Scattered and Reactive Current, Voltage, and Power in Circuits with Nonsinusoidal Waveforms and Their Compensation

Leszek S. Czarnecki, *Senior Member, IEEE*

***Abstract*—This paper discusses properties of the current and power decomposition into the active, scattered, and reactive components. The relative character of this decomposition is analyzed along with orthogonal decomposition of the load voltage. It is shown that this relative character emphasizes the change of the power flow with the cross section observed. It emphasizes the difference in efficiency of the source apparent power compensation possible to achieve with a shunt and with a series compensator. It is also shown in the paper that not only the reactive power Q_r, but also the scattered power D_s can be wholly compensated by a linear reactive compensator, so that the apparent power S of the load can be minimized to its active power P.**

I. Introduction

Power properties of circuits with nonsinusoidal waveforms originally attracted attention primarily for purely scientific reasons. The sources of waveform distortion at that time were not numerous in power systems. This has changed with the development of power electronics. Now, the number of switched power semiconductor devices has increased drastically and the power transmitted at distorted waveforms in some situations reaches hundreds of megawatts, and power properties of systems with nonsinusoidal waveforms are not only an academic problem. Their understanding may have important practical implications for power transmission efficiency and for improvement in its quality.

By 1980 various definitions of powers had been suggested for such circuits in papers [1]–[7]. In 1984 they were followed [8] by the author's concept of the power quantities definitions based on decomposition of the current into the active i_a, scattered i_s, and reactive i components, such that

$$i = i_a + i_s + i_r. \tag{1}$$

It enabled us to decompose the apparent power S of the source into the active P, scattered D_s, and reactive Q_r powers, such that

$$S^2 = P^2 + D_s^2 + Q_r^2. \tag{2}$$

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The author is with the Department of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA 70803.
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Power theory which resulted in (1) and (2) has enabled the achievement of substantial progress toward methods for the minimization of the apparent power [12], [13] and in understanding and describing the power properties of

- three-phase asymmetrical nonlinear circuits [9],
- circuits with nonperiodic waveforms of a finite energy [10],
- circuits with a bidirectional flow of active power [11].

This concept has not been challenged by any published critics to date. Nonetheless, it appears from discussions that there still is a misunderstanding of the concept. Therefore, further elucidation of the vague points of the power theory suggested seems to be necessary. This should be beneficial for a better comprehension of energy-related phenomena in circuits with nonsinusoidal waveforms.

Let us compile the confusing features of the current and power decomposition into the active, scattered, and reactive components and some questions related to it.

1) Currents i_a, i_s, i_r and their rms values change even in an unsplit current loop.

2) The value of powers Q_r and D_s depends on which quantity, voltage or current is chosen as the reference.

3) Can the scattered power D_s be compensated by a linear reactance compensator?

4) Do the active, scattered and reactive currents fulfill Kirchhoff's law and the superposition principle?

Since these are the only problems addressed in this paper, it can be considered as supplementary to paper [8]. The assumptions as to the circuit properties and the symbols used here are also the same.

II. Some Properties of the Active, Scattered, and Reactive Currents

According to [8], the current i of a nonsinusoidal voltage source of voltage u, with complex rms values of harmonics $U_n \triangleq U_n e^{j\alpha_n}$, applied to a linear load of admittance $Y_n \triangleq G_n + jB_n$, can be decomposed into its active, scattered, and reactive components, namely,

$$i_a \triangleq G_e u, \qquad \text{with } G_e \triangleq P/\|u\|^2 \tag{3}$$

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$$i_s \triangleq (G_o - G_e) U_o + \sqrt{2}\, \mathrm{Re} \sum_{n=1}^{\infty} (G_n - G_e) U_n e^{jn\omega_1 t} \quad (4)$$

$$i_r \triangleq \sqrt{2}\, \mathrm{Re} \sum_{n=1}^{\infty} jB_n U_n e^{jn\omega_1 t}. \quad (5)$$

This decomposition at cross sections xx, yy of an unsplit circuit (Fig. 1) results, however, in currents i_a, i_s, i_r of different values, i.e.,

$$i_{ax} \neq i_{ay}; \quad i_{sx} \neq i_{sy}; \quad i_{rx} \neq i_{ry}. \quad (6)$$

It has this strange and often unrealized property because currents i_a, i_s, i_r are not absolute quantities, but relative ones, that are referenced to the voltage u at the cross section considered. This property is not, however, unique to this decomposition. Decompositions suggested by Fryze [2], Shepherd and Zakikhani [4], Depenbrock [6], and Kusters and Moore [7] also behave in the same way. Even the well-known decomposition of a sinusoidal current into the active and reactive components behaves similarly. If the current waveform of the one-port A in Fig. 1 is sinusoidal, then the result of this current $i(t)$ decomposition into the active and reactive components depends on which voltage: u_{xx}, u_{yy}, or u_{xy}, is taken as the reference.

In one sense it means that this decomposition of the current seems to lose its meaning as an absolute basis for interpreting power properties. In another sense, however, this relative character of decomposition reflects a real difference in the power flow which is characterized by differences in the instantaneous powers p_{xx}, p_{xy}, p_{yy} at each of the cross sections: xx, xy, yy. This means that the effect of current minimization with compensators connected at these three cross sections may also be different.

Since the currents i_a, i_s, and i_r are only relative quantities, the conclusion may be drawn that Kirchhoff's current law cannot be applied to these currents. However, if all currents i_k of a node, shown in Fig. 2, with K branches, which have the active power P_k are decomposed into the active, scattered, and reactive currents at the same reference voltage u, then

$$\sum_{k=1}^{K} i_{ka} = \sum_{k=1}^{K} \frac{P_k}{\|u\|^2} u = \frac{u}{\|u\|^2} \sum_{k=1}^{K} P_k = 0 \quad (7)$$

since the power P_k fulfills the balance principle. Similarly,

$$\sum_{k=1}^{K} i_{kr} = \sum_{k=1}^{K} \sqrt{2}\, \mathrm{Re} \sum_{n=1}^{\infty} jB_{kn} U_n e^{jn\omega_1 t}$$
$$= \sqrt{2}\, \mathrm{Re} \sum_{n=1}^{\infty} j \sum_{k=1}^{K} Q_{kn} \frac{U_n}{U_n^2} e^{jn\omega_1 t} = 0 \quad (8)$$

since the balance principle also holds for harmonic reactive powers Q_{kn}. Finally, for the scattered current we obtain

$$\sum_{k=1}^{K} i_{ks} = \sum_{k=1}^{K} (i_k - i_{ka} - i_{kr})$$
$$= \sum_{k=1}^{K} i_k - \sum_{k=1}^{K} i_{ka} - \sum_{k=1}^{K} i_{kr} = 0. \quad (9)$$

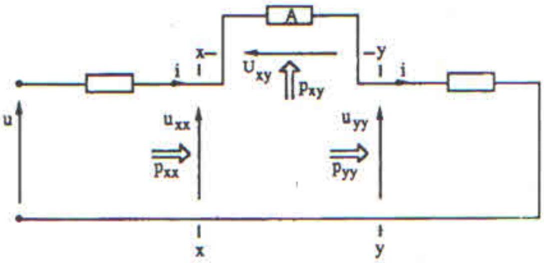

Fig. 1. The unsplit current loop.

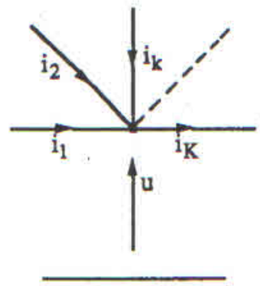

Fig. 2. The node with a reference voltage.

This means the Kirchhoff's current law as regards the active, scattered, and reactive currents holds for parallel branches. Now let us consider the question of whether these currents fulfill the superposition principle. If the reference voltage is

$$u \triangleq \alpha u_1 + \beta u_2 \quad (10)$$

can the currents i_a, i_s, i_r be expressed as

$$i = \alpha i_1 + \beta i_2 \quad (11)$$

for any coefficients α, β? To answer this question, let us note that the active and scattered currents contain the equivalent conductance G_e in their definitions. Since

$$P \triangleq (u, i) = \alpha^2 P_1 + \alpha\beta(u_1, i_2) + \alpha\beta(u_2, i_1) + \beta^2 P_2 \quad (12)$$

where P_1 and P_2 are the active powers at voltages u_1 and u_2, and

$$\|u\|^2 = \alpha^2 \|u_1\|^2 + 2\alpha\beta(u_1, u_2) + \beta^2 \|u_2\|^2. \quad (13)$$

Thus their quotient, $P/\|u\|^2 \triangleq G_e$, is constant and independent of α and β, only if $\alpha = 0$ or $\beta = 0$. Thus the superposition principle does not hold true for the active and scattered currents. As to current i_r let us note that susceptances B_n of a linear load do not depend on the voltage. Moreover, if the voltage is given by (10), then complex rms values U_n of harmonics are equal to

$$U_n = \alpha U_{1n} + \beta U_{2n} \quad (14)$$

and hence

$$i_r = \alpha i_{1r} + \beta i_{2r} \quad (15)$$

so that the superposition principle is fulfilled.

III. Active, Scattered and Reactive Components of the Load Voltage

If, instead of the load voltage, the load current i is taken as the reference quantity, then the load voltage u can be decomposed into the active, scattered, and reactive components. This requires that the equivalent resistance R_e of the load is defined as the resistance of a resistive load that at the same current i has the same active power P as the load considered, i.e.,

$$R_e \triangleq P/\|i\|^2. \tag{16}$$

With this resistance, the active voltage is defined as

$$u_a \triangleq R_e i. \tag{17}$$

If $\mathbf{Z}_n \triangleq R_n + jX_n$ is the load impedance for a harmonic frequency, then the load voltage

$$u = R_o I_o + \sqrt{2}\,\text{Re} \sum_{n=1}^{\infty} (R_n + jX_n) I_n e^{jn\omega_1 t} \tag{18}$$

can be decomposed into the active voltage u_a and voltages

$$u_s \triangleq (R_o - R_e) I_o + \sqrt{2}\,\text{Re} \sum_{n=1}^{\infty} (R_n - R_e) I_n e^{jn\omega_1 t} \tag{19}$$

$$u_r \triangleq \sqrt{2}\,\text{Re} \sum_{n=1}^{\infty} jX_n I_n e^{jn\omega_1 t} \tag{20}$$

such that

$$u = u_a + u_s + u_r \tag{21}$$

where u_s and u_r may be termed the "scattered" and "reactive" voltages. Similarly, as it was shown for current components in paper [8], voltage components u_a, u_s, and u_r are mutually orthogonal, i.e.,

$$(u_a, u_r) = (u_a, u_s) = (u_s, u_r) = 0 \tag{22}$$

so that their rms values fulfill the relationship

$$\|u\|^2 = \|u_a\|^2 + \|u_s\|^2 + \|u_r\|^2. \tag{23}$$

Multiplying this equation by the square of the current rms value $\|i\|^2$ results in the power equation

$$S^2 = P^2 + D_{si}^2 + Q_{ri}^2 \tag{24}$$

where

$$D_{si} \triangleq \|u_s\|\,\|i\|; \quad Q_{ri} \triangleq \|u_r\|\,\|i\| \tag{25}$$

are the scattered and reactive powers, respectively. Index i emphasizes that these powers are defined with the load current i taken as the reference quantity. To emphasize that powers in (2) are defined at the voltage u taken as the reference quantity, their symbols are modified here to D_{su} and Q_{ru}.

If the resistance R_n of a load does not change with the harmonic frequency, then the equivalent resistance R_e is equal to R_n. In such a case, according to (19), the load voltage does not contain any scattered component u_s. The scattered power D_{si} of such a load is equal to zero. However, even if R_n is constant, the conductance G_n of such a load may not be constant. Therefore, the load current contains a scattered component i_s and the scattered power D_{su} is not equal to zero. Thus comparing power equations (2) and (24) for such a load, one can conclude that, in general,

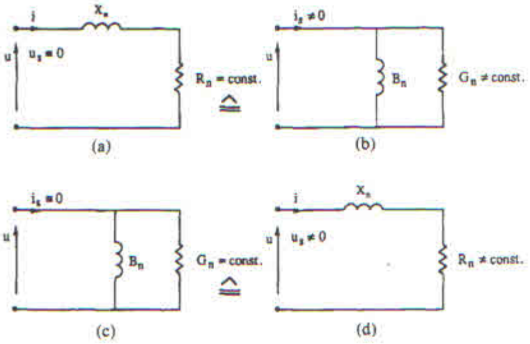

Fig. 3. Two pairs of equivalent circuits (a), (b) and (c), (d).

$$D_{su} \neq D_{si}; \quad Q_{ru} \neq Q_{ri}. \tag{26}$$

These differences occur as a result of a different dependence of the load resistance R_n and the load conductance G_n on harmonic frequency. This difference is illustrated in Fig. 3 for two pairs of equivalent loads. The same refers to the load reactance X_n and its susceptance B_n. A difference in the voltage and current spectra also affects the results of the apparent power decompositions with either the voltage or current taken as the reference quantity. The relative character of these decompositions seems to undermine their sense, but instead provides important information regarding the power factor improvement. The current decomposition provides fundamentals for the design of shunt LC compensators, while the voltage decomposition does the same for series compensators. They affect the circuit in a different way and with different results. A shunt compensator (Fig. 4(b)) compensates the reactive current i_r and the reactive power Q_{ru} entirely, if for each voltage harmonic the susceptance B_{pn} of the compensator fulfills the condition

$$B_{pn} = -B_n. \tag{27}$$

The scattered current i_s is not affected, however.

The reactive voltage u_r and the reactive power Q_{ri} are wholly compensated if the reactance X_{sn} of the series LC compensator shown in Fig. 4(a) fulfills the condition

$$X_{sn} = -X_n \tag{28}$$

but it does not affect the scattered voltage u_s.

Both compensators do not affect scattered powers. Then, if they are not equal to zero, the apparent power S of the source remains higher than the active power P after compensation. If the load conductance G_n is constant, however, $i_s \equiv 0$ and the source current i is minimized to the active current i_a value by a shunt LC compensator. If the load resistance R_n is constant, $u_s \equiv 0$ and the load voltage u is minimized to the active voltage u_a. In both cases the load is wholly compensated and $S = P$.

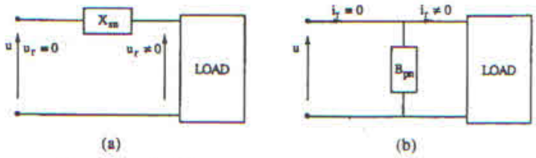

Fig. 4. Load with series (a) and shunt (b) compensators.

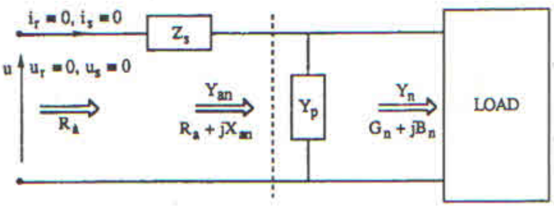

Fig. 5. Structure of a reactive compensator for the reactive and scattered power compensation.

IV. Whole Compensation of Linear Loads Supplied from Nonsinusoidal Voltage Sources

The practical meaning of the decomposition of the nonactive current $i - i_a$ into the scattered i_s and reactive i_r components lies in the separation of components which mutually differ in the possibility of their being compensated by a shunt LC compensator. The scattered current i_s stands for the part of the current $i - i_a$ which cannot be compensated in this way. The problem of its compensation is one of the most frequently recurring questions in discussions of the suggested power theory.

The answer to this question may be based on the observation that a load with a constant resistance R_n or a constant conductance G_n may be wholly compensated. A shunt reactance connected at the load terminals does not affect the conductance G_n seen by the source. However, it affects the resistance R_n of the equivalent circuit. Thus, it affects the scattered voltage u_s of the load. Similarly, a series compensator affects the conductance G_n of the equivalent circuit and the scattered current i_s.

Let us assume that a reactance one-port of susceptance B_{pn} is connected in parallel to a load of admittance $Y_n \triangleq G_n + jB_n$, as it is shown in Fig. 5. The impedance of the circuit obtained is equal to

$$Z_{an} = R_{an} + jX_{an} = \frac{G_n}{G_n^2 + (B_n + B_{pn})^2} - j\frac{B_n + B_{pn}}{G_n^2 + (B_n + B_{pn})^2}. \quad (29)$$

In particular, its resistance R_{an} for harmonic frequencies has a constant value $R_{an} \triangleq R_a$ if the susceptance B_{pn} of the reactance one-port is equal to

$$B_{pn} = -B_n \pm G_n\sqrt{\frac{G_a}{G_n} - 1} \quad (30)$$

which requires that $G_a \geq G_n$ for each order of n of the voltage harmonic. This condition can be fulfilled for any load and a reactance one-port that has susceptance B_{np} may be built if its susceptance is specified at a finite number of harmonic frequencies. Thus, any load with a variable conductance G_n can be converted into a load with a constant resistance R_n. Its voltage does not contain any scattered component. The reactance X_{an} of such a load is equal to

$$X_{an} = \pm R_a\sqrt{\frac{G_a}{G_n} - 1}, \quad R_a \triangleq 1/G_a \quad (31)$$

so that the source is loaded with the reactive power. This may be compensated by a series LC compensator (Fig. 5) with the reactance

$$X_{sn} = -X_{an}. \quad (32)$$

Thus, these two reactance branches modify any linear load to the equivalent load of a constant impedance $Z_n = R_a$, and the source is only loaded with the active power. Its current is equal to

$$i = G_a u = (G_a/G_e)\, i_a. \quad (33)$$

If P' denotes the active power of the compensated load, then

$$P' = (G_a/G_e)P > P \quad (34)$$

since $G_a > G_e$. P' is higher than the active power P of the uncompensated load. Let us note also that no limitation is imposed on the level of the voltage distortion, so that the method developed is valid even for highly distorted voltages.

Example: Let us suppose that the load shown in Fig. 6 is supplied from a source of nonsinusoidal voltage that apart from the fundamental also contains the third and fifth harmonics. The load admittances and impedances for these harmonics are equal to

$$Y_1 = 0.8385 - j0.2077\,S$$
$$Z_1 = 1.124 + j0.2784\,\Omega$$
$$Y_3 = 0.5724 - j0.0207\,S$$
$$Z_3 = 1.745 + j0.0621\,\Omega$$
$$Y_5 = 0.6379 + j0.1552\,S$$
$$Z_5 = 1.480 - j0.3600\,\Omega.$$

With the value of G_a chosen to be equal to G_1, namely,

$$G_a = G_1 = 0.8385\,S$$

the following values are calculated from (30)

$$B_{p1} = 0.2077\,S, \quad B_{p3} = -0.3699\,S$$
$$B_{p5} = 0.2025\,S$$

and may be chosen as the shunt LC compensator susceptances. A one-port of admittance

$$Y_p(s) = \frac{0.0737s^3 + 1.0628s}{s^2 + 5.7622}$$

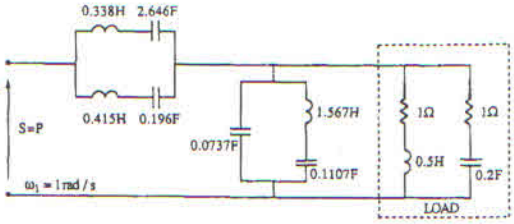

Fig. 6. Example of a load with a scattered and reactive power compensator.

has such susceptances. It modifies the load impedance for harmonic frequencies to values:

$$Z_{a1} = 1.193\Omega$$

$$Z_{a3} = 1.193 + j0.813\Omega$$

$$Z_{a5} = 1.193 - j0.669\Omega$$

i.e., to an impedance with a constant real part. Its imaginary part may be compensated by a series LC one-port with reactance X_{sn} equal to:

$$X_{s1} = 0, \quad X_{s3} = -0.813\Omega, \quad X_{s5} = 0.669\Omega.$$

An LC one-port with impedance

$$Z_s(s) = \frac{0.198s^4 + 2.622s^2 + 2.424}{s^3 + 6.891s}$$

has such a reactance. Some structures and parameters of these two reactance one-ports are shown in Fig. 6. They modify the load for the harmonics considered to a resistive load with a resistance $R_a = 1/G_a = 1.193\Omega$.

V. Conclusions

The relative character of the voltage, current, and apparent power decomposition into the active, scattered, and reactive components in linear circuits with nonsinusoidal waveforms reflects the differences in the power flow at various cross sections of the circuit. It also reflects differences in compensation effects that may be achieved with series or shunt compensators. The conclusion that not only the reactive power, but also the scattered power can be compensated by a reactive circuit, and apparent power S can be reduced to the active power P, is the most important result of the considerations presented.

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