example of such an energy pump is shown in Fig. 2.

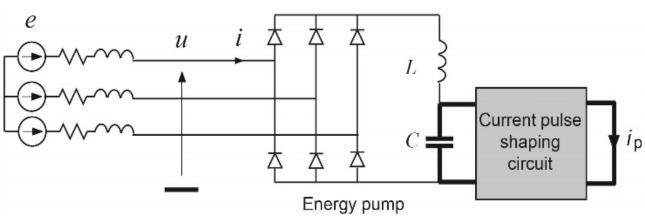


Fig. 2. Example of energy pump for a pulsing load.

Waveforms of the supply current and the voltage of such an energy pump are shown in Fig. 1. The supply current of the pump increases over several periods of the supply voltage and declines to zero when the capacitor voltage approaches its maximum value, so that the energy needed for the pulsing load is stored in the capacitor. Due to the voltage drop on the supply source impedance, there is also a variation of the pump voltage. It causes the voltage non-periodicity.

Some of the most remarkable devices with non-periodic currents are arc furnaces in uneasy phase of melting. Taking into account that the power of an arc furnace could be in the range of hundreds MVA, non-periodic phenomena caused by such a furnace could be remarkable.

The presence of non-periodic voltages and currents in distribution systems raises two issues. First, a cognitive question occurs: *how can power related phenomena be explained and described in a situation where the concepts developed for systems with periodic quantities are no longer valid*? The second question is practical: *how can loads with non-periodic voltages and current be compensated, and in particular, how should reference signals for switching compensator control be generated?* This paper is focused on the first of these two questions, namely, on describing power properties of loads with semi-periodic voltages and currents. The study is confined to single-phase systems, but the approach and results can be generalized easily to three-phase systems.

Voltages and currents in situations described above are non-periodic and consequently, well established definitions in electrical engineering such as the active power

$$P \stackrel{\text{df}}{=} \frac{1}{T}\int_0^T u(t)\, i(t)\, dt = U\, I \cos\varphi \,, \tag{1}$$

the voltage and current rms values

$$||u|| \stackrel{\text{df}}{=} \sqrt{\frac{1}{T}\int_0^T u^2(t)\, dt}\,, \qquad ||i|| \stackrel{\text{df}}{=} \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt} \,, \tag{2}$$

as well as the complex rms (crms) value of harmonics

$$\boldsymbol{X}_n = X_n e^{j\alpha_n} = \frac{\sqrt{2}}{T}\int_0^T x(t) e^{-jn\omega_1 t} dt \,, \tag{3}$$

i.e., the main functionals in the CPC – based power theory, cannot be defined, because non-periodic quantities simply do not have the period $T$.

To calculate powers, non-periodic quantities are sometimes approximated by periodic ones. Nonetheless, when powers in systems with non-periodic quantities are not defined, the error of such an approximation cannot even be evaluated.

Non-periodic voltages and currents in electrical power systems usually have some particular properties that enable us to categorize them as semi-periodic quantities and the CPC-based power theory can be extended to systems with such quantities.

**Semi-periodic voltages and currents**

The main sources of electric energy in power systems, synchronous generators, create a voltage which is with high accuracy sinusoidal and just this generated voltage is the driving force for the energy delivery to the power system. The period $T$ of this generated voltage can be detected by filtering and be used as a sort of "time-frame" for the energy flow analysis.

Due to time variance of the load parameters, the load current can be non-periodic. This non-periodicity can be permanent or transient, so that after some time, the load current could be periodic with the period $T$ again. The same applies to voltages that contain a response to the load currents. The mechanism of semi-periodic distortion is illustrated for a single-phase circuit in Fig. 3.

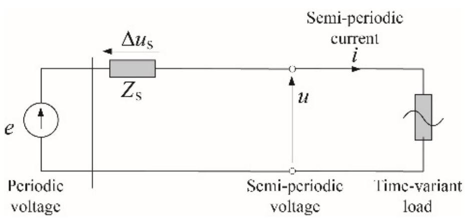


Fig. 3. The mechanism of semi-periodic distortion.

Voltages and currents in systems with time-varying loads, but with the energy delivered by generators of a sinusoidal voltage, are referred to in this paper as semi-periodic voltages and currents.

In mathematics there exists [2] a concept of "quasi-periodic" functions. The meaning of "a semi-periodic" quantity or function, as defined in this paper, differs from the meaning of a "quasi-periodic" function, however.

The ratio of the supply source impedance to the load impedance is usually confined in such a way that the load voltage rms value $U$ does not decline at the maximum load power by more than 5%. Therefore, the semi-periodic component of the load voltage should not be higher than a few per cents of the periodic component. The load current does not have this sort of limitation and consequently, semi-periodicity can be much more visible in currents than in voltages.

Fundamental harmonic $u_1$ of the voltage periodic component, separated by a filter, can be used for detecting the period $T$. A time interval of $T$ duration will be referred to as an ***observation window.*** It is shown in Fig. 4.

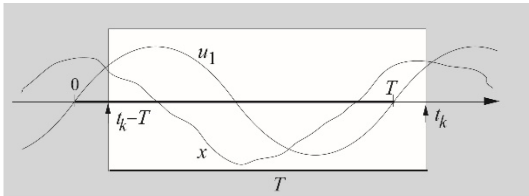


Fig. 4. Observation window.

Such a window starts at some instant of time $t = t_k - T$ and ends at instant $t_k$.

If the quantity $x(t)$ as observed in the window, shown in Fig. 4, is reproduced every period $T$ from minus to plus infinity, then ***a periodic extension***

$$x^{\approx}(t) \stackrel{\text{df}}{=} \sum_{n=-\infty}^{n=\infty} x(t-nT) \tag{4}$$

is created, shown in Fig. 5, which is a fictitious periodic quantity, identical with quantity $x(t)$ only in the observation window, but not outside of it.

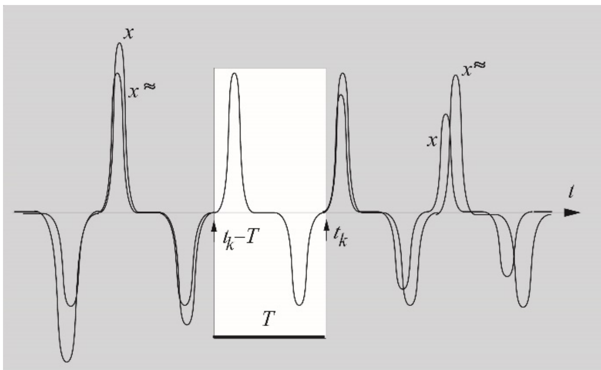


Fig. 5. Semi-periodic quantity $x(t)$ and periodic extension $x^{\approx}(t)$ .

The energy delivered to a load during the observation window preceding the instant $t_k$ is equal to

$$W(t_k) = \int_{t_k-T}^{t_k} u(t)\, i(t)\, dt \,. \tag{5}$$

When the voltage and current at the load terminals are periodic with period $T$, this energy is constant, meaning independent on the beginning of integration. It is not constant when the voltage and current are semi-periodic.

Although interval $T$ is not a period of semi-periodic quantities, it can be used for calculating the value of the average rate of energy flow over the observation window. At the end of this flow observation, at instant $t_k$, this average value is equal to

$$\frac{W(t_k)}{T} = \frac{1}{T}\int_{t_k-T}^{t_k} u(t)\,i(t)\,dt = P^{\approx}(t_k)\ . \tag{6}$$

The power calculated in such a way can be regarded as the active power of a load with a semi-periodic voltage and current. When the voltage and current are not periodic, then the active power defined in such a way is not constant, but it is a function of time. This is emphasized with the wave symbol "$\approx$". It will be referred to as a ***running active power***.

The periodic extension $x^{\approx}(t)$ of what is observed in the observation window $x(t)$ has the rms value, which can be calculated at the instant $t = t_k$, namely

$$\|x^{\approx}\|_{t_k} \stackrel{\text{df}}{=} \sqrt{\frac{1}{T}\int_{t_k-T}^{t_k} x^2(t)\,dt}\ . \tag{7}$$

If the quantity is not periodic, then this value is not constant, but changes with the observation window. It will be referred to a ***running rms value*** of a semi-periodic quantity. When a semi-periodic current $i(t)$ flows through a resistor of resistance $R$, then the running active power of this resistor at the instant $t = t_k$, is equal to

$$P_k^{\approx} = \frac{1}{T}\int_{t_k-T}^{t_k} u(t)\,i(t)\,dt = \frac{1}{T}\int_{t_k-T}^{t_k} R\,i^2(t)\,dt = R\|i^{\approx}\|_{t_k}^2\ . \tag{8}$$

Thus, the running rms value has exactly the same meaning as the conventional rms value. It enables calculation of the energy loss on the resistor in the preceding observation window. It does not allow calculation outside of this window, however.

For two semi-periodic quantities $x(t)$ and $y(t)$, which have periodic extensions with the same period $T$, a scalar product can be defined and calculated at the end of observation window, namely

$$(x^{\approx}, y^{\approx})_{t_k} \stackrel{\text{df}}{=} \frac{1}{T}\int_{t_k-T}^{t_k} x(t)\,y(t)\,dt\ . \tag{9}$$

The running rms value of the sum of two semi-periodic quantities $x(t)$ and $y(t)$ is equal to

$$\begin{aligned}\|x^{\approx}+y^{\approx}\|_{t_k} &= \sqrt{\frac{1}{T}\int_{t_k-T}^{t_k} [x(t)+y(t)]^2 dt} = \\ &= \sqrt{\|x^{\approx}\|_{t_k}^2 + 2(x^{\approx},y^{\approx})_{t_k} + \|y^{\approx}\|_{t_k}^2}\ .\end{aligned} \tag{10}$$

This rms value can be expressed in terms of only running rms value of the sum components, as

$$\|x^{\approx}+y^{\approx}\|_{t_k} = \sqrt{\|x^{\approx}\|_{t_k}^2 + \|y^{\approx}\|_{t_k}^2} \tag{11}$$

on the condition that

$$(x^{\approx}, y^{\approx})_{t_k} = 0 \tag{12}$$

i.e., when these quantities are mutually orthogonal. Property (11), at condition (12), applies only to the observation window of $T$ duration just preceding instant $t = t_k$.

The periodic extension $x^{\approx}(t)$ can be expressed, moreover, as a Fourier series

$$x^{\approx}(t) = X_0^{\approx} + \sqrt{2}\,\mathrm{Re}\sum_{n=0}^{\infty} \boldsymbol{X}_n^{\approx} e^{jn\omega_1 t} \tag{13}$$

with crms values of its harmonics

$$\boldsymbol{X}_n^{\approx} = \boldsymbol{X}_n^{\approx}(t_k) \stackrel{\text{df}}{=} \frac{\sqrt{2}}{T}\int_{t_k-T}^{t_k} x(t)\,e^{-jn\omega_1 t}\,dt \tag{14}$$

calculated at instant of time $t_k$. It means that quantity $x(t)$ can reconstructed from its harmonics, assuming that it does not have points of discontinuity, with full accuracy. This reconstruction applies only to the observation window of $T$ duration, however, but not beyond it.

**Discrete identification of semi-periodic quantities**

Although the running active power and rms value of semi-periodic voltages and currents, as well as the crms values of their harmonics were defined above as functionals of continuous quantities, discrete methods implemented in digital meters are needed for their measurement. Such a meter performs arithmetic operations on digital samples of the load voltage and current, provided by analog to digital (A/D) converters. The sequence of current samples is shown in Fig. 6.

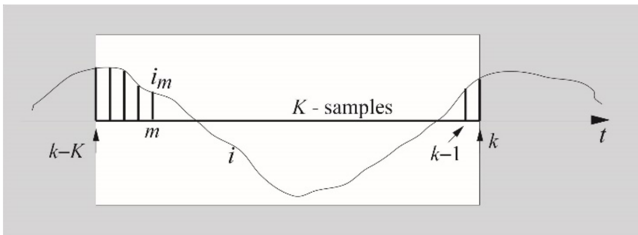


Fig. 6. Current samples in observation window.

To avoid confusion with symbols of harmonics, denoted usually by $u_n$ and $i_n$, these samples will be denoted by $u_m$ and $i_m$ in this paper. The last sample, i.e., at the instant of observation, has index $k$. Assuming that there are $K$ samples of the voltage and current in the observation window, as shown in Fig. 6, then such a digital meter can calculate at the instant $t = t_k$ the running active power according to formula:

$$P(t_k) \stackrel{\text{df}}{=} P_k^{\approx} = \frac{1}{K}\sum_{m=k-K+1}^{m=k} u_m\, i_m \tag{15}$$

The number of samples $K$ has to obey the Nyquist criterion. If $n_{\max}$ is the highest harmonic order of the periodic extension, then the minimum number $K$ of samples in the period $T$ has to be higher than the double value of $n_{\max}$, i.e., $K > 2n_{\max}$. This is the condition for preserving full information on the waveform of a periodic quantity.

A discrete formula for running rms value of semi-periodic quantity has the form

$$\|x^{\approx}\|_k = \sqrt{\frac{1}{K}\sum_{m=k-K+1}^{m=k} x_m^2} \tag{16}$$

while for the scalar product

$$(17)\qquad (x^{\approx},y^{\approx})_k = \frac{1}{K}\sum_{m=k-K+1}^{m=k} x_m\, y_m\ .$$

Two semi-periodic quantities are orthogonal on the condition that

$$(18)\qquad \frac{1}{K}\sum_{m=k-K+1}^{m=k} x_m\, y_m = 0.$$

The complex rms (crms) value of the $n^{\text{th}}$ order harmonic periodic extension of a semi-periodic quantity in observation window $x(t)$ can be obtained from a discrete formula

$$(19)\qquad \boldsymbol{X}^{\approx}_{nk} = \frac{\sqrt{2}}{K}\sum_{m=k-K+1}^{m=k} x_m\, e^{-jn\frac{2\pi}{K}m}\ .$$

To implement a Fast Fourier Transform (FFT) algorithm, an integer power of two is usually selected for $K$ value.

The crms values (19) are not constant, but they change with time $t_k$. They specify harmonics of only a single periodic extension $x^{\approx}(t)$, which changes with the change of the observation instant to a new periodic extension. Therefore, semi-periodic quantities, as non-periodic, cannot be described in terms of harmonics, but by entities referred in this paper as ***quasi-harmonics***, with varying amplitude and phase. Only inside of the observation window quasi-harmonics are identical with the common harmonics.

Situations when two semi-periodic quantities are orthogonal do not differ from those for periodic quantities. There are five such situations. Four of them apply to single-phase quantities, and the last one applies to three-phase quantities. Two semi-periodic quantities are orthogonal, if:

1 - for each pair of samples $x_m$ and $y_m$ in the observation window, their product

$$x_m y_m = 0$$

meaning, only one sample in the pair is different from zero.

2 - one of these quantities is a derivative or integral of the other quantity.

3 - these quantities are quasi-harmonics of different order.

4 - these quantities are quasi-harmonics of the same order, but they are shifted mutually by $\pi/2$.

5 – these quantities are three-phase symmetrical quantities of a different sequence.

When a semi-periodic quantity is composed of a sum of mutually orthogonal components, i.e., components that satisfy any of these five conditions, then the running rms value of such a quantity can be calculated as a root of sum of squares of rms values of its components.

Formulae (15) – (19) along with the concept of orthogonality provide a basic mathematical tool for extension of the concept of the Currents' Physical Components (CPC) to electrical systems with semi-periodic voltages and currents. At the instant of time $t = t_k$, data on voltages and current at the load terminals, collected over the observation window of $T$ duration, are sufficient for the load current decomposition into the Physical Components that uniquely identify physical phenomena in the load over that window. The only concern that could arise is the amount of calculation needed for that between the preceding observation instant of time $t = t_{k-1}$ and the current instant $t = t_k$. This computational burden can be, as shown in the following section, only apparent, however.

### Recursive calculations of running quantities

The running active power at instant $t = t_k$ can be calculated [7] as follows

$$(20)\qquad \begin{aligned} P^{\approx}_k &= \frac{1}{K}\sum_{m=k-K+1}^{m=k} u_m i_m = \\ &= \frac{1}{K}\sum_{m=k-K+1}^{m=k}(u_m i_m + u_{k-K} i_{k-K} - u_{k-K} i_{k-K}) = \\ &= \frac{1}{K}\sum_{m=k-M}^{m=k-1} u_m i_m + \frac{1}{K}(u_k i_k - u_{k-K}\, i_{k-K}) = \\ &= P^{\approx}_{k-1} + \frac{1}{K}(u_k i_k - u_{k-K}\, i_{k-K}) \end{aligned}$$

it means that it can be calculated by updating its value calculated previously, at instant $t = t_{k-1}$. When the voltage and current are periodic, then

$$u_k = u_{k-K}\,,\qquad i_k = i_{k-K}$$

and updating is not needed, since $P^{\approx}_k = P^{\approx}_{k-1}$.

Similarly, the running rms value of a semi-periodic $x(t)$ quantity is

$$(21)\qquad \begin{aligned} \|x^{\approx}\|_k &= \sqrt{\frac{1}{K}\sum_{m=k-K+1}^{m=k} x_m^2} = \\ &= \sqrt{\frac{1}{K}\sum_{m=k-K+1}^{m=k} x_m^2 + x_{k-K}^2 - x_{k-K}^2} = \\ &= \sqrt{\frac{1}{K}\sum_{m=k-K}^{m=k-1} x_m^2 + \frac{1}{K}(x_k^2 - x_{k-K}^2)} = \\ &= \sqrt{\|x^{\approx}\|_{k-1}^2 + \frac{1}{K}(x_k^2 - x_{k-K}^2)} \end{aligned}$$

Thus it is enough to update the previously calculated value.

The same applies to the complex rms values of quasi-harmonics, namely

$$(22)\qquad \begin{aligned} \boldsymbol{X}^{\approx}_{nk} &= \frac{\sqrt{2}}{K}\sum_{m=k-K+1}^{m=k} x_m\, e^{-jn\frac{2\pi}{K}m} = \\ &= \frac{\sqrt{2}}{K}\sum_{m=k-K+1}^{m=k}[x_m e^{-jn\frac{2\pi}{K}m} + (x_{k-K} - x_{k-K})e^{-jn\frac{2\pi}{K}(k-K)}] = \\ &= \frac{\sqrt{2}}{K}\sum_{m=k-K}^{m=k-1} x_m e^{-jn\frac{2\pi}{K}m} + \frac{\sqrt{2}}{K}(x_k - x_{k-K})e^{-jn\frac{2\pi}{K}k} \end{aligned}$$

and eventually

$$(23)\qquad \boldsymbol{X}^{\approx}_{nk} = \boldsymbol{X}^{\approx}_{nk-1} + \frac{\sqrt{2}}{K}(x_k - x_{k-K})e^{-jn\frac{2\pi}{K}k}.$$

Thus, only the difference of the latest sample $x_k$ and that, taken the period earlier $x_{k-K}$, has to be taken into account when updating the crms value of the quasi-harmonic.

Recursive calculation reduces drastically the amount of calculations, but their results can be affected by accumulation of rounding error. Therefore, rounding should be as random as possible, and this should be taken into account in the calculation algorithm construction. Moreover, special procedures, such as periodic reset of calculated functionals might be considered.

The theoretical frame presented above makes descripttion and interpretation of power properties of distribution systems with semi-periodic voltages and currents possible.

This can apply to all distribution system structures and load properties, but equations and definitions developed for semi-periodic voltages and currents are valid only in the $T$-long observation window, just at the instant $t = t_k$, when the last samples of voltages and currents in that window are provided for calculation.

Semi-periodic currents occur in distribution systems due to semi-periodic supply voltage, and/or due to load parameters variability. This variability could be fast, causing current waveform distortion or slow, observed rather as a sort of amplitude or phase modulation. The load current distortion due to periodic variability of the load parameters can be interpreted as caused by generation of quasi-harmonics in the load. At slow variability of these parameters it can be assumed that load properties can be approximated and interpreted as properties of linear loads

**CPC of slowly varying single-phase linear loads**

Let us assume that a linear load is supplied with a semi-periodic voltage

(24) $$u^{\approx}(t) = U_{0k}^{\approx} + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_{nk}^{\approx}\, e^{jn\omega_1 t}$$

If this load is not a source of current quasi-harmonics, then it draws the current

(25) $$i^{\approx}(t) = I_{0k}^{\approx} + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_{nk}^{\approx}\, e^{jn\omega_1 t}$$

The ratio

(26) $$\frac{\boldsymbol{I}_{nk}^{\approx}}{\boldsymbol{U}_{nk}^{\approx}} = \boldsymbol{Y}_{nk}^{\approx} = G_{nk}^{\approx} + jB_{nk}^{\approx}$$

specifies the running admittance of the load for harmonic frequencies.

With respect to the running active power at instant $t_k$, such a load is equivalent to a resistive load of conductance

(27) $$G_{ek}^{\approx} = \frac{P_k^{\approx}}{||u^{\approx}||_k^2}$$

The current of such a resistive load at instant $t = t_k$

(28) $$i_{ak}^{\approx} = G_{ek}^{\approx}\, u_k^{\approx} = G_{ek}^{\approx}\, U_{0k}^{\approx} + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} G_{ek}^{\approx}\, \boldsymbol{U}_{nk}^{\approx}\, e^{jn\frac{2\pi}{K}k}$$

is the active current of the load, as defined by Fryze [3]. The remaining part of the load current

(29) $$i^{\approx} - i_a^{\approx} = (G_{0k}^{\approx} - G_{ek}^{\approx})U_{0k}^{\approx} + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} (\boldsymbol{Y}_{nk}^{\approx} - G_{ek}^{\approx})\boldsymbol{U}_{nk}^{\approx}\, e^{jn\frac{2\pi}{K}k}$$

can be decomposed into the scattered current

(30) $$(G_{0k}^{\approx} - G_{ek}^{\approx})U_{0k}^{\approx} + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} (G_{nk}^{\approx} - G_{ek}^{\approx})\boldsymbol{U}_{nk}^{\approx} e^{jn\frac{2\pi}{K}k} \overset{\mathrm{df}}{=} i_{sk}^{\approx}$$

and into the reactive current

(31) $$\sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_{nk}^{\approx}\, \boldsymbol{U}_{nk}^{\approx}\, e^{jn\frac{2\pi}{K}k} \overset{\mathrm{df}}{=} i_{rk}^{\approx}\,.$$

Thus, the load semi-periodic current can be decomposed exactly as in the case of periodic currents into three components such that at instant $t = t_k$

(32) $$i_k^{\approx} = i_{ak}^{\approx} + i_{sk}^{\approx} + i_{rk}^{\approx}$$

which are Physical Components of this current. Inside of the observation window i.e., for $t_k - T < t < t_k$, this decomposition of the semi-periodic current is identical with the common decomposition

(33) $$i(t) = i_a(t) + i_s(t) + i_r(t)$$

but not outside of that window. Decomposition (32) cannot be found before the instant $t = t_k$, i.e., at the instant when component of the decomposition (31) can be calculated.

These components are mutually orthogonal, so that

(34) $$||i^{\approx}||_k^2 = ||i_a^{\approx}||_k^2 + ||i_s^{\approx}||_k^2 + ||i_r^{\approx}||_k^2\,.$$

Multiplying (34) by the square of the supply voltage running rms value, the power equation is obtained

(35) $$S_k^{\approx 2} = P_k^{\approx 2} + D_{sk}^{\approx 2} + Q_k^{\approx 2}$$

with

(36) $$S_k^{\approx} \overset{\mathrm{df}}{=} ||u^{\approx}||_k ||i^{\approx}||_k$$
$$D_{sk}^{\approx} \overset{\mathrm{df}}{=} ||u^{\approx}||_k ||i_s^{\approx}||_k\,.$$
$$Q_k^{\approx} \overset{\mathrm{df}}{=} ||u^{\approx}||_k ||i_r^{\approx}||_k\,.$$

Power equation (35) describes the relationship between powers of single-phase loads supplied with a semi-periodic voltage, however confined to loads that do not generate quasi-harmonics. It is valid in the interval of $T$ duration preceding instant $t = t_k$. It could be written, for convenience, in the traditional form

(37) $$S^2 = P^2 + D_s^2 + Q^2\,.$$

but it should be remembered, that these powers may not have a constant value and this equation is valid only in the window of $T$ duration preceding time instant $t = t_k$, but not outside of that time window.

**CPC of harmonics generating single-phase loads**

Nonlinear loads or loads with fast varying parameters or quasi-periodic switches can generate quasi-harmonics. The running active power of such quasi-harmonics could be negative, which means that they convey the energy from the load back to the supply source. This phenomenon, revealed in systems with periodic voltages and currents [6], is a distinctive physical phenomenon, which has to be taken into account when the power properties of electrical circuits are analyzed. Let us describe such loads in terms of CPC when the voltage and current are semi-periodic.

Let us consider the equivalent circuit shown in Fig. 7.

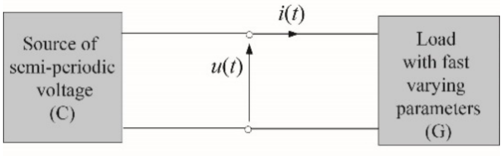


Fig. 7. Equivalent circuit of a load and the supply source.

The running active power of the $n^{\mathrm{th}}$ order quasi-harmonic

(38) $$P_{nk}^{\approx} = U_{nk}^{\approx}\, I_{nk}^{\approx} \cos\varphi_{nk}^{\approx}$$

depending on the phase-shift, can be positive, negative or zero.

The set $N$ of quasi-harmonics orders $n$ can be decomposed into two sub-sets.

When the energy flow is caused by a quasi-harmonic of the $n^{\text{th}}$ order in the distribution voltage, i.e., the power $P^{\approx}_{nk}$ is positive, this quasi-harmonic order belongs to sub-set $N_{\mathrm{C}}$. When this flow occurs because the quasi-harmonic is generated in the load, i.e., the power $P^{\approx}_{nk}$ is negative, its order belongs to sub-set $N_{\mathrm{G}}$. The sign of the active power depends on the phase-shift between the voltage and current quasi harmonic, thus

(39)
$$\begin{aligned} &\text{if } \ |\varphi^{\approx}_{nk}| \leq \pi/2, \ \text{ then } n \in N_{\mathrm{C}} \\ &\text{if } \ |\varphi^{\approx}_{nk}| > \pi/2, \ \text{ then } n \in N_{\mathrm{G}}\,. \end{aligned}$$

It enables the voltage and current decomposition into components with harmonics from sub-sets $N_{\mathrm{C}}$ and $N_{\mathrm{G}}$. At the instant $t = t_k$

(40)
$$i^{\approx}_{k} = \sum_{n\in N} i^{\approx}_{nk} = \sum_{n\in N_{\mathrm{C}}} i^{\approx}_{nk} + \sum_{n\in N_{\mathrm{G}}} i^{\approx}_{nk} = i^{\approx}_{\mathrm{C}k} + i^{\approx}_{\mathrm{G}k}\,,$$

(41)
$$u^{\approx}_{k} = \sum_{n\in N} u^{\approx}_{nk} = \sum_{n\in N_{\mathrm{C}}} u^{\approx}_{nk} + \sum_{n\in N_{\mathrm{G}}} u^{\approx}_{nk} = u^{\approx}_{\mathrm{C}k} - u^{\approx}_{\mathrm{G}k}\,.$$

The voltage $u_{\mathrm{G}}$ is defined as the negative sum of voltage quasi-harmonics because, as a supply source response to load generated current, $i_{\mathrm{G}}$, it has the opposite sign as compared to the sign of the distribution system originated voltage quasi-harmonics. The same applies to quasi-harmonic active power, thus

(42)
$$P^{\approx}_{k} = \sum_{n\in N} P^{\approx}_{nk} = \sum_{n\in N_{\mathrm{C}}} P^{\approx}_{nk} + \sum_{n\in N_{\mathrm{G}}} P^{\approx}_{nk} = P^{\approx}_{\mathrm{C}k} - P^{\approx}_{\mathrm{G}k}\,.$$

Sub-sets $N_{\mathrm{C}}$ and $N_{\mathrm{G}}$ do not contain common quasi-harmonic orders $n$, thus currents $i_{\mathrm{C}}$ and $i_{\mathrm{G}}$ are mutually orthogonal. Hence their running rms values satisfy the relationship

(43)
$$||i^{\approx}||^2 = ||i^{\approx}_{\mathrm{C}}||^2 + ||i^{\approx}_{\mathrm{G}}||^2\,.$$

The same applies to the voltage rms values, namely

(44)
$$||u^{\approx}||^2 = ||u^{\approx}_{\mathrm{C}}||^2 + ||u^{\approx}_{\mathrm{G}}||^2\,.$$

Decomposition (39) of quasi-harmonic orders and the voltage and current according to (40) and (41), mean that the system, as presented in Fig. 7, can be described as superposition of two systems. The first, shown in Fig. 8a, has an LTI load and the second, shown in Fig. 8b, has only a current source on the customer side while the distribution system is a passive energy receiver.

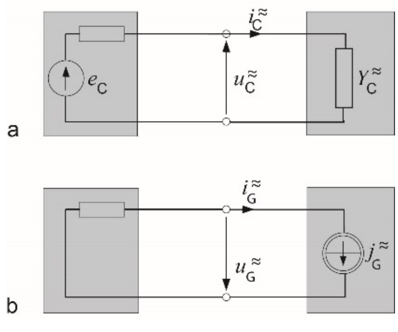


Fig. 8 (a) Equivalent circuit for harmonics $n \in N_{\mathrm{C}}$ and (b) equivalent circuit for harmonics $n \in N_{\mathrm{G}}$.

The circuit in Fig. 8a, as a system with a linear load, can be described according to the CPC approach. Namely, if the equivalent admittance is

(45)
$$\boldsymbol{Y}^{\approx}_{nk} = G^{\approx}_{nk} + jB^{\approx}_{nk} = \frac{\boldsymbol{I}^{\approx}_{nk}}{\boldsymbol{U}^{\approx}_{nk}}\,.$$

and the equivalent conductance

(46)
$$G^{\approx}_{\mathrm{e}k} = \frac{P^{\approx}_{\mathrm{C}k}}{||u^{\approx}_{\mathrm{C}}||^2_k}\,,$$

with

(47)
$$||u^{\approx}_{\mathrm{C}}||_k = \sqrt{\sum_{n\in N_{\mathrm{C}}} ||u^{\approx}_{n}||^2_k}\,,$$

then the current $i^{\approx}_{\mathrm{C}}$ can be decomposed at instant $t = t_k$ into the active, scattered and reactive components, namely

(48)
$$i^{\approx}_{\mathrm{C}k} = i^{\approx}_{\mathrm{Ca}k} + i^{\approx}_{\mathrm{Cs}k} + i^{\approx}_{\mathrm{Cr}k}\,,$$

where

(49)
$$i^{\approx}_{\mathrm{Ca}k} \stackrel{\mathrm{df}}{=} G^{\approx}_{\mathrm{e}k}\, u^{\approx}_{\mathrm{C}k}$$

is the active current

(50)
$$i^{\approx}_{\mathrm{Cs}k} \stackrel{\mathrm{df}}{=} (G^{\approx}_{nk} - G^{\approx}_{\mathrm{e}k})U^{\approx}_{0k} + \sqrt{2}\,\mathrm{Re}\sum_{n\in N_{\mathrm{C}}} (G^{\approx}_{nk} - G^{\approx}_{\mathrm{e}k})\boldsymbol{U}^{\approx}_{nk}\, e^{jn\frac{2\pi}{K}k}$$

is the scattered current and

(51)
$$i^{\approx}_{\mathrm{Cr}k} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\sum_{n\in N_{\mathrm{C}}} jB^{\approx}_{nk}\, \boldsymbol{U}^{\approx}_{nk}\, e^{jn\frac{2\pi}{K}k}$$

is the reactive current. Eventually, the load current can be decomposed into four physical components,

(52)
$$i^{\approx}_{k} = i^{\approx}_{\mathrm{Ca}k} + i^{\approx}_{\mathrm{Cs}k} + i^{\approx}_{\mathrm{Cr}k} + i^{\approx}_{\mathrm{G}k}\,,$$

where

(53)
$$i^{\approx}_{\mathrm{G}k} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\sum_{n\in N_{\mathrm{G}}} \boldsymbol{I}^{\approx}_{nk}\, e^{jn\frac{2\pi}{K}k}$$

is the load generated current at instant $t = t_k$. These currents are mutually orthogonal, hence at instant $t = t_k$

(54)
$$||\,i^{\approx}||^2_k = ||\,i^{\approx}_{\mathrm{Ca}}||^2_k + ||\,i^{\approx}_{\mathrm{Cs}}||^2_k + ||\,i^{\approx}_{\mathrm{Cr}}||^2_k + ||\,i^{\approx}_{\mathrm{G}}||^2_k\,.$$

A diagram which geometrically illustrates this relationship is shown in Fig. 9.

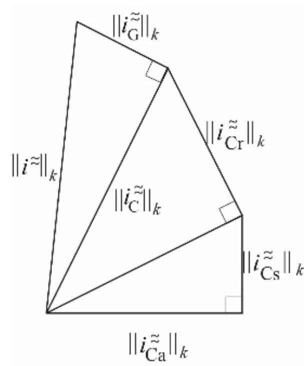


Fig. 9. Diagram of running rms values of the supply current physical components of a HGL.

Loads with fast varying parameters or HGLs are often supplied with sinusoidal or quasi-sinusoidal voltage, or such approximation is quite sufficient for analysis of the power properties in such situations. Fast variance of the load parameters could be the most dominating feature of the system. In such a situation, the set $N_C$ of distribution system originated harmonic orders $n$ is confined to only $n = 1$, i.e., $N_C = \{1\}$, while all other orders belong to the set $N_G$, meaning, the set of the load originated harmonic orders. The current $i_C(t)$ in such a case is composed only of the active and reactive currents, which are quasi-sinusoidal, while the current $i_G(t)$ is composed of all quasi-harmonics. The scattered current does not occur, of course, in the supply current and consequently, it can be decomposed into the active, reactive and the load generated currents. The formula (52), could be reduced in such a situation to

$$i_k^{\approx} = i_{\mathrm{Ca}k}^{\approx} + i_{\mathrm{Cr}k}^{\approx} + i_{\mathrm{G}k}^{\approx}\,. \tag{55}$$

## Extrapolation in the future

The approach presented above enables decomposition of a semi-periodic current into physical components, which is valid only in the observation window, which directly precedes the instant of the present measurements $t = t_k$. Although it provides full information on the power phenomena in the load, it is, unfortunately, to some degree useless for compensation. Assuming that a switching compensator is used for the power factor improvement, then properties of the load at the instant $t = t_{k+1}$, i.e., in a direct future have to be known. Currents that are to be compensated, of the value just at that instant, $t = t_{k+1}$, have to be injected by the compensator into the system.

Their value can be predicted by extrapolation. The most simple is a linear extrapolation, based on the assumption that changes in the direct future, in the time of a single sampling interval $\Delta t$, are equal to changes in the direct past. This assumption means that if $x_k^{\approx}$ is a value of a semi-periodic quantity at the instant $t = t_k$, then in the direct future, at $t = t_{k+1}$, is assumed to be

$$x_{k+1}^{\approx} = x_k^{\approx} + \Delta x^{\approx} = x_k^{\approx} + (x_k^{\approx} - x_{k-1}^{\approx}) = 2\,x_k^{\approx} - x_{k-1}^{\approx}\,. \tag{56}$$

The measurement at the instant $t = t_{k+1}$, can update this extrapolated value to a true one.

Such extrapolation can provide fundamentals for quasi-instantaneous generation of reference signals for switching compensator control.

## Conclusions

The paper demonstrates that the Currents' Physical Components – based power theory, originally developed for systems with periodic voltages and currents, can be generalized to systems where these quantities are semi-periodic. It means that also in such conditions, the interpretation of power phenomena in electrical circuits, as provided by the CPC, remains valid.

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