J159. Rebuttal written at the request of the IEEE Transactions on Instrumentation and Measurements, 2020

Rebuttal to "Comments on „Scattered and Reactive Current, Voltage, and Power in Circuits with Nonsinusoidal Waveforms and Their Compensation"

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The title of this paper has nothing in common with its contents. There is no even a single comment on the results or opinions presented in my paper (www.czarnecki.study >> Research >> Journal Papers >> J42). For example, the term "scattered current" is a key term in my paper. This key term occurs in Comments only once. Even worse: at the beginning of section III, the authors of these Comments write: "To prove that $||i_r||$ = ... fulfills KCL, the author (it means, me) implied in [1] that Q_r = ...fulfilled TT (Tellegen Theorem)..." is not true. Nothing like this was written in my paper. There are three reasons for this, namely: (i) the rms value $||i_r||$ ..... does not fulfill KCL, (ii) the reactive power defined as $Q_r = ||v||\ ||i_r||$ does not fulfill Tellegen Theorem, and (iii) the Tellegen Theorem is not even mentioned in my paper. Thus, this Castro-Nunez and his co-authors' statement is pure mystification. This mystification combined with the lack of any comment on results and opinions expressed in my paper might indicate that the authors of Comments even did not read that paper. The circuits discussed in the Comments, shown in Fig. 1 and Fig. 2, are also not taken from my paper. Properties of such circuits were not the issue of it.

Just below equation (3) the authors write: "Equation (3) reveals that we cannot arrive to $||i||$ from $||i_C||$ and $||i_L||$ using KCL". This sentence reveals quite an amazing fact, that the authors of Comments, probably professors of electrical engineering, do not know that Kirchoff Current Law (KCL) holds only for instantaneous values of branch currents but not for their rms values(!). Rms values cannot be negative, thus their sum can be equal to zero only if all of them are zero. A student of a university course on circuit analysis, who does not know that, will probably fail the course.

Similarly, as with the KCL, the Comments shows that the authors are not aware that the Tellegen Theorem holds for instantaneous values of voltages and currents and consequently, for instantaneous or active powers but not for powers that are defined as non-negative quantities, such as, for example, the apparent power S, thus such non-negative powers cannot satisfy the Balance Principle.

The Comments are mainly on the application of geometric algebra for circuits analysis. Unfortunately, because of the lack of symbols and operations definitions and very poor mathematical standards, this part of Comments would be almost illegible for a reader. One should also observe that geometric algebra is usually not taught at electrical engineering departments. Electrical engineers are accustomed to using the algebra of complex numbers for circuits analysis but not geometric algebra. References to the Comments do not provide much help in reading these Comments because they are written in the same very obscure language. Moreover, if the authors use the paper:

> M. Castro-Nunez, R. Castro-Puche, "The IEEE Standard 1459, the CPC Power Theory, and Geometric Algebra in Circuits with Nonsinusoidal Sources and Linear Loads", *IEEE Trans. on Circuits and Systems-I*, Vol. 59, No. 12, pp. 2980-2990, 2012.

as a reference to Comments, then to keep professional standards, the readers should be also informed by authors on the paper:

> L.S. Czarnecki, M. Almousa, "What is Wrong with the Paper "The IEEE Standard 1459, the CPC Power Theory and Geometric Algebra in Circuits with Nonsinusoidal Sources and Linear Loads?", *Przegląd Elektrotechnicny,* R. 96, Nr. 7, pp. 1-7, 2020.

which demonstrates that there are major errors in the cited paper.

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Comments on: "Scattered and Reactive Current, Voltage, and Power in Circuits with Nonsinusoidal Waveforms and Their Compensation"

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Abstract— **In the above article the author assured that the proposed definition of reactive current fulfilled Kirchhoff's current law (KCL). We point out that this is not always true and consequently the proposed definition of reactive power does not always fulfil Tellegen's theorem (TT) as KCL and TT are interdependent. We pinpoint also the flaw in the demonstration given in the paper and show how the recent circuit analysis approach base on geometric algebra (GA) solves the issue.**

Index Terms—**power system harmonics, reactive power, Kirchhoff's current law, Tellegen's theorem, geometric algebra.**

I. Introduction

According to [1], the definitions of active, scattered and reactive current fulfil KCL. However, this statement does not hold true with the circuit in Fig. 1 where (1) is the excitation source with $\omega = 1 rad/s$ for simplicity, and (2) details the proposed current and power decompositions.

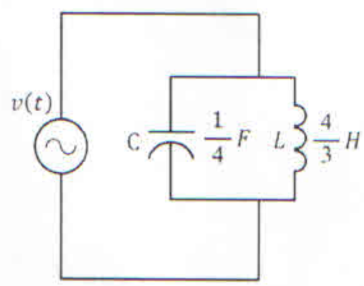

Fig. 1. Circuit used to show that the definition of reactive current proposed in [1] does not satisfy KCL nor the proposed definition of reactive power fulfils TT.

$$v(t) = \sqrt{2}[100\ \sin(\omega t) + 100 \sin(3\omega t)]V \quad (1)$$

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$$\begin{aligned}
S &\overset{def}{=} \|v\|\|i\| = \sqrt{P^2 + D_s^2 + Q_r^2} & G_e &\overset{def}{=} \frac{P}{\|v\|^2} \\
\|i_a\| &\overset{def}{=} \frac{P}{\|v\|} & P &\overset{def}{=} \|v\|\|i_a\| \\
\|i_r\| &\overset{def}{=} \sqrt[2]{\textstyle\sum_{n\in\mathcal{M}} {B_n}^2 V_n^2} & Q_r &\overset{def}{=} \|v\|\|i_r\| \\
\|i_s\| &\overset{def}{=} \sqrt[2]{\textstyle\sum_{n\in\mathcal{M}_0} (G_n - G_e)^2 V_n^2} & D_s &\overset{def}{=} \|v\|\|i_s\|
\end{aligned} \quad (2)$$

II. Analysis of the Circuit in Fig. 1 using [1]

Since in Fig 1, $P = 0$ and $G_n = G_e = 0$ then $\|i_a\| = 0$ and $\|i_s\| = 0$. Thus, only reactive current flows in the circuit. Since $Y_{c1} = j0.25S$, $Y_{L1} = -j0.75S$, $Y_{c3} = j0.75S$ and $Y_{L3} = -j0.25S$ then the reactive current through each branch is,

$$\begin{aligned}
\|i_c\| &= \sqrt{0.25^2 \cdot 100^2 + 0.75^2 \cdot 100^2} = 25\sqrt{10}A \\
\|i_L\| &= \sqrt{0.75^2 \cdot 100^2 + 0.25^2 \cdot 100^2} = 25\sqrt{10}A \\
\|i\| &= \sqrt{0.5^2 \cdot 100^2 + 0.5^2 \cdot 100^2} = 50\sqrt{2}A
\end{aligned} \quad (3)$$

Equation (3) reveals that we cannot arrive to $\|i\|$ from $\|i_c\|$ and $\|i_L\|$ using KCL. Using (3) in (2) yields the following results for the reactive power at each branch.

$$\begin{aligned}
Q_{r(C)} &= (100\sqrt{2})(25\sqrt{10}) = 5\sqrt{5}kVAr \\
Q_{r(L)} &= Q_{r(C)} \\
Q_{r(S)} &= (100\sqrt{2})(50\sqrt{2}) = 10kVAr \neq Q_{r(L)} + Q_{r(C)}
\end{aligned} \quad (4)$$

III. The Flaw in the Demonstration in [1]

To prove that $\|i_r\| \overset{def}{=} \sqrt[2]{\sum_{n\in\mathcal{M}} {B_n}^2 V_n^2}$ fulfils KCL the author implied in [1] that $Q_r \overset{def}{=} \|v\|\|i_r\|$ fulfilled TT using the fact that the balance principle of the reactive power must hold also in harmonic conditions. Although it is true that a sound definition for the harmonic reactive power Q_n must fulfil TT; this does not prove that $Q_r \overset{def}{=} \|v\|\|i_r\|$ fulfils TT as (4) shows.

IV. GA, the Master Key to Understanding Power Properties in Harmonic Conditions

Since the successful use of GA in circuit and power analysis is well documented [2-5]; here we only apply the technique. The reader is encouraged to examine [2-6] for deeper theoretical background. The central idea in analysing circuit using GA is to represent sinusoidal signal by k-vectors. These are elements of the GA of dimension $\mathcal{N} = h + 1$ where h is the harmonic of highest value in a circuit. Equation (5) provides the transformation operation among the time domain (TD) and the $\mathcal{G}_{\mathcal{N}}$-domain while (6) provides Ohm's law when n harmonics are present where $\langle \boldsymbol{V} \rangle_h$ is the harmonic voltage, $\boldsymbol{Y}_h = \frac{\widetilde{Z}_h}{\widetilde{Z}_h Z_h}$ is the harmonic admittance and $\boldsymbol{Z}_h$ is the harmonic impedance. Thus, for an inductor $\boldsymbol{Z}_h = \boldsymbol{Z}_l = \boldsymbol{\omega} L$ & $\widetilde{\boldsymbol{Z}_l} = -L\boldsymbol{\omega}$ where $\boldsymbol{\omega} = h\omega\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2$. For a capacitor $\boldsymbol{Z}_h = \boldsymbol{Z}_c = \frac{1}{\boldsymbol{\omega} C}$ & $\widetilde{\boldsymbol{Z}_c} = \frac{-1}{\boldsymbol{\omega} C}$ [2-6]. It is apparent that $\widetilde{\boldsymbol{Z}}_h = -\boldsymbol{Z}_h$.

$$\begin{array}{|lcl} x_{c1}(t) = X\sqrt{2}\cos\omega t & \leftrightarrow & \boldsymbol{X}_{c1} = X\boldsymbol{\sigma}_1 \\ x_{s1}(t) = X\sqrt{2}\sin\omega t & \leftrightarrow & \boldsymbol{X}_{s1} = -X\boldsymbol{\sigma}_2 \\ x_{ch}(t) = X\sqrt{2}\cos(h\omega t) & \leftrightarrow & \boldsymbol{X}_{ch} = X\bigwedge_{i=2}^{h+1}\boldsymbol{\sigma}_i \\ x_{ch}(t) = X\sqrt{2}\sin(h\omega t) & \leftrightarrow & \boldsymbol{X}_{sh} = X\bigwedge_{i=1,i\neq 2}^{h+1}\boldsymbol{\sigma}_i \end{array} \tag{5}$$

$$\boldsymbol{I}_s = \sum_{h=1}^{n}\langle \boldsymbol{V}\rangle_h \boldsymbol{Y}_h = \sum_{h=1}^{n}\widetilde{\boldsymbol{Y}_h}\langle \boldsymbol{V}\rangle_h = \sum_{h=1}^{n}\langle \boldsymbol{I}_s\rangle_h \tag{6}$$

Applying (5) to (1) yields $\boldsymbol{V} = (V_1\boldsymbol{\sigma}_2 + V_3\boldsymbol{\chi}_{s3})V$ where $V_3 = -100$, $V_3 = 100$, $\boldsymbol{\chi}_{s3} = \boldsymbol{\sigma}_1\boldsymbol{\sigma}_3\boldsymbol{\sigma}_4$ and $\|\boldsymbol{V}\| = \sqrt{V_1^2 + V_3^2} = 100\sqrt{2}\,V$. Now, since $\boldsymbol{Y}_{c1} = \frac{\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2}{4}S, \boldsymbol{Y}_{L1} = \frac{-3\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2}{4}S, \boldsymbol{Y}_{c3} = \frac{3\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2}{4}$ & $\boldsymbol{Y}_{L3} = \frac{-\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2}{4}$ then applying (6) yields (7) where $\boldsymbol{\chi}_{c3} = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_3\boldsymbol{\sigma}_4$ and all currents are in [A].

$$\begin{array}{|ll} \boldsymbol{I}_{s1} = -100\boldsymbol{\sigma}_2\left[\frac{\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2}{4} - \frac{3\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2}{4}\right] = \underbrace{25\boldsymbol{\sigma}_1}_{I_{C1}}\underbrace{-75\boldsymbol{\sigma}_1}_{I_{L1}} & \\ \boldsymbol{I}_{s3} = 100\boldsymbol{\chi}_{s3}\left[\frac{3\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2}{4} - \frac{\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2}{4}\right] = \underbrace{75\boldsymbol{\chi}_{c3}}_{I_{C3}}\underbrace{-25\boldsymbol{\chi}_{c3}}_{I_{L3}} & \\ \boldsymbol{I}_C = 25\boldsymbol{\sigma}_1 + 75\boldsymbol{\chi}_{c3} & \|\boldsymbol{I}_C\| = 25\sqrt{10} \\ \boldsymbol{I}_L = -75\boldsymbol{\sigma}_1 - 25\boldsymbol{\chi}_{c3} & \|\boldsymbol{I}_L\| = 25\sqrt{10} \\ \boldsymbol{I}_s = \boldsymbol{I}_C + \boldsymbol{I}_L & \\ \boldsymbol{I}_s = -50\boldsymbol{\sigma}_1 + 50\boldsymbol{\chi}_{c3} & \|\boldsymbol{I}_s\| = 50\sqrt{2} \end{array} \tag{7}$$

In (7) the product $100\boldsymbol{\chi}_{s3}\left[\frac{3\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2}{4}\right]$ yields $\boldsymbol{I}_{C3} = 75\boldsymbol{\chi}_{c3}$ because $(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_3\boldsymbol{\sigma}_4)(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2) = \boldsymbol{\sigma}_1^2\boldsymbol{\sigma}_3\boldsymbol{\sigma}_4\boldsymbol{\sigma}_2 = \boldsymbol{\sigma}_2\boldsymbol{\sigma}_3\boldsymbol{\sigma}_4 = \boldsymbol{\chi}_{c3}$ as $\boldsymbol{\sigma}_1^2 = 1$ [6]. As the reader might have noticed, (5) creates an isomorphism between the current and voltage quantities in the TD and their counterparts in the $\mathcal{G}_{\mathcal{N}}$-domain. This isomorphism is key as it guarantees the fulfilment of KCL in the $\mathcal{G}_{\mathcal{N}}$-domain and consequently the fulfilment of TT as [2-5] show. The general expressions for the current through the inductor, the capacitor and the source are: $\boldsymbol{I}_L = [I_{1L}\boldsymbol{\sigma}_1 + I_{3L}\boldsymbol{\chi}_{c3}]A$ where $I_{1L} = \frac{V_1}{h\omega L} = \frac{V_1}{L}$ & $I_{3L} = \frac{-V_3}{h\omega L} = \frac{-V_3}{3L}$; $\boldsymbol{I}_C = [I_{1C}\boldsymbol{\sigma}_1 + I_{3C}\boldsymbol{\chi}_{c3}]A$ where $I_{1c} = -V_1\omega hC = -V_1 C$ and $I_{3c} = V_3\omega hC = V_3 3C$, and $\boldsymbol{I}_s = [I_{1s}\boldsymbol{\sigma}_1 + I_{3s}\boldsymbol{\chi}_{c3}]A$ where $I_{1s} = I_{1L} + I_{1C}$ and $I_{3s} = I_{3L} + I_{3C}$ so $I_{1s} = V_1\left[\frac{1-\omega^2h^2LC}{\omega hL}\right] = V_1\left[\frac{1-LC}{L}\right]$ and $I_{3s} = V_3\left[\frac{\omega^2h^2LC-1}{\omega hL}\right] = V_3\left[\frac{9LC-1}{3L}\right]$. Thus, $\boldsymbol{I}_S = \boldsymbol{I}_L + \boldsymbol{I}_C$ and multiplying at the left, on both sides by $\boldsymbol{V} = (V_1\boldsymbol{\sigma}_2 + V_3\boldsymbol{\chi}_{s3})V$ yields the power at each one of the three branches as (8) shows.

$$\begin{array}{l} \boldsymbol{M}_L = \boldsymbol{V}\boldsymbol{I}_L = \lambda_1\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \lambda_2\boldsymbol{\sigma}_3\boldsymbol{\sigma}_4, \text{ where} \\ \lambda_1 = -(V_1I_{1L} + V_3I_{3L}) \quad \lambda_2 = (V_1I_{3L} + V_3I_{1L}) \\ \boldsymbol{M}_c = \boldsymbol{V}\boldsymbol{I}_c = \gamma_1\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \gamma_2\boldsymbol{\sigma}_3\boldsymbol{\sigma}_4, \text{ where} \\ \gamma_1 = -(V_1I_{1c} + V_3I_{3c}) \quad \gamma_2 = (V_1I_{3c} + V_3I_{1c}) \\ \boldsymbol{M}_s = \boldsymbol{V}\boldsymbol{I}_s = \mu_1\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \mu_2\boldsymbol{\sigma}_3\boldsymbol{\sigma}_4, \text{ where} \\ \mu_1 = -(V_1I_{1s} + V_3I_{3s}) \quad \mu_2 = (V_1I_{3s} + V_3I_{1s}) \end{array} \tag{8}$$

As expected, $\boldsymbol{M}_s = \boldsymbol{M}_L + \boldsymbol{M}_C$ and writing $\boldsymbol{M}_s$ in terms of the input variables V_1, V_3, L, C *and* h yields,

$$\begin{array}{l} \boldsymbol{M}_s = \left[\frac{V_3^2 - 3V_1^2 + 3LC(V_1^2 - 3V_3^2)}{3L}\right]\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \boldsymbol{M}_{34} \\ \text{where,}\quad \boldsymbol{M}_{34} = \left[\frac{6LC+2}{3L}\right]\boldsymbol{\sigma}_3\boldsymbol{\sigma}_4 \end{array} \tag{9}$$

Thus, the norm of $\boldsymbol{M}_s$ given by $\|\boldsymbol{M}_s\| = \sqrt{\langle \widetilde{\boldsymbol{M}}_s\boldsymbol{M}_s\rangle_0}$ where $\widetilde{\boldsymbol{M}}_s = -\boldsymbol{M}_s$ and $\langle\ \rangle_0$ denotes the 0-vector (scalar) part of the product yields,

$$\begin{array}{l} \|\boldsymbol{M}_s\| = \sqrt{\underbrace{\|v\|^2\|i_r\|^2}_{Q_r^2 = S^2} + \mu_s} \quad \text{where,} \\ \|v\|^2 = \|\boldsymbol{V}\|^2 = (V_1^2 + V_3^2) \\ \|i_r\|^2 = \|\boldsymbol{I}_r\|^2 = \underbrace{V_1^2\left(\frac{1-LC}{L}\right)^2}_{I_{s1}^2} + \underbrace{V_3^2\left(\frac{9LC-1}{3L}\right)^2}_{I_{s3}^2} \\ \mu_s = (4V_1V_3)\underbrace{V_1\left(\frac{1-LC}{L}\right)}_{I_{s1}}\underbrace{V_3\left(\frac{9LC-1}{3L}\right)}_{I_{s3}} \end{array} \tag{10}$$

It is clear from (10) that $\|\boldsymbol{M}_s\|$, *the norm of* $\boldsymbol{M}_s$, *contains:*

1. *The square of the voltage's RMS value* $\|v\|$;
2. *The square of the proposed definition of reactive current* $\|i_r\|$ *in [1];*
3. *The square of the proposed definition of reactive power* Q_r *in [1];*
4. *The square of the traditional definition of S; and,*
5. *The equivalent impedance of the circuit;*

When $V_3 = 0$, $I_{3s} = 0$ and (10) reduces to $\|\boldsymbol{M}_s\| = \frac{V_1}{\omega L} - V_1\omega C$ as $\omega = 1$ which is the classical definition of reactive power for a capacitor and an inductor in sinusoidal conditions; i.e., $\|\boldsymbol{M}_s\| = Q_{rs} = Q_{rL} - Q_{rc}$. When $V_1 = 0$, $I_{1s} = 0$ and (10) reduces to $\|\boldsymbol{M}_s\| = V_3\omega C - \frac{V_3}{\omega L}$ which is the classical definition of reactive power for a capacitor and an inductor when $\omega = 3rad/s$; i.e., $\|\boldsymbol{M}_s\| = Q_{rs3} = Q_{rc3} - Q_{rL3}$. In either case $\|\boldsymbol{M}_s\| = Q_{rs}$. When $L \to \infty$ the circuit's load is only a capacitor and when $C \to 0$ the circuit's load is only an inductor and, in each case, (10) reduces to,

$$\begin{array}{l} \|\boldsymbol{M}_s\|_{L\to\infty} = \sqrt{V_1^4C^2 - 2V_1^2V_3^2C^2 + 9V_3^4C^2} \\ \|\boldsymbol{M}_s\|_{C\to 0} = \sqrt{\frac{V_1^4}{L^2} - \frac{2V_1^2V_3^2}{9L^2} + \frac{V_3^4}{9L^2}} \end{array} \tag{11}$$

Equations (10) and (11) reveal that certain expressions; e.g., $S \stackrel{\text{def}}{=} \|v\|\|i\|$ *and* $Q_r \stackrel{\text{def}}{=} \|v\|\|i_r\|$ which are appropriate in sinusoidal conditions, aren't automatically valid in non-sinusoidal situations. The following example is crucial for understanding this point. Notice, since $S \stackrel{\text{def}}{=} \|v\|\|i_r\| = Q_r$ then if $L \to \infty$ then $S_{L\to\infty} \stackrel{\text{def}}{=} \sqrt{V_1^4C^2 + 10V_1^2V_3^2C^2 + 9V_3^4C^2} = Q_{r(L\to\infty)}$ and as $V_1 = V_3$ then $S_{L\to\infty} = 2\sqrt{5}V_1^2C = Q_{r(L\to\infty)}$. Thus, if $C = 1/16F$ then the capacitor's reactive power is $S_{L\to\infty} = 2.8kVA = Q_{r(L\to\infty)}$. Now, if this capacitor alone is used as the compensator for the RLC load of the circuit in Fig 2 then $S \stackrel{\text{def}}{=} \sqrt{P^2 + \left(Q_{r(RLC)} - Q_{r(L\to\infty)}\right)^2} = 12.33kVA$ since $P = 10kW$ and $Q_{r(RLC)} = 10KVAr$. Thus, the power factor (pf) is improved to 0.81. However, as $\|i_s\| = 95.61A$ and $\|v\| = 100\sqrt{2}V$ then $S \stackrel{\text{def}}{=} \|v\|\|i_s\| = 13.52kVA$. Thus, $pf = 0.74$ which conflicts the prior result.

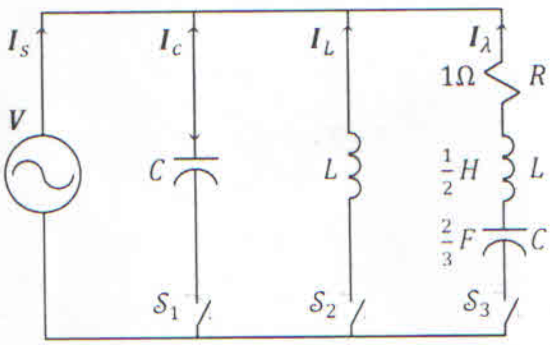

Fig. 2. Circuit used to show how the use of valid expressions in sinusoidal conditions may yield to contradictions in harmonic conditions.

On the other hand, if $C \to 0$ then $S_{C\to 0} \stackrel{\text{def}}{=} \sqrt{\frac{V_1^4}{L^2} + \frac{10V_1^2V_3^2}{9L^2} + \frac{V_3^4}{9L^2}} = Q_{r(C\to 0)}$ and as $V_1 = V_3$ then $S_{C\to 0} = \frac{2\sqrt{5}V_1^2}{3L} = Q_{r(L\to\infty)}$. Thus, if $L = 80/21H$ then the inductor's reactive power is $S_{C\to 0} = 3.91kVA = Q_{r(L\to\infty)}$. Now, if this inductor alone is used as the compensator for the RLC load of the circuit in Fig 2 then $S \stackrel{\text{def}}{=} \sqrt{P^2 + \left(Q_{r(RLC)} - Q_{r(L\to\infty)}\right)^2} = 11.71kVA$ and $pf = 0.85$. However, as $\|i_s\| = 94.95A$ and $\|v\| = 100\sqrt{2}V$ then $S \stackrel{\text{def}}{=} \|v\|\|i_s\| = 13.43kVA$ and $pf = 0.74$ which conflicts the previous results. According to [1], when $S \stackrel{\text{def}}{=} \|v\|\|i_s\|$ is used, both compensators, C and L, yield $pf = 0.74$ but using $S \stackrel{\text{def}}{=} \sqrt{P^2 + \left(Q_{r(RLC)} - Q_{r(L\to\infty)}\right)^2}$ yields $pf = 0.8$ with C and 0.85 with L. Judging by the compensator's reactive power capacity, $2.8kVA$ for C and $3.91kVA$ for L, the inductor is more efficient which explains the results 0.8 *and* 0.85 above. However, this implies that: 1) the result 0.74 is erroneous and 2) the inductor is a better compensator than the LC compensator which yields $pf = 0.82$ using $S \stackrel{\text{def}}{=} \|v\|\|i_s\|$. However, the LC compensator is more efficient; thus, according to the scientific literature, it should yield a higher pf. Worse yet, if $C = 3/16F$ and $L = 16/3H$ then, even though this compensator reduces the reactive current by $50A$, the $pf = 0.82$. In contrast, the current and power on each branch using the $\mathcal{G}_N$-domain is,

$$\begin{cases} \boldsymbol{V} = 100(-\boldsymbol{\sigma}_2 + \boldsymbol{\chi}_{s3})V & \|\boldsymbol{V}\| = 100\sqrt{2}\,V \\ \boldsymbol{I}_\lambda = 50[\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2 + \boldsymbol{\chi}_{s3} - \boldsymbol{\chi}_{c3}] & \|\boldsymbol{I}_\lambda\| = 100 \\ \boldsymbol{I}_C = 6.25\boldsymbol{\sigma}_1 + 18.75\boldsymbol{\chi}_{c3} & \|\boldsymbol{I}_c^b\| = 19.76 \\ \boldsymbol{I}_L = -26.25\boldsymbol{\sigma}_1 - 8.75\boldsymbol{\chi}_{c3} & \|\boldsymbol{I}_L^b\| = 27.67 \\ \boldsymbol{I}_s^C = \boldsymbol{I}_\lambda + \boldsymbol{I}_C & \|\boldsymbol{I}_s^C\| = 95.61 \\ \boldsymbol{I}_s^L = \boldsymbol{I}_\lambda + \boldsymbol{I}_L & \|\boldsymbol{I}_s^L\| = 94.95 \\ \boldsymbol{I}_s^{LC} = \boldsymbol{I}_\lambda + \boldsymbol{I}_C + \boldsymbol{I}_L & \|\boldsymbol{I}_s^{LC}\| = 50\sqrt{3} \end{cases} \quad (12)$$

$$\begin{cases} \boldsymbol{M}_\lambda = 10 + 10[\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3\boldsymbol{\sigma}_4] & \|\boldsymbol{M}_\lambda\| = 10\sqrt{3} \\ \boldsymbol{M}_C = -1.25[\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3\boldsymbol{\sigma}_4] & \|\boldsymbol{M}_c^b\| = 1.25\sqrt{2} \\ \boldsymbol{M}_L = -1.75[\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3\boldsymbol{\sigma}_4] & \|\boldsymbol{M}_L^b\| = 1.75\sqrt{2} \\ \boldsymbol{M}_s^C = 10 + 8.75[\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3\boldsymbol{\sigma}_4] & \|\boldsymbol{M}_\alpha\| = 15.91 \\ \boldsymbol{M}_s^L = 10 + 8.25[\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3\boldsymbol{\sigma}_4] & \|\boldsymbol{M}_{\alpha\beta}\| = 15.34 \\ \boldsymbol{M}_s^{LC} = 10 + 7[\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3\boldsymbol{\sigma}_4] & \|\boldsymbol{M}_s\| = 3\sqrt{22} \end{cases} \quad (13)$$

According to (13) the pf is: 0.58 without compensation, 0.62 with the compensator $C = 1/16F$, 0.65 using $L = 80/21H$, 0.71 using $L\,\&\,C$ and 0.82 using $C = \frac{3}{16F}$ *and* $L = 16/3H$ *—whether using* $\|\boldsymbol{M}_s\| = \sqrt{\langle\widetilde{\boldsymbol{M}}_s\boldsymbol{M}_s\rangle_0}$ *or* $\|\boldsymbol{M}_s\| = \sqrt{P^2 + (\|\boldsymbol{CN}_r\| - \|\boldsymbol{M}_{cm}\|)^2}$ where $\boldsymbol{CN}_r$ is the reactive power multivector [2-5] and $\boldsymbol{M}_{cm} = \{\boldsymbol{M}_C, \boldsymbol{M}_L, \boldsymbol{M}_C + \boldsymbol{M}_L\}$. *In summary, while a holistic view of the results attained with [1] conflict with the scientific literature; the results attained in the $\mathcal{G}_N$-domain agree —the higher the compensator's reactive current capacity the higher the pf.*

REFERENCES

[1] Czarnecki, L.S. Scattered and Reactive Current, Voltage, and Power in Circuits with Nonsinusoidal Waveforms and Their Compensation. IEEE Trans. Instrum. Meas. 40, 563-574 (1991).

[2] Castro-Núñez, M., Castro-Puche, R. Advantages of Geometric Algebra Over Complex Numbers in the Analysis of Networks with Nonsinusoidal Sources and Linear Loads. IEEE Trans. Circuits Syst. 59, 2056-2064 (2012).

[3] Castro-Núñez, M., Castro-Puche, R. The IEEE Standard 1459, the CPC Power Theory, and Geometric Algebra in Circuits with Nonsinusoidal Sources and Linear Loads. IEEE Trans. Circuits Syst. 59, 2980-2990 (2012).

[4] Castro-Núñez, M., Londoño-Monsalve, D. Castro-Puche, R. M, the conservative power quantity based on the flow of energy. The Journal of Engineering, 2019, 269-276, (2016).

[5] Castro-Núñez, M., Castro-Puche, R., Londoño-Monsalv, D. Theorems of Compensation and Tellegen in Non-sinusoidal Circuits via Geometric Algebra. The Journal of Engineering, 2019, 3409-3417, (2019)

[6] Doran, C., Lasenby, A. Geometric Algebra for Physicists Cambridge Univ. Press, (2005).