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Currents' Physical Components (CPC) – based Power Theory A Review
Part I: Power Properties of Electrical Circuits and Systems

Abstract. *The CPC-based power theory (PT) of electrical circuits and systems provides an interpretation of the energy transfer-related physical phenomena in such systems and fundamentals for their compensation. It has been developed, step-by-step, with partial results published in Polish, German, English and American journals and in conference proceedings, often not reported on the main databases and consequently, difficult to be found. Its development was a reaction for the lack of progress in long-lasting attempts aimed at explanation of the energy transfer-related physical phenomena and the lack of fundamentals for compensation in electrical systems with nonsinusoidal voltages. This Review provides a draft of the whole CPC concept with references to more detailed results and a concise historical background and critical comments on other power theories.*

Streszczenie. *Teoria mocy obwodów i systemów elektrycznych, oparta na koncepcji Składowych Fizycznych Prądów, tworzy podstawę teoretyczną dla interpretacji zjawisk fizycznych towarzyszących przesyłowi energii w takich układach i ich kompensacji. Była ona rozwijana stopniowo, z cząstkowymi wynikami publikowanymi w czasopismach polskich, niemieckich, angielskich i amerykańskich oraz w materiałach konferencyjnych, często trudnych do odnalezienia. Rozwój CPC był odpowiedzią na brak postępu w badanich nad teorią mocy i na brak podstaw kompensacji w układach z napięciami niesinusoidalnymi. Przegląd ten jest skrótem CPC, z odnośnikami do bardziej szczegółowych wyników oraz z tłem historycznym i krytycznymi uwagami dotyczącymi wyników innych teorii mocy.* **(Teoria mocy Składowych Fizycznych Prądów (CPC). Artykuł przeglądowy, Część I: Właściwości energetyczne obwodów i systemów elektrycznych).**

Keywords: Current decomposition, unbalanced loads, asymmetrical systems, power definitions, power theory.
Słowa kluczowe: Rozkład prądu, odbiorniki niezrównoważone, systemy niesymetryczne, definicje mocy, teoria mocy.

Introduction

The development of the power theory of electrical circuits and systems has more than century-long history with a great variety of different attempts aimed at its formulation, with a lot of discussions and controversy.

The Currents' Physical Components (CPC) – based power theory is one of these attempts. It describes and clarifies power properties of single-phase and three-phase systems with linear and nonlinear loads supplied with a non-sinusoidal voltage. It also creates fundamentals for compen-sation of such systems.

It has been developed, as drafted in [51], step-by-step, starting [15] in 1983, with partial results published in Polish, German, English and in American journals, and in confe-rence proceedings. The development of the CPC-based PT was interlaced with critical studies on other PTs. Unfortu-nately, some results could not be easy to find. A prospective reader might not be even aware of their existence. Therefore, Part I of this Review is to provide fundamentals and be a major reference for the CPC-based power theory of electrical circuits and systems. This power theory creates also theore-tical fundamentals for the synthesis of reactive compensators and for the development of algorithms for control of switching compensators in electrical systems of any complexity. Fundamentals of the CPC-based methods of compensation will be drafted in Part II of this Review.

A short history of the power theory

Although the concepts of the active, reactive, and appa-rent powers, P, Q, and S, have been known earlier, the present debate on power properties of electrical systems was initiated in 1892 by Steinmetz measurement [1] of powers in a circuit with an electric arc. He concluded that the power equation

$$S^2 = P^2 + Q^2 \tag{1}$$

was not valid. Steinmetz observed, that in spite of zero reactive power Q, the apparent power S was higher than the load active power P, thus

$$S \geq P\,. \tag{2}$$

The cost of electric energy production and delivery by its provider is associated with the apparent power, while the value of that energy for the customer is associated with the active power P, therefore, the inequality (2) is very important for the power system economy. This inequality raises two questions. The first one is cognitive in nature, namely

> ***Why can the apparent power S be higher than the active power P? What physical phenomena in the load are responsible for this inequality?***

The second question has practical implications, namely

> ***How can the difference between the apparent power S and the active power P be reduced by a compen-sator?***

Apparently easy questions inspired by the Steinmetz observation, have occurred to be ones of the most difficult questions of the electrical engineering. Hundreds of scien-tists have attempted to explain and describe the power pro-perties of loads with nonsinusoidal voltages and currents and develop methods of compensation. Hundreds of papers were published. Several "schools" of power theory [7], [8],

[12], [13], [19], [27], [35] were established. These "schools" of the power theory and comments to them, [14], [23], [31], [36], [37], [47], [50], [53], [54], represent only a part of much wider debate. A reader is encouraged to look also into papers [9], [11], [20], [22], [25], [28], [30], [32], [34], [38 - 45], [51], [55] on the power theory.

Schools of the power theory

Budeanu's power theory formulated in 1927 [7] in the frequency domain, was the first major attempt aimed at des-cription of power properties of electrical loads supplied with nonsinusoidal voltage. It is probably the most widely distribu-ted power theory of single-phase circuits with nonsinusoidal voltages and currents. The correctness of this theory was challenged in 1987 [23], where it was demonstrated that

- There is no physical phenomenon associated with the Budeanu's reactive power Q.
- There is no association between Budeanu's distortion power D and the voltage and current mutual distortion.
- There is no relation between the power factor improve-ment and the Budeanu's reactive power Q reduction.

Fryze's power theory, formulated in 1931 [8] in the time-domain, introduced the concept of the active current and the load current decomposition into orthogonal components. It introduced the concept of the reactive current but did not provide its physical interpretation other than that it is a useless current. It does not create fundamentals [31] for com-pensator synthesis or control.

Shepherd and Zakikhani's power theory formulated in 1972 in the frequency-domain in [12] provided a right defi-nition of the reactive current $i_r(t)$. It raised and solved the issue of calculation of the optimal capacitance C_{opt}, which in the presence of the supply voltage harmonics increases the power factor to the maximum possible value. Unfortunately, the active power P was lost in the power equation they deve-loped.

Kuster and Moore's power theory formulated in 1980 [13] in the time-domain, solved the problem of the optimal capacitance calculation without using the concept of harmo-nics. The solution was valid, however, as shown in [14], [17], only at the ideal supply source.

Instantaneous Reactive Power (IRP) p-q Theory, for-mulated by Nabae, Akagi and Kanazawa in 1984 [19], in the time-domain, and generalized in [30], provides fundamen-tals for switching compensator control. This control algo-rithm is valid only at sinusoidal [41] and symmetrical [42] supply voltage, however. Moreover, the IRP p-q PT misinter-prets power phenomena [36], [37], [46] in electrical systems.

FBD Method developed by Depenbrock [27] in the time-domain, is a generalization of Fryze's power theory for three-phase systems. It correctly defines, after Buchholtz [3], the apparent power S, but has all disadvantages of the Fryze's power theory.

The Conservative Power Theory (CPT), developed by Tenti and others in [35], is formulated in the time-domain and has a strong analogy to Budeanu's power theory and its defi-ciencies, although it follows Fryze's concept of the current orthogonal decomposition. The current components in the CPT do not have physical interpretation, however. Moreover, the CPT does not provide, as shown in [47, 50], right funda-mentals for a capacitive compensator design. It also wrongly defines, as shown in [54], the unbalanced power in three-phase systems.

Definitions of apparent power in three-phase circuits

Since most of the electric energy is transferred by three-phase systems, power properties of such systems are of the crucial importance for the power systems economy. There-fore, there was a lot of research [3 - 5], [9 - 11], [19 - 20], [22], [27 - 30], [32 - 35], [38 - 40], [43], focused on the development of the power theory of three-phase systems in the presence of harmonics and asymmetry.

There were two major obstacles which made some of these efforts doomed to fail. First, three-phase systems have as their sub-set all single-phase systems. As long as the power properties of single-phase systems were not correctly described and comprehended, it was not possible to correctly describe such properties of three-phase systems. Second, all studies were carried on using the wrong definition of the apparent power S of three-phase systems. In 1920 a commit-tee of AIEE suggested [2] two definitions of the apparent power S, known as the arithmetical definition

$$S = S_A = U_R I_R + U_S I_S + U_T I_T \tag{4}$$

and the geometrical definition

$$S = S_G = \sqrt{P^2 + Q^2}\,. \tag{5}$$

A debate [6] on these two definitions in the twenties was inconclusive and both of them were adopted as standard definitions. They can be found in the IEEE Standard Dictio-nary of Electrical and Electronics Terms. There was also a definition of the apparent power suggested in 1922 [3] by Buchholtz,

$$S = S_B = \sqrt{U_R^2 + U_S^2 + U_T^2}\,\sqrt{I_R^2 + I_S^2 + I_T^2} \tag{3}$$

but it was dominated by the arithmetic and the geometric definitions, endorsed by the IEEE, and it was not used.

When voltages and currents are sinusoidal and symmet-rical these three definitions result in the same numerical value of the apparent power S and consequently, the power factor λ. When these conditions are not satisfied, the value of the power factor λ depends on the apparent power define-tion. This is illustrated on the circuit in Fig. 1, with purely resistive load and an ideal transformer with the turn ratio 1:1.

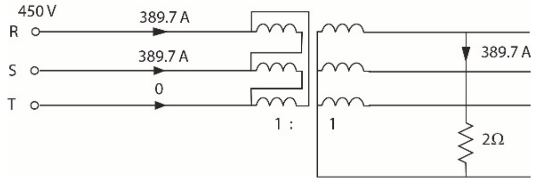

Fig. 1. A circuit with unbalanced purely resistive load.

The load active power is equal to P = 304 kW, the reactive power Q is zero, because the load is purely resistive, while the apparent power S, depending on its definition, is

$$S_A = 351 \text{ kVA}, \qquad S_G = 304 \text{ kVA}, \qquad S_B = 430 \text{ kVA}$$

and consequently, depending on the selected definition, the power factor has the value

$$\lambda_A = \frac{P}{S_A} = 0.86, \quad \lambda_G = \frac{P}{S_G} = 1, \quad \lambda_B = \frac{P}{S_B} = 0.71\,.$$

and it is not clear which value is the right one.

The reasoning in [33], presented in 1999, has demonstra-ted that the right value of the power factor is obtained when the apparent power is calculated according to the Buchholtz definition. This selection of the right definition of the appa-rent power S eventually paved the road for describing unba-lanced three-phase systems in power terms in the frame of the CPC.

Main approaches to power theory development

The main difference in various approaches to the power theory development boils down to two questions:

- *Should the power properties of a system be described in the **time-domain** or in the **frequency-domain**?*
- *Should the power properties of a system be described in terms of **instantaneous** or in terms of **averaged** over the period values?*

Mathematically, due to the Fourier Transforms, descript-tions of continuous quantities in the time-domain and in the frequency-domain are mutually equivalent. The question whether the power theory should be formulated in the frequ-ency-domain, as suggested in [7], or in the time-domain, as suggested in [8], boils down to the question: *should the concept of harmonics be used for that purpose or not?*

To answer such a question two aspects of the issue should be taken into account. The first of them is the metro-logical availability of harmonics. Such availability can be cru-cial for technical implementations of a harmonics-based PT. When the concept of harmonics was introduced, it was pos-sible to measure, using tuned filters, only the rms value of harmonics. Their phase was practically beyond measure-ment possibility. Now, sampling and a digital signal process-sing (DSP) are capable of providing complex rms (crms) values of harmonics up to relatively high order in real time.
Thus, the availability of harmonics values is not an issue.

The second aspect of the harmonics issue is more cru-cial: does the concept of harmonics contribute to our com-prehension of power phenomena or hinder it?

Thinking of voltages and currents as sums of harmonics when applied to the energy flow studies may lead to unac-ceptable conclusions. This was demonstrated by Fryze in [8] who analyzed the energy flow in the circuit shown in Fig. 2, with dc supply voltage and a periodic switch.

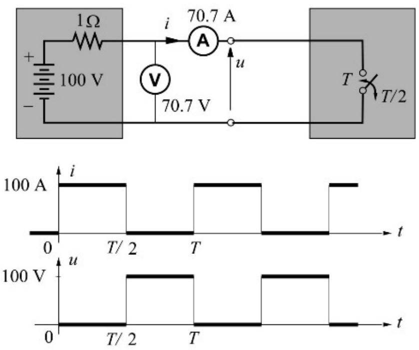

Fig. 2. A circuit with a periodic switch.

According to the frequency-domain approach, the load voltage and current can be expressed in terms of harmonics in the forms

$$u(t) = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n) = \sum_{n=0}^{\infty} u_n \tag{6}$$

$$i(t) = I_0 + \sqrt{2}\sum_{n=1}^{\infty} I_n \cos(n\omega_1 t + \beta_n) = \sum_{n=0}^{\infty} i_n\,. \tag{7}$$

The instantaneous power $p(t)$ at the load terminals, i.e., the rate of energy, $W(t)$, flows from the supply source to the load is equal to

$$p(t) = \frac{dW}{dt} = u(t)\, i(t) = \sum_{r=0}^{\infty} u_r \sum_{s=0}^{\infty} i_s = \sum_{n=0}^{\infty} S_n \cos(n\omega_1 t + \psi_n)\,. \tag{8}$$

Thus, it is equal to an infinite sum of oscillating components.

This formula suggests the existence of an infinite number of oscillating components in the instantaneous power. To find the instantaneous power at any instant of time, this series has to be calculated.

In the time-domain it is directly visible, however, from the voltage and current waveforms that, apart from discontinuity points, their product

$$p(t) = u(t)\, i(t) \equiv 0\,. \tag{9}$$

At discontinuity points, a tiny amount of energy is only stored in a stray inductance and capacitance of conductors.

Thus there are no flow of energy and no energy oscilla-tion in such a circuit. The time-domain approach is clearly superior to the frequency-domain in such a situation.

The power theory of single-phase circuits with LTI loads at nonsinusoidal supply voltage was formulated, however, using the CPC concept, in the frequency-domain. Two main components of the load current, namely, the reactive current $i_r(t)$ and the scattered current $i_s(t)$, are defined in [15], [16], using the concept of harmonics, meaning in the frequency-domain. Some attempts aimed at defining them in the time-domain have failed.

Let us discuss the second question related to instan-taneous versus averaged approach.

The most fundamental power quantity in electrical circuits and systems, the instantaneous power $p(t)$, specifies the rate
of the electric energy flow at each instant of time. Due to this interpretation, it is the most unquestionable power quantity. This can imply a conclusion that the whole power theory should be based on quantities defined as instantaneous ones.

There are situations in power systems where indeed instantaneous values of voltages and currents are crucial, as it is during disturbances or faults. Performance of electrical systems with periodic voltages and currents is specified at normal operation entirely in terms of quantities defined as some integrals over the supply voltage period T, however. These are the active, reactive and apparent powers, the voltage and current rms value, the power factor, rms value of harmonics, harmonic distortion, or the voltage and current symmetrical components. The instantaneous power $p(t)$ usually is not a matter of interest for system designers and operators. At the same time, terms like "*rms*", "*apparent power*", "*harmonic*" and "*symmetrical component*", are alien for theories that describe instantaneous properties of elec-trical systems.

The major difficulty of power theories that claim to be "instantaneous" stems from the fact that power properties of the load cannot be identified instantaneously. This is illustrated in Fig. 3, with an unknown load and a pair of instantaneous values of the load voltage and current. As shown in Fig. 4. There could be a resistor, an inductor or a capacitor in this "black box".

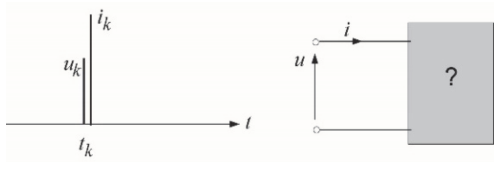

Fig. 3. Unknown load and pair of instantaneous values of the load voltage and current.

In fact, an infinite number of different loads can have identical pairs of voltage and current samples. Taking into account that the load voltage and current are in general nonsinusoidal, the instantaneous values of the voltage and current over the whole period T have to be measured to draw conclusions on the load power properties.

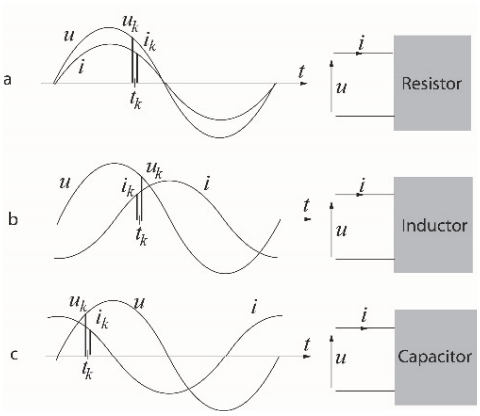

Fig. 4. Different loads with identical pairs of the voltage and current instantaneous value.

Quantities obtained by averaging could be added to the instantaneous power theory, but this would undermine the claim that the theory is indeed instantaneous. It is just the case of the IRP *p-q* Theory, where the active power P can be found only by averaging the instantaneous active power p, but even with this averaging, the concepts of the apparent power S and consequently, the concept of the power factor does not exist inside of this theory.

The CPC concept

Explanation of the power properties of electrical systems in terms of physical phenomena in the load was the main idea behind the development of the Currents' Physical Compo-nents - based power theory. This development was initiated for linear time-invariant (LTI) loads in Polish [15] and in English [18]. Initially, in [15], [16], [18], [21], or [24], this approach was referred to as a "***current orthogonal decom-position***". The term "***Currents' Physical Components***" was used for the first time in [36]. Identification of the load current components associated with distinctive physical phenomena in the load, which gave the name to this theory, has served this goal. These components can be identified by measure-ment of voltages and currents at the load terminals.

It is crucially important for the load current decomposition into CPC, that these components are mutually orthogonal. Being orthogonal, they contribute to the supply current rms value independently of each other. In such a way, the CPC reveals the effect of individual physical phenomena in the load upon the supply current rms value. Only one of these components is useful for the transfer of the energy from its provider to a customer; remaining ones are useless for that. Equally important for this theory development was finding relations of the CPC with the load parameters, that could be calculated based on measurements of the voltages and currents at the load terminals. These relations create very fundamentals for the development of reactive compensators.

Single-phase circuits with LTI loads

With respect to the active power P at the voltage u, a single-phase LTI load is equivalent to a purely resistive load, on the condition that its conductance is

(10) $$G_{\rm e} = \frac{P}{||u||^2}\,.$$

This conductance was called in [8] an ***equivalent conduc-tance*** of the load. It draws an ***active current***,

(11) $$i_{\rm a} = G_{\rm e}\, u$$

from the supply source. The CPC-based PT requires that voltages and currents are expressed by Fourier Series, which are used, as initiated in [15], in a complex form

(12) $$u = U_0 + \sqrt{2}\,{\rm Re}\sum_{n=1}^{\infty}\boldsymbol{U}_n\, e^{jn\omega_1 t}, \quad \boldsymbol{U}_n = U_n\, e^{j\alpha_n}\,.$$

Symbol $\boldsymbol{U}_n$ denotes the ***complex rms*** (***crms***) [18] value of the $n^{\rm th}$ order load voltage harmonic. The load has for harmo-nic frequencies the admittance $\boldsymbol{Y}_n = G_n + jB_n$, thus, the load current can be expressed by a Fourier series as follows

(13) $$i = I_0 + \sqrt{2}{\rm Re}\sum_{n=1}^{\infty}\boldsymbol{I}_n\, e^{jn\omega_1 t} = Y_0 U_0 + \sqrt{2}{\rm Re}\sum_{n=1}^{\infty}\boldsymbol{Y}_n\boldsymbol{U}_n\, e^{jn\omega_1 t}.$$

The remaining part of the load current, $i - i_{\rm a}$, is useless for the energy transfer. It can be presented in the form

(14) $$\begin{aligned} i - i_{\rm a} &= G_0 U_0 + \sqrt{2}{\rm Re}\sum_{n=1}^{\infty}(G_n + jB_n)\boldsymbol{U}_n\, e^{jn\omega_1 t} - \\ &- G_{\rm e}(U_0 + \sqrt{2}{\rm Re}\sum_{n=1}^{\infty}\boldsymbol{U}_n\, e^{jn\omega_1 t}). \end{aligned}$$

Two different components can be identified [15] in this useless current. One of them is "in-phase" or in "counter-phase" with the voltage harmonics:

(15) $$(G_0 - G_{\rm e})U_0 + \sqrt{2}{\rm Re}\sum_{n=1}^{\infty}(G_n - G_{\rm e})\boldsymbol{U}_n\, e^{jn\omega_1 t} = i_{\rm s}\,.$$

It occurs when the load conductance for harmonics G_n differs from the equivalent conductance $G_{\rm e}$. The values G_n are usu-ally scattered around $G_{\rm e}$, therefore, this component was called in [18], ***a scattered current***.

The remaining component of the useless current

(16) $$\sqrt{2}{\rm Re}\sum_{n=1}^{\infty} jB_n\boldsymbol{U}_n\, e^{jn\omega_1 t} = i_{\rm r}$$

occurs due to a phase shift of the load current harmonics with respect to the load voltage harmonic. It was called ***a reactive current***. Consequently, the load current can be decomposed into three components

(17) $$i = i_{\rm a} + i_{\rm s} + i_{\rm r}\,.$$

Each of these components is associated with a distinctively different physical phenomenon in the load and therefore, they stand for the ***Current Physical Components***.

The active current is associated with a permanent trans-fer of the energy from the supply source to the load. Transfer with the rate, averaged over period T, equal to the active power P.

The scattered current is associated with the phenomenon of the load conductance G_n change with the order of harmo-nic. This change is a common property of electrical loads.

The reactive current is associated with the phenomenon of a phase-shift between the load current and the supply voltage harmonics.

The reactive current $i_{\rm r}$ is orthogonal to the active and to the scattered currents, $i_{\rm a}$ and $i_{\rm s}$, because (Appendix A) har-monics of these currents are shifted mutually by $\pi/2$. Ortho-gonality of the active and the scattered currents is not directly visible, however. It was proven in [15], that their scalar product is zero. Namely, with (A4), we obtain

$$(i_s, i_a) = \mathrm{Re}\sum_{n=0}^{\infty} \boldsymbol{I}_{sn}\boldsymbol{I}^*_{an} = \mathrm{Re}\sum_{n=0}^{\infty} (G_n - G_e)\boldsymbol{U}_n G_e \boldsymbol{U}^*_n =$$

(18) $$= G_e \sum_{n=0}^{\infty} (G_n - G_e)U_n^2 = G_e (\sum_{n=0}^{\infty} G_n U_n^2 - G_e \sum_{n=0}^{\infty} U_n^2) =$$

$$= G_e (P - G_e \|u\|^2) = G_e (P - P) = 0.$$

Thus, all components in the decomposition (17) are mutually orthogonal, hence their rms values satisfy the relationship

(19) $$\|i\|^2 = \|i_a\|^2 + \|i_s\|^2 + \|i_r\|^2 .$$

Multiplying (19) by the square of the supply voltage rms value, $\|u\|$, a power equation of LTI loads supplied with a nonsinusoidal voltage is obtained, namely

(20) $$S^2 = P^2 + D_s^2 + Q^2$$

where

(21) $$P = \|u\|\,\|i_a\|, \quad D_s = \|u\|\,\|i_s\|, \quad Q = \|u\|\,\|i_r\|.$$

are the active, scattered and reactive powers. Decomposition (17) is not valid, however, when the load, due to nonlinearity or time-variance, is a source of current harmonics. Such loads are referred to as Harmonic Generating Loads (HGL).

Single-phase circuits and systems with HGL

In paper [26] there was considered a purely resistive circuit, shown in Fig. 5, with a supply voltage

$$e(t) = 100\sqrt{2}\,\sin\omega_1 t \ \ \text{[V]}$$

and a load, which is a source the third order current harmonic

$$j(t) = 50\sqrt{2}\,\sin 3\omega_1 t \ \ \text{[A]}.$$

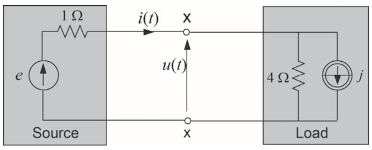

Fig.5. An example of a circuit.

The voltage and the current in the cross-section x-x, have the waveforms

$$u(t) = 80\sqrt{2}\,\sin\omega_1 t - 40\sqrt{2}\,\sin 3\omega_1 t \ \ \text{V}$$

$$i(t) = 20\sqrt{2}\,\sin\omega_1 t + 40\sqrt{2}\,\sin 3\omega_1 t \ \ \text{A}.$$

Their rms value is equal to, respectively

$$\|u\| = \sqrt{80^2 + 40^2} = 89.5\ \text{V}, \quad \|i\| = \sqrt{20^2 + 40^2} = 44.7\ \text{A}$$

so that, the apparent power S in the cross-section x-x is

$$S = \|u\|\,\|i\| = 89.5 \times 44.7 = 4000\ \text{VA}.$$

The active power P in this cross-section is equal to

$$P = \frac{1}{T}\int_0^T u(t)\,i(t)\,dt = 80\times 20 - 40\times 40 = 0 .$$

There is no phase-shift between the voltage and current harmonics thus, there is no reactive power in the cross-section x-x. The conductance in this cross-section does not change with the harmonic order, hence there is no scattered power D_s. Thus, a question was asked in [26]: "*how can the power equation be written when the active, scattered and the reactive powers are equal to zero, while the apparent power has a non-zero value?"* We face the question: "*S = to what*?".

Analysis of circuits with HGL revealed, [26], that the active power of some harmonics P_n can be negative, thus at the frequency of such harmonics the energy is transferred from the load back to the supply source, where it is dissipated on the source resistance. This is a physical phenomenon, that should be taken into account by the power theory.

Depending on the sign of the harmonic active power

(22) $$P_n = U_n I_n \cos\varphi_n$$

at the load terminals, the set N of harmonic orders n can be divided into two sub-sets N_C and N_G, namely

(23) $$\begin{aligned} &\text{if } P_n \geq 0, \text{ i.e., } |\varphi_n| \leq \pi/2, \text{ then } n \in N_C \\ &\text{if } P_n < 0, \text{ i.e., } |\varphi_n| > \pi/2, \text{ then } n \in N_G . \end{aligned}$$

It enables the voltage and the current decomposition into components with harmonics from sub-sets N_C and N_G, namely

(24) $$i = \sum_{n\in N} i_n = \sum_{n\in N_C} i_n + \sum_{n\in N_G} i_n = i_C + i_G$$

(25) $$u = \sum_{n\in N} u_n = \sum_{n\in N_C} u_n + \sum_{n\in N_G} u_n = u_C - u_G .$$

Voltage u_G is a negative sum of harmonics that occur at the load terminals as a response to the current i_G. The same applies to harmonic active powers P_n, thus

(26) $$P = \sum_{n\in N} P_n = \sum_{n\in N_C} P_n + \sum_{n\in N_G} P_n = P_C - P_G .$$

Sub-sets N_C and N_G do not contain common harmonic orders thus, currents i_C and i_G are mutually orthogonal. Hence, their rms values satisfy the relationship

(27) $$\|i\|^2 = \|i_C\|^2 + \|i_G\|^2 .$$

The HGL could be nonlinear, but in a fixed working point, specified by the voltage u, the state of the circuit can be regarded as a sum of its responses to the voltage e_C and the current j_G separately, i.e., as a linear circuit. The Superpo-sition Principle allows us to analyze the original circuit with two equivalent circuits shown in Fig. 6.

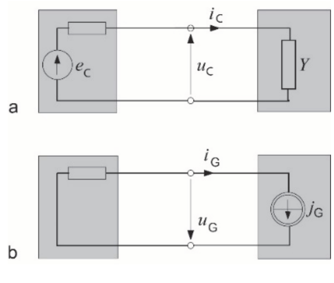

Fig. 6. (a) An equivalent circuit for harmonics orders $n \in N_C$, and (b) an equivalent circuit for harmonics orders $n \in N_G$.

The circuit in Fig. 6a can be described according to the CPC approach. First of all, the current contains an active current

(28) $$i_{Ca} = G_{Ce}\, u_C , \qquad G_{Ce} = \frac{P_C}{\|u_C\|^2} .$$

It differs from the active current as defined by Fryze, because the equivalent conductance G_{Ce} is defined by (28) in a diffe-rent way than in the Fryze's power theory [8].

For each harmonic of the order n from the set N_C the load admittance can be measured, so that the scattered and reac-tive components

(29) $$i_{Cs} = (G_0 - G_{Ce})U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N_C}(G_n - G_{Ce})\boldsymbol{U}_n e^{jn\omega_1 t}$$

(30) $$i_{Cr} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N_C} jB_n\,\boldsymbol{U}_n e^{jn\omega_1 t}$$

can be calculated. In such a way the current of an HGL can be decomposed into four components, namely

(31) $$i = i_{Ca} + i_{Cs} + i_{Cr} + i_G.$$

The last component in this decomposition i_G, referred to as a ***load generated current***, is associated with the phenomenon of a permanent energy transfer at some harmonic frequen-cies from the load back to the supply source.

The active, scattered and the reactive currents, i_{Ca}, i_{Cs}, and i_{Cr} are mutually orthogonal for the same reasons as in circuits with LTI load. Consequently, the rms values of the load current Physical Components satisfy the relationship

(32) $$\|i\|^2 = \|i_{Ca}\|^2 + \|i_{Cs}\|^2 + \|i_{Cr}\|^2 + \|i_G\|^2.$$

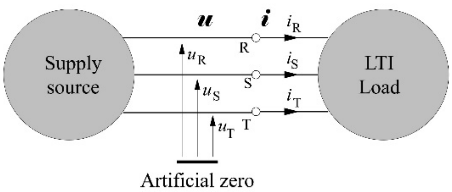

Fig. 7. A structure of three-phase, three-wire system.

Three-phase circuits and systems with LTI loads at sinusoidal voltage

An increase in complexity required [24] that three-phase systems, shown in Fig. 7, was described by more compact symbols than single-phase systems. In particular, three-phase vectors, of voltages and currents, defined as follows:

(33) $$\boldsymbol{u} = \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_S \\ \boldsymbol{U}_T \end{bmatrix} e^{j\omega t} = \sqrt{2}\,\mathrm{Re}\,\{\boldsymbol{U} e^{j\omega t}\}$$

(34) $$\boldsymbol{i} = \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{I}_R \\ \boldsymbol{I}_S \\ \boldsymbol{I}_T \end{bmatrix} e^{j\omega t} = \sqrt{2}\,\mathrm{Re}\,\{\boldsymbol{I} e^{j\omega t}\}.$$

were needed for that. The active power of the load is

(35) $$P = \frac{1}{T}\int_0^T (u_R i_R + u_S i_S + u_T i_T)\,dt = \frac{1}{T}\int_0^T (\boldsymbol{u}^T \boldsymbol{i})\,dt = (\boldsymbol{u}, \boldsymbol{i})$$

where symbol $(\boldsymbol{u}, \boldsymbol{i})$ denotes a scalar product of the voltage and current vectors. Formula (35) is valid irrespective of the voltages and currents waveform or their asymmetry. When these quantities are sinusoidal, then the active power is

(36) $$P = \sum_{p=R,S,T} U_p I_p \cos\varphi_p.$$

The reactive power Q, by analogy to the active power, is

(37) $$Q = \sum_{p=R,S,T} U_p I_p \sin\varphi_p.$$

The Buchholz definition of the apparent power is a special case of the definition of this power for systems with nonsinu-soidal voltages and currents, introduced [24] in 1988:

(38) $$S = \|\boldsymbol{u}\|\,\|\boldsymbol{i}\|$$

where symbols $\|\boldsymbol{u}\|$ and $\|\boldsymbol{i}\|$ denote a ***three-phase rms value,*** introduced in [24], of the supply voltage and the load current vectors. Its meaning is explained in Appendix C.

One of the most important conclusions which can be drawn from the reasoning presented in [33] on the right definition of the apparent power S is the conclusion that if the apparent power S is defined correctly, i.e., by (38), then the commonly used power equation (1) is not valid. The valid equation can be obtained in the frame of the CPC concept.

At the assumption that the supply voltage is sinusoidal, symmetrical of the positive sequence, the vector of the load currents (34) can be expressed [24] in terms of the line-to-line admittances of the equivalent load as follows

(39) $$\boldsymbol{i} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{I}_R \\ \boldsymbol{I}_S \\ \boldsymbol{I}_T \end{bmatrix} e^{j\omega t} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}_e \begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_S \\ \boldsymbol{U}_T \end{bmatrix} + \boldsymbol{Y}_u \begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_T \\ \boldsymbol{U}_S \end{bmatrix}\} e^{j\omega t}$$

where

(40) $$\boldsymbol{Y}_e = G_e + jB_e = \boldsymbol{Y}_{RS} + \boldsymbol{Y}_{ST} + \boldsymbol{Y}_{TR}$$

is referred to [24] as an ***equivalent admittance***, and

(41) $$\boldsymbol{Y}_u = Y_u e^{j\psi} = -(\boldsymbol{Y}_{ST} + \alpha\,\boldsymbol{Y}_{TR} + \alpha^*\boldsymbol{Y}_{RS}), \quad \alpha = 1e^{j2\pi/3}$$

is referred to as an ***unbalanced admittance*** of the load. The first term of (39) can be rearranged to the form

(42) $$\sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}_e \boldsymbol{U} e^{j\omega t}\} = \sqrt{2}\,\mathrm{Re}\{(G_e + jB_e)\boldsymbol{U} e^{j\omega t}\} = \boldsymbol{i}_a + \boldsymbol{i}_r$$

where

(43) $$\boldsymbol{i}_a = \sqrt{2}\,\mathrm{Re}\{G_e \boldsymbol{U} e^{j\omega t}\} = \sqrt{2}\,\mathrm{Re}\{G_e \mathbf{1}^p \boldsymbol{U}_R e^{j\omega t}\}$$

is an ***active current***, and

(44) $$\boldsymbol{i}_r = \sqrt{2}\,\mathrm{Re}\{jB_e \boldsymbol{U} e^{j\omega t}\} = \sqrt{2}\,\mathrm{Re}\{jB_e \mathbf{1}^p \boldsymbol{U}_R e^{j\omega t}\}$$

is a ***reactive current***. The symbol $\mathbf{1}^p$ denotes ***a unite three-phase vector of the positive sequence***

(45) $$\mathbf{1}^p = [1, \alpha^*, \alpha]^T.$$

Formulas (43) and (44) enable us to relate the active and reactive currents to the crms value $\boldsymbol{U}_R$ of the line R voltage. They emphasize the positive sequence symmetry of these currents.

The second term of (39) has a negative sequence and it is zero when the load is balanced, i.e., $\boldsymbol{Y}_{ST} = \boldsymbol{Y}_{TR} = \boldsymbol{Y}_{RS}$. It can be rearranged to the form

(46) $$\sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}_u \begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_T \\ \boldsymbol{U}_S \end{bmatrix}\} e^{j\omega t} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}_u \mathbf{1}^n \boldsymbol{U}_R\} e^{j\omega t} = \boldsymbol{i}_u$$

and stands for an ***unbalanced current***. Symbol $\mathbf{1}^n$ denotes ***a unite three-phase vector of the negative sequence***

(47) $$\mathbf{1}^n = [1, \alpha, \alpha^*]^T.$$

In such a way, the vector of the load currents was decom-posed into three components

(48) $$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_r + \boldsymbol{i}_u$$

associated with three distinctively different physical pheno-mena: - with the permanent energy transfer from the supply source to the load ($\boldsymbol{i}_a$); - with the phase-shift between load voltages and currents ($\boldsymbol{i}_r$); and - with the load current asym-metry ($\boldsymbol{i}_u$). Thus, these are Physical Components of three-phase LTI loads current.

All these three components are mutually orthogonal. The active and reactive are orthogonal because they are shifted mutually by $\pi/2$. The unbalanced current is orthogonal to the remaining ones because (Appendix D) they are of the oppo-site sequence. Consequently, the three-phase rms values of the current components of LTI loads satisfy the relationship

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{r}\|^2 + \|\boldsymbol{i}_\mathrm{u}\|^2 \,. \tag{49}$$

Multiplying (49) by the square of the load voltage rms value $\|\boldsymbol{u}\|$, the power equation

$$S^2 = P^2 + Q^2 + D_\mathrm{u}^2 \tag{50}$$

of three-phase LTI loads is obtained, with

$$P = \|\boldsymbol{u}\|\,\|\boldsymbol{i}_\mathrm{a}\| = G_\mathrm{e}\,\|\boldsymbol{u}\|^2, \quad Q = \pm\|\boldsymbol{u}\|\,\|\boldsymbol{i}_\mathrm{r}\| = -B_\mathrm{e}\,\|\boldsymbol{u}\|^2 \tag{51}$$

and

$$D_\mathrm{u} = \|\boldsymbol{u}\|\,\|\boldsymbol{i}_\mathrm{u}\| = Y_\mathrm{u}\,\|\boldsymbol{u}\|^2 \tag{52}$$

which is a new power quantity called in [24] an ***unbalanced power***.

Three-phase systems with LTI loads and nonsinusoidal supply voltage

The current decomposition presented in the previous section was expanded for similar systems as previously, but with nonsinusoidal supply voltage in [24]. When this voltage is symmetrical, then particular voltage harmonics are also symmetrical. Depending on the harmonic order, they have positive, negative, or zero sequence symmetry. Since the load current in three-wire systems cannot contain harmonics of the zero sequence, the load is affected only by the voltage harmonics of the positive and of the negative sequence. It means that the voltage harmonics of the zero sequence should be eliminated from all quantities that describe the power properties of three-wire systems. It can be done by referencing the load voltage not to a ground, but to an artifi-cial zero. Such a voltage can be presented in the form

$$\boldsymbol{u} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}\begin{bmatrix}\boldsymbol{U}_{\mathrm{R}n}\\ \boldsymbol{U}_{\mathrm{S}n}\\ \boldsymbol{U}_{\mathrm{T}n}\end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}\boldsymbol{U}_n\,e^{jn\omega_1 t} \tag{53}$$

where N is a set of harmonic orders of the positive and nega-tive sequence harmonics. The active power of the load is

$$P = \frac{1}{T}\int_0^T \boldsymbol{u}(t)\bullet\boldsymbol{i}(t)\,dt = (\boldsymbol{u},\boldsymbol{i}) = \mathrm{Re}\sum_{n\in N}\boldsymbol{U}_n\bullet\boldsymbol{I}_n^* \,. \tag{54}$$

With respect to the active power P, the load is equivalent to a balanced resistive load at the same voltage $\boldsymbol{u}$, configu-red in star, referred to as the **equivalent conductance** of three-phase loads, if this conductance is equal to

$$G_\mathrm{e} = \frac{P}{\|\boldsymbol{u}\|^2}. \tag{55}$$

This is because the active power of such a load is

$$P = G_\mathrm{e}\,(\|u_\mathrm{R}\|^2 + \|u_\mathrm{S}\|^2 + \|u_\mathrm{T}\|^2) = G_\mathrm{e}\,\|\boldsymbol{u}\|^2 \,. \tag{56}$$

Such an equivalent resistive load draws the current

$$\boldsymbol{i}_\mathrm{a}(t) = G_\mathrm{e}\,\boldsymbol{u}(t) \tag{57}$$

which is referred to as the active current of the load. Its physi-cal interpretation is the same as discussed previously.

Since the load is linear, its current can be calculated using the Superposition Principle, i.e., harmonic-by-harmo-nic, and for an individual harmonic the current decompose-tion, obtained in the previous Section, remains valid. Thus, a load current harmonic of the n^th order can be decomposed into

$$\boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n} \tag{58}$$

with

$$\boldsymbol{i}_{\mathrm{a}n}(t) = G_{\mathrm{e}n}\,\boldsymbol{u}_n(t) = \frac{P_n}{\|\boldsymbol{u}_n\|^2}\,\boldsymbol{u}_n(t) \,. \tag{59}$$

The current harmonic components in (58), depending on the order n, can be of the positive or of the negative sequence. To express them in a compact analytical form, a unite three-phase vector for the n^th order harmonic can be defined, namely

$$\mathbf{1}_n = \begin{bmatrix}1\\ 1e^{-jn\frac{2\pi}{3}}\\ 1e^{jn\frac{2\pi}{3}}\end{bmatrix} = \begin{bmatrix}1\\ \beta_n\\ \beta_n^*\end{bmatrix} = \begin{cases}\mathbf{1}^\mathrm{p}, \text{ for } n=3k-2\\ \mathbf{1}^\mathrm{n}, \text{ for } n=3k-1,\ k=1,2...\\ \mathbf{1}^\mathrm{z}, \text{ for } n=3k\end{cases} \tag{60}$$

where

$$\beta_n = 1e^{-jn\,2\pi/3} = (\alpha^*)^n \,. \tag{61}$$

With such a vector

$$\boldsymbol{i}_{\mathrm{r}n} = \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{e}n}\,\mathbf{1}_n\boldsymbol{U}_{\mathrm{R}n}\,e^{jn\omega_1 t}\} \tag{62}$$

$$\boldsymbol{i}_{\mathrm{u}n} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{Y}_{\mathrm{u}n}\,\mathbf{1}_n^*\boldsymbol{U}_{\mathrm{R}n}\,e^{jn\omega_1 t}\} \,. \tag{63}$$

In these formulas, the $B_{\mathrm{e}n}$ is a load equivalent susceptance for the n^th order harmonic. It is an imaginary part of the load equivalent admittance for that harmonic

$$\boldsymbol{Y}_{\mathrm{e}n} = G_{\mathrm{e}n} + jB_{\mathrm{e}n} = \boldsymbol{Y}_{\mathrm{RS}n} + \boldsymbol{Y}_{\mathrm{ST}n} + \boldsymbol{Y}_{\mathrm{TR}n} \,. \tag{64}$$

If we return to a sinusoidal situation, discussed in the previous Section, it will occur that the equivalent admittance is independent on the harmonic sequence. The unbalanced admittance for that harmonic is dependent on this sequence, however, and it is equal to

$$\boldsymbol{Y}_{\mathrm{u}n} = Y_{\mathrm{u}n}\,e^{j\psi_n} = -(\boldsymbol{Y}_{\mathrm{ST}n} + \beta_n\boldsymbol{Y}_{\mathrm{TR}n} + \beta_n^*\boldsymbol{Y}_{\mathrm{RS}n}). \tag{65}$$

Having components of the load current harmonic $\boldsymbol{i}_n$, the load current can be expressed as

$$\boldsymbol{i} = \sum_{n\in N}\boldsymbol{i}_n = \sum_{n\in N}(\boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n}) = \sum_{n\in N}\boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_\mathrm{r} + \boldsymbol{i}_\mathrm{u} \,. \tag{66}$$

Because the equivalent conductance $G_{\mathrm{e}n}$ of the load can change with the harmonic orders, the sum of active currents of individual harmonic $i_{\mathrm{a}n}$ is not equal to the active current $\boldsymbol{i}_\mathrm{a}$ of the load. It differs from this current by the scattered current

$$\boldsymbol{i}_\mathrm{s} = \sum_{n\in N}\boldsymbol{i}_{\mathrm{a}n} - \boldsymbol{i}_\mathrm{a} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}(G_{\mathrm{e}n} - G_\mathrm{e})\,\mathbf{1}_n\boldsymbol{U}_{\mathrm{R}n}\,e^{jn\omega_1 t} \,. \tag{67}$$

When (58) and (66) are combined, a decomposition of the load current into the Currents' Physical Components is obtained

$$\boldsymbol{i} = \boldsymbol{i}_\mathrm{a} + \boldsymbol{i}_\mathrm{s} + \boldsymbol{i}_\mathrm{r} + \boldsymbol{i}_\mathrm{u} \tag{68}$$

where

$$\boldsymbol{i}_\mathrm{r} = \sum_{n\in N}\boldsymbol{i}_{\mathrm{r}n} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}jB_{\mathrm{e}n}\,\mathbf{1}_n\boldsymbol{U}_{\mathrm{R}n}\,e^{jn\omega_1 t} \tag{69}$$

is the load reactive current, and

$$\boldsymbol{i}_\mathrm{u} = \sum_{n\in N}\boldsymbol{i}_{\mathrm{u}n} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}\boldsymbol{Y}_{\mathrm{u}n}\,\mathbf{1}_n^*\boldsymbol{U}_{\mathrm{R}n}\,e^{jn\omega_1 t} \tag{70}$$

is the load unbalanced current.

The current components in (58), as it was proven in the previous Section, are mutually orthogonal. Harmonics of different order are also orthogonal, hence, all components on the right side of (66) are mutually orthogonal. Orthogonality of the active and scattered currents in three-phase systems was proven in [24]. Thus, all components of the load current in (68) are mutually orthogonal, so that their three-phase rms values satisfy the relationship

(71) $$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\rm a}||^2 + ||\boldsymbol{i}_{\rm s}||^2 + ||\boldsymbol{i}_{\rm r}||^2 + ||\boldsymbol{i}_{\rm u}||^2 .$$

Three-phase systems with HGL at nonsinusoidal voltage

The voltage and current distortion in three-phase industrial systems can occur not only because of the distribution voltage distortion but also because of the presence of the harmonics generating loads (HGLs). The main idea of the load current decomposition in such a system into the Physical Components is identical to these applied to single-phase circuits with HGL, developed in [26]. It requires that the sign of the active power of individual harmonics

(72) $$P_n = U_{{\rm R}n} I_{{\rm R}n} \cos\varphi_{{\rm R}n} + U_{{\rm S}n} I_{{\rm S}n} \cos\varphi_{{\rm S}n} + U_{{\rm T}n} I_{{\rm T}n} \cos\varphi_{{\rm T}n}$$

is identified. At a symmetrical voltage, it is enough to identify the sign of the term

(73) $$I_{n{\rm a}} = I_{{\rm R}n} \cos\varphi_{{\rm R}n} + I_{{\rm S}n} \cos\varphi_{{\rm S}n} + I_{{\rm T}n} \cos\varphi_{{\rm T}n} .$$

This sign is used next for decomposition, according to (23), the set of all harmonic orders N into sets $N_{\rm C}$ and $N_{\rm G}$. Having these sets, the vector of the load current can be decomposed into the Physical Components as follows

(74) $$\boldsymbol{i} = \boldsymbol{i}_{\rm Ca} + \boldsymbol{i}_{\rm Cs} + \boldsymbol{i}_{\rm Cr} + \boldsymbol{i}_{\rm Cu} + \boldsymbol{i}_{\rm G}$$

with

(75) $$\boldsymbol{i}_{\rm Ca} = \frac{P_{\rm C}}{||\boldsymbol{u}_{\rm C}||^2} \boldsymbol{u}_{\rm C}, \qquad \boldsymbol{i}_{\rm G} = \sum_{n\in N_{\rm G}} \boldsymbol{i}_n$$

while the remaining components are defined according to (67), (69), (70), only the set N has to be replaced by $N_{\rm C}$.

Three-phase systems with neutral

Loads in three-phase systems with a neutral conductor are composed usually of balanced three-phase devices and aggregates of single-phase loads as shown in Fig. 8. The LTI
load current decomposition into CPC was developed in [49]. A draft of this decomposition is presented below.

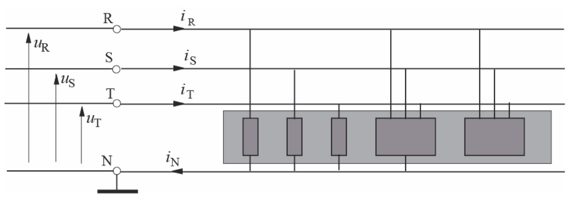

Fig. 8. A load supplied by a three-phase, four-wire line.

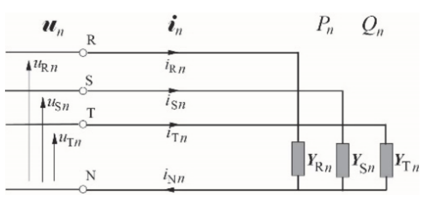

Fig. 9. An equivalent load for the $n^{\rm th}$ order harmonic.

When the load is LTI, then, the Superposition Principle can be applied to its analysis, i.e., it can be analyzed by a harmo-nic-by-harmonic approach.

The load in Fig. 8 has for each individual harmonic of the $n^{\rm th}$ order an equivalent circuit shown in Fig. 9, with the line-to-neutral admittances equal to

(76) $$\boldsymbol{Y}_{{\rm R}n} = \frac{\boldsymbol{I}_{{\rm R}n}}{\boldsymbol{U}_{{\rm R}n}}, \quad \boldsymbol{Y}_{{\rm S}n} = \frac{\boldsymbol{I}_{{\rm S}n}}{\boldsymbol{U}_{{\rm S}n}}, \quad \boldsymbol{Y}_{{\rm T}n} = \frac{\boldsymbol{I}_{{\rm T}n}}{\boldsymbol{U}_{{\rm T}n}}.$$

The load current harmonic of the $n^{\rm th}$ order is composed of an active current

(77) $$\boldsymbol{i}_{{\rm a}n}(t) = G_{{\rm e}n}\, \boldsymbol{u}_n(t) = \sqrt{2}\,{\rm Re}\{G_{{\rm e}n} \boldsymbol{U}_n e^{jn\omega_1 t}\}$$

and the reactive current

(78) $$\boldsymbol{i}_{{\rm r}n}(t) = \sqrt{2}\,{\rm Re}\{ jB_{{\rm e}n} \boldsymbol{U}_n e^{jn\omega_1 t}\}$$

where G_{en} and B_{en} are the real and imaginary parts of the load equivalent admittance for the $n^{\rm th}$ order harmonic

(79) $$\boldsymbol{Y}_{{\rm e}n} = G_{{\rm e}n} + jB_{{\rm e}n} = \frac{1}{3}(\boldsymbol{Y}_{{\rm R}n} + \boldsymbol{Y}_{{\rm S}n} + \boldsymbol{Y}_{{\rm T}n}) .$$

When the load is unbalanced, then the $n^{\rm th}$ order current harmonic contains moreover an unbalanced current

(80) $$\boldsymbol{i}_{{\rm u}n} = \boldsymbol{i}_n - \boldsymbol{i}_{{\rm a}n} - \boldsymbol{i}_{{\rm r}n} .$$

The unbalanced component of the load current harmonic of the $n^{\rm th}$ order can be asymmetrical so that it can be decom-posed into symmetrical components of the positive, negative and the zero sequences. It was found in [49] that these components of the unbalanced current can be specified in terms of three unbalanced admittances

(81) $$\boldsymbol{Y}^{\rm p}_{{\rm u}n} = \frac{1}{3}[(\boldsymbol{Y}_{{\rm R}n} + \alpha\beta\boldsymbol{Y}_{{\rm S}n} + \alpha^*\beta^*\boldsymbol{Y}_{{\rm T}n}) - \boldsymbol{Y}_{{\rm e}n}(1 + \alpha\beta + \alpha^*\beta^*)]$$

(82) $$\boldsymbol{Y}^{\rm n}_{{\rm u}n} = \frac{1}{3}[(\boldsymbol{Y}_{{\rm R}n} + \alpha^*\beta\boldsymbol{Y}_{{\rm S}n} + \alpha\beta^*\boldsymbol{Y}_{{\rm T}n}) - \boldsymbol{Y}_{{\rm e}n}(1 + \alpha^*\beta + \alpha\beta^*)]$$

(83) $$\boldsymbol{Y}^{\rm z}_{{\rm u}n} = \frac{1}{3}[(\boldsymbol{Y}_{{\rm R}n} + \beta\boldsymbol{Y}_{{\rm S}n} + \beta^*\boldsymbol{Y}_{{\rm T}n}) - \boldsymbol{Y}_{{\rm e}n}(1 + \beta + \beta^*)]$$

such that

(84) $$\boldsymbol{i}_{{\rm u}n} = \sqrt{2}{\rm Re}\{(\boldsymbol{Y}^{\rm p}_{{\rm u}n}\mathbf{1}^{\rm p} + \boldsymbol{Y}^{\rm n}_{{\rm u}n}\mathbf{1}^{\rm n} + \boldsymbol{Y}^{\rm z}_{{\rm u}n}\mathbf{1}^{\rm z})\boldsymbol{U}_{{\rm R}n} e^{jn\omega_1 t}\} .$$

The admittances (81) can have a non-zero value only for harmonics of the negative or the zero order, but it is zero for the positive sequence harmonics. Thus, the unbalanced current of the positive sequence in (84) can occur only if the supply voltage has harmonics of the negative or the zero sequence.

The admittances (82) can have a non-zero value only for harmonics of the positive or the zero order, but it is zero for the negative sequence harmonics. Thus, the unbalanced current of the negative sequence in (84) can occur only if the supply voltage has harmonics of the positive or the zero sequence.

The admittances (83) can have a non-zero value only for harmonics of the positive or the negative order, but it is zero for the zero sequence harmonics. Thus, the unbalanced current of the zero sequence in (84) can occur only if the supply voltage has harmonics of the positive or the negative sequence.

The equivalent conductance of the load for harmonic fre-quencies G_{en} can differ from the equivalent conductance $G_{\rm e}$, so that the load current can also contain a scattered current, as defined by (67). Thus, the load current in a three-phase system with a neutral conductor can be decomposed exactly as in the systems without the neutral conductor, i.e., as follows

(85) $$\boldsymbol{i} = \boldsymbol{i}_{\rm a} + \boldsymbol{i}_{\rm s} + \boldsymbol{i}_{\rm r} + \boldsymbol{i}_{\rm u}$$

with the unbalanced current

$$(86)\quad \boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\mathrm{Re}\sum_{n\in N}(\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}\mathbf{1}^{\mathrm{p}} + \boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n}\mathbf{1}^{\mathrm{n}} + \boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}\mathbf{1}^{\mathrm{z}})\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}$$

which in four-wire systems has a more complex form as com-pared to this current in three-wire systems.

When a system with a neutral conductor supplies not only LTI loads but also HGLs, then similarly as in three-wire systems, the load current contains moreover, as it was shown previously, a load generated harmonic current $\boldsymbol{i}_{\mathrm{G}}$. It means, that the load current decomposition into CPC (74), developed for three-wire systems, and repeated once again

$$(87)\quad \boldsymbol{i} = \boldsymbol{i}_{\mathrm{Ca}} + \boldsymbol{i}_{\mathrm{Cs}} + \boldsymbol{i}_{\mathrm{Cr}} + \boldsymbol{i}_{\mathrm{Cu}} + \boldsymbol{i}_{\mathrm{G}}$$

is valid also for three-phase systems with a neutral conductor and a nonsinusoidal, but symmetrical supply voltage. Thus, there are only five physical phenomena in three-phase sys-tems, both three-, and four-wire, with linear and/or harmonic generating loads, which affect the load current and its three-phase rms value, since it satisfies the relationship:

$$(88)\quad ||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{Ca}}||^2 + ||\boldsymbol{i}_{\mathrm{Cs}}||^2 + ||\boldsymbol{i}_{\mathrm{Cr}}||^2 + ||\boldsymbol{i}_{\mathrm{Cu}}||^2 + ||\boldsymbol{i}_{\mathrm{G}}||^2 .$$

Summary

This Review demonstrates that five and only five physical phenomena determine currents of three-phase loads sup-plied by a nonsinusoidal voltage. These are:

1. A permanent flow of the energy from the supply source to the load,
2. A permanent flow of the energy from the load to the supply source,
3. A phase-shift of the load current harmonics with respect to the supply voltage harmonics,
4. A change of the load conductance with the harmonic order,
5. An asymmetry of the line currents, caused by the load imbalance.

We should be aware, however, that the concept of ***"a physi-cal phenomenon"*** is not completely clear. What is and what is not a physical phenomenon can be a matter of a subjective judgement, [53], [55]. In spite of their name, the Currents' Physical Components do not exist as physical, but only as mathematical entities. They are only an effect of a mathema-tical decomposition of the load current. Nonetheless, these Components are ***associated*** with distinctive physical pheno-mena that affect energy transfer in electrical systems. Therefore, this decomposition provides fundamentals for interpreting the energy transfer in terms of physical pheno-mena in electrical systems.

The development of the CPC is not jet completed. This Review describes the power properties of electrical systems at the assumption that the supply voltage is symmetrical. These properties at asymmetrical voltage were described in [48], [52] only for three-wire, but not for four-wire systems.

This decomposition is well suited to the present measure-ment technology, based on digital signals processing (DSP) so that all complex rms values needed for decomposition are easily available by measurements at the load terminals.

The CPC can be specified in terms of the circuit para-meters therefore, the CPC approach creates fundamentals for reactive compensator design and for control of switching compensators, known mainly as the active power filters.

Appendix A

The rms value of the sum of periodic quantities $x(t)$ and $y(t)$ of the same period T, is equal to

$$(\mathrm{A.1})\quad ||z|| = ||x+y|| = \sqrt{\frac{1}{T}\int_0^T [x(t)+y(t)]^2 dt} = \sqrt{||x||^2 + 2\,(x,y) + ||y||^2} .$$

Their rms values satisfy the relationship

$$(\mathrm{A.2})\quad ||z||^2 = ||x||^2 + ||y||^2$$

on the condition that their scalar product

$$(\mathrm{A.3})\quad (x,y) = \frac{1}{T}\int_0^T x(t)\,y(t)\,dt$$

is zero. Quantities with zero scalar product are referred to as mutually ***orthogonal***. The orthogonality of some quantities can be concluded without calculation. Orthogonal are sinu-soidal quantities shifted mutually by $\pi/2$; harmonics of differ-rent order n, and quantities related mutually by integration.

When quantities $x(t)$ and $y(t)$ are specified in terms of the crms values $\boldsymbol{X}_n$ and $\boldsymbol{Y}_n$, then their scalar product is equal to

$$(\mathrm{A.4})\quad (x,y) = \frac{1}{T}\int_0^T x(t)\,y(t)\,dt = \mathrm{Re}\sum_{n=0}^{\infty}\boldsymbol{X}_n\boldsymbol{Y}_n^{*} .$$

Appendix B

A scalar product of three-phase vectors $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ is defined as

$$(\mathrm{B.1})\quad (\boldsymbol{x},\boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}(t)\bullet\boldsymbol{y}(t)\,dt .$$

When the vectors $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ are specified in terms of the three-phase vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$ of the crms values of harmonics, then their scalar product can be calculated as

$$(\mathrm{B.2})\quad (\boldsymbol{x},\boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}(t)\bullet\boldsymbol{y}(t)\,dt = \mathrm{Re}\sum_{n=0}^{\infty}\boldsymbol{X}_n\bullet\boldsymbol{Y}_n^{*} .$$

Appendix C

Let us consider a three-phase, purely resistive transmis-sion device, shown in Fig. C1(a), with the current specified by the vector $\boldsymbol{i}$.

The energy dissipated by line currents in this device in one period T is equal to

$$(\mathrm{C.1})\quad W = \int_0^T (R_{\mathrm{R}}i_{\mathrm{R}}^2 + R_{\mathrm{S}}i_{\mathrm{S}}^2 + R_{\mathrm{T}}i_{\mathrm{T}}^2)\,dt = (R_{\mathrm{R}}||i_{\mathrm{R}}||^2 + R_{\mathrm{S}}||i_{\mathrm{S}}||^2 + R_{\mathrm{T}}||i_{\mathrm{T}}||^2)T.$$

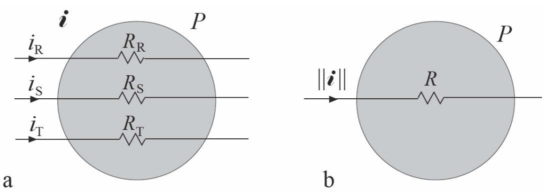

Fig. C1. A physical interpretation of three-phase current rms value.

Such devices are built in such a way that their symmetry is as high as possible, meaning that $R_{\mathrm{R}} = R_{\mathrm{S}} = R_{\mathrm{T}} = R$, thus, the active power of the device is

$$(\mathrm{C.2})\quad P = \frac{W}{T} = R\,(\,||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2\,) = R\,||\boldsymbol{i}||^2$$

where the quantity

$$(\mathrm{C.3})\quad ||\boldsymbol{i}|| = \sqrt{||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2}$$

is referred to as a ***three-phase rms value*** of the three-phase current. It is equal to a dc current of a single-phase device, shown in Fig. C1(b), with resistance R, which is equivalent as to energy loss caused by the current vector ***i*** in a three-phase symmetrical device, shown in Fig. 10(a), of the same line resistance R.

A three-phase rms value of a symmetrical nonsinusoidal quantity

(C.4) $$\boldsymbol{x} = \sum_{n\in N} \boldsymbol{x}_n = \sqrt{2}\mathrm{Re}\sum_{n\in N} \boldsymbol{X}_n e^{jn\omega_1 t}$$

has a three-phase rms value

(C.5) $$\|\boldsymbol{x}\| = \sqrt{(\boldsymbol{x}, \boldsymbol{x})} = \sqrt{\mathrm{Re}\sum_{n\in N} \boldsymbol{X}_n \bullet \boldsymbol{X}_n^*} = \sqrt{\sum_{n\in N} \|\boldsymbol{x}_n\|^2}\ .$$

Appendix D

Let a vector $\boldsymbol{x}(t)$ has a positive sequence, i.e.,

(D.1) $$\boldsymbol{x} = \sqrt{2}\mathrm{Re}\{\mathbf{1}^{\mathrm{p}} X_{\mathrm{R}}\} e^{j\omega t}$$

and a vector $\boldsymbol{y}(t)$ has a negative sequence, i.e.,

(D.2) $$\boldsymbol{y} = \sqrt{2}\mathrm{Re}\{\mathbf{1}^{\mathrm{n}} Y_{\mathrm{R}}\} e^{j\omega t}\ .$$

Their scalar product is zero, since

$$(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{Re}\{\boldsymbol{X}\bullet\boldsymbol{Y}^*\} = \mathrm{Re}\{\mathbf{1}^{\mathrm{p}} X_{\mathrm{R}}\bullet(\mathbf{1}^{\mathrm{n}} Y_{\mathrm{R}})^*\} =$$

(D.3) $$= \mathrm{Re}\{(1+\alpha+\alpha^*) X_{\mathrm{R}} Y_{\mathrm{R}}^*\} = 0.$$

Thus, such vectors are mutually orthogonal.

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