PHYSICAL FUNDAMETALS OF THE POWER THEORY OF ELECTRICAL SYSTEMS

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Dostawa enegii od jej producentów do odbiorców jest opisywana za pomocą różnych mocy elektrycznych, definiowanych w ramach teorii mocy obwodów elektrycznych. Niestety, istnieje obecnie kilka takich teorii. Wszystkie one są matematycznie poprawne, mogą się jednak zasadniczo różnić co do interpretacji zjawisk towarzyszących przesyłowi energii oraz ich wpływowi na skuteczność tego przesyłu. Zjawiska te i ich interpretacje są właśnie przedmiotem tego artykułu. Wyjaśnienie i poprawna interpretacja zjawisk fizycznych związanych z przesyłem energii elektrycznej jest krytycznie ważna dla poprawnego definiowania mocy elektrycznych i metod kompensacji.

Physical Fundamentals of the Power Theory of Electrical Systems

Delivery of the electric energy from its producers to customers is described in terms of powers, defined in a frame of power theories of electrical systems. Currently, there are several of such theories, however. All of them are mathematically correct but differ as to the interpretation of the physical phenomena which accompany the energy transfer and affect its effectiveness. These phenomena are discussed in this paper. Explanation and credible interpretation of the physical phenomena that accompany the energy transfer is crucially important for the right definition of the electrical powers and methods of compensation.

1. INTRODUCTION

Three principal powers used to describe power properties of electrical systems: the active power P, the reactive power Q, and the apparent power S. These powers were introduced into the electrical circuit power theory at the end of the 19th century. Back in 1892, Steinmetz noticed [1] that even if the reactive power were equal to zero, the apparent power S could be greater than the active power P, thus, fulfilling the following inequality:

$$S \geq P\,. \tag{1}$$

Since the energy supplier must deliver the energy to the customer with the apparent power S, while the customer consumes the energy at a rate equal to the active power P, this inequality is of fundamental importance for the power system economy. It also provokes questions as to the physical reasons for this inequality and the methods of its reduction.

Steinmetz's observations initiated research focused on finding the physical causes of this inequality. Its explanation has proved, however, to be one of the most difficult issues in the electrical circuits power theory in the 20th century. Hundreds of scientists worked on this problem. Hundreds of papers and lots of doctoral theses were devoted to it. Lots of theories explaining this inequality were propounded. Among the most recognized are the power theories suggested by Budeanu (1927) [3], Fryze (1931) [4, 13], Shepherd and Zakikhani (1972) [5], Kusters and Moore (1980) [6], Nabae and Akagi (1984) [9], Depenbrock (1993) [12], and Tenti (2003) [15]. Some of these theories were even adopted as international standards; they were published in papers as well as in textbooks on electrical circuits. "Schools" of scientists centered around a particular theory flourished; within these schools, power effects in circuits or in power systems were described in a similar manner. Some power definitions were supported by decisions of scientific/technical committees of international standing. This relates e.g. to the definition of the apparent power S in three-phase systems. In 1920, the American Institute of Electrical Engineers (AIEE) was not able to decide which definition of this power, arithmetic or geometric, was correct [2] and introduced both definitions into the Standard Dictionary of Electrical and Electronics Terms [14]. In the same dictionary of standard terms, we can find definitions of the reactive power Q and the distortion power D given in accordance with the Budeanu's power theory. Budeanu's powers, as well as completely different powers defined by Fryze, are supported by German standard DIN 40100. Power definitions suggested by Kusters and Moore, as well as a compensation method they developed, were supported by the International Electrotechnical Commission (IEC). The Institute of Electrical and Electronics Engineers (IEEE) established even a special Committee for Definition of the Reactive Power in Nonsinusoidal Systems, which created a document devoted to power definitions, known as Standard 1459.

Mathematically, all these power theories are correct. The slightest mathematical error in the power theory would, of course, eliminate such a theory from further discussion. In spite of this correctness, no power theory explains why the apparent power S can be greater than the load's active power P and what physical phenomena are responsible for this inequality. So, after almost a century of research, no known power theory can explain Steinmetz's observation (1).

This state of things in power theory development did not advance progress in the methods of compensating circuits with a nonsinusoidal supply voltage. First results on the reactive compensation were obtained by Shepherd and Zakikhani [5] in 1970 and by Kusters and Moore [6] in 1980; this was, however, solely capacitative compensation. A method for calculating the compensating capacitance at nonsinusoidal supply voltage worked out by Kusters and Moore was recommended by the IEC. Still, it turned out [8] that under real supply conditions, i.e. when the supply source had an inductive impedance, the method did provide the right results. The Instantaneous Reactive Power p-q Theory, elaborated by Nabae and Akagi [9], makes it possible to control a switching compensator, referred usually to as an "active power filter", on the condition, however, that the supply voltage is sinusoidal [17] and symmetrical [18].

The entire scientific effort and research focused on explaining power phenomena in electrical circuits can be summarized in a negative way by the fact that since 1983 no power theory was able to explain power phenomena in such a simple circuit, as that shown in Fig. 1, containing only a series RL load and a supply source of a nonsinusoidal voltage. Moreover, no power theory was able to suggest a method of compensation of such a load and even to settle the issue, whether compensation of such a load to the unity power factor, $\lambda = P/S$, is possible or not.

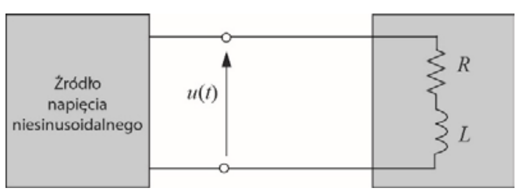

Fig. 1. A circuit with RL load and a supply voltage.

Physical phenomena in such a circuit were explained, together with the development of a compensation method, in 1983 [7, 10], within the framework of a new power theory, based on the concept of the Currents' Physical Components (CPC). This theory reveals and describes the energy transfer-related physical phenomena in single-phase and three-phase circuits with stationary, linear and non-linear loads, as well as loads with periodically variable parameters, i.e. with harmonics generating loads (HGLs). It provides fundamentals for a synthesis of the reactive compensators and for development of algorithms for the switching compensators control. Nowadays, the CPC-based power theory is the most advanced power theory of electrical circuits and systems. Therefore, it is worthwhile to list and explain the physical phenomena constituting the foundations of this theory. These are the phenomena determining the effectiveness of the energy transfer in electrical circuits and systems.

When we deliberate or discuss on the physical phenomena, the conclusions are subjective, however. Our opinions can differ as to what is and what is not a physical phenomenon and what are its characteristics. Such opinions cannot have practical merits, however. A synthesis of compensators and their control based on these conclusions and opinions can bring about some very practical results: compensator can function correctly or not. The application of these conclusions can constitute a real test of their correctness.

2. INSTANTANEOUS POWER

The power properties of electrical circuits and systems are usually described by powers. The most basic of these powers is the instantaneous power $p(t)$. It is defined as a rate of flow of the electric energy $W(t)$ from the supply source to the load. If the supply voltage of a single-phase load is equal to $u(t)$, and its current is $i(t)$, then the instantaneous power at the load terminals is defined as:

$$p(t) = \frac{d}{dt} W(t) = u(t)\, i(t) \ . \tag{2}$$

In three-phase circuits, with symbols specified in Fig. 2,

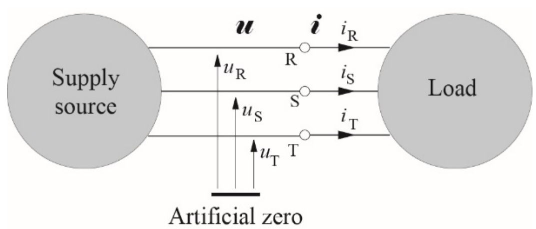

Fig. 2. A stricture of three-wire, three-phase circuit

the instantaneous power is defined as:

$$p(t) = \frac{d}{dt} W(t) = u_R i_R + u_S i_S + u_T i_T = \boldsymbol{u}^T \boldsymbol{i} \tag{3}$$

thus, it is a vector product of three-phase vectors of voltages and currents

$$\boldsymbol{u} = [u_R,\ u_S,\ u_T]^T, \qquad \boldsymbol{i} = [i_R,\ i_S,\ i_T]^T \ . \tag{4}$$

at load terminals.

Among all electrical powers, the instantaneous power, defined as a rate of the energy flow, enjoys the most obvious physical interpretation.

The active power P is defined as the instantaneous power averaged over one period T of the voltage variability, namely

$$P = \frac{1}{T} \int_0^T p(t)\, dt \tag{5}$$

It is regarded to be a power with a physical interpretation as distinct as that of the instantaneous power. We must note, however, that the active power is defined only for periodic voltages and currents, with a defined period T. Periodic waveforms only approximate real waveforms, however. It is debatable, moreover, whether any quantity averaged over a time interval, is a physical quantity or not.

Since the physical interpretation of the instantaneous power is so obvious, some researchers tend to express the view that just the instantaneous power $p(t)$ should be the principal power quantity, constituting the foundation of the power theory of electrical circuits. To question this viewpoint, we must observe that inequality (1), which should be explained by the power theory, cannot be expressed by means of the instantaneous power.

Nonetheless, the instantaneous power $p(t)$ is one of two powers on which Nabae and Akagi founded their Instantaneous Reactive Power (IRP) p-q Theory [9]. It has been re-named in that theory to "the ***instantaneous active power***". It is calculated within the IRP p-q theory, together with the second power quantity "*the **instantaneous reactive power***" q, with three-phase load voltages and currents, transformed into the orthogonal α - β coordinates with the Clarke Transform:

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} x_R \\ x_S \end{bmatrix}. \tag{6}$$

In accordance with the IRP p-q theory, the instantaneous powers of three-phase load supplied by a three-wire line are defined in the α - β coordinates as follows

$$p = u_\alpha\, i_\alpha + u_\beta\, i_\beta \tag{7}$$

$$q = u_\alpha\, i_\beta - u_\beta\, i_\alpha\,. \tag{8}$$

This theory was developed from the viewpoint of controlling switching compensators. According to the authors, a compensator should wholly compensate the instantaneous reactive power q and the alternating component $\tilde{p}$ of the instantaneous active power p. This means that the compensated load can burden the supply source with only the constant component of the instantaneous power, $\bar{p}$. However, this is an erroneous conclusion, arising from ignorance on properties of the instantaneous power $p(t)$ and leading to a faulty control of the compensator. To demonstrate this, let us discuss an ideal three-phase load, i.e. a balanced and purely resistive load, shown in Fig. 3.

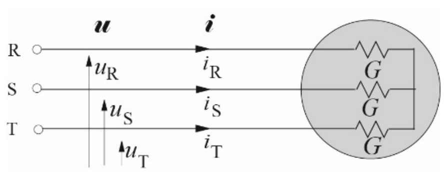

Fig. 3. An ideal three-phase resistive load.

The power factor of this load is equal, of course, to $\lambda = P/S = 1$. If this ideal load is supplied by a distorted voltage, then its power factor is not degraded. Let us examine, however, the impact of the supply voltage distortion upon the load's instantaneous power. Let us assume that the supply voltage is symmetrical and distorted by the 5$^{\text{th}}$ order harmonic, i.e.

$$\boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_5\,. \tag{9}$$

Let us also assume that the voltage harmonics of the load's line R are:

$$u_{R1} \triangleq \sqrt{2}\, U_1 \cos \omega_1 t\,, \qquad u_{R5} \triangleq \sqrt{2}\, U_5 \cos 5\omega_1 t\,. \tag{10}$$

The load's instantaneous power can be expressed as:

$$p(t) = \boldsymbol{u}^{\mathrm{T}} \boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}} G\, \boldsymbol{u} = G[\boldsymbol{u}_1 + \boldsymbol{u}_5]^{\mathrm{T}}[\boldsymbol{u}_1 + \boldsymbol{u}_5] = G(\boldsymbol{u}_1^{\mathrm{T}} \boldsymbol{u}_1 + \boldsymbol{u}_5^{\mathrm{T}} \boldsymbol{u}_5 + \boldsymbol{u}_1^{\mathrm{T}} \boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}} \boldsymbol{u}_1)\,. \tag{11}$$

The first two terms of this formula are equal to, respectively,

$$G\,\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_1 = G\|\boldsymbol{u}_1\|^2 = P_1\,, \quad G\,\boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_5 = G\|\boldsymbol{u}_5\|^2 = P_5 \tag{12}$$

where P_1 and P_5 denote the active powers of the fundamental and the 5^{th} order harmonic, while symbol $\|.\|$ denotes the three-phase rms value of these harmonics. The last two terms in formula (11) can be expressed in the form

$$G\,(\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_1) \;=\; 6GU_1U_5\cos 6\omega_1 t\,. \tag{13}$$

So, the instantaneous power of the load is equal to

$$p(t) = \frac{dW}{dt} = P_1 + P_5 + 6GU_1U_5\cos 6\omega_1 t \tag{14}$$

Thus, in spite of this fact, that the load's power factor is equal to one, and there is no need to compensate such a load, the instantaneous power contains an alternating component. The IRP – based control algorithm of the compensator requires compensation of this component, however, and ensures this compensation, but the attained result is directly opposite to the expected result: the compensator causes an increase of the supply current distortion [17] and deteriorates the power factor.

Similar result as to the instantaneous power properties is obtained when the supply voltage is asymmetrical, even if it is sinusoidal. If $\boldsymbol{U}^{\mathrm{p}}$ is a crms value of the positive sequence component of the supply voltage, $\boldsymbol{U}^{\mathrm{n}}$ is a crms value of the negative sequence component of the supply voltage, and the active powers of these components at the terminals of balanced resistive load are equal to P^{p} and P^{n}, respectively, then the instantaneous power is equal to [18]

$$p(t) = \frac{dW}{dt} = P^{\mathrm{p}} + P^{\mathrm{n}} + 6GU^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega_1 t\,. \tag{15}$$

Compensation of the oscillating component of this power in accordance with the control algorithm, based upon the IRP p-q theory, causes a distortion of supply current [18] and decrease of the power factor.

In both described cases, due to a distortion or asymmetry of the supply voltage, the instantaneous power $p(t)$, contains an alternating component, but it does not affect the load's power factor.

The power factor can be reduced by a reactive power Q and its presence is habitually explained by the energy oscillation between the supply source and the load. A question can be asked: is it a correct explanation? The question: ***does the reactive power Q result from the energy oscillations between the supply source and the load? has cognitive merit.*** It applies even to circuits and systems with sinusoidal voltages and currents.

This issue can be resolved with the help of Currents' Physical Components – based power theory applied to the linear, time-invariant three-phase loads, supplied with a symmetrical sinusoidal voltage. The supply current in such circuit

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}} \tag{16}$$

contains three Physical Components: an active current $\boldsymbol{i}_{\mathrm{a}}$, a reactive current $\boldsymbol{i}_{\mathrm{r}}$ and an unbalanced current $\boldsymbol{i}_{\mathrm{u}}$, defined as follows:

$$\boldsymbol{i}_{\mathrm{a}} = \sqrt{2}\,\mathrm{Re}\,\{G_{\mathrm{e}}\,\mathbf{1}^{\mathrm{p}}\boldsymbol{U}_{\mathrm{R}}\,e^{j\omega t}\} \tag{17}$$

$$\boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re}\,\{\,jB_{\mathrm{e}}\,\mathbf{1}^{\mathrm{p}}\boldsymbol{U}_{\mathrm{R}}\,e^{j\omega t}\} \tag{18}$$

$$\boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\,\mathrm{Re}\,\{\boldsymbol{Y}_{\mathrm{u}}\,\mathbf{1}^{\mathrm{n}}\boldsymbol{U}_{\mathrm{R}}\,e^{j\omega t}\} \tag{19}$$

where G_e is the load's equivalent conductance, B_e is the load's equivalent susceptance. Symbols $\mathbf{1}^p$, $\mathbf{1}^n$ denote three-phase unite versors of the positive and the negative sequence, respectively:

$$\mathbf{1}^p = [1, \alpha^*, \alpha]^T, \qquad \mathbf{1}^n = [1, \alpha, \alpha^*]^T \tag{20}$$

These versors are visualized in Fig. 4.

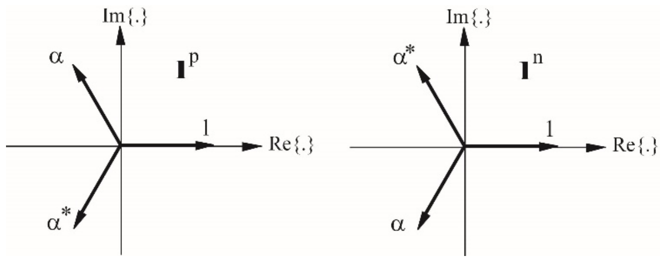

Fig. 4. Three-phase unite vectors of the positive and the negative sequence

Under these conditions, the instantaneous power of the load

$$p(t) = \boldsymbol{u}^T\boldsymbol{i} = \boldsymbol{u}^T(\boldsymbol{i}_a + \boldsymbol{i}_r + \boldsymbol{i}_u) = p_a(t) + p_r(t) + p_u(t) \tag{21}$$

can be expressed [16] as the sum of three instantaneous powers associated with the Currents' Physical Components.

The instantaneous power associated with the active current

$$p_a(t) = \boldsymbol{u}^T\boldsymbol{i}_a = \boldsymbol{u}^T G_e\,\boldsymbol{u} = G_e\,||\boldsymbol{u}||^2 = P \tag{22}$$

is constant. If we assume that the voltage at the load terminals is equal to

$$\boldsymbol{u} \overset{\text{df}}{=} \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix} = \sqrt{2}U_R \begin{bmatrix} \cos\omega t \\ \cos(\omega t - 120^0) \\ \cos(\omega t + 120^0) \end{bmatrix} \tag{23}$$

then the reactive current can be expressed as

$$\boldsymbol{i}_r = \sqrt{2}\,\text{Re}\{jB_e\mathbf{1}^p\boldsymbol{U}\,e^{j\omega t}\} = -\sqrt{2}B_eU_R \begin{bmatrix} \sin\omega t \\ \sin(\omega t - 120^0) \\ \sin(\omega t + 120^0) \end{bmatrix}. \tag{24}$$

When the vector product of these quantities is calculated, we obtain

$$p_r(t) = \boldsymbol{u}^T\boldsymbol{i}_r = 0 \tag{25}$$

In other words, the presence of reactive current and consequently, the reactive power Q, does not cause any flow of the energy between the supply source and the load. So, the question put above is answered in the negative: *the* ***reactive power Q does not result from the energy oscillations.*** The energy oscillations accompany another phenomenon in this circuit, namely, the load current asymmetry, caused by the load imbalance, i.e. the presence of an unbalanced current. If we assume that the voltage at the load terminals is expressed by (23), then, the unbalanced current can be expressed as:

$$\boldsymbol{i}_u = \sqrt{2}\,\text{Re}\{\boldsymbol{Y}_u\mathbf{1}^n\boldsymbol{U}\,e^{j\omega t}\} = \sqrt{2}Y_uU_R \begin{bmatrix} \cos(\omega t + \Psi) \\ \cos(\omega t + \Psi + 120^0) \\ \cos(\omega t + \Psi - 120^0) \end{bmatrix} \tag{26}$$

where Ψ is the phase of the unbalance admittance $\boldsymbol{Y}_u$. The instantaneous power associated with the unbalanced current can be expressed as

$$p_u(t) = \boldsymbol{u}^T \boldsymbol{i}_u = 3Y_u U_R^2 \cos(2\omega t + \Psi) \tag{27}$$

meaning, it causes the energy oscillation between the supply source and the load.

3. POWER BALANCE

The reactive power Q in circuits with a nonsinusoidal supply voltage is not defined in any other way than as a product of three-phase rms values of the supply voltage and the load's reactive current, namely

$$Q = ||\boldsymbol{u}||\,||\boldsymbol{i}_r|| \tag{28}$$

so that it is defined in a manner similar to that of the apparent power S

$$S = ||\boldsymbol{u}||\,||\boldsymbol{i}|| \tag{29}$$

and similarly, it does not hold any physical interpretation.

If the load voltages and currents are sinusoidal, or at least, if they can be approximated by sine waveforms, which is common in the power systems analysis, then the reactive power Q in single-phase systems can be defined by a formula similar to the one which defines the active power P, namely

$$Q = \frac{1}{T}\int_0^T u(t)\, i(t - \frac{T}{4})\, dt\,. \tag{30}$$

This formula suggests a physical interpretation of the reactive power Q similar to the interpretation of active power P. In a case of the active power, however, it is the averaged instantaneous power $p(t)$, while there is no phenomenon in the circuit that could be calculated by multiplying the voltage at one time instant t and the current at another time instant, $t - 4T/4$.

This opinion that the reactive power Q is, after all, a physical quantity is sometimes supported by an important property of this power; namely, if the load voltages and currents are sinusoidal, then the reactive power fulfills the power balance principle, in the same way as the active power P. The balance principle of the active power P results from one of the fundamental laws of physics: the law of the energy conservation (LEC). The balance principle of the reactive power Q cannot be derived from the LEC, however. It can be deduced from the Tellegen's theorem. This theorem is an outcome of Kirchhoff's laws and yet quite often it is not taught in university electrical circuit courses. The theorem can be formulated as follows: let us consider two different networks with identical topologies, i.e. with an identical number of nodes connected in the identical way by K branches, as shown in Fig. 5. The corresponding branches of both circuits can have entirely different structures, voltages and currents.

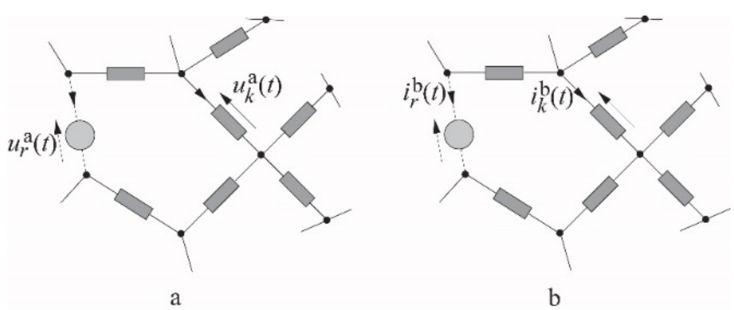

Fig. 5. Two circuits with identical topologies

The Tellegen's theorem states that the sum of all K products of the branch voltages of the first circuit and the branch currents of the second circuit, at each instant of time, is equal to zero, namely

$$\sum_{k=1}^{K} u_k^{\mathrm{a}}(t)\, i_k^{\mathrm{b}}(t) \equiv 0\,. \tag{31}$$

The law of the reactive power Q conservation can be derived from the Tellegen's theorem in the following way. If two circuits are identical, but the source voltages in circuit 5.b are shifted with respect to the source voltages in circuit 5.a, by $T/4$, then

$$i_k^{\mathrm{b}}(t) \equiv i_k^{\mathrm{a}}(t-\frac{T}{4}) \tag{32}$$

Averaging the sum of these products over the period T, results in:

$$\frac{1}{T}\int_0^T \sum_{k=1}^{K} u_k^{\mathrm{a}}(t)\, i_k^{\mathrm{b}}(t)\, dt = \sum_{k=1}^{K} \frac{1}{T}\int_0^T u_k^{\mathrm{a}}(t)\, i_k^{\mathrm{a}}(t-\tfrac{T}{4})\, dt = \sum_{k=1}^{K} Q_k^{\mathrm{a}} = 0\,. \tag{33}$$

i.e. we achieve the balance principle for the reactive power Q. The Tellegen's theorem does not describe, however, any physical phenomenon, since the currents and the voltages in (31) belong to different circuits. Fulfilling the balance principle by the reactive power Q cannot, therefore, endorse the argument in support of the physical meaning of this power.

4. PHYSICAL PHENOMENA DETERMINING EFFECTIVENESS OF ENERGY TRANSFER IN ELECTRICAL CIRCUITS

The discussion presented above shows that time variability of the load's instantaneous power $p(t)$ does not make it possible to explain the physical phenomena accountable for the effectiveness of the energy transfer in electrical circuits. All physical phenomena responsible for this effectiveness are disclosed by the power theory based on the concept of the Currents' Physical Components. These phenomena can appear in three-phase circuits of the structure shown in Fig. 6, with a nonsinusoidal supply voltage and nonlinear loads or loads with periodically variable parameters.

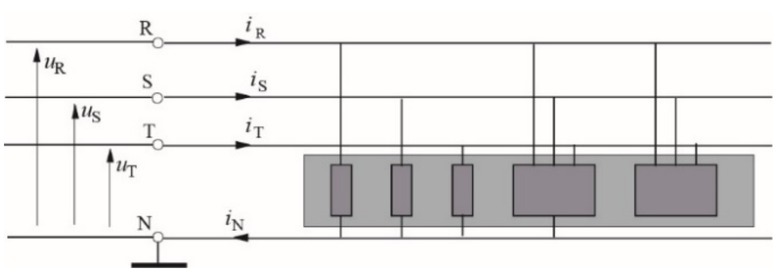

Fig. 6. Three-phase loads supplied by four-wire line.

To simplify the load current decomposition, it has been assumed that the supply voltage, albeit non-sinusoidal, is still symmetrical. Let us assume that N is the set of n orders of the voltage and current harmonics. The crms values of these harmonics, $\boldsymbol{U}_n$ and $\boldsymbol{I}_n$, can be measured with an acceptable accuracy at the load terminals. If so, then the load's voltages and currents can be expressed as:

$$\boldsymbol{u} = \sum_{n\in N} \boldsymbol{u}_n = \sqrt{2}\mathrm{Re}\sum_{n\in N} \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\mathrm{Re}\sum_{n\in N} U_n e^{jn\omega_1 t} \tag{34}$$

$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \sqrt{2}\mathrm{Re}\sum_{n\in N} \begin{bmatrix} \boldsymbol{I}_{\mathrm{R}n} \\ \boldsymbol{I}_{\mathrm{S}n} \\ \boldsymbol{I}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_n e^{jn\omega_1 t}. \tag{35}$$

For each voltage harmonic of n^{th} order from the set N we can calculate parameters of a circuit, shown in Fig. 7, equivalent to the original one.

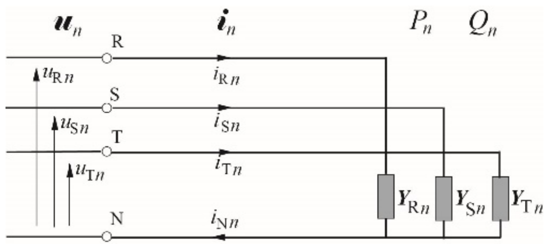

Fig. 7. A load equivalent to original load for n_{th} order harmonic

where

$$\boldsymbol{Y}_{\mathrm{R}n} = \frac{\boldsymbol{I}_{\mathrm{R}n}}{\boldsymbol{U}_{\mathrm{R}n}}, \quad \boldsymbol{Y}_{\mathrm{S}n} = \frac{\boldsymbol{I}_{\mathrm{S}n}}{\boldsymbol{U}_{\mathrm{S}n}}, \quad \boldsymbol{Y}_{\mathrm{T}n} = \frac{\boldsymbol{I}_{\mathrm{T}n}}{\boldsymbol{U}_{\mathrm{T}n}}. \tag{36}$$

The energy in circuits with nonsinusoidal voltages and currents is transferred by individual harmonics with active power equal to

$$P_n = \mathrm{Re}\{\boldsymbol{Y}_{\mathrm{R}n} + \boldsymbol{Y}_{\mathrm{S}n} + \boldsymbol{Y}_{\mathrm{T}n}\}\,||\boldsymbol{u}_n||^2\,. \tag{37}$$

Nonlinear loads and loads with periodically, with a period T, varying parameters, meaning the loads that generate current harmonics, can cause [11] flow of the energy in the direction opposite to the standard one, i.e. from the load back to the supply source. This is a physical phenomenon, which can be identified by the sign of the harmonic active power P_n. The harmonics can, therefore, be classified on the basis of this sign, into harmonics transferring the energy to the load and harmonics transferring the energy to the supply source. This means that the set N can be decomposed into two subsets, denoted by symbols N_{C} and N_{G}:

$$\begin{cases} \text{when} \quad P_n \geq 0, \quad \text{then} \quad n \in N_{\mathrm{C}} \\ \text{when} \quad P_n < 0, \quad \text{then} \quad n \in N_{\mathrm{G}}\,. \end{cases} \tag{38}$$

When subsets N_{C} and N_{G} are known, then the load's currents, voltages, and the active power can be decomposed into two components:

$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \sum_{n\in N_{\mathrm{C}}} \boldsymbol{i}_n + \sum_{n\in N_{\mathrm{G}}} \boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{C}} + \boldsymbol{i}_{\mathrm{G}} \tag{39}$$

$$\boldsymbol{u} = \sum_{n\in N} \boldsymbol{u}_n = \sum_{n\in N_{\mathrm{C}}} \boldsymbol{u}_n + \sum_{n\in N_{\mathrm{G}}} \boldsymbol{u}_n = \boldsymbol{u}_{\mathrm{C}} - \boldsymbol{u}_{\mathrm{G}} \tag{40}$$

$$P = \sum_{n\in N} P_n = \sum_{n\in N_{\mathrm{C}}} P_n + \sum_{n\in N_{\mathrm{G}}} P_n = P_{\mathrm{C}} - P_{\mathrm{G}}\,. \tag{41}$$

The load current can be decomposed into five components:

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{Ca}} + \boldsymbol{i}_{\mathrm{Cs}} + \boldsymbol{i}_{\mathrm{Cr}} + \boldsymbol{i}_{\mathrm{Cu}} + \boldsymbol{i}_{\mathrm{G}} \tag{42}$$

Each of these components is associated with a different physical phenomenon. The current

$$\boldsymbol{i}_{\rm G} = \sum_{n\in N_{\rm G}} \boldsymbol{i}_n \tag{43}$$

is called the load's ***generated current. It can occur*** due to a generation of the current harmonics by the load nonlinearity and/or periodic variability of its parameters. This current is associated with a permanent transfer of the energy from the load to the supply source. The current

$$\boldsymbol{i}_{\rm Ca} = \frac{P_{\rm C}}{||\boldsymbol{u}_{\rm C}||^2}\,\boldsymbol{u}_{\rm C} = G_{\rm Ce}\,\boldsymbol{u}_{\rm C} \tag{44}$$

is associated with the phenomenon of a permanent transfer of the energy from the supply source to the load. This is the ***active current.***

For each n^{th} order harmonic from the set $N_{\rm C}$ we can calculate the equivalent admittance

$$\boldsymbol{Y}_{\rm e\mathit{n}} = G_{\rm e\mathit{n}} + jB_{en} = \frac{P_n - jQ_n}{||\boldsymbol{u}_n||^2} = \frac{1}{3}(\boldsymbol{Y}_{{\rm R}n} + \boldsymbol{Y}_{{\rm S}n} + \boldsymbol{Y}_{{\rm T}n}) \tag{45}$$

and define the ***scatter current***

$$\boldsymbol{i}_{\rm Cs} = \sqrt{2}\,{\rm Re}\sum_{n\in N_{\rm C}} (G_{{\rm e}n} - G_{\rm Ce})\,\mathbf{1}_n \boldsymbol{U}_{{\rm R}n}\, e^{jn\omega_1 t}\ . \tag{46}$$

This current is associated with the phenomenon of a change of the load's equivalent conductance for harmonic frequencies G_{en} around the equivalent conductance $G_{\rm Ce}$. The ***reactive current*** is defined as:

$$\boldsymbol{i}_{\rm Cr} = \sqrt{2}\,{\rm Re}\sum_{n\in N_{\rm C}} jB_{{\rm e}n}\,\mathbf{1}_n \boldsymbol{U}_{{\rm R}n}\, e^{jn\omega_1 t}\ . \tag{47}$$

This current is associated with the phenomenon of the phase shift of the load current harmonics with respect to the supply voltage harmonics. Symbol $\mathbf{1}_n$ denotes a three-phase unite vector, defined as follows:

$$\mathbf{1}_n = \begin{bmatrix} 1 \\ 1e^{-jn\frac{2\pi}{3}} \\ 1e^{jn\frac{2\pi}{3}} \end{bmatrix} = \begin{bmatrix} 1 \\ \beta_n \\ \beta_n^* \end{bmatrix} = \begin{cases} \mathbf{1}^{\rm p}, \text{ dla } n = 3k-2 \\ \mathbf{1}^{\rm n}, \text{ dla } n = 3k-1, \\ \mathbf{1}^{\rm z}, \text{ dla } n = 3k \end{cases} \quad k = 1, 2..., \quad \beta_n = 1e^{-jn\frac{2\pi}{3}} = (\alpha^*)^n, \quad \alpha = 1e^{j\frac{2\pi}{3}}\ . \tag{48}$$

The supply current of unbalanced loads contains also the component

$$\boldsymbol{i}_{\rm Cu} = \sqrt{2}{\rm Re}\sum_{n\in N_{\rm C}} (\boldsymbol{Y}^{\rm p}_{{\rm u}n}\,\mathbf{1}^{\rm p} + \boldsymbol{Y}^{\rm n}_{{\rm u}n}\mathbf{1}^{\rm n} + \boldsymbol{Y}^{\rm z}_{{\rm u}n}\,\mathbf{1}^{\rm z})\boldsymbol{U}_{{\rm R}n}\, e^{jn\omega_1 t} \tag{49}$$

termed the ***unbalance current.*** This current is associated with the phenomenon of asymmetry of the load's line currents. The admittances in this formula are equal [19], respectively, to:

$$\boldsymbol{Y}^{\rm p}_{{\rm u}n} = \frac{1}{3}[(\boldsymbol{Y}_{{\rm R}n} + \alpha\beta\boldsymbol{Y}_{{\rm S}n} + \alpha^*\beta^*\boldsymbol{Y}_{{\rm T}n}) - \boldsymbol{Y}_{{\rm e}n}(1 + \alpha\beta + \alpha^*\beta^*)] \tag{50}$$

$$\boldsymbol{Y}^{\rm n}_{{\rm u}n} = \frac{1}{3}[(\boldsymbol{Y}_{{\rm R}n} + \alpha^*\beta\boldsymbol{Y}_{{\rm S}n} + \alpha\beta^*\boldsymbol{Y}_{{\rm T}n}) - \boldsymbol{Y}_{{\rm e}n}(1 + \alpha^*\beta + \alpha\beta^*)] \tag{51}$$

$$\boldsymbol{Y}^{\rm z}_{{\rm u}n} = \frac{1}{3}[(\boldsymbol{Y}_{{\rm R}n} + \beta\boldsymbol{Y}_{{\rm S}n} + \beta^*\boldsymbol{Y}_{{\rm T}n}) - \boldsymbol{Y}_{{\rm e}n}(1 + \beta + \beta^*)] \tag{52}$$

so that they can be calculated based on the results of measurements of the voltages and currents at the load terminals.

5. CONCLUSIONS

This paper demonstrates that five and only five physical phenomena determine currents of three-phase loads supplied by a nonsinusoidal voltage. These are:

1. A permanent flow of the energy from the supply source to the load,
2. A permanent flow of the energy from the load to the supply source,
3. A phase-shift of the load current harmonics with respect to the supply voltage harmonics,
4. A change of the load conductance with the harmonic order,
5. An asymmetry of the line currents, caused by the load imbalance.

Therefore, these components are referred to as the Currents' Physical Components (CPC). We should be aware, however, that the concept of ***"a physical phenomenon"*** is not completely clear. What is and what is not a physical phenomenon can be a matter of a subjective judgement.

It has been demonstrated in the paper, moreover, that oscillation of the energy between the supply source and the load, or its storage in the load, does not affect the supply current of three-phase loads and the effectiveness of the energy transfer.

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